# Characteristics of the Service Provided by Communications Satellites in Uncontrolled Orbits

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*In a communications system which uses satellites in uncontrolled orbits, there would be times when the communication between two stations would be interrupted because no satellites would be in view. This paper provides material which can be used to describe the service which a random system of satellites would provide. An example system which could be used to provide initial world-wide service is also discussed. This system has certain nonrandom characteristics, and the effect of these characteristics on the service to various parts of the world is examined by computer simulation.*

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## I. INTRODUCTION

One of the several types of systems which have been proposed for world-wide communications would use numerous satellites in circular orbits 3000–8000 miles* above the earth's surface. The satellites would contain active repeaters through which any pair of stations on earth could communicate whenever a satellite is mutually visible to the stations. In this system, the satellites would move independently of each other, and the motions of the satellites relative to the earth's rotation would not be controlled. The manner in which the satellites are distributed about the earth at a time chosen at random would be much the same as if the satellites were randomly placed into position at that time.

Two ground stations will be able to communicate a large part of the time if a random system of satellites is used, but there will be times when no satellites will be mutually visible. This paper describes the service which satellites in random, or uncontrolled, orbits would provide. Service, in this context, refers to the availability of satellites for communication and does not pertain to the type or subjective quality of communication. Some of the material in this paper is an extension of a study reported earlier by Pierce and Kompfner.[1]

Sections II and III describe the geometrical properties of satellite orbits and satellite visibility. Section IV presents some methods and results which have been used to describe the service provided by a random system. Section V discusses a particular system of satellites, which approximates a random system, and describes the service which would be furnished to points throughout the world. The nonrandom

* The nautical mile is used throughout this paper.

characteristics which this system might have and the effect which these characteristics would have on the service are studied in Section VI.

## II. SATELLITE ORBITS

This section describes some basic geometrical properties of satellite orbits. The discussions of service in later sections are based on the orbit properties defined here.

### 2.1 *Orbit Altitude*

The general orbit of an earth satellite is an ellipse which has the earth centered at one focus. Two quantities which are popularly used to describe the altitude of the elliptical orbit are the height of perigee and the height of apogee. As shown in Fig. 1, the perigee height is the distance of closest approach to the earth's surface, and the apogee height is the maximum altitude reached by the satellite. The circular orbit, which is of prime interest here, is a special case in which the apogee and perigee heights are equal. The circular-orbit altitude will be designated by $H$.

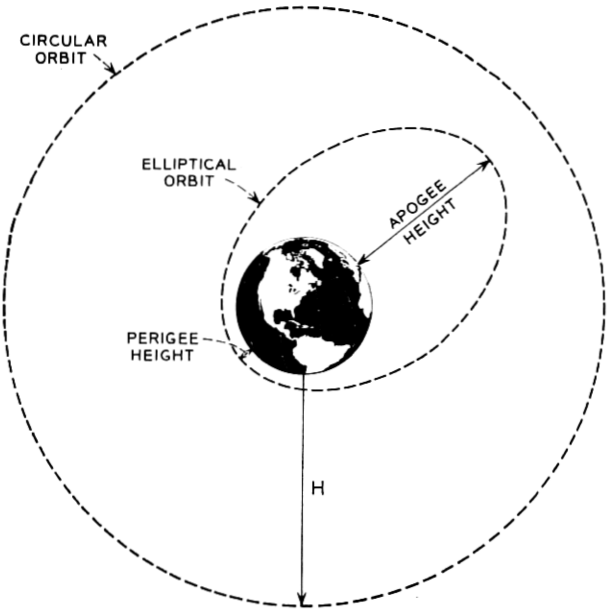


Fig. 1 — Elliptical and circular orbits.

### 2.2 *Satellite Period*

The period of an earth satellite, i.e., the time required for a satellite to move through 360° in its orbit, is given by

$$P = 2\pi \sqrt{\frac{a^3}{MG}} \tag{1}$$

where $M$ is the mass of the earth, $G$ is the universal gravitational constant, and $a$ is the semimajor axis of the orbit. The period is plotted as a function of $H$ in Fig. 2.

### 2.3 *Orientation of the Orbit Relative to the Earth*

A satellite in an equatorial orbit moves in a plane which contains the earth's equator, and the plane of a polar orbit contains the earth's

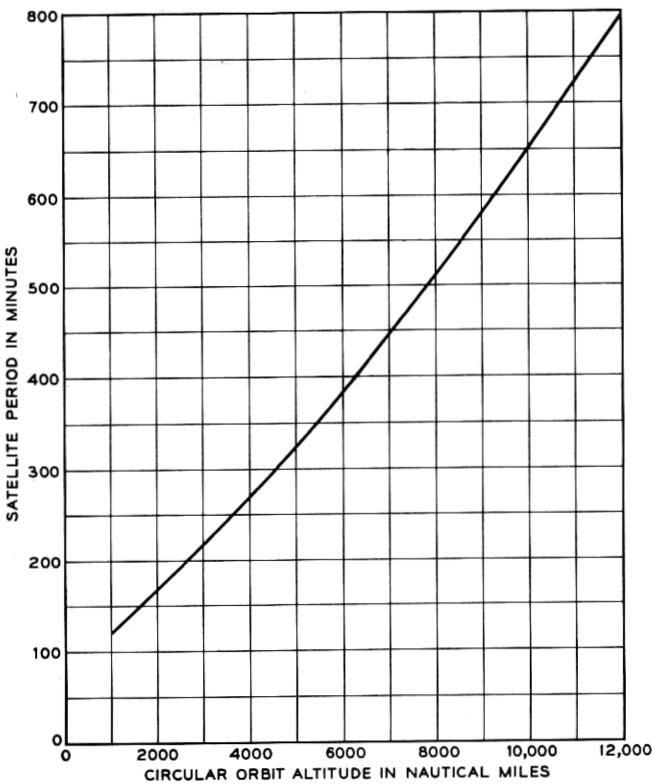


Fig. 2 — Satellite period vs circular orbit altitude. The period of elliptical orbits is found by using a circular orbit altitude which is the average of the apogee height and the perigee height of the elliptical orbit.

rotation axis. The orientation of other orbits relative to the earth is defined by the inclination angle between the equatorial plane and the orbit plane.

As shown in Fig. 3, the inclination angle is measured counter-clockwise from the equatorial plane to the orbit plane. The direction of the motion of the satellite is important, so the measurement is specified at the point at which the satellite crosses the equator from south to north (the ascending node). Satellites in orbits with inclinations less than 90° move in the direction of the earth's rotation. These orbits are easier to achieve than retrograde orbits (inclination larger than 90°) because of the inertial velocity added by the rotation of the earth.

