Magnetization and Pull Characteristics of Mating Magnetic Reeds

By R. L. PEEK, JR.

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Expressions are presented for the magnetization and pull relations of the mating flat magnetic reeds used as contacting members in reed relays. The results of an experimental study expressed in dimensionless form give the force or pull between the reeds in terms of their dimensions, the gap between them, and the flux density. Since the attainable flux density is limited by saturation, the pull expression leads to conditions which must be satisfied by the reed and gap dimensions to provide desired levels of contact and retractile force.

The ampere-turn sensitivity of a relay using mating magnetic reeds depends on the flux required and on the reluctance of the magnetic path through the reeds. Expressions are given for the reluctance in the case of an air return path and in that where the air return is partially replaced by a shielding member.

Expressions are also given for the operate time of such reed relays. This depends on the concurrent flux development and motion of the reeds. The time of flux development varies inversely with the power input to the coil, but the motion time cannot be less than that required when the flux density is raised abruptly to its maximum value at the start of motion.

I. INTRODUCTION

A sealed reed relay comprises one or more sealed contacts assembled with a coil and shielding and supporting members. The distinctive feature is the construction of a sealed contact.[1,2] In its simplest form, as shown in Fig. 1, it consists of a glass envelope in which are sealed a pair of magnetic reeds, which serve as contacting members, actuated by the magnetic field induced in them by the coil current.

In another form,[2] there is only one moving reed, the other magnetic member being short and nearly rigid. This construction, using two of these short members to provide both back and front contacts, has been

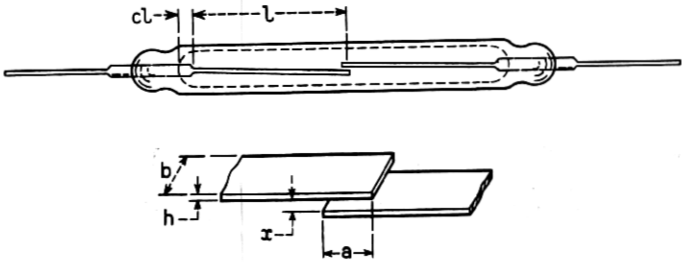

Fig. 1 — Dimensions of mating magnetic reeds.

used with mercury-wetted contact surfaces.[3,4] Both forms have been used in single- and multiple-contact relays, both neutral and with permanent magnet bias used to give polar or locking performance.[5] Recent work has included development of a miniature sealed contact.[6]

As an aid to the design of sealed contacts, and to the understanding of their characteristics, an analysis has been made of their performance relations, including the dependence of the pull, contact force, speed and sensitivity on the dimensions of the reeds. This analysis is presented here in the form applying to the simple case of equal mating reeds, but is, with some modification, applicable to other forms of magnetic reed contacts.

The major controlling factor in the performance of sealed contacts is the flux-carrying capacity of the reeds, as limited by the saturation density of the reed material. The treatment given here therefore starts from the relation between the reed flux and the attractive force at the gap, which must deflect the reeds and provide the contact force. This relation is determined essentially by the reed dimensions alone. The sensitivity, as measured by the ampere turns for operation and release, depends upon both the flux and the reluctance of the magnetic circuit, and hence upon the dimensions of the coil and shielding members, as well as upon those of the reeds. Relations are given for the estimation of reluctance and hence of sensitivity. Finally, expressions are given for the estimation of the speed of operation, which depends upon the times of field development and of reed motion, with the latter establishing a lower limit to the attainable time of operation.

II. PULL RELATION

Using the notation of Fig. 1 for the reed dimensions, the attractive force F between the reeds is given by Maxwell's law as

$$F = \frac{\varphi_G^2}{8\pi ab}, \tag{1}$$

where φ_G is the flux in the gap where the reeds overlap. The total flux Φ is the sum of φ_G and the fringing flux which passes from one reed to the other by air paths around the gap. As shown below in the discussion of reluctance, approximate estimates may be made of the fringing flux, but the relations involved are not adapted to a simple direct treatment of the relation between the pull F and the total flux Φ. A direct experimental study was therefore made to determine the approximate form of this relation.

In this study, pull and flux measurements were made of four different sets of reeds, having the dimensions listed in Table I. Each set of reeds was assembled with overlap values, a, of 25, 50 and 100 milli-inches, and the pull and flux measured for values of gap x in the range from 1 to 10 milli-inches over the range of coil energization from 0 to 300 ampere turns.

These measurements were all made with the reeds supported in a brass fixture which could be adjusted to make the overlap surfaces parallel and to set the overlap and the gap at desired values. The coil used had inside and outside diameters of 0.39 and 0.86 inch, and was 1.64 inches long. It was centered over the gap in making the measurements, and was provided with a central search coil located on its inner diameter. Measurements were also made of the flux in the coil alone, and this air-core flux was subtracted from the flux readings to give the reed flux Φ.

If the reeds are long compared with the overlap, as in practice and in these measurements, the pull is independent of the reed length. If the permeability of the reed material is high enough for the reeds to be essentially equipotential surfaces near the gap, the pull F is a function of Φ, x, a, b and h only. For dimensional consistency, therefore, the pull must be given by an equation of the form

$$\frac{8\pi abF}{\Phi^2} = f\left(\frac{x}{a}, \frac{a}{b}, \frac{h}{b}\right). \tag{2}$$

This is a convenient dimensionless form, since the left-hand term must, from (1), approach unity for $x = 0$. (In evaluating this term, consistent

TABLE I — DIMENSIONS OF REEDS TESTED

Thickness, h (milli-inches)	Width, b (milli-inches)	Length (inches)
10	100	1.75
20	100	1.75
30	100	1.75
21	60	1.75

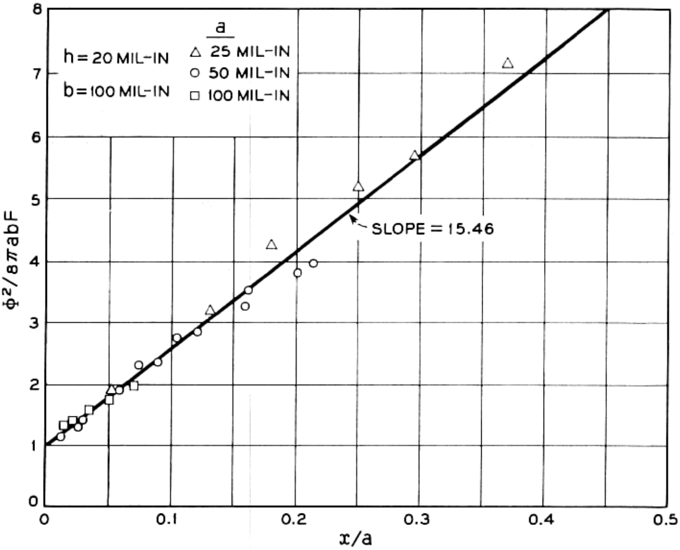

Fig. 2 — Results of pull measurements.

