Diffusion Length Measurement by Means of Ionizing Radiation

By W. ROSENZWEIG

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Penetrating radiation in the form of high-energy electrons, heavy particles, and gamma rays may be used to determine minority-carrier diffusion lengths in semiconductor materials containing junctions by measuring the radiation-induced short circuit current. The electron-beam method yields an accurate absolute determination of diffusion length once the carrier-generation rate as a function of depth in the material has been measured. A series of such experiments is described for silicon solar cells utilizing electrons ranging in energy from 0.61 Mev to 1.16 Mev. A resultant maximum generation rate of $2.25 \times 10^6 \pm 5$ per cent carriers/cm per incident 1 Mev electron is obtained at a depth of 0.096 gm/cm^2. Measurements with 16.8 Mev and 130 Mev protons, and Co^{60} gamma rays are found to be in good agreement with the electron-beam measurements. An experimental arrangement is described which yields rapid and accurate diffusion-length measurements of solar cells under conditions in which radiation damage is negligible.

I. INTRODUCTION

The minority-carrier diffusion length is an important quantity characterizing the properties of a semiconductor material or device, particularly in any situation involving the transport of charge by minority carriers. It is related to the steady-state minority carrier lifetime through the expression $L = \sqrt{D\tau}$, in which D is the diffusion coefficient and τ is the lifetime. A variety of methods have been employed to measure diffusion length in the steady state, or lifetime under transient conditions and are reviewed by Bemski.[1] The steady-state methods have a particular advantage in that the measurement is uncomplicated by trapping effects. One particular steady-state method suggested by Gremmelmaier[2] involves the use of gamma radiation to generate excess carriers at a known rate in a pn junction device. The radiation-generated short-circuit current may then be used to calculate diffusion lengths when the device geometry is known.

It is the purpose of this paper to discuss, more generally, the use of penetrating ionizing radiations for diffusion-length measurement. Experimental results will be presented for energetic electrons and protons as well as gamma rays. It will be shown that of the three, electrons lend themselves readily to an accurate absolute determination, with an accuracy of ±5 per cent having been achieved. Even though the method is quite general it will be treated in terms of one device, the silicon solar cell. It is the radiation damage study of this device which led to the development of the technique.

In the text which follows, Section II will be devoted to the theoretical background for the diffusion-length determination. Section III will be a description of the energetic electron measurements and the analysis which yields the absolute calibration. Sections IV and V will deal with protons and gamma rays, respectively, and give experimental results which are compared with the electron beam measurements. A brief discussion of the radiation damage problem will be given in Section VI along with some quantitative information which shows that measurements can be made in most practical cases with negligible damage.

II. THEORY

Radiation incident on the surface of a material having a shallow diffused junction, such as a solar cell shown schematically in Fig. 1, will generate excess carriers at a rate which is a function of the depth from the surface. The function depends on the nature of the radiation. That portion of the carriers which is produced deep in the bulk of the material can be collected only if the carriers diffuse to the junction. Thus, for deeply generated carriers, the bulk diffusion length is of primary importance in determining the collection efficiency. In fact, because of the very shallow junction in solar cells, the diffusion length is the only parameter which governs the collection efficiency for penetrating radiation, the front layer becoming important only for nonpenetrating radiation. For the case in which excess carriers are generated uniformly throughout the material at a rate of g_0 carriers per cm^3 per second, the collected current density, J_{sc}, under short circuit conditions is given by