The plane which contains a satellite orbit remains fixed in space

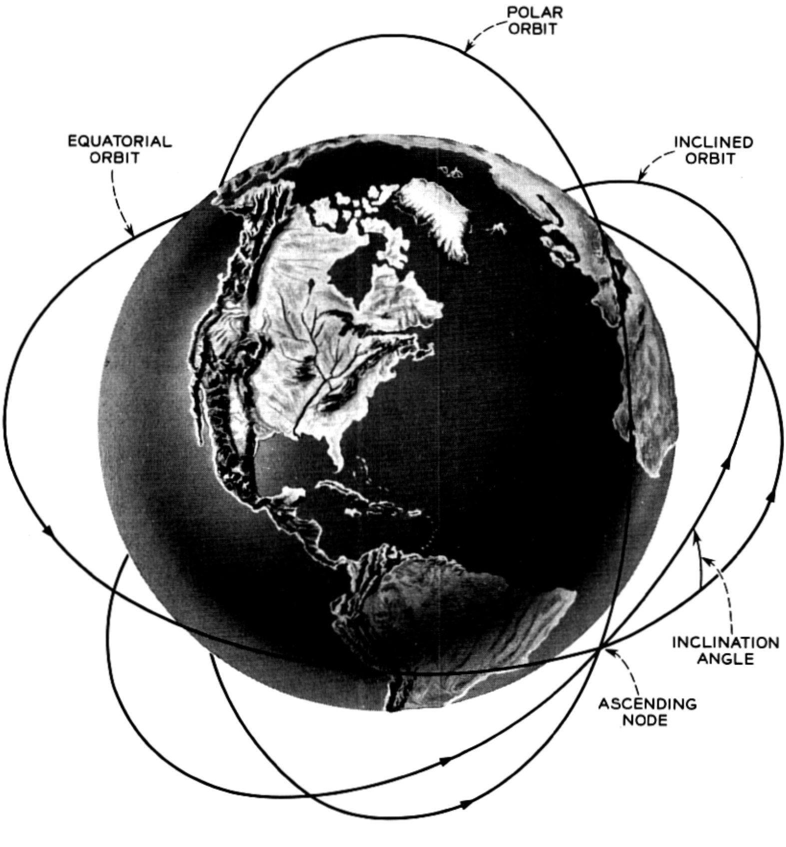


Fig. 3 — Polar, inclined, and equatorial orbits. The orientation of the orbit plane relative to the earth is given by the inclination angle.

while the earth rotates under the orbit.* The orientation with the earth's axis is constant, but the orientation of the orbit relative to a point on the earth continually changes.

### 2.4 *Relative Orientation of Orbit Planes and Relative Positions of Satellites*

The orientation of a system of several orbit planes is not adequately described by the inclination angle, for the planes can have any relative orientation around the earth's axis. This orientation will be specified by giving the angular position of the ascending nodes. The angle is measured in the equatorial plane from an arbitrary reference. Thus, if the ascending nodes of three polar orbits are at 0°, 120°, and 240°, the planes are uniformly spaced.

The position of a satellite in its orbit at a particular time will be given by a phase angle. This phase is the angular separation between the ascending node and the satellite. The angle is measured at the center of the orbit and is positive in the direction of satellite motion. A satellite over the north pole would thus have a phase of 90°.

The relative positions of several satellites can be completely described by giving an altitude, inclination, relative ascending node, and phase for each satellite. In a random system, all satellites usually have the same altitude, and one or several inclinations may be specified. At a time chosen at random, all ascending nodes and phases would be equally probable. In some systems one degree of randomness is removed by specifying the relative ascending nodes. Also, several satellites with the same inclination may have the same ascending node and would therefore be in the same plane.

## III. SATELLITE VISIBILITY

Satellite communication can take place between two ground stations only if a satellite is visible simultaneously from both stations. Thus, mutual visibility conditions determine the service which a system of satellites provides to the stations.

### 3.1 *Visibility Geometry*

A satellite is considered usable for communication if its elevation angle above the theoretical horizon is larger than some specified $\alpha$.† This

* This is true only for polar and equatorial orbits. The effect of the small motion of inclined orbits will be taken up in Section VI.

† For convenience in terminology, the satellite will be said to be *visible* only when this condition is met.

angle limitation is influenced by local terrain, performance of the radio system,[1] and considerations of interference with terrestrial systems.[2] A value of 7.5° will be used for $\alpha$ in the studies presented here, since this value has been used in most previous discussions.

The angle $\alpha$ defines a cone of satellite visibility as shown in Fig. 4a. The orbit of a satellite at altitude $H$ intersects this cone in the two points at which visibility begins and ends. If the orientation of the orbit relative to the ground station is arbitrary, the locus of all possible extremes of visibility is a circle on the cone. Projecting these points radially onto the earth results in a circle with angular-great-circle radius $\psi$ as shown in Fig. 4b, where

$$\Psi = \cos^{-1}\left(\frac{R}{R+H}\cos\alpha\right) - \alpha \tag{2}$$

A station could see a satellite which is at an altitude $H$ whenever it is over this circular region. Two stations can communicate only if their circles of visibility intersect and form a mutual visibility region.

Some mutual visibility regions for 6000-mile orbits are shown in Fig.

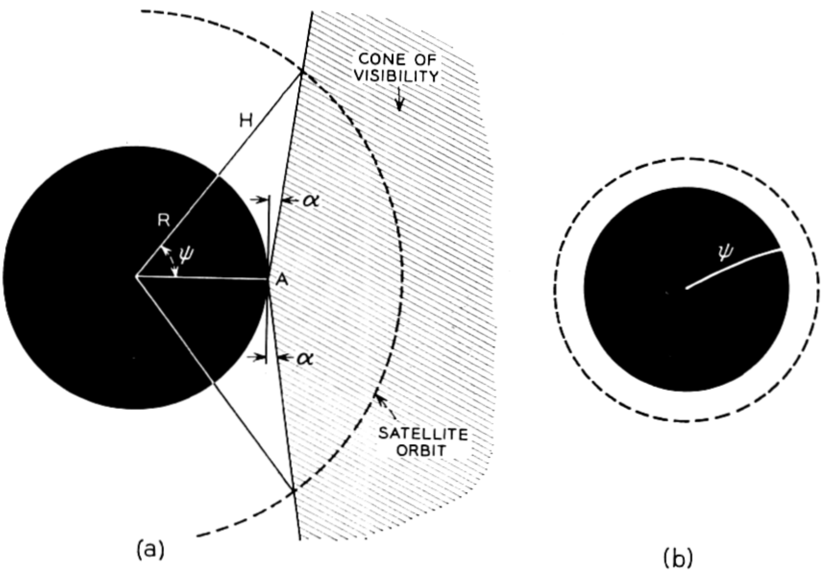


Fig. 4 — Satellite visibility geometry: (a) Plane view of the cone of visibility for a ground station at A. $\alpha$ is the minimum usable elevation angle. (b) Circle of visibility. A satellite of altitude $H$ would be more than $\alpha$ degrees above the horizon at the ground station whenever it is over this circular region centered on the ground station.