units must be used: in C.G.S. units, Φ is in maxwells, a and b in centimeters, and F in dynes.) From direct plots of the measured values of F and Φ there were read values of F for $\Phi = 10{,}000\ bh$ (i.e. for a flux density of 10,000 gauss). For each set of reeds and each overlap, the corresponding values of $\Phi^2/8\pi abF$ were plotted against x/a, as illustrated in Fig. 2, which shows the results for $h = 10$ milli-inches, $b = 100$ milli-inches and three values of a. As the points for all three values of a fall on the same curve, the right-hand side of (2) is independent of a/b, and reduces to a function of x/a and h/b only. The experimental relation of Fig. 2 is linear, and similar linear plots, independent of a/b, were obtained for the other three sets of reeds. Thus the functional relation (2) was found to be of the form

$$\frac{\Phi^2}{8\pi abF} = 1 + k\,\frac{x}{a}, \tag{3}$$

where k is a function of h/b. The observed values of k for the four sets of reeds were plotted against the corresponding values of h/b and a linear plot was obtained, conforming to the equation

$$k = 6.66 + 44.4\,\frac{h}{b}. \tag{4}$$

These results were obtained for values of Φ corresponding to a density of 10,000 gauss. Similar plots were made for other values of density, and substantially the same relation was found to apply for densities ranging from 20 per cent to 80 per cent of the saturation density, which was about 15,000 gauss for the material used in these tests.

Within this range of reed flux density, therefore, all the observed values of pull conformed approximately to (3), with k given by (4). These expressions may therefore be used to estimate the pull of mating magnetic reeds at densities up to 80 per cent of saturation, when each reed is long compared with the overlap. The expressions are approximate, and minor deviations, particularly in the value of k, are produced by changes in coil length and in the return path and shielding configuration.

A further, and more important, deviation from these relations occurs in the closed gap condition, $x = 0$, where the pull is given by (3) as $\Phi^2/8\pi ab$. For release, this pull equals the retractile force of the reeds, shown in Fig. 3 as sX. Thus the flux at which release occurs should be given by $\sqrt{8\pi absX}$. Observed values of release flux show considerable variation, with the upper limit close to this computed value. This corresponds to the fact that surface irregularities or lack of parallelism of the mating surfaces concentrate the closed gap flux over a smaller area than ab, and increase the pull over that given by (3) for $x = 0$. The actual closed gap pull is therefore variable, and in general higher than the computed closed gap pull.

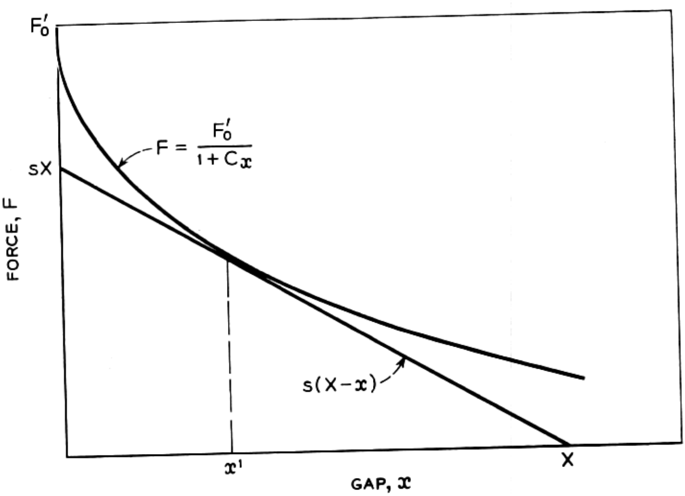

Fig. 3 — Pull and retractile force relations.

III. CAPABILITY

For operation, the pull curve of the reeds must exceed their stiffness load, shown in Fig. 3 as the force $s(X - x)$. Here X is the open gap separation, and s is the combined stiffness, or half the stiffness of one reed if the two are alike. (For unequal reeds, $1/s$ is the sum of the compliances of the two reeds.) Let F_0' be the closed gap pull, $\Phi^2/8\pi ab$, for the minimum value of Φ required for operation. Then from (3) the pull for this value of Φ is given by

$$F = \frac{F_0'}{1 + Cx}, \tag{5}$$

where $C = k/a$. At the point of tangency, $x = x'$, the pull and load curves must be equal and have equal slopes, so that:

$$\frac{F_0'}{1 + Cx'} = s(X - x'),$$

$$\frac{CF_0'}{(1 + Cx')^2} = s.$$

From these two conditions:

$$\frac{x'}{X} = \frac{CX - 1}{2CX}, \tag{6}$$

and, if $CX > 1$,

$$\frac{F_0'}{sX} = \frac{(1 + CX)^2}{4CX}. \tag{7}$$

If $CX \lesseqgtr 1$, the tangency condition no longer applies, the pull and load are equal at $x = 0$, and $F_0' = sX$.

After operation, the flux exceeds the just-operate value. Let F_0 be the value of the closed gap pull $\Phi^2/8\pi ab$ for the final operated flux. Then the contact force is equal to $F_0 - sX$, and the retractile force, tending to open the contact on release, is sX. The contact behavior varies with the contact force, so that one design requirement is that the contact force should exceed some specified minimum value F_C.

A second requirement is that the ratio of retractile force to the contact force should exceed some minimum value n (of the order of unity) to minimize the danger of sticking. The area of actual intimate contact, and hence the force required to break cold welds over this area, varies directly with the contact force. In formulating this requirement, the operated pull is taken as given by F_0. As stated above, the actual operated pull is variable and in general exceeds F_0. Hence the actual contact force is usually in excess of its computed value, and the actual value of the retractile force ratio n less than its computed value.

