Winding Tolerances in Helix Waveguide

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In a perfect helix waveguide the circular electric wave loss is increased by eddy currents, finite pitch of the helix, radiation through the wire spacing and effects of the wire coating. Only the contributions from eddy currents and pitch are large enough to limit wire size and spacing.

Experimental helix waveguides have tilted turns. These tilts cause coupling between circular electric and unwanted modes. From the coupling between modes in curved and in offset helix waveguide, the coupling in a helix waveguide with tilted turns is found. For helix waveguide with slightly irregular winding of arbitrary form, generalized telegraphist's equations are derived.

Tilts and other irregularities in the winding increase the circular electric wave loss. The average increase is a function of the covariance of irregularities. Winding tilts with an exponential covariance and an rms value of 0.6° increase the TE_{01} loss in 2-inch inside diameter waveguide at 55 kmc at the most by 10 per cent of the loss in a perfect copper pipe with smooth walls. Present fabrication procedures insure a smaller wire tilt than this.

I. INTRODUCTION

Helix waveguide consisting of closely wound insulated copper wire covered with an electrically absorbing or reactive jacket is a good transmission medium for circular electric waves.[1] In long distance communication with circular electric waves it is useful as a mode filter, for negotiating bends or particularly as transmission line proper instead of a plain metallic waveguide.

As in metallic waveguide, the loss of circular electric waves decreases steadily with frequency only in a perfect helix waveguide. Any deviations from a round and straight guide and from a uniform and low pitch will add to the loss of circular electric waves.

Deviations from straightness and deformations of the cross section of helix waveguide have been analyzed before and their effect on circular electric wave transmission has been determined.[2,3] When these imperfections are caused in the manufacturing process they are statistically distributed over the guide length with a small correlation distance. Then they add nearly the same average loss to the circular electric wave as they do in a plain metallic waveguide.[4] Manufacturing tolerances for straightness and for cross-sectional deformations are therefore the same for helix waveguide as they are for metallic waveguide.

Deviations of the winding from a low-pitch uniform spiral are imperfections peculiar to the helix waveguide. Their effect on circular electric wave transmission will be analyzed here and tolerances on the winding of the helix waveguide for low-loss transmission will be determined.

II. WIRE SIZE AND PITCH

This section reviews results of earlier work.

Helix waveguide is usually wound from round wire with an insulating layer. Even when such a helix is perfectly accurate and uniform its differences from a smooth metallic waveguide add to the circular electric wave loss. The various effects can be listed as follows:

2.1 *Eddy Current Losses in the Spaced Wires*[5,6]

The circumferential wall currents of circular electric waves are uniformly distributed in a smooth wall. In the spaced wires of the helix waveguide their distribution is nonuniform. The heat loss is therefore increased over the smooth wall loss. In Fig. 1 this loss increase is plotted over the spacing for a wire size small compared to the wavelength, using Morrison's calculations.

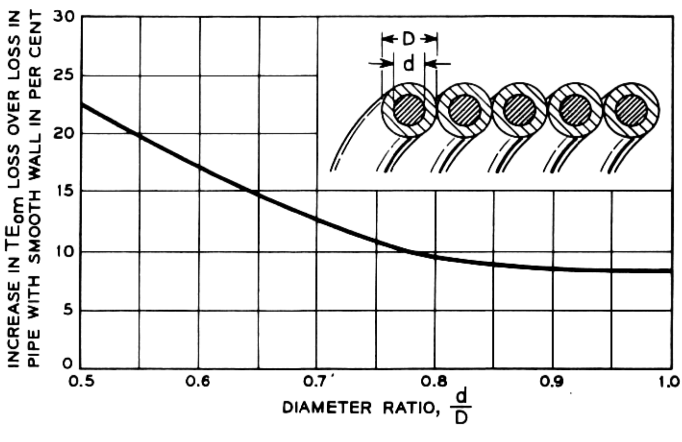

Fig. 1 — Eddy current losses in spaced helix wires (Ref. 5).

2.2 *Pitch*[1]

If the helix of radius a is closely wound from a single wire of insulation diameter D, then the pitch angle ψ is given by

$$\tan \psi = \frac{D}{2\pi a}.$$

If for a faster manufacturing process n wires are wound simultaneously,

$$\tan \psi = \frac{nD}{2\pi a}. \qquad (1)$$

The wall currents of circular electric waves are strictly circumferential. In the helix waveguide their path is disturbed by the finite pitch. Power of circular electric waves is dissipated into the wall impedance Z, which the surrounding jacket presents to the waveguide interior. The added circular electric wave loss due to finite pitch is

$$\alpha_p = \frac{k_m^2}{kh_m a^3} \operatorname{Re}\left(\frac{Z}{Z_0}\right) \sin^2 \psi, \qquad (2)$$

where $Z_0 = \sqrt{\mu/\epsilon}$ is the wave impedance and $k = \omega\sqrt{\mu\epsilon}$ the propagation constant of free space. k_m is the mth root of $J_1(x) = 0$, and $h_m = \sqrt{k^2 - (k_m^2/a^2)}$ the phase constant of the TE_{0m} wave.

