System Calculations

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*This paper describes system calculations made in preparation for the Project Echo communication experiment and their adaptation to the problem of interpreting the results of the experiment.**

I. INTRODUCTION

Many factors, such as power output, frequency, antenna gain, free-space path loss, receiver noise temperature, and method of modulation, influence the performance of a satellite communication system such as the Project Echo experiment. In addition to those mentioned, which are also common to point-to-point microwave systems, three other factors must be considered in the design of satellite communication systems. All three are functions of satellite position in the region of mutual visibility. They are:

(a) variations in free-space path loss,
(b) variations in sky noise temperature,
(c) loss in the earth's atmosphere.

In this paper these system parameters are used to predict the performance of the voice circuits that constitute the Echo communications experiment. The performance is discussed for the case of normal propagation conditions, and does not take into account attenuation due to rainfall or multipath fading. These are statistical occurrences, and are beyond the scope of this paper.

II. FREE-SPACE PATH LOSS FORMULA

Assume a transmitting antenna of actual gain G_1 radiating power of P_T watts. The power density at a distance d_1 will then be

* As discussed elsewhere in this issue, the Project Echo communication experiment was a joint operation of the National Aeronautics and Space Administration, Jet Propulsion Laboratories, Naval Research Laboratory, and Bell Telephone Laboratories.

$$\Phi_1 = G_1 \frac{P_T}{4\pi d_1^2}.$$

The amount of power intercepted by an object of projected area σ will then be

$$P_1 = \Phi_1 \sigma.$$

A sphere, in effect, radiates this energy isotropically; hence the power density a distance d_2 from the sphere will be

$$\Phi_2 = \frac{P_1}{4\pi d_2^2}.$$

The amount of power received by an antenna having effective aperture area A_2 from this field is

$$P_R = \Phi_2 A_2 = \Phi_2 \frac{\lambda^2 G_2}{4\pi},$$

where

$$G_2 = \frac{4\pi A_2}{\lambda^2}.$$

After suitable substitutions the received power is:

$$P_R = G_1 G_2 \frac{\lambda^2 \sigma P_T}{(4\pi)^3 d_1^2 d_2^2}. \tag{1}$$

Rearranging (1) gives the free-space path loss, L:

$$L = \frac{P_T}{P_R} = \frac{(4\pi)^3 d_1^2 d_2^2}{G_1 G_2 \lambda^2 \sigma}. \tag{2}$$

This expression serves to calculate the expected free-space path loss, providing the various parameters can be determined to sufficient accuracy. The path loss is a function of satellite position as indicated by the presence of $d_1^2 d_2^2$ in (2).

III. SYSTEM PARAMETERS

In order to compute the free-space path loss, antenna gains and frequencies of operation are required. These constants are detailed below:

Antenna	*Gain, db*	*Line loss, db*	*Net gain, db*
BTL 2390 mc horn	43.3 ± 0.16	0	43.3 ± 0.16
BTL 960 mc dish	43.1 ± 0.1	0.5	42.6 ± 0.1
BTL 961 mc dish	32.6 ± 0.2	0	32.6 ± 0.2
JPL 2390 mc dish	53.7*	0.4	53.3
JPL 960 mc dish	45.8 ± 0.6	0.2	45.6 ± 0.6
NRL 2390 mc dish	50.2*	1.6*	48.6

* Estimated values, not measured.

IV. FREE-SPACE PATH LOSS COMPUTATIONS

The free-space path loss has been computed for the Echo balloon as a function of position for the two-way path between Goldstone and Holmdel. The balloon scattering cross section, σ, was assumed to be that of a 100-foot-diameter sphere, perfectly conducting and many wavelengths in diameter, so that

$$\sigma = \frac{\pi(100)^2}{4} = 7854 \text{ square feet.}$$

The frequency in the east-west direction was 960 mc; in the west-east direction it was 2390 mc. Equation (2) was used to compute path loss; the results are presented in Figs. 1 and 2. Fig. 1 is a plot of path loss versus satellite altitude when the balloon was midway between the terminals. Fig. 2 shows contours of constant free-space path loss relative to the loss at midpath for a satellite height of 1000 miles and assuming the radius of the earth to be 3950 miles. The contours are in steps of 1 db, and are plotted on a stereographic projection. The inclination of the orbit of Echo I is 47.27°, which limits the northern extent of mutual visibility. The necessary equations for these computations are derived in Appendix A.