5. Because of the polar projection used, the parts of the regions which are south of the equator are not shown. Satellites in polar orbits could pass over any part of these regions, but those with lower inclinations would not move further north than the latitude corresponding to the inclination angle. Thus if the motion of the satellites is also considered, the area of mutual visibility would be reduced for lower-inclined orbits, and it would degenerate to a line for equatorial orbits.

Fig. 5 shows a region in which a satellite in a polar orbit could be

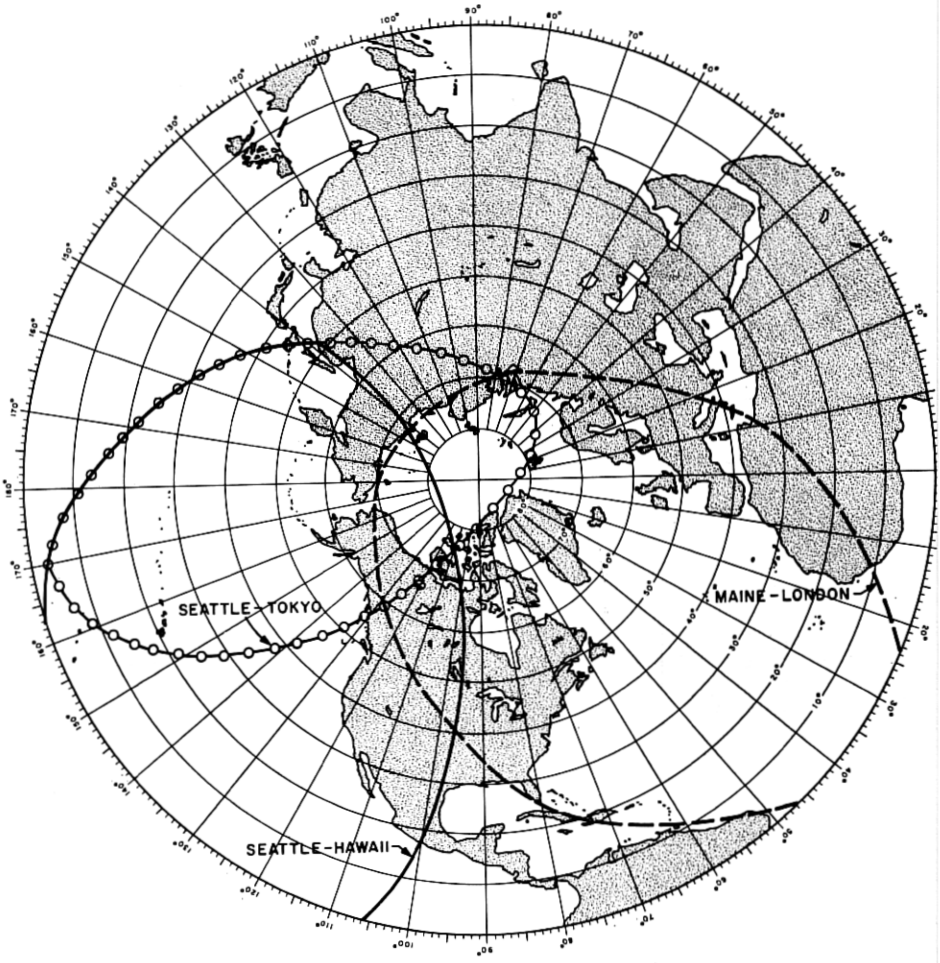


Fig. 5 — Mutual visibility regions for 6000-mile polar orbits. $\alpha = 7.5°$. Portions of the visibility regions below the equator are not shown on this projection.

seen simultaneously by all three pairs of stations. This region and the other regions of overlap present a problem of competition for the use of satellites which are over these regions. A discussion of this problem is given in Section 4.1.3.

### 3.2 *The Probability that a Satellite is Mutually Visible*

The description of random-orbit service requires a knowledge of the probability that a satellite in a particular orbit is mutually visible to two stations at a randomly chosen time. A numerical method is used to obtain an estimate of this probability, which will be designated as $p$.

#### 3.2.1 *Numerical Determination of* $p$

Fig. 5 can be used to determine $p$ for Seattle and Tokyo for 6000-mile polar orbits. The earth rotates under the orbit, so at a randomly-chosen time the orbit plane can intersect the earth along any meridian. The meridian lines in the figure thus represent a uniform sample of all orientations of the plane. All phases are equally likely for the satellite at the chosen time. For a particular orientation of the plane, the fraction of the corresponding meridian line which is contained in the mutual visibility region is the probability that the satellite can be seen. The average of the fractions determined for all meridians in the sample is an estimate of $p$.

For an equatorial orbit, $p$ is simply the fraction of the earth's equator which is contained in the visibility region. The value of $p$ for the general inclined orbit was determined by a computer program which uses a method similar to the one described for polar orbits. A method which uses numerical integration of an expression for $p$ is given in Ref. 3.

#### 3.2.2 *Variations in* $p$

For a given orbit altitude and inclination, $p$ is a function of the latitudes of the two ground stations and the distance between them. For pairs of points whose mutual visibility regions include one of the earth's poles, polar orbits provide a higher $p$ than do other orbits. Regions which are centered on the equator are best served by equatorial orbits. Intermediate inclinations give the highest $p$ for regions which are between these extremes.