$$J_{sc} = eg_0L \tag{1}$$

where e is the electronic charge. (Derivation can be found in the Ap-

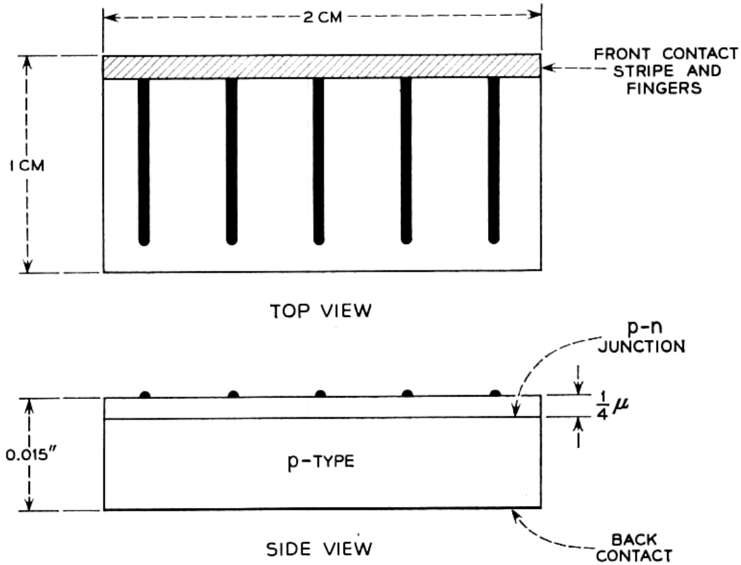

Fig. 1 — Schematic representation of the silicon solar cell.

pendix.) Equation (1) indicates that the quantitative determination of L requires a measurement of current, under short circuit conditions, when a known area of the device is "illuminated" with radiation which is capable of producing excess carriers uniformly throughout the bulk of the material and at a *known* rate. Light which has been filtered with material which is identical to that of which the cell is made (silicon in this instance) would satisfy the requirement of uniform generation rate. However, the requirement of knowing quantitatively the generation rate is difficult to meet, because the amount of reflection of the incident light depends on the condition of the surface, and the generation rate due to the transmitted beam depends strongly on the exact width of the energy band gap of the filter relative to that of the device.

The types of radiation which satisfy both needs are energetic nuclear particles, electrons, and gamma rays. As these radiations penetrate the material, a major portion of their energy is spent in causing transitions of valence electrons to the conduction band, i.e., the generation of hole-electron pairs. A small fraction of the energy produces damage by creating lattice vacancies and interstitials through nuclear collisions resulting in a reduction of carrier lifetime and change in carrier concentration and mobility. The rate at which silicon is damaged by the various types of radiation will be briefly discussed in Section VI with the conditions necessary to produce negligible damage during a measurement of diffusion length.

The average amount of energy required to produce a hole-electron pair in silicon has been measured to be 3.6 ev, a number which appears to hold well over a wide range of particle energy and types.[3] The determination of g_0 for a particular case thus requires a knowledge of the rate at which energy is given up to the material. Moreover, to assure oneself of the uniformity of the generation rate, it is also necessary to know its rate of change with depth into the material. If the latter quantity be designated by g_1, then it is shown in the Appendix that the nonuniformity can be neglected to the extent that $(g_1/g_0)L$ is small compared to one.

For the cases of the charged particles it is convenient to introduce one other quantity, the average specific ionization, which is the average spatial rate of hole-electron pair production per incident particle. If the generation rate, at some depth in the material, is g, due to N charged particles normally incident per second per cm² of surface, then the average specific ionization, S, at that depth is given by

$$S = \frac{g}{N} \text{ pairs/cm.} \tag{2}$$

For the situation of uniform carrier generation, (1) may be rewritten in the form

$$J_{sc} = JS_0L \tag{3}$$

where J is the incident current density for singly charged particles, and S_0 is the uniform specific ionization.

III. ELECTRONS

Of the charged particle radiations, electrons can be handled most easily experimentally. This is due to the fact that accelerators for 1 Mev electrons, which are sufficiently penetrating for the purpose at hand, are relatively simple in design and can be operated quite reliably. Beam intensity calibrations can be carried out accurately, for electrons, with Faraday cups of not too complicated design. This situation is in contrast to heavy particle accelerators which are large and complex installations.

The theoretical calculation of the average specific ionization as a function of depth per incident electron is difficult.[4] However, it is easy to evaluate this function experimentally, as was done, in the following way: A pre-bombarded silicon solar cell was exposed to a broad beam of monoenergetic electrons normally incident to its surface. The reason for the pre-bombardment was two-fold: (*i*) the diffusion length of the minority carriers was reduced to only a few microns so that the spatial variation of ionization in the sensitive layer was negligible, and (*ii*) the diffusion length did not change appreciably during the measurements. The solar cell short-circuit current density, J_{sc}, was measured and the electron beam calibrated to obtain the incident electron current density, J_e. By forming the ratio J_{sc}/J_e one has determined the average ionization per incident electron in the spatial interval a distance L from the junction. This quantity was then obtained as a function of depth by placing absorbers, approximately equivalent in atomic number to the cell material, on top of the cell and measuring the resultant short circuit current. The placing of the absorbers on the cell was equivalent to moving the sensitive layer, in which the ionization was measured, through the volume of the material. Care was taken in this procedure that the absorbers overlapped the edges of the cell sufficiently so that electrons scattered out at the periphery of the cell were compensated by equivalent electrons scattered in.