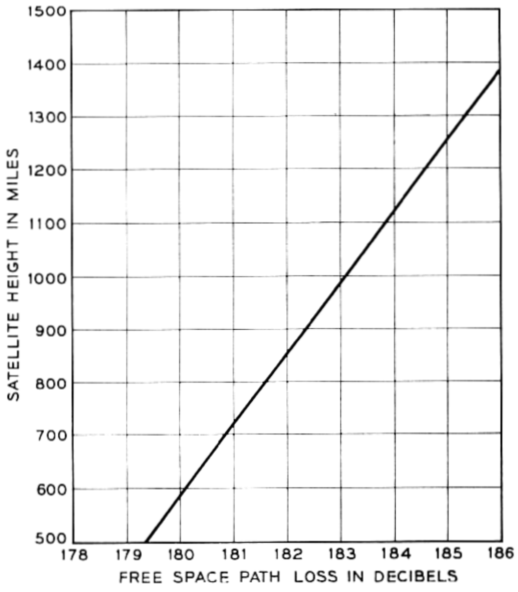


Fig. 1 — Path lost in east-west direction, Holmdel to Goldstone; for west-east direction, subtract 0 5 db.

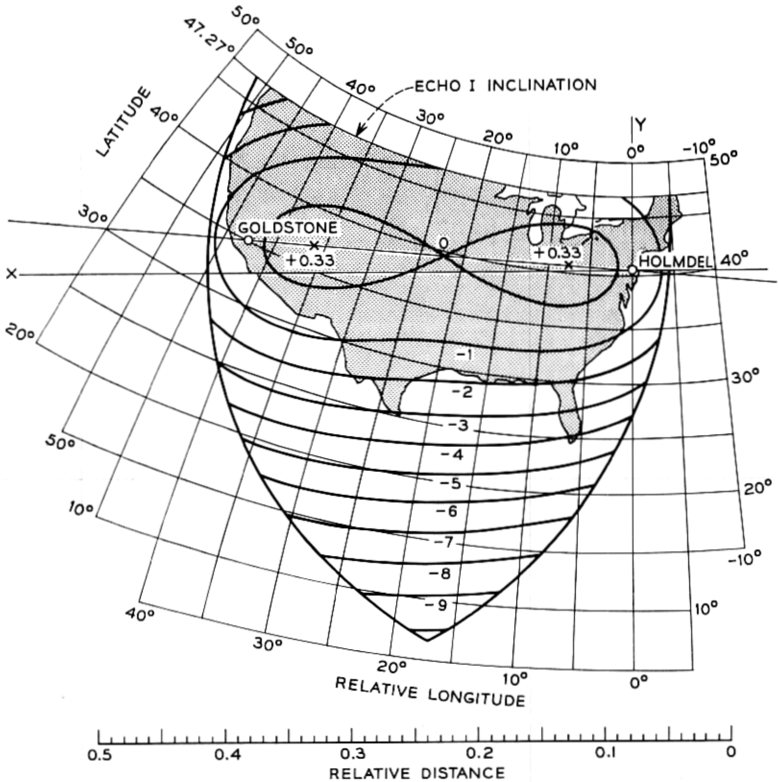


Fig. 2 — Contours of constant free-space path loss, Holmdel to Goldstone.

Stump Neck is only 200 miles from Holmdel, so the path loss from NRL-BTL can be computed as the round-trip loss from either terminal to the balloon. The error in this assumption is less than 0.6 db (Appendix B) for any position in the area of mutual visibility. The contours of constant path loss for this case are circles centered on Holmdel; they are shown in Fig. 3. The path loss at 2390 mc and a satellite altitude of 1000 miles is 178.7 db.