For a given pair of ground stations and a given inclination, $p$ increases with the altitude of the orbit. An example of the nature of this increase is shown in Fig. 6 which gives $p$ vs altitude for Maine–London for polar orbits. The rate of increase of p with altitude begins to diminish

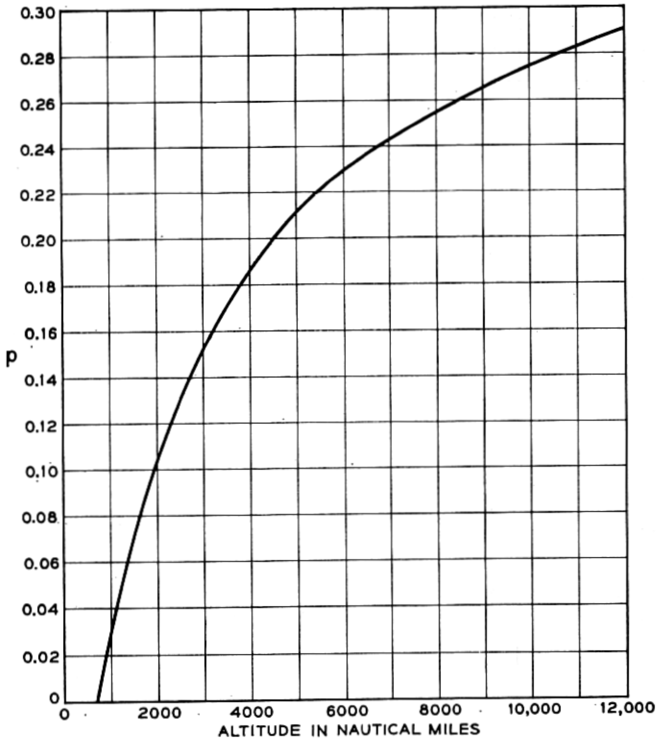


Fig. 6 — Fraction of time ($p$) a satellite in a circular polar orbit of altitude $H$ is visible to Maine and London. $\alpha = 7.5°$.

rapidly at altitudes above about 8000 miles. The reason for this is apparent from (2). For high altitudes in this equation, the radius of the visibility region is approaching $(\pi/2) - \alpha$ which is the maximum value which can be attained, and the size of the visibility region directly influences the value of $p$. The maximum value of $p$ for this pair of points is 0.368.

Equation (2) shows that the size of the visibility circle is also influenced by the value of $\alpha$. Thus the minimum elevation angle affects the value of $p$ for a given orbit and a given pair of ground stations. Fig. 7 gives $p$ vs $\alpha$ for Maine–London and 6000-mile polar orbits. The value of $p$ is read from the left scale. If $\alpha$ is zero, i.e., if the satellite were used whenever it was above the theoretical horizon, $p$ would be 0.28. An $\alpha$ of 20° would result in a $p$ of about half of this maximum value. So if $\alpha$ were 20°, the satellite could be used during only half of the time it is visible. The use of an $\alpha$ of 7.5° results in about 82 per cent utiliza-

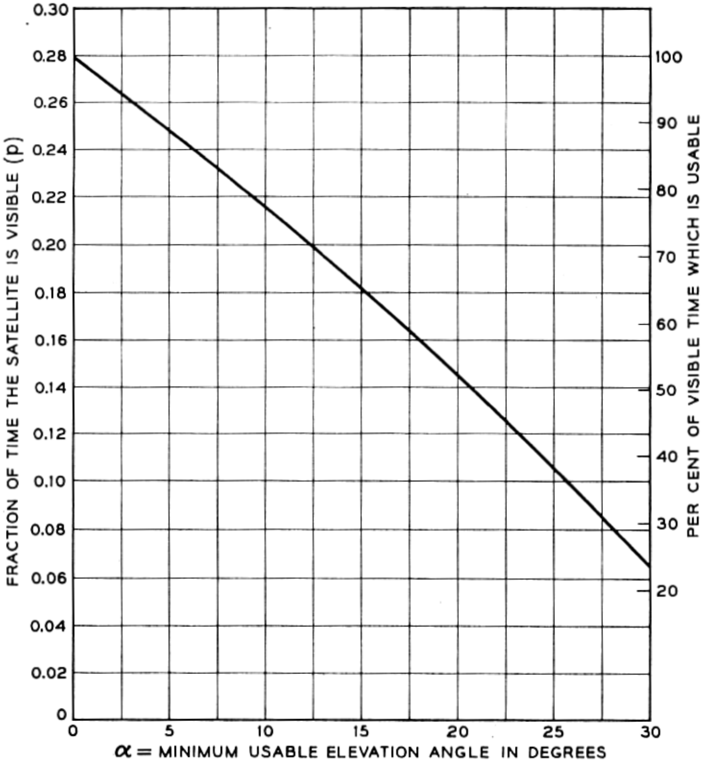


Fig. 7 — The effect of the minimum elevation angle on the usefulness of a satellite in a 6000-mile polar orbit. Ground stations at Maine and London.

tion of the visible time, so there is something to be gained in reducing this angle if it is possible.

### 3.2.3 *A Partial Table of p*

During the course of previous studies, values of $p$ have been determined numerically for many ground-station locations, orbit altitudes, and inclinations. These values are given in the table in the Appendix. The table is by no means complete, but many of the pairs of points which are of interest are included.*

The first two columns in the table are the colatitudes of the ground stations. These numbers may be measured from the north pole or the south pole of the earth as long as the same convention is used for both points. The third column is the longitude separation of the two stations.

* Ref. 4 gives values of $p$ in a more general form.

The other columns are headed by altitudes and inclinations. Linear interpolation may be used between successive altitudes and successive inclinations with reasonable accuracy.

## IV. SERVICE PROVIDED BY SATELLITES IN RANDOM ORBITS

### 4.1 *Quality of Service, q*

A random-orbit system will use a multiplicity of satellites to provide service to many pairs of ground stations. For a particular pair of stations, the quality of service is defined as the fraction of time the required number of satellites is available for use. The symbol $q$ is used to denote the quality of service.

#### 4.1.1 *Satellites in Orbits of the Same Altitude and Inclination*

*(i) Single-Satellite Visibility*

Since $p$ is the probability that a satellite in a certain orbit will be visible to a pair of ground stations at a time chosen at random, $(1 - p)$ is the probability that the satellite will not be visible at that time. If there are $n$ satellites with identical $p$'s, and if the satellites have random ascending nodes and phases, the probability that none of these satellites would be useful at a random time is $(1 - p)^n$. The fraction of time at least one satellite would be useful is then

$$q_1 = 1 - (1 - p)^n \tag{3}$$

Fig. 8 gives values of $q_1$ for a range of values of $n$ and $p$. If $p$ is known, the number of satellites needed to furnish a certain quality of service may be found. For example, for stations in Maine and London, and 6000-mile polar satellites, $p = 0.23$. For a quality of service of 0.999, Fig. 8 shows 27 satellites would be required. For $q_1 = 0.99$, $n = 18$; and for $q_1 = 0.90$, $n = 9$.

Equation (3) may also be written

$$n = \frac{\log (1 - q_1)}{\log (1 - p)}$$

If $n$ is changed to $n'$, $q_1$ would change to $q_1'$, and the ratio of $n'/n$ is given by

$$\frac{n'}{n} = \frac{\log (1 - q_1')}{\log (1 - q_1)}$$

which is independent of $p$. This equation was used to derive a table (next page) which contains the number of satellites needed for various $q_1$ relative to the number needed for $q_1 = 0.999$.