A reactive jacket will not dissipate any power. A helix waveguide, designed for transmitting the circular electric wave around bends, has a quarter wave jacket with a very large wall impedance.[2] In this case, to keep α_p low, ψ has to be chosen small.

2.3 *Power Dissipation Through Wire Spacing*

Even though the helix is closely wound the wires are spaced by the wire insulation. With the electric field of circular electric waves parallel to the wires, the space between acts as waveguide below cutoff. Being short, this cutoff waveguide will transmit some circular electric wave power, which is then absorbed by the jacket. The circular electric wave loss caused by this power absorption has been investigated for various forms of wire cross section.[7] For round helix wires this loss is so small compared to the eddy current losses of Fig. 1 that it may be entirely neglected for any wire spacing. Consequently the increase in eddy current losses, rather than the power dissipation through the gaps, limits the wire spacing.

2.4 *Effect of Wire Insulation*

The insulating layer of the helix wires adds to the circular electric wave loss in two different ways. Its dielectric constant tends to concentrate the electric field into the layer. Thus the wall currents and the wall current losses are increased. In addition, the finite loss factor of any insulating material causes dielectric losses in the small but finite electric field of the circular electric wave. Both of these effects can be calculated with a sufficient approximation from attenuation formulas for the round waveguide with a dielectric lining.[8]

Again, it is found that the effects of the insulating layer are so small compared to the eddy current losses that they may be neglected.

Number of wires, wire size, and wire spacing through insulation are therefore determined by the pitch effect of (2) and the eddy current loss of Fig. 1. To speed the winding the numbers of wires should be large. To increase the effects of a reactive or resistive jacket on unwanted modes the wires should be widely spaced.[2] The increase in circular electric wave loss from Fig. 1 and equation (2) sets a limit, however, to number of wires and their spacing.

III. TILTED WINDING

The preceding discussion has considered only the loss in a perfectly wound helix waveguide. A practical helix will not have perfectly uniform windings. One imperfection in particular has been most notable in research models of helix waveguide made by Bell Telephone Laboratories. This imperfection is tilts in the winding.

Aside from the finite pitch, a single turn of the helix is usually not in a transverse plane, but is slightly inclined and forms a small angle θ with the axis. Even an improved winding method with an automatic feed control has not entirely eliminated this inclination.[9]

Such inclined helix turns give rise to mode conversion. There is a simple way to analyze circular electric wave propagation in helix waveguide with nonuniformly tilted winding, in which the results of previous calculations are used. Consider a perfect helix waveguide, a section of which between $z = 0$ and $z = L$ has been deflected in an arbitrary manner by $x(z)$, as shown in Fig. 2. With this deflection is associated a change of guide direction dx/dz and for gentle deflections a curvature $1/R = d^2x/dz^2$.

One way to calculate propagation through this deflected section is to use the formulas for wave propagation in the curved helix waveguide[2] and evaluate them for the curvature distribution d^2x/dz^2. Thus, when propagation is described by generalized telegraphist's equations,[10] there is curvature coupling between a circular electric mode m and the modes n

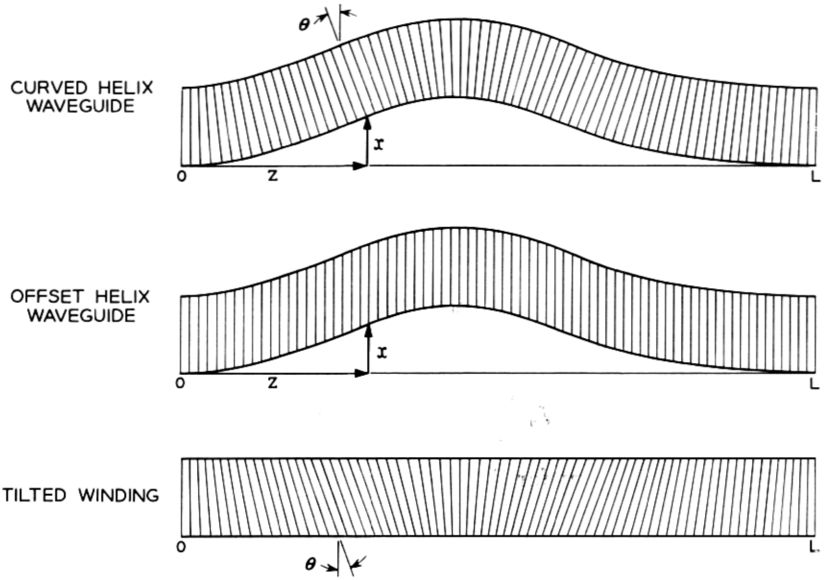

Fig. 2 — Curved helix waveguide as superposition of offset and tilted winding.