It can be seen from Figs. 2 and 3 that the difference between maximum and minimum path loss on the JPL-BTL path is about 10 db, while for the NRL-BTL path this difference is 19 db. It should also be noted that between NRL and BTL this maximum difference is encountered twice on every pass, while the maximum difference almost nevers occurs on the JPL-BTL path.

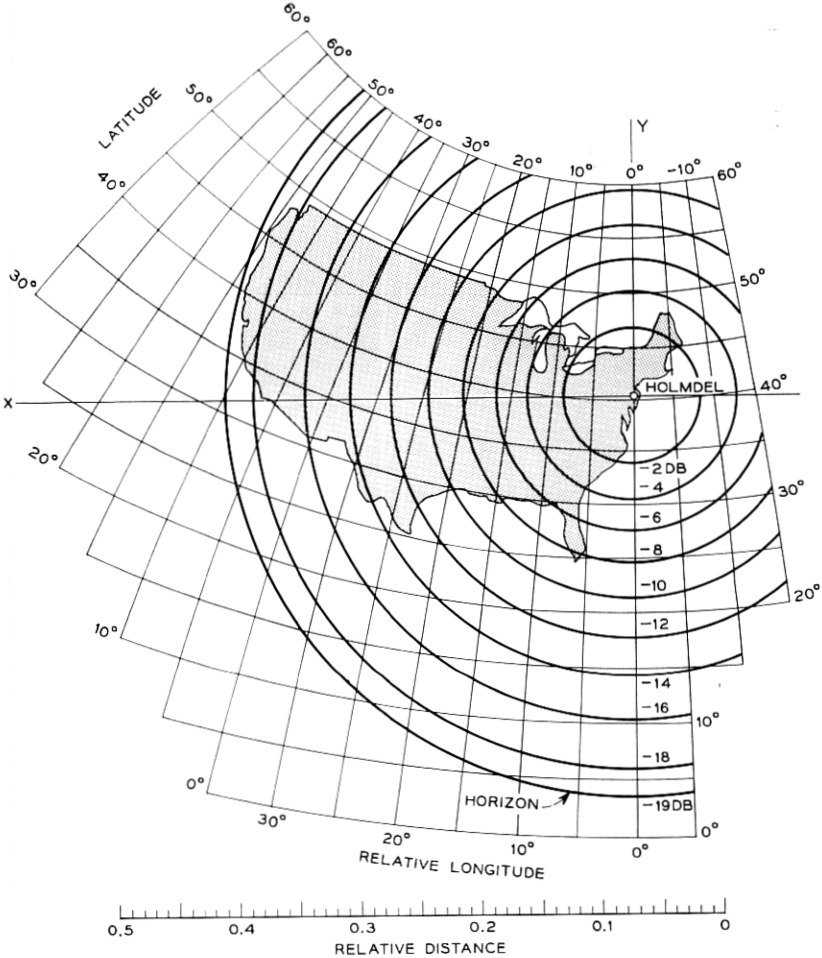


Fig. 3 — Contours of constant free-space path loss, Stump Neck to Holmdel.

V. EXPECTED SIGNAL-TO-NOISE RATIOS IN VOICE CIRCUITS

The signal-to-noise power ratios to be expected depend upon the type of modulation technique that is employed, as well as upon the received carrier-to-noise power ratio. The first step is to compute the carrier-to-noise ratio at the receiver for suitable conditions, and then to discuss the modulation methods and voice band signal-to-noise performance. This has been done for the two-way voice path between JPL and BTL, and

TABLE I — SATELLITE MIDWAY BETWEEN TERMINALS

Parameter	East-to-West	West-to-East
Transmitted power:	+70 dbm (10 kw)	+70 dbm (10 kw)
Frequency:	960.05 mc	2390 mc
Transmitting antenna net gain:	42.6 db	53.3 db
Receiving antenna net gain:	45.6 db	43.3 db
Free-space path loss:	183.1 db	182.6 db
Loss through atmosphere:	0 db	0 db
Received carrier power:	−113.1 dbm	−112.6 dbm
Receiver system noise temperature:	350° K	25° K
Receiver noise power in 6-kc band:	−135.4 dbm	−146.8 dbm
Carrier-to-noise ratio at receiver (C/N):	22.3 db	34.2 db

the results are tabulated in Table I. The satellite is assumed to be midway between the terminals.