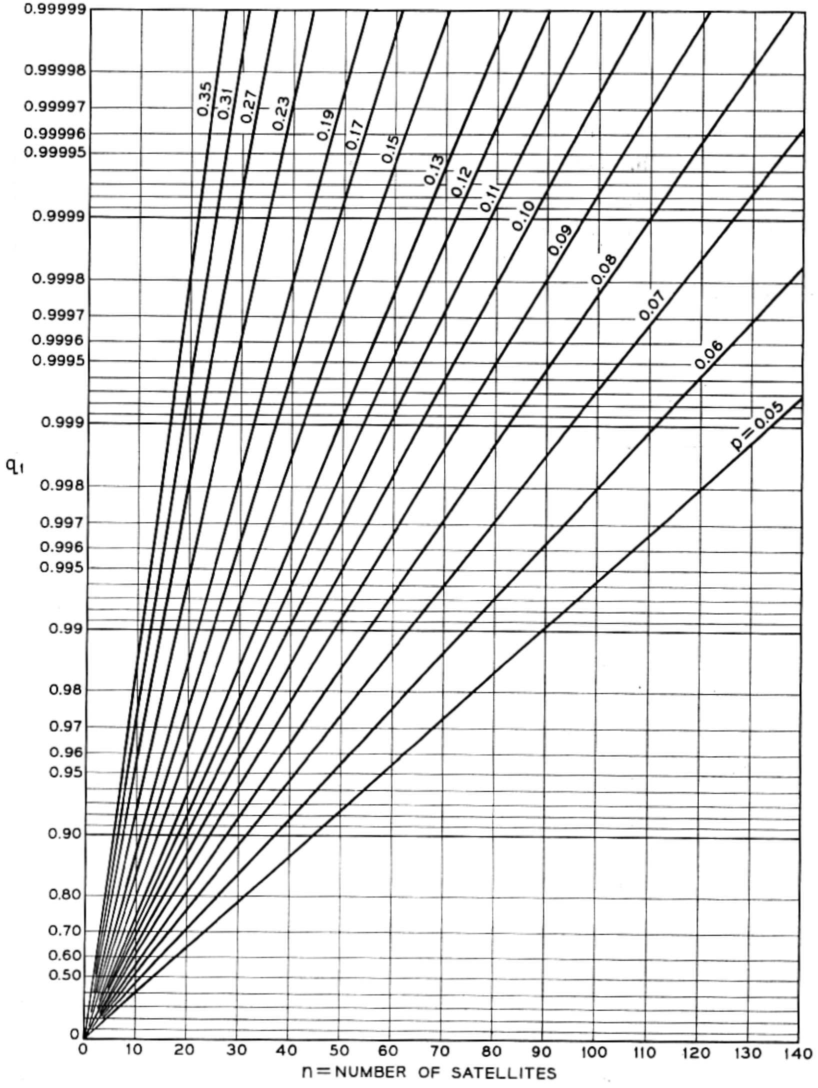


Fig. 8 — Quality of service for single satellite visibility, $q_1$, for a range of values of $p$ and $n$.

| $q_1$ | Number of Satellites Needed |
|---|---|
| 0.999 | $n$ |
| 0.99 | $0.67n$ |
| 0.95 | $0.43n$ |
| 0.90 | $0.33n$ |
| 0.75 | $0.20n$ |
| 0.50 | $0.10n$ |

*(ii) Multiple-Satellite Visibility*

Some pairs of ground stations may require more than one satellite at a time if the traffic between the two stations exceeds the capacity of one satellite. If $m$ ground antennas were used at each station, $m$ communication paths could be established by way of separate satellites. The quality of service on path $m$, which is the fraction of time $m$ or more satellites are usable by the ground stations, is given by

$$q_m = 1 - \sum_{i=0}^{m-1} \binom{n}{i} p^i (1-p)^{n-i} \tag{4}$$

Figure 9 gives values of $q_m$ as a function* of $q_1$. For a value of $q_1 = 0.999$ on the abscissa, the intersections of the ordinate line with the curves give $q_2 = 0.991$, $q_3 = 0.964$, etc. $q_1$, $q_2$, and $q_3$ would be the qualities of service for three paths between Maine and London if the paths were operated on a priority basis, i.e., if path 3 operated only when three or more satellites were visible, path 2 only when two or more were visible, and path 1 when one or more was visible. Figs. 8 and 9 may be used to determine the number of satellites needed for a particular $q_m$. Suppose $q_2 = 0.98$ is desired. From Fig. 9, $q_1 = 0.9975$. If we again use the example of $p = 0.23$, Fig. 8 shows 23 satellites would be needed.

Multiple paths between two points may be operated with equal priority simply by assigning periods of no service to the paths in turn. The lengths of these periods are not equal, but over a long period of time all paths would get equal qualities of service. Suppose there are three paths ($m = 3$). One of the paths would be out of service whenever only two satellites are visible, two would be out whenever only one satellite is visible, and all would be out when no satellites are visible. The fractions of these times for which a particular path would be out of service are $\frac{1}{3}$, $\frac{2}{3}$, and 1 respectively. The general fraction is $(m - i)/m$, where $m$ is the number of paths and $i$ is the number of satellites visible ($i < m$). The average quality of service on each of $m$ paths with equal priority is

$$\bar{q}_m = 1 - \sum_{i=0}^{m-1} \frac{m-i}{m} \binom{n}{i} p^i (1-p)^{n-i} \tag{5}$$

* This relationship varies slightly with $p$. The curves in Fig. 9 result from using $p = 0.18$. The values of $p$ which are of interest in this work range between 0.05 and 0.30. The changes in the curves of Fig. 9 for these extreme values of $p$ would be very small, so the curves can be used with reasonable accuracy for any $p$ in this range.