The results of a series of such experiments are shown in Figs. 2–5 for

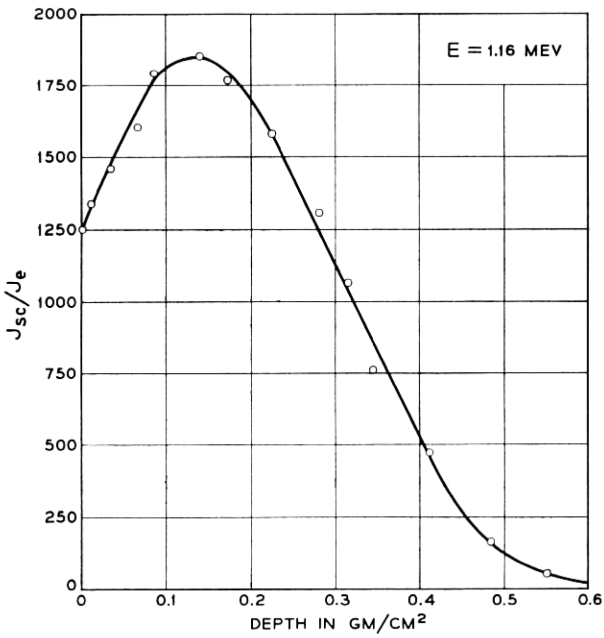

Fig. 2 — Ratio of collected current density to incident electron current density vs depth for 1.16 Mev electrons in a silicon solar cell of 8.75 μ diffusion length.

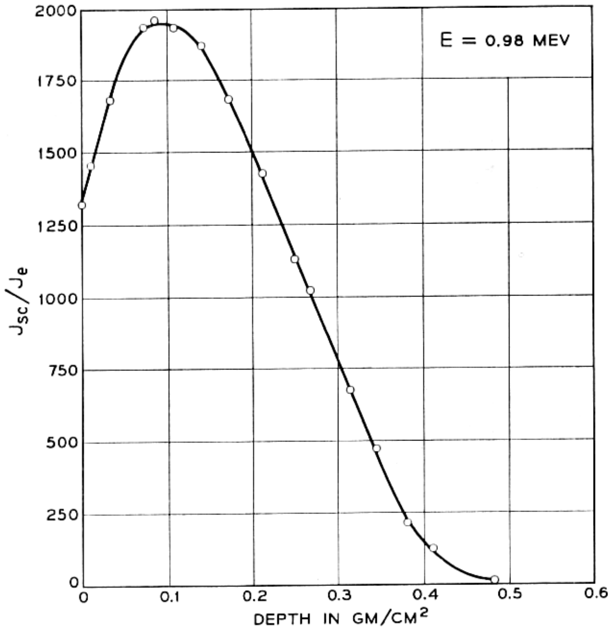

Fig. 3 — Ratio of collected current density to incident electron current density vs depth for 0.98 Mev electrons in a silicon solar cell of 8.75 μ diffusion length.

incident electrons of 1.16 Mev, 0.98 Mev, 0.80 Mev and 0.61 Mev energies respectively. They are plots of the ratio J_{sc}/J_e versus depth in silicon. (The actual absorbers used were made of aluminum, which is sufficiently close in atomic number to silicon to require only a density correction to yield equivalent thickness of silicon.) Since the integral over-all depths of the average specific ionization, S_e, per incident electron is equal to the total ionization which the electron produces in the silicon, then by means of (3) one gets the following result:

$$\int_0^\infty S_e \, dx = \frac{1}{L} \int_0^\infty \frac{J_{sc}}{J_e} \, dx = \frac{E}{\epsilon} \tag{4}$$

where E is the average energy absorbed in the silicon per incident electron and $\epsilon = 3.6$ ev/pair.