For other positions of the balloon in the region of mutual visibility, the C/N ratio is modified by three effects:

(a) variations in free-space path loss;

(b) variations in sky noise temperature;

(c) loss in the earth's atmosphere.

The first item has been discussed in Section IV, and the correction for position can be made for any satellite position from Fig. 2. The remaining two items have been discussed by Hogg[1] and by DeGrasse, Hogg, Ohm, and Scovil.[2] For example, the sky noise temperature and atmospheric loss can be calculated when the antennas are pointed at the horizon. The loss through the atmosphere and system noise temperature are then 3.2 db and 435°K for the east-west circuit, and 4.2 db and 110°K for the west-east circuit. This would be the worst case. For elevation angles above 10°, however, these effects are essentially negligible.

The audio signal-to-noise ratio depends to a considerable extent on the modulation technique. Three techniques, single-sideband (SSB), FM, and FM with feedback (FMFB), are considered here. The transmitters are assumed to be peak-power-limited and the audio signal to be the maximum rms sine wave obtainable. The audio bandwidth is 3 kc, and the noise bandwidth is assumed to be 6 kc.

5.1 *Single-Sideband (SSB)*

The maximum transmitted rms sine wave power is 3 db less than the transmitter peak, C. However, for SSB the noise bandwidth may be reduced to 3 kc, resulting in an audio signal-to-noise ratio of

$$S/N = C/N.$$

5.2 *Frequency Modulation (FM)*

The audio signal-to-noise ratio for this case is given by the standard FM formula, which applies when the receiver input is above the threshold:

$$S/N = 3M^2C/N,$$

where M is the index of modulation.

The index for the Echo experiment was 10, so when the receiver is operated above the threshold the signal-to-noise ratio is 25 db better than that of SSB. However, the threshold for this receiver occurs at a C/N of approximately 22 db, because the noise bandwidth required to accommodate this signal is about 66 kc.

5.3 *Frequency Modulation with Feedback (FMFB)*

The audio signal-to-noise ratio for this receiver is the same as that for FM when the C/N is above the threshold. However, this receiver has a threshold near C/N = 13 db.[3] At any C/N equal to or greater than 13 db, the audio S/N exceeds that of SSB by 25 db.

When the balloon is midway between the terminals the expected S/N, based on the formulae above, would be as follows:

	Audio S/N for SSB	*Audio S/N for FM, FMFB*
East-to-west:	22.3 db	47.3 db
West-to-east:	34.2 db	59.2 db (57 db measured*)

The same general comments apply to the NRL-BTL circuit; the path loss being modified according to the free-space path loss differences shown in Fig. 3 and the difference in free-space path loss when the balloon is directly above the terminals. When the balloon is directly overhead the audio S/N would be

	Audio S/N for SSB	*Audio S/N for FM, FMFB*
NRL-BTL	38.6 db	63.6 db (>57 db measured*)

VI. RECEIVED SIGNAL STRENGTH USING ORBITAL PARAMETERS

The foregoing material was based on the assumption that the satellite orbit was circular and the altitude was 1000 miles. After the experiment was underway it was necessary to compute the loss for the known posi-

* The maximum S/N obtainable in this receiver is limited by the audio amplifier noise. This begins to be significant at a S/N of about 50 db and accounts for the difference between the computed and measured signal-to-noise ratios.

tion of the balloon in order to compare the measured and theoretical received signal amplitudes.