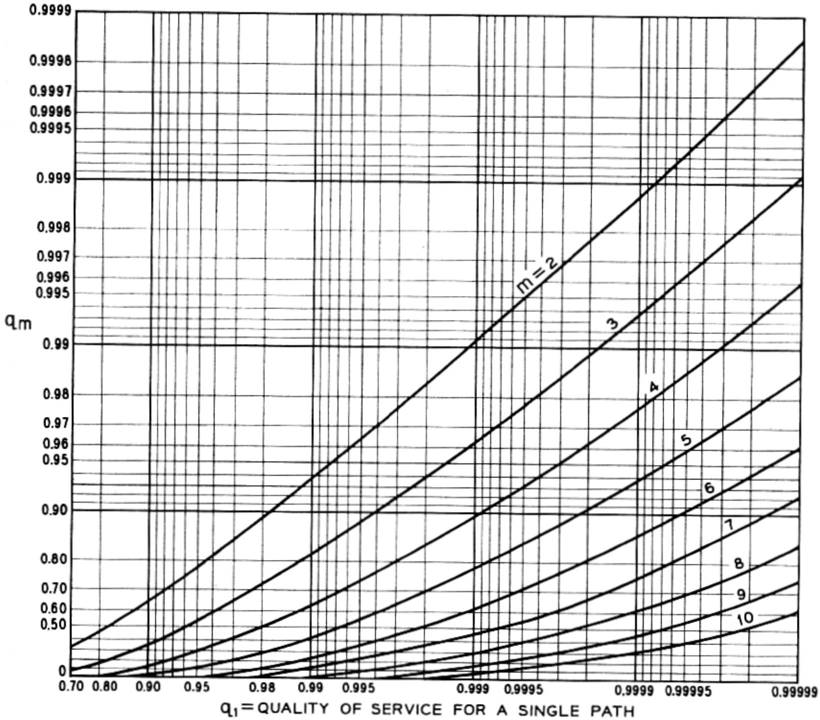


Fig. 9 — Fraction of time $q_m$ that $m$ or more satellites are visible. In a system with multiple paths between two ground stations $q_m$ is the quality of service on a path with priority $m$. The number of satellites needed to furnish $q_m$ is found by entering Fig. 8 with $q_1$ and $p$.

Fig. 10 gives values $\bar{q}_m$ as a function* of $q_1$. For $q_1 = 0.999$, $\bar{q}_2 = 0.995$, $\bar{q}_3 = 0.985$, etc. A previous example showed 27 satellites in 6000-mile polar orbits would provide $q_1 = 0.999$ between Maine and London. The above results show three equal-priority paths between these points would have qualities of service of 0.985. If 0.999 service were desired on each path, Fig. 10 shows $q_1 = 0.99996$, so 39 satellites would be needed (Fig. 8 with $p = 0.23$ and $q_1 = 0.99996$). This procedure can be used to determine the service from one point to several other points when the mutual visibility regions are almost identical. For example, stations in England, France, and Germany could be considered to be at about the same latitude and longitude, so 39 satellites would provide a

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* The previous comment on the effect of different values of $p$ also applies here.

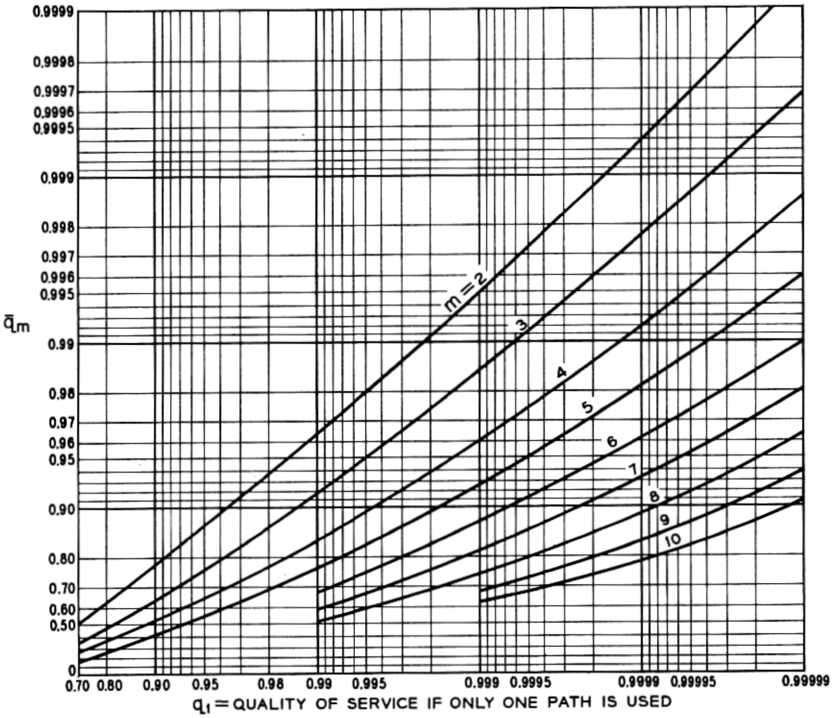



Fig. 10 — Quality of service $\bar{q}_m$ on each of $m$ paths with equal priority. The number of satellites needed to furnish $\bar{q}_m$ is found by entering Fig. 8 with $q_1$ and $p$.

$q_1$ of about 0.999 to each of these points with a separate satellite for each path.

### 4.1.2 *Mixed Orbits*

A previous section indicated that no single orbit inclination is best for all pairs of stations. This may be verified by examination of the table in the Appendix. Miami–Rio de Janeiro, for example, would have a quality of service of only 0.80 if the 27 polar satellites discussed before were used. Equatorial orbits are best for this pair of stations, but these orbits provide little service to Maine–London. This implies that satellites should be placed in both kinds of orbits if both pairs of stations are to be served equally well. It is also possible that a single intermediate inclination would be best.

Suppose a system consists of a total of $n$ satellites distributed in $k$ different orbits. Let the $j$th orbit contain $n_j$ satellites, and let $p_j$ be the fraction of time a satellite in this orbit is visible to a pair of stations. The quality of service for a single satellite is then

$$q_1 = 1 - \prod_{j=1}^{k} (1 - p_j)^{n_j} \tag{6}$$

$$\text{where} \quad n = \sum n_j$$

A solution of (6) for the above Miami–Rio de Janeiro example shows that 22 satellites in 6000-mile equatorial orbits would need to be added to the polar satellites to make $q_1 = 0.999$. However, Maine–London also gets some service from the equatorial satellite, so less than 27 polar satellites are now required to give 0.999 service to this path. Thus the solution must be adjusted until it converges to the point where 0.999 service results for both paths.

The problem of mixed orbits is concerned with the specification of the number of satellites which should be placed in each of $k$ different orbits in order to satisfy the usability requirements of several ground stations. If the satellite altitudes are the same, the best set of orbits might be considered to be the one which requires the smallest number of satellites.* The solution may be approximated by determining the number of satellites in each orbit individually and in sequence as in the above example. This method is difficult and time-consuming for complex problems, and a more direct approach is to use a linear-programming model. Such a model has been developed, but it is beyond the scope of this paper.

A more general problem of mixed orbits involves the determination of the set of satellites, in orbits of all possible altitudes and inclinations, which will furnish required qualities of service for a minimum cost. The linear-programming model represents a large step toward the solution of this problem; however, other problems related to costs and functional relationships between other system parameters must be thoroughly investigated before the problem can be solved.