A satisfactory design must meet the requirements for operation, contact force and retractile force through the range of dimensional variation occurring in manufacture. The present discussion is confined to the case where only variations in the gap X need be considered. Let the minimum and maximum gap values be X_1 and X_2, and let $c_1 = X_2/X_1$. The final flux must be below saturation, and for purposes of estimation may be assumed to correspond to a density B'', of the order of 90 per cent of the saturation density. For operation to occur without excessive reed reluctance, the operating density B' should be of the order of 80 per cent of saturation. Let $c_2 = F_0/F_0' = (B''/B')^2$. Then the operate condition (7) will be satisfied for all values of X if

$$\frac{F_0}{sX_2} = \frac{c_2(1 + CX_2)^2}{4CX_2}.$$

The contact force requirement is satisfied for all values of X if

$$F_0 = F_C + sX_2 .$$

The retractile force requirement that $sX > n(F_0 - sX)$ is satisfied for all values of X if

$$\frac{F_0}{sX_1} = \frac{n + 1}{n}.$$

From the preceding three equations, the design requirements can be written as

$$\frac{4CX_2}{(1 + CX_2)^2} = \frac{c_1c_2n}{n + 1}, \tag{8}$$

$$F_0 = \frac{F_c}{1 - \dfrac{nc_1}{n + 1}}, \tag{9}$$

$$s = \frac{F_0 - F_C}{X_2}. \tag{10}$$

The reed dimensions must be chosen to satisfy these three equations for given values of X_1, c_1, F_C and n (c_2 being approximately fixed by the reed material used). The dimensions to be selected are a, b, h and whatever other dimension determines s: the free length l if the reeds are of uniform cross section. If the section ratio b/h is tentatively taken as fixed by manufacturing considerations, the reed dimensions can be evaluated as follows:

The value of C is computed from (8), and since $a = k/C$ and k is given by (4), the required value of a is thereby determined.

Since the value of Φ for F_0 is bhB'',

$$F_0 = \frac{bh^2B''^2}{8\pi a}. \tag{11}$$

Then, with F_0 evaluated from (9), and a, B'' and b/h known, the thickness h (and hence the width b) can be determined from (11). The stiffness s is given by (10). For a reed of uniform section $b \times h$ the length l can be determined from the required value of s. As (11) determines the value of bh^2, and the stiffness varies as bh^3, the shortest reed meeting the requirements will be that for the largest value of b/h consistent with manufacturing considerations.

The relations outlined above allow for variations in the gap X, but not in the other dimensions. In practice, allowance must also be made for variations in the reed thickness h. This can be done by applying the treatment outlined above to the case where h has upper and lower limits, leading to expressions similar to (8), (9) and (10).

IV. ILLUSTRATIVE COMPUTATIONS

Using the relations given above, reed dimensions have been computed to give the three values of minimum contact force F_C shown as parameters for the curves of Fig. 4, over the range of gap dimensions shown. These illustrative cases were computed for the values of b/h, X_2/X_1, n and flux density shown in the legend of this figure. The values of flux density are those applying to iron-nickel alloys of about 50 per cent nickel. These have a Young's modulus value of about 25×10^6 psi, which was taken as applying in computing the length required to meet the stiffness requirement.

The computed dimensions shown in Fig. 4 are the reed thickness h, length l' and overlap a. The overlap varies inversely with the gap, as shown by (8) when b/h, n, c_1 and c_2 are fixed, as in these computed cases. The thickness and length both increase as the gap and the required contact force are increased. The gap is the major factor controlling the length and hence the over-all size of the sealed contact. The choice of gap is fixed in part by voltage breakdown requirements, and in part by manufacturing considerations which determine the variation in gap for which allowance must be made. The comparisons of Fig. 4 are somewhat idealistic in that a constant ratio c_1 ($= X_2/X_1$) is assumed. Actually, the ratio c_1 is usually larger for small gaps than large ones, in which case the reduction in reed thickness and length resulting from reducing the gap is less than that shown in Fig. 4.

In the computations for Fig. 4 the retractile force ratio n is taken as unity. This is the computed minimum ratio of retractile force to contact

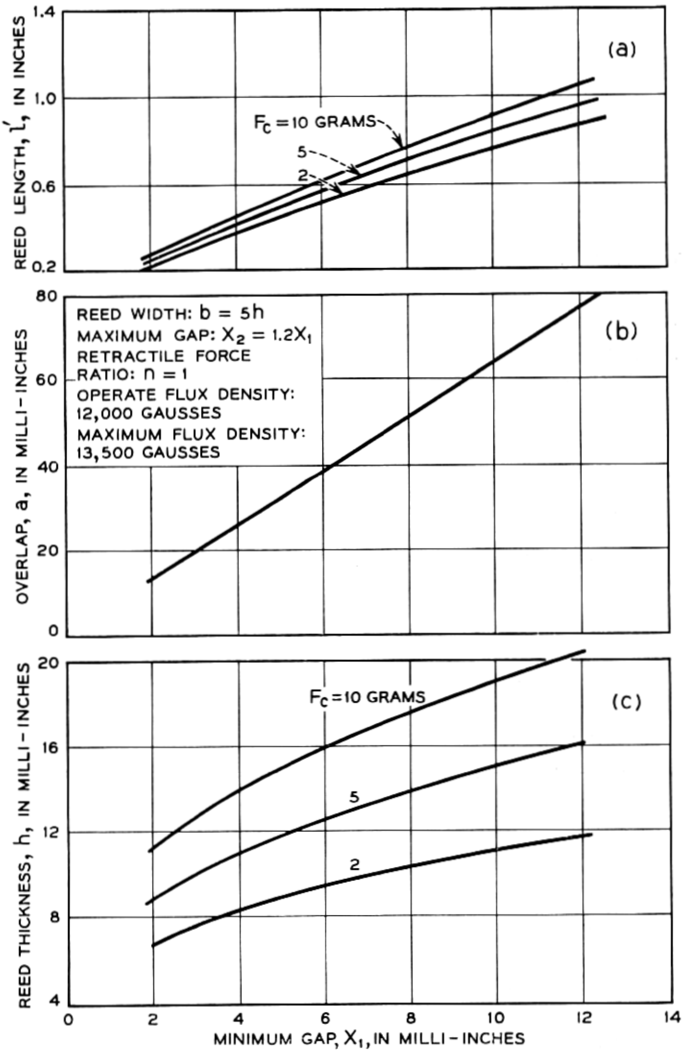

Fig. 4 — Illustrative computed reed dimensions.

force. As discussed previously, the actual contact force may be increased by flux concentration in the closed gap, and the actual ratio may be materially below the computed minimum, which serves therefore as a basis for comparing one design with another, rather than as an absolute measure of the liability to sticking. From (9), increasing n increases F_0, and thus increases the thickness h and the required length l'.