This average absorbed energy is very nearly equal to the energy of the incident electron. The latter quantity was measured with a combination vacuum Faraday cup-calorimeter, under scattering conditions of 2 mils of aluminum and 2 inches of air. The absorption measurements were made with additional scattering of 3.5 mils of aluminum and 3

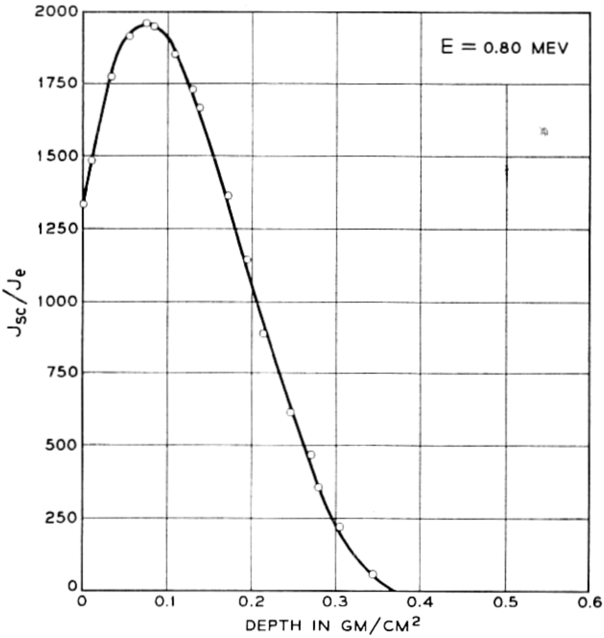

Fig. 4 — Ratio of collected current density to incident electron current density vs depth for 0.80 Mev electrons in a silicon solar cell of 8.75 μ diffusion length.

inches of air. The extrapolated ranges obtained from these curves, when corrected for the additional air and aluminum, give initial electron energies, through the range-energy relationship,[5] which agree with the calorimeter measurements within 3 per cent. Using the energies obtained by the extrapolated ranges and evaluating the integrals under the smooth curves, one obtains a set of L values for the cell used in these measurements as in Table I under the heading Uncorrected values. One observes that the calculated L values decrease with decreasing energy. The cause for this is not clearly understood, but it is possible to

Table I — Diffusion Length Values Computed from Absorption Curves for Electrons of Various Energies

Electron Energy (Mev)	Uncorrected $L(\mu)$	Backscatter Corrected $L(\mu)$
1.16	8.34	8.54
0.98	8.25	8.48
0.80	7.84	8.15
0.61	7.64	8.04

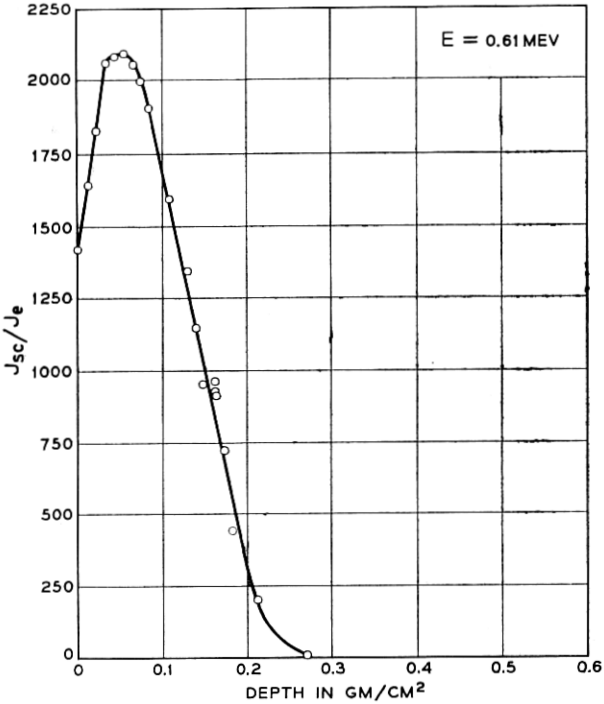

Fig. 5 — Ratio of collected current density to incident electron current density vs depth for 0.61 Mev electrons in a silicon solar cell of 8.75 μ diffusion length.