#### 4.1.3 *The Effect of Overlap of Mutual Visibility Regions*

In Fig. 5, nearly all of the mutual visibility region for Seattle–Tokyo is contained within the regions for Maine–London and Seattle–Hawaii. When a satellite is mutually visible to Seattle and Tokyo, this satellite

* This is really an oversimplification. Depending on the location of the launching sites, the payload which a given vehicle can put in orbit depends on the inclination angle. With the possibility of launching several satellites with a single vehicle, minimizing the number of satellites does not necessarily minimize the total cost.

may already be in use by one of the other pairs of points. The previous discussion on multiple-satellite visibility indicates the problem could be solved most of the time by using other satellites which are also visible. The effect of this overlap on the qualities of service for each pair of points will be determined to illustrate the magnitude of the problem. For simplification, the overlap of the Seattle–Hawaii region with the Maine–London region will be ignored.

The Seattle–Tokyo path would be out of service if either of the following three conditions prevails:

(*i*) There is no satellite in the Seattle–Tokyo region. The fraction of time this would occur is given by $(1 - p_{\text{S-T}})^n$ where the subscript identifies the region.

(*ii*) There is exactly one satellite in region A (the area common to the Maine–London and Seattle–Tokyo regions) *and* there are no satellites in the remainder of the Maine–London region *and* there are no satellites in the remainder of the Seattle-Tokyo region. The fraction of time the event occurs is

$$[np_{\text{A}}\,(1 - p_{\text{A}})^{n-1}][1 - (p_{\text{M-L}} - p_{\text{A}})]^{(n-1)}[1 - (p_{\text{S-T}} - p_{\text{A}})]^{(n-1)}$$

If all paths have equal priority, Seattle-Tokyo would get the satellite in one-half of the situations. Thus one-half of the above expression is the fraction of time the path is out of service because of overlap with Maine–London.

(*iii*) Same as condition (*ii*) except region B (the area common to the Seattle–Hawaii and Seattle–Tokyo regions) replaces region A and the Seattle–Hawaii region replaces the Maine–London region.

As a numerical example, a system of 27 satellites in 6000-mile polar orbits was considered. Table I summarizes the effect of the overlap.

The effect of overlap is difficult to determine in a more complex situation; but because the effect is small, one can usually account for it by nominally reducing the quality of service or nominally increasing the number of satellites. In this example, the addition of one satellite would compensate for the overlap, i.e., $q_1$ for 28 satellites with overlap considered would be as good as $q_1$ for 27 satellites with overlap not in-

TABLE I

| Path | $q_1$ if Overlap is not Considered | $q_1$ including the Effect of Overlap | Fraction of Service Lost Because of Overlap |
|---|---|---|---|
| Seattle–Tokyo | 0.9876 | 0.9841 | 0.0035 |
| Seattle–Hawaii | 0.9904 | 0.9874 | 0.0030 |
| Maine–London | 0.9991 | 0.9987 | 0.0004 |

cluded. Complex overlap situations can better be evaluated by a computer simulation which has been developed to describe the service furnished by systems of satellites.

### 4.2 *Description of the Periods of Service and No-Service by Simulation*

Knowledge of the fraction of time a path is out of service is not sufficient to evaluate the service provided by a system of satellites. The description of the length and frequency of these periods of no service is at least equally important. For a given quality of service, the no-service periods could be seconds long and occur frequently, or they could be days long and occur only occasionally. Certainly the way the no-service periods would be evaluated and administered would be different for these two extremes.

To describe the periods of service and no service, a computer program which simulates a world-wide satellite communications system was used. The program assigns satellites to the several ground-station pairs in a system in a way which maximizes the service on a priority basis (the paths may have equal or ordered priority). The motions of the system may be simulated for any length of time, and periods of several years may be studied with modest amounts of computer time. The periods of no service are recorded for each ground-station pair, and a statistical description of the service periods and periods of no service is provided. Both random and nonrandom systems can be studied. The results presented in this section assume random ascending nodes and phases for the satellites.

#### 4.2.1 *Description of Service for Various Qualities of Service*

Fig. 11 displays the periods of no service on a path from Maine to London for a system of 12 satellites in 6000-mile random polar orbits (a quality of service of 0.95). This figure covers a representative 30-day period from a one-year simulation. The darkened areas, which are drawn to scale, represent the times when the path is out of service because no satellites are visible. The average length of the periods of no service ($t_o$) and the average time between these periods ($t_i$) are given for the one-year period. Fig. 12 presents the periods of no service for the Maine–London path if the 6000-mile polar orbit system has 18 satellites ($q = 0.99$), and Fig. 13 gives the same information for a system of 27 satellites ($q = 0.999$).

The communication system will probably need to be operated in such a way that periods of no service do not interrupt communication which is in progress. This would be accomplished by keeping new traffic off