of first order circumferential dependence (i.e., TE_{1n} and TM_{1n}). The coupling coefficient is

$$c_c = C_c \frac{d^2x}{dz^2}, \tag{3}$$

with

$$C_c = N_n \frac{\sqrt{\pi}}{2ka} \sqrt{\frac{h_n}{h_m}} \frac{k_m k_n^2}{k_m^2 - k_n^2} J_1(k_n) \left(1 + \frac{h_m}{h_n} + \frac{h_m + h_n}{h_m - h_n} d_n\right) \tag{4}$$

where k, k_m and h_m have the same meanings as in (2). k_n is the radial propagation constant of the coupled mode n and $h_n = \sqrt{k^2 - (k_n^2/a^2)}$ is the axial propagation constant of the coupled mode n. N_n is a normalization factor for the coupled mode n [given by (35) below] and

$$d_n = \frac{pJ_p(k_n)}{k_n J_p'(k_n)}, \tag{5}$$

where $p = 1$ in the present case.

A mode m of unit amplitude incident at $z = 0$ converts power in the deflected section to the coupled modes n. For gentle deflection the amplitude and phase of mode n at $z = L$ is given by

$$A_n = -je^{-jh_nL} \int_0^L C_c \frac{d^2x}{dz^2} e^{-j(h_m-h_n)z} \, dz. \tag{6}$$

Another way to calculate propagation through the deflected section is to consider it as a continuously offset waveguide with a continuously varying tilt θ of the winding. Both the offset x and the tilt $\theta = dx/dz$ will then cause coupling between a circular electric mode m and modes n of first-order circumferential dependence. The coupling coefficient for offset has been calculated[3] before:

$$c_0 = C_0 x, \tag{7}$$

with

$$C_0 = N_n \frac{\sqrt{\pi}}{2} \sqrt{\frac{h_n}{h_m}} \frac{k_m k_n^2}{ka^3} J_1(k_n) \, d_n. \tag{8}$$

The coupling coefficient for tilted winding is still unknown:

$$c_t = C_t \frac{dx}{dz}. \tag{9}$$

Amplitude and phase of a coupled mode n are now given by

$$A_n = -je^{-jh_nL} \int_0^L \left(C_0 x + C_t \frac{dx}{dz} \right) e^{-j(h_m-h_n)z} \, dz. \tag{10}$$

Equation (10) should give the same result as (6). Integrating by parts brings (10) into a form that can be directly compared with (6):

$$A_n = -je^{-jh_nL} \int_0^L - \left(\frac{C_0}{(h_m - h_n)^2} + j \frac{C_t}{h_m - h_n} \right) \frac{d^2x}{dz^2} e^{-j(h_m-h_n)} \, dz. \tag{11}$$

Equations (11) and (6) can only given identical results when

$$C_c = - \frac{C_0}{(h_m - h_n)^2} - j \frac{C_t}{h_m - h_n}.$$

Hence the coupling coefficient for tilted winding is

$$jC_t = (h_n - h_m) C_c + \frac{C_0}{h_n - h_m}. \tag{12}$$

Substituting from (4) and (8) into (12), and from (12) into (9):

$$jc_t = N_n \frac{\sqrt{\pi}}{2} \frac{k_m k_n^2}{k\sqrt{h_m h_n} a^3} J_1(k_n) \theta. \tag{13}$$

With these coupling coefficients, generalized telegraphist's equations can be written down for the helix waveguide with tilted winding. Their solution describes propagation in the obliquely wound helix waveguide completely. For example, if a mode m of unit amplitude is incident on a length L of helix waveguide with nonuniformly tilted winding $\theta(z)$, then the output amplitude and phase of this mode m is given to second order by

$$A_m = e^{-jh_mL}\left[1 - \sum_n \int_0^L e^{j(h_m-h_n)z}\,dz \int_0^{L-z} c_{tn}(u)c_{tn}(u+z)\,du\right]. \quad (14)$$

The summation in (14) is to be extended not only over all the modes n but also over their two polarizations according to the orientation of θ; θ may not only be in the plane of Fig. 2, it can also be perpendicular to that plane.