enumerate two contributing factors. The average energy of the incident electrons is less than that determined by the calorimeter under less severe scattering conditions. Similarly, the extrapolated range is only a measure of the range of the most energetic components, the less energetic ones manifesting themselves only in the shallower portion of the absorption curve. Secondly, a certain fraction of the incident energy flux is reflected back by way of secondary electron emission. The latter effect can be taken into account quantitatively by making use of some recently published data by Wright and Trump.[6] They give curves for the fraction of the incident electron beam energy which is scattered backward as a function of energy. When one makes use of the curve for aluminum, one arrives at L values shown in Table I under the heading Backscatter Corrected values. The variation with energy has been reduced but the trend still remains. If one makes the assumption that the correct L value is approached as the electron energy goes to infinity then one can attempt to obtain this value by plotting L against the reciprocal of the energy and extrapolating to zero. Such an extrapolation yields $L = 9.0\ \mu$ which is about 5 per cent higher than the value at 0.98 Mev. If one now takes a value halfway between these two, namely $8.75\ \mu$, any remaining uncertainty will introduce errors of less than ± 3 per cent.

With the L value of the experimental cell thus determined, it is possible to normalize the ordinates of the curves in Figs. 2–5 to give the average specific ionization as a function of depth for the various initial electron energies. For example, 0.98 Mev electrons have a maximum average specific ionization in silicon of 225 pairs/μ and, for the scattering conditions of this experiment, have a surface ionization of 152 pairs/μ. The initial slope of the curve is 2.85 pairs/μ^2 so that for a cell of 100 μ diffusion length, $g_1L/g_0 = 0.3$ (see Appendix).

The above nonuniformity in the carrier generation rate is large enough to make it desirable to make diffusion length measurements in the vicinity of the maximum where the variation in the generation rate is less than 2 per cent over a range of 250 μ. An experimental arrangement for readily doing this has been developed and takes the form of a double-aperture Faraday cup, which is shown schematically in Fig. 6. The cup is placed in the 1 Mev electron beam in a position such that

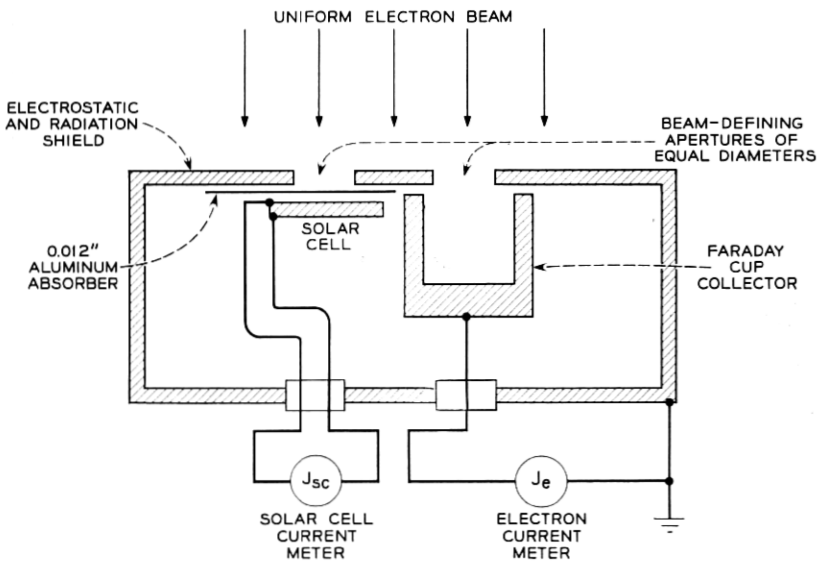

Fig. 6 — Schematic representation of the double-aperture vacuum Faraday cup for measuring J_{sc}/J_e .

equal electron fluxes enter both apertures. This is assured by making repeat measurements with the cup rotated 180° around its own axis. Under one aperture is a 12 mil aluminum foil, bringing the carrier generation rate near the maximum in silicon, which is followed by the cell whose diffusion length is to be measured. The beam entering the other aperture is collected in a Faraday cup. The ratio of the cell short circuit current to the cup current when divided by 225 will give the diffusion length in microns. Such measurements are carried out with sufficiently sensitive amplifiers so that an L determination can be made with an exposure of 10^{10} electrons/cm^2. The ratio is formed electronically and is displayed in digital form. The measurements are reproducible to within ±3 per cent.

IV. HEAVY PARTICLES

For the case of energetic protons, deuterons, and alpha particles the situation is fairly simple. Since the particle mass is much greater than the electron mass, the incident particle passes through the material with only negligible deflection except for the extremely rare instance in which it undergoes a large angle nuclear collision. At any depth in the material, the specific ionization may be obtained by dividing the rate of energy loss, dE/dx, by the average energy required to produce a pair; i.e., 3.6 ev for silicon.