V. STIFFNESS ESTIMATION

The reed length is denoted l' in Fig. 4, as that giving the desired stiffness [$2s$ from (10)] for a uniform reed of length l' and stiffness given by $Ebh^3/4l'^3$. As shown in Fig. 1, convenient construction for the seal requires the use of a cylindrical section extending from the seal for a length cl, with the flat section extending beyond this for a length l. The cylindrical and rectangular sections are of equal area. The stiffness of such two section cantilevers are given in Fig. 6-30 of Peek and Wagar,[7] from which can be derived the following expression for the ratio l'/l of two cantilevers of equal stiffness, one having the dimensions shown in Fig. 1, and one of length l' and the same rectangular section as the other:

$$\left(\frac{l'}{l}\right)^3 = 1 + \left(\frac{\pi h}{3b}\right)[(1 + c)^3 - 1]. \tag{12}$$

Thus if cl and l are known, l' can be evaluated and used to determine the stiffness. Conversely, if l' is determined from the required stiffness, as in Fig. 4, the corresponding flat reed length l can be determined for a given value of cl.

VI. RELUCTANCE FOR AN AIR RETURN PATH

The flux Φ between the reeds required for operation can be estimated by means of the relations given in preceding sections. The corresponding value of coil magnetomotive force $\mathfrak{F}$, or $4\pi NI$, is given by $\mathfrak{R}\Phi$, where $\mathfrak{R}$ is the reluctance of the magnetic circuit. Estimating the sensitivity, or the value of $\mathfrak{F}$ for operation, therefore requires some method for estimating the value of the reluctance $\mathfrak{R}$.

Most reed relays have essentially an air return path, whose reluctance is only slightly reduced by the can or shield placed over the core. For an air return, the flux through the reeds is analogous to the current through a leaky transmission line, as discussed in Section 9-2 of Peek and Wagar.[7] Referring to Fig. 5, let φ be the flux in the reed at a distance x from the plane of symmetry, and let f be the potential difference

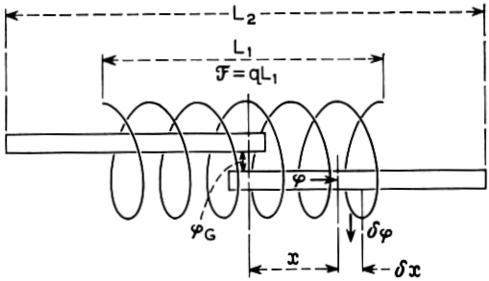

Fig. 5 — Notation for analysis of air return magnetic field.

between the point x and the plane of symmetry. The flux $d\varphi$ leaving the reed over the length dx is given by

$$-pf = \frac{d\varphi}{dx}, \tag{13}$$

where p is the permeance per unit length of reed of the air path from x to the plane of symmetry. The rate of change in f is given by

$$\frac{df}{dx} = q - r\varphi \qquad \text{for } x < \frac{L_1}{2},$$

$$\frac{df}{dx} = -r\varphi \qquad \text{for } \frac{L_1}{2} < x < \frac{L_2}{2},$$

where r is the reluctance of the reed per unit length and $q = \mathfrak{F}/L_1$, where L_1 is the coil length. Substituting (13) in these expressions gives the equations

$$\begin{aligned} \frac{d^2\varphi}{dx^2} - pr\varphi + pq &= 0 \qquad \text{for } x < \frac{L_1}{2}, \\ \frac{d^2\varphi}{dx^2} - pr\varphi &= 0 \qquad \text{for } \frac{L_1}{2} < x < \frac{L_2}{2}. \end{aligned} \tag{14}$$

Subject to limitations discussed below, p and r may be taken as constants, in which case the solution to (14) is given by

$$\begin{aligned} \varphi &= \frac{q}{r} + Ae^{\alpha x} + Be^{-\alpha x} \qquad \text{for } x < \frac{L_1}{2}, \\ \varphi &= Ce^{\alpha x} + De^{-\alpha x} \qquad \text{for } \frac{L_1}{2} < x < \frac{L_2}{2}, \end{aligned} \tag{15}$$

where

$$\alpha = \sqrt{pr}. \tag{16}$$

For continuity, the expressions for φ and $d\varphi/dx$ at $x = L_1/2$ given by the two forms of (15) must be equal, from which expressions are obtained for the coefficients C and D in terms of A and B. The coefficients A and B in turn are determined by the boundary conditions, which require that φ be zero at $x = L_2/2$ and that for a gap flux φ_G at $x = 0$, the potential $f = -\varphi_G\mathcal{R}_G/2$ at $x = 0$, where $\mathcal{R}_G$ is the gap reluctance. Then, from (13), the boundary conditions become

$$\varphi = \varphi_G\,, \quad \frac{d\varphi}{dx} = \frac{p\mathcal{R}_G\varphi_G}{2} \qquad \text{at } x = 0,$$

$$\varphi = 0 \qquad\qquad \text{at } x = \frac{L_2}{2}.$$

Substituting these conditions in (15), there are obtained

$$2A = \left(1 + \frac{p\mathcal{R}_G}{2\alpha}\right)\varphi_G - \frac{q}{r}, \tag{17}$$

$$2B = \left(1 - \frac{p\mathcal{R}_G}{2\alpha}\right)\varphi_G - \frac{q}{r}, \tag{18}$$

$$\varphi_G = \frac{q}{r}\,\frac{\cosh \alpha L_2/2 - \cosh \alpha(L_2 - L_1)/2}{\cosh \alpha L_2/2 + (p\mathcal{R}_G/(2\alpha)\sinh \alpha L_2/2}. \tag{19}$$