From (14) the loss which is added to the mode m by a tilted winding may be calculated. With

$$j(h_m - h_n) = \Delta\alpha_n + j\Delta\beta_n$$

and

$$C_{tn}^2 = P_n + jQ_n\,,$$

the added loss is to second order in θ is

$$\alpha_t = \frac{1}{L}\sum_n \int_0^L e^{\Delta\alpha_n z}(P_n \cos \Delta\beta_n z - Q_n \sin \Delta\beta_n z)\,dz \cdot \int_0^{L-z} \theta(u)\theta(u+z)\,du. \quad (15)$$

IV. IRREGULAR WINDING

The obliquely wound helix of the preceding section is just a special case of a general irregular winding. In Fig. 3 a turn of such an oblique helix has been drawn in more detail. Aside from a small pitch the wire follows the curve

$$z = a \tan \theta(1 - \cos \varphi) \quad (16)$$

around the circumference. Its direction deviates by ψ from the transverse direction, where

$$\tan \psi = \tan \theta \sin \varphi$$

or, to first order for small tilt,

$$\psi = \theta \sin \varphi. \quad (17)$$

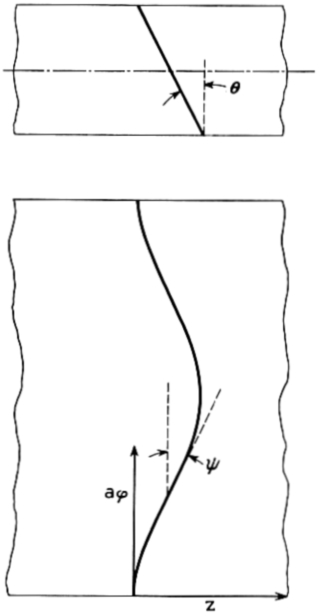

Fig. 3 — Tilted turn in helix waveguide.

A general irregular winding can be described by a Fourier series

$$\psi = \sum_p \theta_p \sin p\varphi. \tag{18}$$

A summation of cos $p\varphi$ would only add identical terms with different polarization; it has been omitted from (18). The boundary conditions at this irregular helix are

$$E_\varphi = -E_z \tan \psi, \tag{19}$$

$$E_z = -ZH_\varphi + (E_\varphi - ZH_z) \tan \psi, \tag{20}$$

where Z is the wall impedance which the outside jacket presents through the helix to the waveguide interior.

Wave propagation in such an irregular structure is best analyzed by converting Maxwell's equations:

$$\frac{1}{r}\frac{\partial E_z}{\partial \varphi} - \frac{\partial E_\varphi}{\partial z} = -j\omega\mu H_r\,, \tag{21}$$

$$\frac{\partial E_r}{\partial z} - \frac{\partial E_z}{\partial r} = -j\omega\mu H_\varphi\,, \tag{22}$$

$$\frac{1}{r}\frac{\partial}{\partial r}(rE_\varphi) - \frac{1}{r}\frac{\partial E_r}{\partial \varphi} = -j\omega\mu H_z \tag{23}$$

$$\frac{1}{r}\frac{\partial H_z}{\partial \varphi} - \frac{\partial E_\varphi}{\partial z} = j\omega\epsilon E_r , \tag{24}$$

$$\frac{\partial H_r}{\partial z} - \frac{\partial H_z}{\partial r} = j\omega\epsilon E_\varphi , \tag{25}$$

$$\frac{1}{r}\frac{\partial}{\partial r}(rH_\varphi) - \frac{1}{r}\frac{\partial H_r}{\partial \varphi} = j\omega\epsilon E_z \tag{26}$$

into generalized telegraphist's equations[10] for the boundary conditions (19) and (20).

An appropriate form of representation is in terms of normal modes of the perfect helix waveguide. With two sets of wave functions

$$\begin{aligned} T_n &= N_n J_p(\chi_n r) \sin p\varphi, \\ T_n' &= N_n J_p(\chi_n r) \cos p\varphi \end{aligned} \tag{27}$$

which satisfy the wave equation

$$\nabla^2 T_n = -\chi_n^2 T_n \tag{28}$$

the normal mode fields of the perfect helix waveguide are the individual terms of the sums:

$$\begin{aligned} E_r &= \sum_n V_n \left(\frac{\partial T_n}{\partial r} + \frac{d_n}{r}\frac{\partial T_n'}{\partial \varphi}\right), \\ E_\varphi &= \sum_n V_n \left(\frac{\partial T_n}{r\partial \varphi} - d_n \frac{\partial T_n'}{\partial r}\right), \\ H_r &= \sum_n I_n \left(-\frac{\partial T_n}{r\partial \varphi} + d_n \frac{h_n^2}{k^2}\frac{\partial T_n'}{\partial r}\right), \\ H_\varphi &= \sum_n I_n \left(\frac{\partial T_n}{\partial r} + \frac{d_n}{r}\frac{h_n^2}{k^2}\frac{\partial T_n'}{\partial \varphi}\right), \end{aligned} \tag{29}$$

and by substituting from (29) into (23) and (26) and using (28):

$$\begin{aligned} H_z &= j\omega\epsilon \sum_n V_n \, d_n \frac{\chi_n^2}{k^2} T_n', \\ E_z &= j\omega\mu \sum_n I_n \frac{\chi_n^2}{k^2} T_n . \end{aligned} \tag{30}$$