As an example, Fig. 7 is a plot of the specific ionization and its spatial rate of change in silicon as a function of proton energy. The values were computed from a tabulation of dE/dx versus energy in aluminum as given by Sternheimer.[7] These curves show that for minority carrier diffusion lengths of the order of 100 microns, a proton of initial energy greater than about 18 Mev will generate carriers uniformly to within 2 per cent in a depth of one diffusion length from the surface. This implies a 2 per cent accuracy in (1) when a uniform generation rate of g_C is used. The uniformity of carrier generation is improved with increased proton energy.

Similar considerations may be applied to other heavy nuclear particles. For quantitative information on dE/dx and range-energy relationships, references such as Bethe and Ashkin[8] or Allison and Warshaw[9] are useful.

Diffusion length measurements were made with 16.8 Mev protons (Princeton Cyclotron) and 130 Mev protons (Harvard Cyclotron). The ionization currents were measured on cells of 1 cm^2 area and the beam current densities measured with Faraday cups available at each of the two installations. When (3) is solved for L with $S_0 = 1.46 \times 10^7$ pairs/

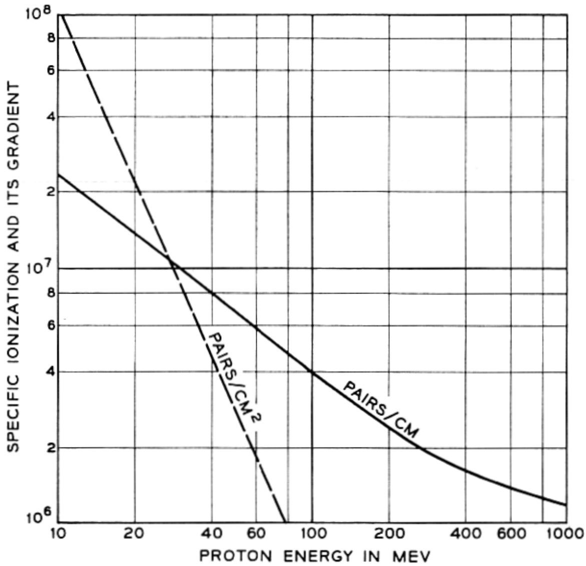

Fig. 7 — Specific ionization and its spatial rate of change vs energy for protons in silicon.

cm at 16.8 Mev and $S_0 = 3.03 \times 10^6$ pairs/cm at 130 Mev, the results in Table II are obtained. For comparison, the table also contains the diffusion length measurements obtained with the electron beam. As may be seen, agreement is obtained between the two methods for a large range of diffusion length values for both conductivity types and at two greatly different proton energies.

V. GAMMA- AND X-RAYS

Measurement with this type of radiation is discussed in the previously mentioned article by Gremmelmaier.[2] He shows that to a good degree of approximation,

$$g_0 = N_\gamma K \frac{\bar{E}}{\epsilon} \tag{5}$$

where

N_γ = incident radiation flux in quanta per cm²-sec,
K = the absorption coefficient,
$\bar{E}$ = the average electron energy per absorbed quantum,
ϵ = average energy per pair (3.6 ev for silicon).

TABLE II — DIFFUSION LENGTH MEASUREMENTS OBTAINED BY MEANS OF PROTONS COMPARED WITH 1 MEV ELECTRON MEASUREMENTS

Cell Type	Cell No.	Proton Energy (Mev)	$L(\mu)$ by Protons	$L(\mu)$ by Electrons
n on p	211	16.8	21.0	21.5
n on p	622	16.8	86.0	88.0
n on p	1112	16.8	29.3	32.5
n on p	1021	16.8	13.8	11.5
p on n	5011	16.8	5.2	4.8
p on n	5021	16.8	22.4	19.5
p on n	280R	16.8	8.7	7.9
p on n	283R	16.8	5.6	3.9
n on p	8112	130	5.4	5.5
n on p	10802	130	5.4	5.35
n on p	B-1	16.8	41.2	40.4
n on p	6112	16.8	12.0	11.5
n on p	C-2	16.8	20.4	21.2
n on p	D-2	16.8	8.2	7.8
n on p	7112	16.8	2.3	2.3
n on p	7111	16.8	2.14	2.36
n on p	6911	16.8	2.2	2.4
n on p	7012	16.8	2.5	2.4

For a given quantum energy, K and $\bar{E}$ can be calculated from tabulated absorption and scattering cross sections.[10] However, the determination of N_γ for a given source and particular geometry is quite difficult.