As the boundary condition for $x = 0$ shows, the flux φ in the reed initially increases with x, and hence the point of maximum flux Φ is at some positive value of x where $d\varphi/dx = 0$. Applying this condition to the expression for φ given by (15), the maximum flux Φ is given by

$$\Phi = \frac{q}{r} - 2\sqrt{AB}.$$

This maximum flux Φ is the total flux between the reeds. The reluctance $\mathcal{R}$, or $\mathfrak{F}/\Phi$, is given by qL_1/Φ. Substituting from the preceding equations, this expression for $\mathcal{R}$ may be written in the form

$$\mathcal{R}pL_2 = \frac{L_1}{L_2}\,\frac{(\alpha L_2)^2}{1 - \left[\left(1 - \frac{r}{q}\,\varphi_G\right)^2 - \left(\frac{Q}{2\alpha L_2}\,\frac{r}{q}\,\varphi_G\right)^2\right]^{\frac{1}{2}}}, \tag{20}$$

where $Q = \mathcal{R}_G pL_2$, and $r\varphi_G/q$ is given by (19). Thus the ratio $\mathcal{R}pL_2$ is a function of the three ratios L_1/L_2, αL_2 and Q, or $\mathcal{R}_G pL_2$. For the limiting case where the reed reluctance per unit length r is so small that α approaches zero, the hyperbolic functions in (19) may be replaced by their series expansions and the higher power in α neglected. Then for $\alpha \to 0$, (20) reduces to the expression:

$$\mathcal{R}pL_2 = \frac{\dfrac{2(4 + Q)L_2}{2L_2 - L_1}}{1 + \dfrac{Q^2(2L_2 - L_1)L_1}{16(4 + Q)L_2^2}}. \tag{21}$$

VII. DISCUSSION OF AIR RETURN RELUCTANCE

Fig. 6 shows $\mathcal{R}pL_2$ for the case $L_1/L_2 = 0.5$ plotted against $\mathcal{R}_G pL_2$ for various values of αL_2. The increase of $\mathcal{R}$ with increasing αL_2 measures the effect of the reed reluctance. Aside from the reed reluctance, $\mathcal{R}$ comprises the air return reluctance in series with the gap reluctance as shunted by the parallel air path across the gap. The curves show how this shunt path limits the increase in $\mathcal{R}$ with increasing $\mathcal{R}_G$.

The effect of increasing L_1/L_2 is to increase $\mathcal{R}$, as can be seen from (21), except for large values of $\mathcal{R}_G$ (and hence of Q). Thus the reluctance is reduced and the sensitivity increased by using a short coil concentrated over the gap.

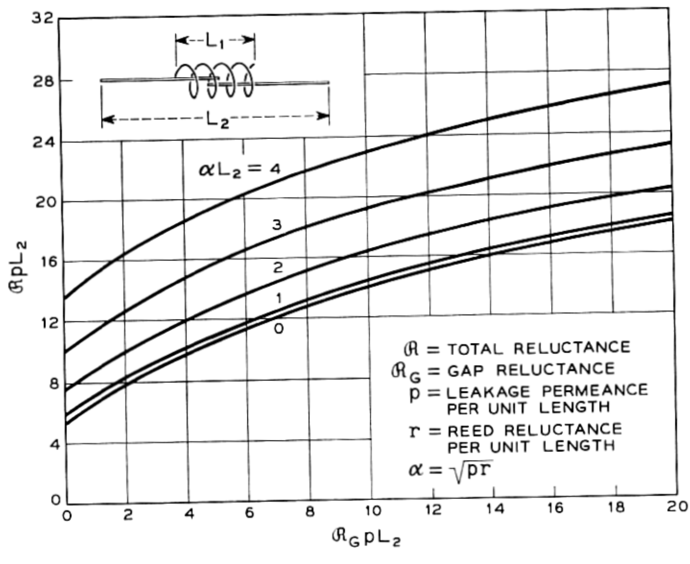

Fig. 6 — Reluctance relation for $L_1/L_2 = 0.5$.

In deriving the preceding equations, p and r have been taken as constants. The estimation of the air path permeance of bar magnets is discussed in a classic paper by Evershed,[8] who showed that satisfactory estimates can be obtained by using the constant value of p applying to a uniformly magnetized ellipsoid. This is the function of the ratio L/D shown in Fig. 7. In applying this to mating rectangular reeds, L is taken as the over-all length L_2, and D as $2(b + h)/\pi$.

The assumed constancy of r, the reluctance per unit length, is at best an approximation, since r is inversely proportional to the permeability of the material, which varies with the flux density, so that r varies along the length of the reeds. If, however, r is small, the error introduced by this approximation is minor.

The effect of the permeability on the reluctance and sensitivity, as related to the size and capability of mating reed contacts, is illustrated by the computed results of Table II. Here the dimensions h, b, a and l have been taken as those shown in Fig. 4 for minimum gap values of 4 and 10 milli-inches and contact force values of 2 and 10 grams. The operate flux values, as in Fig. 4, are for a density of 12,000 gauss. The over-all length L_2 has been taken as 3.75 times the computed reed length. The permeance p is taken from Fig. 7. The critical gap reluctance has been obtained by adding a 2 milli-inch air gap allowance for the closed gap to the critical gap x' computed from (6).

With $\mathcal{R}_G p L_2$ thus computed for each of these two cases, values of αL_2 have been computed for two values of permeability μ, 1000 and 5000. With r given by $1/\mu bh$, corresponding values of αL_2 have been computed, and corresponding values of $\mathcal{R} p L_2$ read from Fig. 6, taking the coil

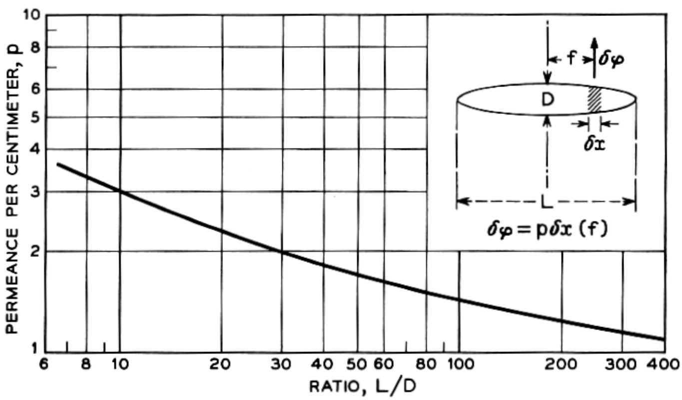

Fig. 7 — Permeance factor for ellipsoidal magnets.

Table II — Illustrative Reluctance and Sensitivity Values

	Gap, X_1 (milli-inches)			
	4		10	
Contact force, F_C (grams)	2		10	
Operate flux, Φ (maxwells)	27		140	
Length, L_2 (cm)	3.65		8.68	
Permeance factor, p	1.76		1.73	
Gap reluctance, $\mathcal{R}_G$ (cm^{-1})	1.30		0.38	
$\mathcal{R}_G p L_2$	8.34		5.77	
	Permeability, μ			
	1000	5000	1000	5000
Reluctance per cm, r (cm^{-2})	0.450	0.090	0.086	0.017
αL_2	3.26	1.30	3.18	1.50
$\mathcal{R} p L_2$	19.0	13.8	17.0	12.5
$\mathcal{R}$ (cm^{-1})	2.96	2.15	1.11	0.83
Operate ampere-turns	63	46	126	93

length L_1 as half of L_2. There were thus obtained the values of reluctance $\mathcal{R}$ shown in Table II, and thus finally the operate values, given (in abampere-turns) by $\mathcal{R}\Phi/4\pi$.