From $E_\varphi = 0$ at the perfect helix:

$$d_n = \frac{p}{k_n} \frac{J_p(k_n)}{J_p'(k_n)}, \tag{31}$$

and from $E_z = -ZH_\varphi$ again for the perfect helix:

$$\frac{p}{k_n^2}\left(\frac{h_n^2}{k^2} d_n - \frac{1}{d_n}\right) = \frac{j}{\omega\epsilon a Z}. \tag{32}$$

Equation (32) is the characteristic equation for the perfect helix waveguide. Its roots $k_n = \chi_n a$ determine the propagation constants

$$h_n^2 = k^2 - \chi_n^2 \tag{33}$$

of the normal modes of the perfect helix waveguide.

The transverse fields of the normal modes are orthonormal in that

$$\frac{1}{V_n I_m} \int_S (E_{tn} \times H_{tm})\, dS = \delta_{nm} \tag{34}$$

when the normalization factor N_n in (27) is chosen so that

$$N_n = \frac{\sqrt{2}}{\sqrt{\pi}\, J_p(k_n)} \left[\frac{h_n^2}{k^2}(k_n^2 - p^2)\, d_n^2 + \frac{p^2}{d_n^2} + k_n^2\left(1 - \frac{p^2}{k^2 a^2}\right) + 2p\left(\frac{1}{d_n} - d_n\right)\right]^{-\frac{1}{2}}. \tag{35}$$

The integral in (34) extends over the cross section of the guide; δ_{nm} is the Kronecker symbol.

The z-dependence of the voltage and current coefficients in (29) is found by substituting the sums of (29) for the transverse field components into Maxwell's equations.

Add

$$-\left(\frac{1}{r}\frac{\partial T_m}{\partial \varphi} - d_m \frac{h_m^2}{k^2}\frac{\partial T_m'}{\partial r}\right)$$

times (21) and

$$\frac{\partial T_m}{\partial r} + d_m \frac{h_m^2}{k^2}\frac{1}{r}\frac{\partial T_m'}{\partial \varphi}$$

times (22) and integrate over the cross section of the guide. Using (34), the result is

$$\begin{aligned} \frac{dV_m}{dz} + j\frac{h_m^2}{\omega\epsilon} I_m = {} & \int_S (\text{grad } E_z)(\text{grad } T_m)\, dS \\ & + d_m \frac{h_m^2}{k^2}\int_S (\text{grad } E_z)(\text{flux } T_m')\, dS - j\omega\mu \sum_n I_n \frac{\chi_n^2}{k^2} \\ & \cdot \int_S \left[(\text{grad } T_n)(\text{grad } T_m) + d_m \frac{h_m^2}{k^2}(\text{grad } T_n)(\text{flux } T_m')\right] dS, \end{aligned} \tag{36}$$

where the gradient and flux of a scalar are defined by:

$$\text{grad}_r\, T = \frac{\partial T}{\partial r}, \qquad \text{grad}_\varphi\, T = \frac{1}{r}\frac{\partial T}{\partial \varphi},$$
$$\text{flux}_r\, T = \frac{1}{r}\frac{\partial T}{\partial \varphi}, \qquad \text{flux}_\varphi\, T = -\frac{\partial T}{\partial r}. \tag{37}$$

After partial integration on the right-hand side of (36),

$$\frac{dV_m}{dz} + j\frac{h_m^2}{\omega\epsilon} I_m = \int_0^{2\pi} E_z\left(\frac{\partial T_m}{\partial r} + \frac{d_m}{a}\frac{h_m^2}{k^2}\frac{\partial T_m'}{\partial \varphi}\right) a\, d\varphi$$
$$+ \chi_m^2 \int_S E_z T_m\, dS - j\omega\mu \sum_n I_n \frac{\chi_n^2}{k^2} \tag{38}$$
$$\cdot\left[\int_0^{2\pi} T_n\left(\frac{\partial T_m}{\partial r} + \frac{d_m}{a}\frac{h_m^2}{k^2}\frac{\partial T_m'}{\partial \varphi}\right) a\, d\varphi + \chi_m^2 \int_S T_n T_m\, dS\right].$$