The use of this radiation does offer definite advantages. The rate of degradation by the radiation is substantially lower than for the case of energetic particles since the secondary electron flux is weighted toward the low energy end, with a sizable portion below the damage threshold energy. It is highly penetrating, so that measurements can be made through appreciable thicknesses of absorber. For the same reason, the carrier generation rate is uniform to great depth. The advantages of a gamma- or X-ray source can be utilized without the evaluation of the various constants in (5) by calibrating with a cell of known diffusion length, the latter quantity having been determined with electrons.

Solar cell short circuit currents were measured in a 10 kilocurie Co^{60} source. The radiation level within the source was about 0.87×10^6 r/hr. A number of cells whose diffusion lengths had previously been determined using the electron beam were placed inside the source and their short circuit currents measured. The ratio of current to diffusion length for each of the cells is shown in Table III. As may be seen, the ratio undergoes variations up to 20 per cent. This is caused primarily because the cells did not occupy positions of equal intensity in the γ-ray source. It is thus reasonable to conclude that there is good agreement between a diffusion length measurement in the γ-ray source and under

TABLE III — GAMMA RADIATION INDUCED SHORT CIRCUIT CURRENT NORMALIZED TO 1 MEV ELECTRON BEAM DETERMINED DIFFUSION LENGTH

Cell Type	Cell No.	$L(\mu)$	I_{sc}/L $(\mu a/\mu)$
n on p	2A	18.6	.174
n on p	2B	19.0	.159
n on p	3A	17.7	.141
n on p	3B	17.9	.154
p on n	5A	4.3	.176
p on n	5B	4.4	.171
p on n	6A	7.0	.163
p on n	6B	5.2	.170

the electron beam, regardless of whether the minority carriers are holes or electrons. Since the area of each of the cells used was 1 cm², the average value of the ratio gives a calibration for the particular γ-ray source of 0.162 $\mu a/\mu$-cm². This number may be converted to an absorbed dose rate[11] and yields 0.88×10^6 rad/hr. By means of the definition of the rad and the roentgen, and using the mass stopping power of air relative to silicon one can convert this number to correspond to 0.81×10^6 r/hr which is in reasonable agreement with the existing calibration.

VI. RADIATION DAMAGE

The damage produced by energetic radiation causes the minority carrier diffusion length to decrease in a well defined way. Fig. 8 gives the experimental results for the measured diffusion length as a function of 1 Mev electron flux for an n-diffused, p-type 1 Ω-cm base silicon solar cell. The solid curve is a plot of the theoretical formula

$$\frac{1}{L^2} = \frac{1}{L_0^2} + K\Phi \tag{6}$$

in which L is the diffusion length after some bombardment flux Φ, and L_0 the initial diffusion length, with $L_0 = 140\ \mu$ and $K = 1.8 \times 10^{-10}$.

The value of K depends on many parameters such as the energy and type of bombarding particle, the resistivity and conductivity type of the material, the temperature, and the impurity concentration. The variation of K with particle type and energy will be the subject of other papers. Table IV summarizes some of the results for the radiations mentioned in this paper and for silicon solar cells made from pulled crystals and irradiated at room temperature.

From this one can determine, for example, the per cent change in

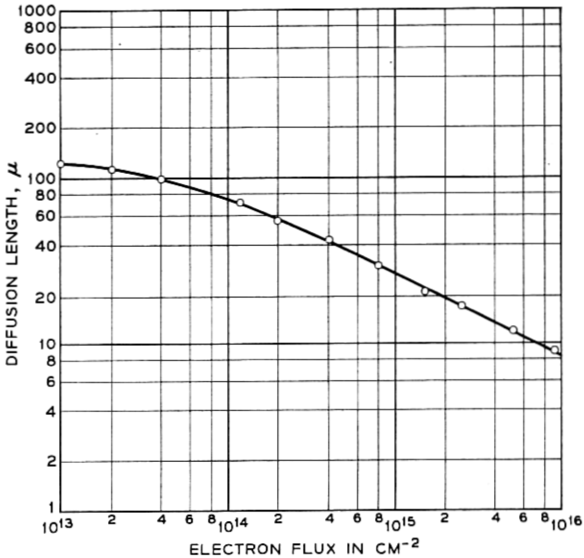

Fig. 8 — Degradation of diffusion length vs electron flux.