In the range of magnetic reed dimensions and contact and retractile forces illustrated by these computations, αL_2 is small and the reed reluctance minor for μ of the order of 5000. The materials used for these reeds have permeabilities of 5000 and higher at densities of the order of 12,000 gauss, or at about 80 per cent of the saturation density. The permeability falls rapidly as the density approaches saturation, and is of the order of 1000 at about 90 per cent of the saturation density. Hence a design requiring an operating density over 80 per cent that for saturation may have a materially increased reluctance and consequent loss of sensitivity.

VIII. RELEASE RELUCTANCE AND SENSITIVITY

As noted in the discussion of the pull relation, the computed value of flux at which release occurs is given by $\sqrt{8\pi absX}$. Since this is necessarily a lower value of flux than that for operation, the density is lower and the permeability higher than in operation, so that in the release case the reed reluctance is minor, and the preceding equations can be used to estimate the reluctance for the closed gap condition. Since flat metallic pole faces in contact have an effective air gap x_0 of about 2 milli-inches (0.005 cm), $\mathcal{R}_G$ for this case is given by x_0/ab. Then the abampere-turns for release are given by $\mathcal{R}\Phi/4\pi$, where $\mathcal{R}$ and Φ are the values applying in release. As previously stated, the actual release flux is variable, and in general less than its computed value, and there is therefore a corresponding variation in release sensitivity.

IX. ALTERNATIVE RELUCTANCE APPROXIMATIONS

The analysis of the two preceding sections is useful in providing an understanding of the field distribution in the reeds and coil and of the way in which the leakage field shunting the gap varies with the gap, the flux density, and the reed and coil dimensions. It is, however, limited to the case of an air return, and does not apply to a configuration such as that shown in Fig. 8, where sleeve members are used to couple the reeds to the return path provided by the cover. In such cases, the reluctance can be approximately estimated in terms of the lumped constants of the magnetic circuit shown in the figure. A similar treatment can be applied to the air return case, giving approximate expressions that are simpler than those developed above.

As indicated in Fig. 8, the coil magnetomotive force is taken as developing an air flux φ_A, which only affects the inductance, and a reed flux Φ, the maximum or total flux Φ of the preceding discussion. The path of Φ comprises the reed reluctance $\mathcal{R}_R$ in series with the sleeve gap reluctance $2\mathcal{R}_S$ and the parallel combination of the gap reluctance $\mathcal{R}_G$ and the shunting leakage reluctance $\mathcal{R}_L$. The path, of course, includes the shielding return members, but these are negligible in reluctance compared with the rest of the path.

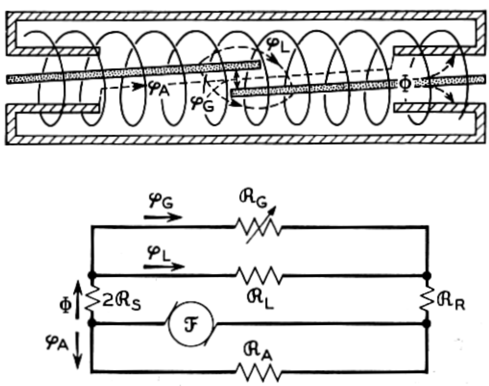

Fig. 8 — Magnetic circuit for reeds coupled to a return path member.

The reed reluctance can be estimated from the reluctance per unit length $r = 1/\mu bh$. The sleeve reluctance can be estimated by an approximation obtained from Fig. 9-8 of Peek and Wagar,[7] from which

$$\mathcal{R}_S = \frac{1}{2\pi L_S} \ln \frac{\pi D}{2(b + h)}, \tag{22}$$

where L_S is the sleeve length, and D its diameter.

As shown in the air return case, the leakage reluctance $\mathcal{R}_L$ varies with the gap reluctance, and cannot readily be expressed in a simple form. An approximate expression for the parallel combination of $\mathcal{R}_G$ and $\mathcal{R}_L$, however, can be derived from the experimentally observed pull relation of (3). The pull given by this relation must equal that given by the general pull equation for a variable reluctance $\mathcal{R}_X$:

$$F = \frac{\Phi^2}{8\pi} \frac{d\mathcal{R}_X}{dx}.$$

If $\mathcal{R}_X$ is the parallel combination of $\mathcal{R}_G$ and $\mathcal{R}_L$, this expression for F/Φ^2 can be equated to that given by (3), and the resulting expression for $d\mathcal{R}_X/dx$ integrated to give the increase in $\mathcal{R}_X$ from its value for $x = 0$. This zero gap value may be taken as x_0/ab, with $x = 0.005$ cm, as in the Table II estimates of $\mathcal{R}_G$. Thus $\mathcal{R}_X$ is given approximately by

$$\mathcal{R}_X = \frac{x_0}{ab} + \frac{1}{kb} \ln \left(1 + k \frac{x}{a}\right), \tag{23}$$

where, as before, k is given by (4).

Then the total reluctance $\mathcal{R}$ for the magnetic circuit of Fig. 8 is given by

$$\mathcal{R} = \mathcal{R}_E + \mathcal{R}_R + \mathcal{R}_X, \tag{24}$$

with $\mathcal{R}_X$ given by (23) and $\mathcal{R}_E$ taken as $2\mathcal{R}_S$, as given by (22). This expression can be used to estimate the reluctance for other relay configurations, using expressions for $\mathcal{R}_E$ corresponding to the return path coupling and configuration. It may be applied as an approximation to the air return path case, with $\mathcal{R}_E$ taken as the air return reluctance. For negligible reed reluctance, this is given by (21) for $\mathcal{R}_G$ and hence Q equal to zero, so that $\mathcal{R}_E$ is given by

$$\mathcal{R}_E = \frac{8}{p(2L_2 - L_1)}. \tag{25}$$

While these expressions for the reluctance are approximate, they are simple in form, and are convenient to use, in conjunction with operate and release flux estimates, in estimating operate and release sensitivity.