In the line integral along the boundary, E_z from the boundary condition (20) is substituted. In the surface integral over the cross section, (30) is substituted for E_z. Subsequently the boundary conditions of the perfect helix waveguide may be used to simplify (38) to first order in ψ:

$$\frac{dV_m}{dz} + j\frac{h_m^2}{\omega\epsilon} I_m = -\sum_n V_n\, d_n \frac{k_n^2 k_m^2}{k^2 a^3}\int_0^{2\pi} T_n' T_m \psi\, d\varphi. \tag{39}$$

For the other set of generalized telegraphist's equations add

$$-\left(\frac{\partial T_m}{\partial r} + \frac{d_m}{r}\frac{\partial T_m'}{\partial \varphi}\right)$$

times (24) and

$$-\left(\frac{1}{r}\frac{\partial T_m}{\partial \varphi} - d_m\frac{\partial T_m'}{\partial r}\right)$$

times (25) and integrate over the cross section. The result is

$$\frac{dI_m}{dz} + j\omega\epsilon V_m = -\int_S (\text{grad}\, H_z)(\text{flux}\, T_m)\, dS$$
$$+ d_m \int_S (\text{grad}\, H_z)(\text{grad}\, T_m')\, dS + j\omega\epsilon \sum_n V_n\, d_n \frac{\chi_n^2}{k^2} \tag{40}$$
$$\cdot \int_S [(\text{grad}\, T_n')(\text{flux}\, T_m) - d_m\, (\text{grad}\, T_n')(\text{grad}\, T_m')]\, dS.$$

After partial integration on the right-hand side of (40):

$$\frac{dI_m}{dz} + j\omega\epsilon V_m = \\ d_m \chi_m^2 \int_S H_z T_m' \, dS - j\omega\epsilon \sum_n V_n \, d_n \, d_m \frac{\chi_n^2 \chi_m^2}{k^2} \int_S T_n' T_m' \, dS. \tag{41}$$

Expression (30) for H_z holds only for the normal modes of the perfect waveguide. It has been obtained by differentiating the sum (29) for E_φ in (23). The individual terms of this sum vanish at $r = a$, while in the present case according to (19) E_φ has a finite value there. Hence the sum (29) for E_φ is nonuniformly convergent and differentiation makes it diverge. To replace H_z in (41), substitute in (23) E_r from (29), multiply (23) by T_m', and integrate over the cross section

$$-j\omega\mu \int_S H_z T_m' \, dS = \int_0^{2\pi} E_\varphi T_m' a \, d\varphi + \sum_n V_n \, d_n \, \chi_n^2 \int_S T_n' T_m' \, dS. \tag{42}$$

With (42), the boundary condition (19) for E_φ, and (30) for E_z, the second set of generalized telegraphist's equations is

$$\frac{dI_m}{dz} + j\omega\epsilon V_m = \sum_n I_n \, d_m \frac{k_n^2 k_m^2}{k^2 a^3} \int_0^{2\pi} T_n T_m' \psi \, d\varphi. \tag{43}$$

These equations represent an infinite set of coupled transmission lines. For the present purpose it is more convenient to write these transmision-line equations not in terms of currents and voltages but in terms of the amplitudes of forward and backward traveling waves A and B. The current and voltage of a typical mode are related to the wave amplitudes by

$$V = \sqrt{K} \, (A + B),$$

$$I = \frac{1}{\sqrt{K}} (A - B),$$

where K is the wave impedance

$$K_m = \frac{h_m}{\omega\varepsilon}.$$

If the currents and voltages in (39) and (43) are replaced by the traveling wave amplitudes, the following equations for coupled traveling waves are obtained:

$$\frac{dA_m}{dz} + jh_m A_m = \sum_n (\kappa_{nm}^+ A_n - \kappa_{nm}^- B_n), \\ \frac{dB_m}{dz} - jh_m B_m = \sum_n (\kappa_{nm}^+ B_n - \kappa_{nm}^- A_n). \tag{44}$$

The coupling coefficients are given by

$$\kappa_{nm}^{\pm} = \frac{k_n^2 k_m^2}{2k^2 a^3}\left(d_m \sqrt{\frac{h_m}{h_n}} \int_0^{2\pi} T_n T_m' \psi \, d\varphi \mp d_n \sqrt{\frac{h_n}{h_m}} \int_0^{2\pi} T_n' T_m \psi \, d\varphi\right). \tag{45}$$