For this purpose a program was written for the IBM 7090 computer for calculation of a path-loss parameter, L, for two given stations using Echo I at the same time:

$$L(t) = 10 \log \left[(4\pi)^3 \frac{d_1^2(t) d_2^2(t)}{\lambda^2 \sigma} \right],$$

where t refers to time. The inputs to this program are the orbital elements, station coordinates, frequency, and balloon cross section. The received power in decibels is then:

$$(P_R)_{\text{db}} = 10 \log (G_1 G_2 P_T) - L(t).$$

To save computer time, $L(t)$ was only calculated for one frequency, 2390 mc, since the values only differed by a constant from those of another frequency. Calling this value $L_0(t)$, and using the antenna gains from the table, the following expressions for received power in dbm were derived:

BTL to JPL at 960 mc:	$P_R = 166.1 - L_0 \pm 0.7$,
JPL to BTL at 2390 mc:	$P_R = 166.6 - L_0$,
NRL to BTL at 2390 mc:	$P_R = 162.9 - L_0$,
BTL to BTL at 961 mc:	$P_R = -122.0 - 40 \log d$.

The last line refers to the BTL radar, and the slant range d is in kilomiles. These numbers must be corrected by any variation in transmitted power from 10 kw.

APPENDIX A

The variation in free-space path loss as a function of the position of the satellite in the region of mutual visibility can be understood by examining the behavior of $d_1^2 d_2^2$ in (2).

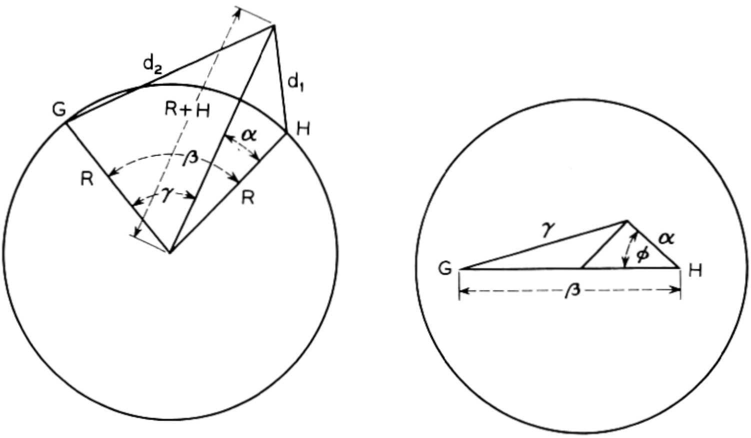
The geometry is shown in Fig. 4. The distances from the satellite to terminals H and G are d_1 and d_2 respectively. The path loss is proportional to $d_1^2 d_2^2$. Using the law of cosines,

$$\begin{aligned} d_1^2 &= R^2 + (R+h)^2 - 2R(R+h)\cos\alpha = A - B\cos\alpha, \\ d_2^2 &= R^2 + (R+h)^2 - 2R(R+h)\cos\gamma = A - B\cos\gamma, \end{aligned} \qquad (3)$$

where $\cos\gamma = \cos\varphi \sin\alpha \sin\beta + \cos\alpha \cos\beta$.

Now let $M = B/A$ and normalize to A:

$$\frac{d_1^2 d_2^2}{A^2} = (1 - M\cos\alpha)(1 - M\cos\varphi\sin\alpha\sin\beta - M\cos\alpha\cos\beta). \qquad (4)$$



(a) (b)

Fig. 4 — Satellite geometry for path-loss calculations.

Solving for $\cos\varphi$ results in

$$\cos\varphi = \frac{(1 - M\cos\alpha)(1 - M\cos\alpha\cos\beta) - \left(\dfrac{d_1^2 d_2^2}{A^2}\right)}{M(1 - M\cos\alpha)(\sin\alpha\sin\beta)}, \tag{5}$$

with

$$M = \frac{2R(R+h)}{R^2 + (R+h)^2},$$

$$\beta = \text{central angle between terminals H, G,}$$

$$\frac{d_1^2 d_2^2}{A^2} = \text{normalized path-loss parameter.}$$

We are interested only in points which do not fall below the horizon. Thus, α has a maximum determined by

$$\cos\alpha_{\max} = \frac{R}{R+h}. \tag{6}$$

The normalized path-loss parameter when the satellite is midway between the terminals is found by noting that $\varphi = 0$ and $\alpha = \gamma = \beta/2$. Thus

$$\left(\frac{d_1^2 d_2^2}{A^2}\right)_0 = (1 - M\cos\beta/2)^2. \tag{7}$$

The maximum value of this parameter occurs when the satellite appears on the horizon to both terminals. For this case, $\gamma = \alpha_{max}$ and

$$\left(\frac{d_1^2 d_2^2}{A^2}\right)_{\max} = \left(1 - \frac{MR}{R + h}\right)^2. \tag{8}$$