X. MOTION TIME

The operate time of magnetic reed contacts is that required for field development and reed motion. These two occur together, since reed motion starts as soon as the field and the resultant pull start to develop. If the power input is high enough for the field to develop rapidly, the reeds may be saturated before any significant motion has occurred. In this case the times of field development t_1 and reed motion t_2 are additive in determining the operate time. The value of t_2 for this case constitutes a lower limit to the operate time, since no shorter motion time is possible than that for full magnetization of the reeds. An expression for this minimum motion time may be obtained by the following approximation.

The pull curve is given by (3), where Φ has its maximum value, which may be taken as about 90 per cent of saturation, as in the capability estimates. This pull exceeds the just-operate pull shown in Fig. 3, and hence always exceeds the stiffness load $s(X - x)$. The area between the pull curve and the load line represents the kinetic energy supplied to the reeds, which may be denoted $2T$, so that T is the kinetic energy supplied to each of the equal reeds. Writing F_0, as before, for the closed gap pull for maximum flux, the kinetic energy is given by

$$2T = \int_0^X \left[\frac{F_0}{1 + Cx} - s(X - x)\right] dx,$$

and hence

$$2T = \frac{F_0}{C} \ln (1 + CX) - \frac{SX^2}{2}. \tag{26}$$

While the acceleration is variable, the motion time is nearly the same as that for uniform acceleration, for which the kinetic energy T for each reed is given by

$$T = 2m \left(\frac{X}{2t_2}\right)^2,$$

where m is the effective mass of each reed, which moves through half the gap X in time t_2. Thus the minimum motion time t_2 is given by:

$$t_2 = X \sqrt{\frac{m}{2T}}, \tag{27}$$

where T is given by (26). To illustrate the magnitude of the quantities involved for mating magnetic reeds, Table III gives the computed values of t_2 for the two cases of Fig. 4 in which the minimum gaps are 4 and 10

TABLE III — MOTION TIME ESTIMATES

X_2 (milli-inches)	X_2 (cm)	F_0 (dynes)	s (dynes/cm)	C (cm^{-1})	$2T$ (ergs)	m (grams)	t_2 (μ seconds)
4.8	0.0122	5,000	246,000	240	10.2	0.0041	245
12.0	0.0305	25,000	491,000	96	128	0.0549	634

milli-inches and the contact forces are 2 and 10 grams respectively. The gap is taken as having its maximum value X_2, and F_0 as corresponding to a density of 13,500 gauss, as before. As the reeds deflect as cantilever beams, the effective mass m of each reed is taken as one quarter of its actual mass, assuming a density of 8.2 grams/cm.^{3} The values of C, or k/a, are those applying for the dimensions given in Fig. 4 for these two cases.

These estimates show that for the range of reed and gap dimensions covered in Fig. 4, the motion times lie in a relatively narrow band. These times are small compared with those of most other devices which open and close contacts in a metallic circuit: they are large compared with the switching times of many electronic devices.

XI. OPERATE TIME

To estimate the total operate time, an expression is required for the time of flux development. As shown in Chapter 4 of Peek and Wagar,[7] an adequate approximation can be obtained from the exponential relation for flux development with the constant reluctance (and hence inductance) for the initial open gap condition. For magnetic reeds, eddy current effects are negligible except for flux development so fast that t_1 is negligible compared with t_2. The flux and coil current develop together with a time constant L/R. The time t_1 at which the ampere-turns equal the just operate value $(NI)_0$ is given by

$$\frac{(NI)_0}{NI} = 1 - e^{-Rt_1/L},$$

where NI is the steady-state value of ampere-turns, Hence:

$$t_1 = \frac{L'N^2}{R} \ln \frac{1}{1 - v},$$

where $v = (NI)_0/(NI)$ and L' is the single-turn inductance, or $4\pi/\mathcal{R}$, where $\mathcal{R}$ is the open gap reluctance for the average flux linked by the coil. If both numerator and denominator in this expression are multiplied by I^2, NI is written as $(NI)_0/v$, and t_2 added to t_1 to give the total operate time t, the resulting expression is

$$t = t_2 + \frac{L'(NI)_0^2}{I^2R}\left(\frac{1}{v^2}\ln\frac{1}{1-v}\right). \tag{28}$$

As shown in the text reference cited above, the function of v appearing in brackets has a minimum value of 2.5 for $v = 0.715$, and departs from this minimum by less than 5 per cent for $0.6 < v < 0.8$. Thus for minimum operate time, $(NI)_0/NI$ should lie in this range, and (28) reduces to

$$t = t_2 + \frac{2.5nL''(NI)_0^2}{I^2R}, \tag{29}$$

where nL'' is written for L'. If there are n sealed contacts (pairs of mating reeds) used in a single relay coil, the single-turn inductance is nL'', where L'' is the single turn inductance of one sealed contact.

If the total time t is materially larger than t_2, the two terms of (29) are not strictly additive, but the approximation has been found to give adequate agreement with experimental observations.

In (28), the term $L'(NI)_0^2$ is proportional to $\Phi(NI)_0$ (where Φ is the operate flux) and hence to the field energy required for operation. Thus the electrical energy input for operation, which is proportional to I^2Rt_1, is proportional to this field energy. As previously noted, (28) also shows that t_1 varies with v, or I_0/I, and that the time is a minimum for a given power input if the coil circuit is designed to give I_0/I a value of about 0.7. It also follows from (29) that increasing the steady-state power reduces the operate time until it approaches the lower limit t_2, the minimum time for reed motion.

To use (29) for preliminary estimates requires estimation of the operate ampere-turns $(NI)_0$ and the single-turn inductance nL''. $(NI)_0$ can be estimated by the procedures described in the sections on reluctance estimates. To estimate the single-turn inductance requires estimating the reluctance for the average field linked by the coil. In general, this is a larger reluctance than that for the maximum reed flux, to which the expressions given in preceding sections apply. The maximum flux is limited by reed saturation, and hence controls the capability.