If m represents a circular electric wave:

$$T_m = 0,$$

$$T_m' = N_m J_0(\chi_m r),$$

$$d_m N_m = \frac{k}{\sqrt{\pi}\, h_m k_m J_0(k_m)},$$

and for the coupling coefficients,

$$\kappa_{nm}^{\pm} = \frac{k_m k_n^2}{2\sqrt{\pi}\, k \sqrt{h_m h_n}\, a^3} \int_0^{2\pi} T_n \psi \, d\varphi. \tag{46}$$

For a tilted winding with ψ from (17),

$$\kappa_{nm}^{\pm} \equiv j c_t \, .$$

In this case circular electric waves interact only with modes of first circumferential order ($p = 1$). Helix irregularities of higher order in ψ will cause coupling to modes of correspondingly higher circumferential order. In general,

$$\kappa_{nm}^{\pm} = \frac{\sqrt{\pi}\, k_m k_n^2 N_n J_p(k_n)}{2k\sqrt{h_m h_n}\, a^3}\, \theta_p \, . \tag{47}$$

V. TOLERANCES

The design of a helix waveguide is started by selecting wire size and spacing. A tolerable amount of added TE_{01} loss is specified, and with Fig. 1 and equation (2) wire size and spacing are determined.

If, for example, eddy current losses in the helix should not be more than 10 per cent of the loss in a guide with smooth walls, then from Fig. 1 the ratio of wire diameter to insulation diameter should be

$$\frac{d}{D} > 0.775. \tag{48}$$

To determine the actual wire size with (2) the wall impedance has to be specified. Different applications of the helix waveguide require different values for the wall impedance. A typical and also very critical example is the helix waveguide for intentional bends. In this case, by surrounding the helix with a quarter-wave jacket and a metallic shield the wall impedance is made very high.

The general formula for the wall impedance of the shielded helix waveguide is[2]

$$Z = j\frac{k_n^{\,e}}{\omega\epsilon_e a}\tan k_n^{\,e}\delta, \tag{49}$$

where ϵ_e is the permittivity of the jacket, $k_n^{\,e} = a\sqrt{\omega^2\mu\epsilon_e - h_n^2}$ the radial propagation constant in it, and δ the relative thickness.

For a quarter-wave jacket of a low-loss material the wall impedance is real and approximately

$$\frac{Z}{Z_0} = \frac{4}{\pi}\frac{(\epsilon' - 1)^{\frac{3}{2}}}{\epsilon'\epsilon''}, \tag{50}$$

with

$$\frac{\epsilon_e}{\epsilon_0} = \epsilon' - j\epsilon''.$$

Substituting (50) into (2), an equation for nD is obtained.

Fiber glass laminated with epoxy resin has a relative permittivity at millimeter wavelengths of $\epsilon_e/\epsilon_0 = 4 - j(0.04)$. The relative wall impedance from (49) is then $Z/Z_0 = 41.4$. In 2-inch inside diameter waveguide with smooth walls, the TE_{01} loss at 55.5 kmc is $\alpha_0 a = 2.77 \times 10^{-6}$. Less than 10 per cent of this figure is added to the TE_{01} loss in the present example when the pitch is

$$\frac{nD}{a} < 4 \times 10^{-3}. \tag{51}$$

No. 37 wire (AWG) with a heavy Formex coat has $d = 0.0045$ and $D = 0.0054$. It very nearly satisfies conditions (48) and (51) when the helix is wound from one wire only ($n = 1$). Lower wall impedance values such as are used for helix mode filters or all helix guide would not require as low a pitch as (51).

For the winding process, tolerances for irregularities must be specified. In (15) the added loss is expressed in terms of the θ of a tilted winding. The loss caused by higher-order irregularities can also be determined by (15) when the corresponding coupling coefficients (47) are substituted.

In the present problem, however, the irregularities are not known, but at best some of their statistical properties are known. Equation (15) can then be used to express the statistics of the loss in terms of the statistics of the winding irregularities.[11,12]

For an oblique winding θ is assumed to be a stationary random process with covariance $R(u)$ and spectral distribution $S(\xi)$:

$$R(u) = \langle \theta(z)\theta(z+u) \rangle, \tag{52}$$

$$S(\xi) = \int_{-\infty}^{+\infty} R(u)\, e^{-j2\pi\xi u}\, du. \tag{53}$$

In (52), $\langle x \rangle$ is the expected value of x.