When the satellite is at midpath for an altitude of 1000 miles the values of these parameters are as follows:

$$\begin{aligned} R &= 3950 \text{ miles}, \\ \alpha &= \beta/2 = 16.89°, \\ A &= 4.01 \times 10^7 \text{ miles}^2, \\ B &= 3.91 \times 10^7 \text{ miles}^2, \\ M &= 0.975, \\ d_1^2 d_2^2 &= 7.2 \times 10^{12} \text{ miles}^4. \end{aligned}$$

A program has been prepared for the IBM 704 computer using expressions (5) through (8), and the path-loss contours of Figs. 2 and 3 were plotted from these data. Negative signs indicate increasing loss.

The points of minimum path loss are found by setting $\varphi = 0$ in (4) and differentiating with respect to α:

$$\begin{aligned} \frac{d}{d\alpha}\left(\frac{d_1^2 d_2^2}{A^2}\right) = {} & M \sin \alpha[1 - M \cos (\beta - \alpha)] \\ & - (1 - M \cos \alpha) M \sin (\beta - \alpha). \end{aligned} \tag{9}$$

The desired points will be solutions of the equation obtained by setting (9) equal to zero; $\alpha = \beta/2$ is a solution, but it is not necessarily a point of minimum loss. By the usual tests we have the following: the midpath point is a maximum loss point if $\cos \beta/2 - M$ is negative. Conversely, if $\cos \beta/2 - M$ is positive, the midpath is a minimum loss point.

APPENDIX B

When the terminals are close together, as are Stump Neck and Holmdel, their central angle β is small, and the path-loss computations are simplified, because $d_1^2 d_2^2 \approx d_1^4$. The purpose of this appendix is to derive the maximum error incurred by using this approximation. From (3),

$$d_2^2 = d_1^2 + 2R(R + h)(\cos \alpha - \cos \gamma), \tag{10}$$

where $\cos \gamma = \cos \varphi \sin \alpha \sin \beta + \cos \alpha \cos \beta$.

Substituting for $\cos \gamma$ in (10) results in

$$d_2^2 = d_1^2 + 2R(R + h)[\cos \alpha \,(1 - \cos \beta) - \cos \varphi \sin \alpha \sin \beta]. \tag{11}$$

If $\beta \ll \pi/2$, this reduces to

$$d_2^2 = d_1^2 - 2R(R + h)\beta \sin \alpha \cos \varphi. \tag{12}$$

The ratio of the two sides of the approximation is

$$\frac{d_1^2 d_2^2}{d_1^4} = \frac{d_2^2}{d_1^2} = 1 - \frac{2\beta R(R + h) \cos \varphi \sin \alpha}{R^2 + (R + h)^2 - 2R(R + h) \cos \alpha}. \tag{13}$$

The path-loss error in decibels is given by 10 log [equation (13)]. The error will be maximum when $\cos \varphi = \pm 1$ and $\alpha = \alpha_{\max}$. Assuming that $R = 3950$ miles, $h = 1000$ miles, and $\cos \varphi = -1$, then $\alpha_{\max} = 37.1°$, and

$$\frac{d_2^2}{d_1^2} = 1 + 2.65\beta, \qquad \beta \ll \pi/2. \tag{14}$$

For the NRL-BTL path, $\beta \approx 200/3950 = 0.0506$ radian, and $d_2^2/d_1^2 = 1.134$. The maximum error is $10 \log 1.134 = 0.546$ db.

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