For a tightly coupled magnetic circuit, as in Fig. 8, the maximum and average flux are nearly the same, and the same reluctance expressions apply, except that some allowance may be made for the air field (φ_A of Fig. 8) in estimating the inductance. For the distributed field of

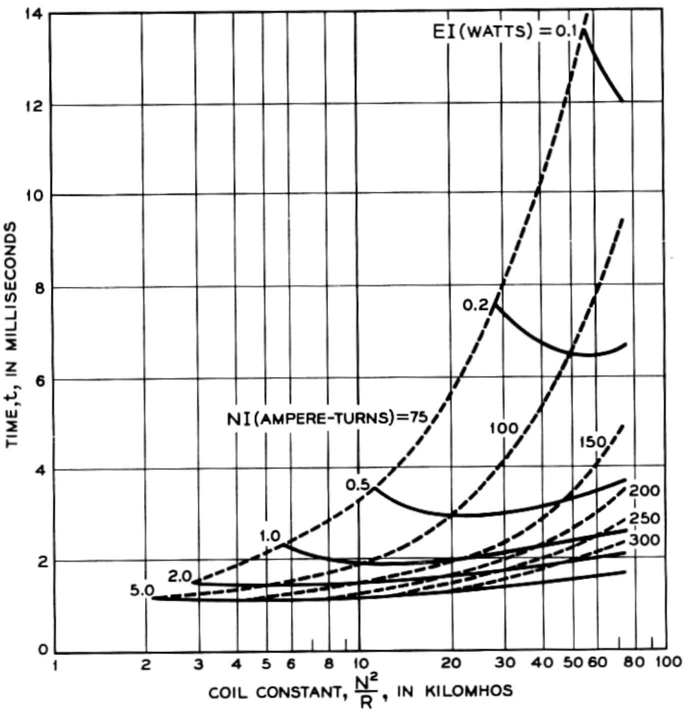

Fig. 9 — Observed operate times for a two-switch reed relay.

an air path return relay, the average flux linked by the coil is materially less than the maximum value. The analysis of this case given above can be used to give expressions for the average flux, but these are complex in form, and are not included here.

The agreement of experimental observations with (28) and (29) is illustrated in Figs. 9 and 10. Fig. 9 shows direct measurements of operate time plotted against the coil circuit constant N^2/R for various values of steady state power input. The dotted curves are for constant values of ampere-turns NI. [$(NI)^2 = I^2R \cdot (N^2/R)$.] The sealed contacts in this relay had a value of $(NI)_0$ of about 100 ampere-turns, for which the minimum times should occur for a value of NI of about 140, which is near that for the observed minima.

The minimum times for this two-switch assembly are plotted against n/I^2R in Fig. 10, together with those for three other cases with one, three and four switches. In agreement with (29) the plots are linear, and have a common intercept t_2, the minimum motion time. These

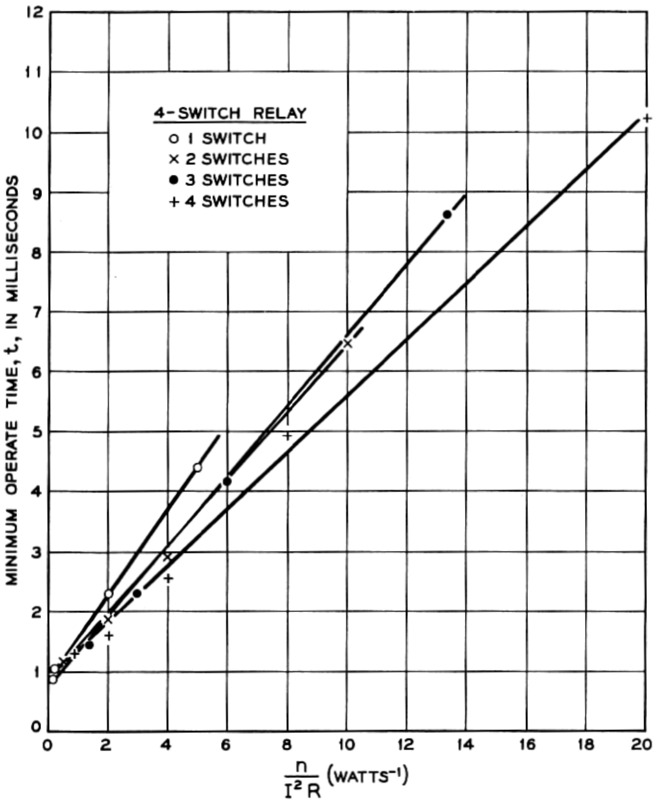

Fig. 10 — Observed minimum operate times for a reed relay.

times were measured to the end of the short chatter interval following initial closure, so that the indicated value of t_2 includes this chatter interval. The slopes of the lines measure the relative value of the single turn inductance per contact L''. The differences in these slopes is ascribed to the considerable air core inductance in this coil, which had space for four switches (sealed contacts). This results in a decrease in the apparent value of L'' as the number of switches increases.

XII. DISCUSSION

The analysis given in this paper is intended for use in development studies of sealed magnetic reed contacts. It can be used to estimate the gap and reed dimensions needed to meet design objectives with respect to contact gap, contact and retractile forces, size, speed and sensitivity. In development studies, models based on the initial estimates can be made and measured, and the measurement results used in conjunction with the analysis as a guide in further work.

In the performance of mating magnetic reeds, the major controlling quantity is the flux between the reeds. This flux, which determines the pull, is limited by saturation and hence by the reed cross section. This total reed flux comprises the gap flux proper and the leakage flux shunting the gap. As shown in the general analysis of the air return reluctance, this leakage field varies with the gap, reed and coil dimensions, and the relation between the pull and the total reed flux has a corresponding variation. Hence the simple pull equation derived from experiment applies rigorously only for the experimental coil length and return path conditions used in deriving it, and minor deviations occur in applying it to other cases.

Corresponding deviations occur for the reluctance given by the expression derived from the pull equation, and for the other approximate reluctance equations. All these approximations, however, are adequate for preliminary estimates, and may be applied with greater accuracy in detailed studies by using values of the constants that are experimentally determined for the case in question. While the general analysis can be used to estimate the air return reluctance, and the treatment can be extended to give expressions for the pull and the inductance, it is limited to the air return case, and is too complex for convenient application. It serves, however, to show the character and extent of the deviations from the simpler approximations resulting from the distributed character of the leakage field.

As the times of flux development and reed motion depend respectively on the inductance corresponding to the coil flux linkages and on the pull for maximum flux, timing estimates based on the simple approximations given here are also subject to some minor variation. This can be reduced in development studies by using experimentally determined values of the constants.

The expressions given here apply primarily to mating reeds of equal size, but are approximately applicable to unequal reeds provided both are long compared with the gap overlap. In such cases the effective stiffness of the reed combination must be taken as the reciprocal of the sum of the compliances of the two reeds.

XIII. ACKNOWLEDGMENTS

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