Taking the expected value on both sides of (15) the average added loss is obtained in terms of the covariance $R(u)$:

$$\langle \alpha_t \rangle = \frac{1}{L} \sum_n \int_0^L e^{\Delta\alpha_n z} R(z)(L - z)(P_n \cos \Delta\beta_n z - Q_n \sin \Delta\beta_n z)\, dz. \tag{54}$$

For a mere estimate the covariance is assumed to be exponential to simplify the calculation:

$$R(z) = \frac{\pi S_0}{L_0}\, e^{(-2\pi|z|/L_0)}. \tag{55}$$

Then the spectral distribution of θ becomes

$$S(\xi) = \frac{S_0}{1 + L_0^2\xi^2}, \tag{56}$$

with $S(\xi)$ nearly flat with spectral density S_0 for mechanical frequencies smaller than $\xi_0 = 1/L_0$. L_0 may be regarded as the cutoff mechanical wavelength according to (56) or as a correlation distance according to (55).

The average added loss is for $L \gg L_0$

$$\langle \alpha_t \rangle = \pi S_0 \sum_n \frac{P_n(2\pi - \Delta\alpha_n L_0) - Q_n \Delta\beta_n L_0}{\Delta\beta_n^2 L_0^2 + (2\pi - \Delta\alpha_n L_0)^2}. \tag{57}$$

To evaluate (57) the characteristic equation (32) of the perfect helix waveguide has to be solved for all the coupled modes n, and propagation constants and coupling coefficients have to be calculated.

The helix waveguide for intentional bends is again a typical and critical example. In this case $Z = \infty$ at the design frequency and the characteristic equation (32) simplifies to

$$d_n = \pm \frac{k}{h_n}. \tag{58}$$

The roots k_{n0} of $J_{p+1}(x) = 0$ and $J_{p-1}(x) = 0$ are good approximations for the roots of (58). With $k_n = k_{n0} \pm x$, where

$$x = \frac{p}{k_{n0}} \left(1 - \sqrt{1 - \frac{k_{n0}}{k^2 a^2}} \right), \tag{59}$$

the approximations can be sufficiently improved. The coupling coefficients in this case are given by

$$\kappa_{nm} = \frac{k_m k_n \theta_p}{2k\sqrt{h_m h_n}\, a^3}\left(1 - \frac{p^2}{k^2 a^2} \mp \frac{p}{k h_n a^2}\right)^{-\frac{1}{2}}. \tag{60}$$

With these relations, (57) has been evaluated for a helix waveguide with a nonuniformly tilted winding ($p = 1$).

Fig. 4 shows the rms value of θ as a function of the correlation distance L_0 for an added average TE_{01} loss of 10 per cent of the loss in a copper pipe with smooth walls. The waveguide diameter is 2 inches and the frequency 55.5 kmc. The tolerance is most critical for a correlation distance of 1 inch. But even then an rms tilt of 0.6° can be tolerated. In experimental models of helix waveguide the maximum tilt has, with some care, been kept below 0.3°.

VI. CONCLUSION

In a perfectly wound helix waveguide the circular electric wave loss is significantly increased only by the eddy current losses in spaced wires. The finite pitch contributes to the circular electric wave loss only when the wall impedance is very high or when the helix is wound from more than one wire.

Of all irregularities in the helix a changing tilt of the winding has been observed to be most significant. Assuming this tilt to be randomly

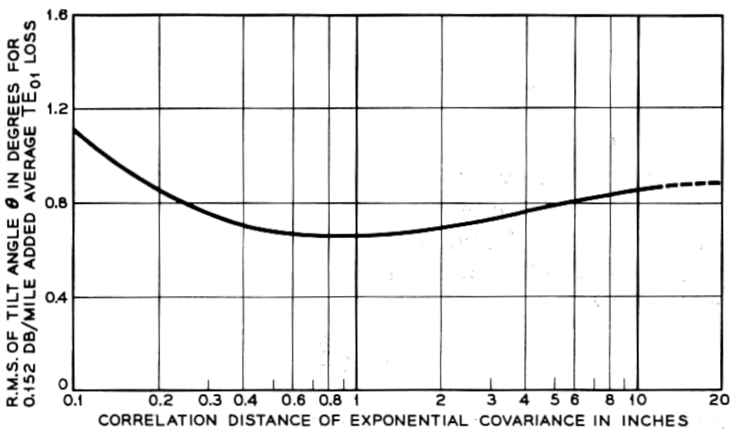

Fig 4. — TE_{01} loss in helix waveguide with random tilt of winding, 2-inch inside diameter, at 55.5 kmc. Design for intentional bends with infinite wall impedance.

distributed along the waveguide with an exponential covariance, the increase in circular electric wave loss can be calculated. In a 2-inch inside diameter helix waveguide at 55.5 kmc an rms tilt angle of 0.6° adds at the most 10 per cent of the loss in a perfect copper pipe to the average TE_{01} loss. In experimental models of helix waveguide the maximum tilt deviation has been kept below 0.3°.

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