diffusion length of an n-type base solar cell (1 Ω cm) during a measurement with 1 Mev electrons for an exposure of 10^{10} cm^{-2} when the initial L value is 100 μ. Differentiation of (6) yields

$$\frac{dL}{L} = -\frac{K}{2} L^2 \, d\Phi \tag{7}$$

which gives, for the above case, a change of less than 0.2 per cent.

VII. CONCLUSION

It has been demonstrated that the method for measuring diffusion length by means of ionizing radiations is particularly suitable for the case of the solar cell and is expected to be useful for devices of similar geometry. Measurements with three types of radiations—electrons, pro-

TABLE IV — RADIATION DAMAGE PARAMETER $K = (d/d\Phi)(1/L^2)$ FOR SOME RADIATIONS AND CONDUCTIVITY TYPES IN SILICON

Base Conductivity Type and Resistivity		1 Mev Electrons	16.8 Mev Protons	130 Mev Protons	Co^{60} γ-rays
p-type	1 Ω cm	1.8×10^{-10}	8.3×10^{-7}	3.3×10^{-7}	$2.6 \times 10^{-3} r^{-1} cm^{-2}$
p-type	10 Ω cm	5.8×10^{-11}			
n-type	1 Ω cm	2.6×10^{-9}	5.1×10^{-6}	2.0×10^{-6}	$4.0 \times 10^{-2} r^{-1} cm^{-2}$

tons and gamma rays—are shown to give consistent results over a wide range of diffusion lengths and for both conductivity types in silicon. The electron beam technique is shown to lend itself most readily to an absolute calibration and to routine measurements of diffusion length. By this technique the average specific ionization of electrons as a function of penetration depth is measured. For 0.98 Mev electrons in silicon this ionization reaches a maximum value of 225 pairs/μ $\pm$ 5 per cent at a depth of 0.096 gm/cm^2.

The consistency found among the various types of radiation allows one to reverse the argument and suggest that calibrated solar cells be used as solid-state ionization chambers for measuring radiation intensity. A range of 10^3 rad/hr to 10^9 rad/hr should be achievable.

VIII. ACKNOWLEDGMENT

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APPENDIX

For a solar cell of the type shown in Fig. 1, for which the junction depth is much smaller than the diffusion length of the minority carriers, one needs to solve the diffusion equation only in the semi-infinite region beyond the junction for the case of penetrating radiation. If the junction is assumed to be located at $x = 0$ and the carrier generation rate as a function of distance from the junction to be given by $g(x)$, then the steady-state excess carrier concentration will be given by the solution of the equation:

$$D \frac{d^2n}{dx^2} - \frac{n}{\tau} = -g(x) \tag{8}$$

where

D = minority carrier diffusion coefficient,
τ = minority carrier lifetime,
n = excess minority carrier concentration.

Under short circuit condition, i.e., zero bias on the junction, the equation is solved with boundary conditions $n = 0$, $x = 0$, and $dn/dx = 0$, $x = \infty$. If the carrier generation rate is reasonably uniform within a diffusion length, $L = \sqrt{D\tau}$, of the junction, then $g(x)$ may be represented by the first two terms of its Taylor series expansion as follows:

$$g(x) = g_0 + g_1 x. \tag{9}$$

The solution to (8) with $g(x)$ given by (9) under the above boundary conditions is given by:

$$n(x) = g_0\tau(1 - e^{-x/L}) + g_1 x\tau \tag{10}$$

which yields for the magnitude of the short circuit current density:

$$J_{sc} = eg_0 L\left(1 + \frac{g_1}{g_0} L\right). \tag{11}$$

From (11) it may be seen that under strictly uniform carrier generation, i.e., $g_1 \equiv 0$, the short circuit current density is given by $J_{sc} = eg_0 L$. This magnitude is increased or diminished in the nonuniform case by an amount $eg_1 L$ depending on whether the generation rate increases or diminishes with depth in the material, i.e., depending on whether g_1 is positive or negative.

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