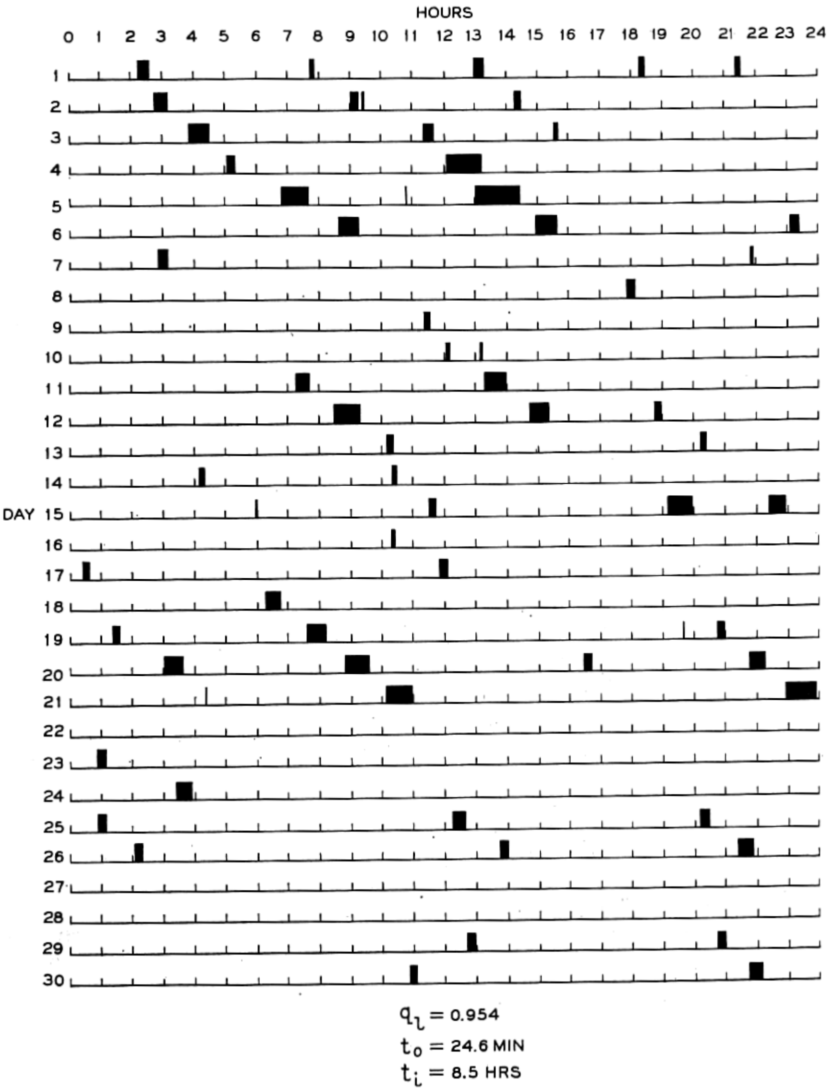



Fig. 11 — Maine–London service from 12 satellites in 6000-mile polar orbits.

the system during a "guard interval" which precedes the no-service period. The guard interval would be long enough to insure, with a reasonable probability, that all customers would be finished by the time the no-service period starts.

With this method of operation, the no-service periods can be interpreted simply as delays in service to new customers. The periods are predictable weeks in advance, and they are short; therefore, they are not the same as catastrophic system failures such as cable breaks and repeater failures.* As far as ordinary toll customers are concerned, a short no-service period is the same as a temporary saturation of the system by heavy traffic.† In view of the above comments, the no-service periods, which are popularly called "outages," might more appropriately be called "service delays."

The daily period of heavy traffic between most countries is only several hours long. The service delays which would occur during the busy period would probably be of primary importance, and those outside this period would be secondary. As an example assume the busy period is four hours long. Then, for the 0.99 service shown in Fig. 12, there would be no days in which more than one service delay would occur during the busy period. The absolute time in the figure is arbitrary, so the busy period could occur at any time. If all possible locations of the busy period are considered, the 30-day period shown could have as many as eight or as few as zero service delays during busy periods. The maximum number for the 0.95 service shown in Fig. 11 would be between six and twenty for that 30-day period, and the 0.999 service of Fig. 13 would have between zero and two delays during busy periods. In comparing these three services, the numbers of satellites needed in each case should also be considered since the cost of satellites in orbit will be a large part of the total system cost.

#### 4.2.2 *The Effect of Orbit Altitude on the Service*

Fig. 14 is a representative display of the delays which would occur on the Maine–London path if a system of 18 satellites in 3000-mile random polar orbits were used. The quality of service is 0.95, which is the same quality as that of Fig. 11. In the 3000-mile system the service delays would be more frequent but of shorter duration. These differences are caused by the shorter orbital period and shorter visibility times of the 3000-mile satellites.

If the operational method discussed previously is used, a fixed guard interval would be added to each of the delays in the figures. The length of the guard interval is determined by lengths of telephone calls, so it would be the same for all cases. It is apparent that the effect of adding

* There will also be repeater failures in satellites, but the effect of these failures is only to reduce the quality of service. For example, if a system started with 27 satellites and 0.999 service (Fig. 13), failure of one-third of the satellites would only reduce the service to 0.99 (Fig. 12).

† The effect may be more severe for private line customers who require a circuit continuously.

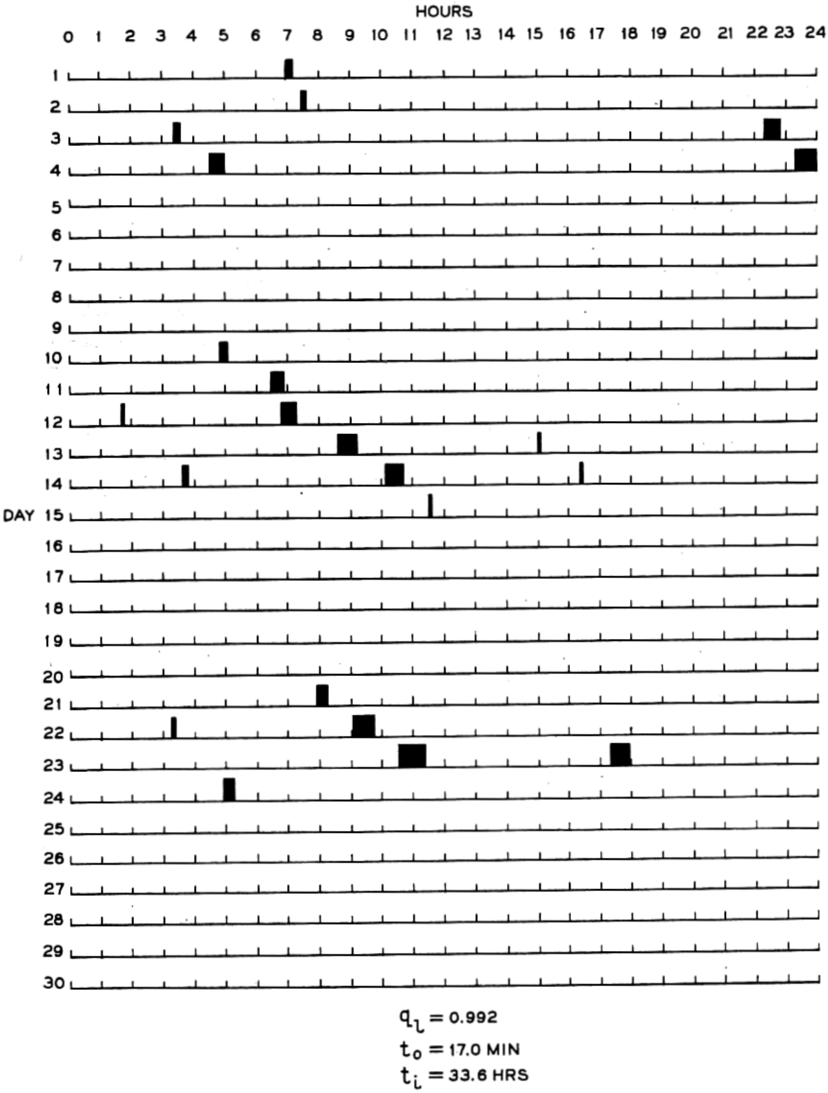



Fig. 12 — Maine–London service from 18 satellites in 6000-mile polar orbits.

the guard interval to each of the delays in Fig. 14 would be more severe than if it were added to the delays of Fig. 11.

Although it may appear that for a given quality of service a high orbit would always give better service than a lower orbit, this is not strictly true. As the satellite periods and visibility times become longer,

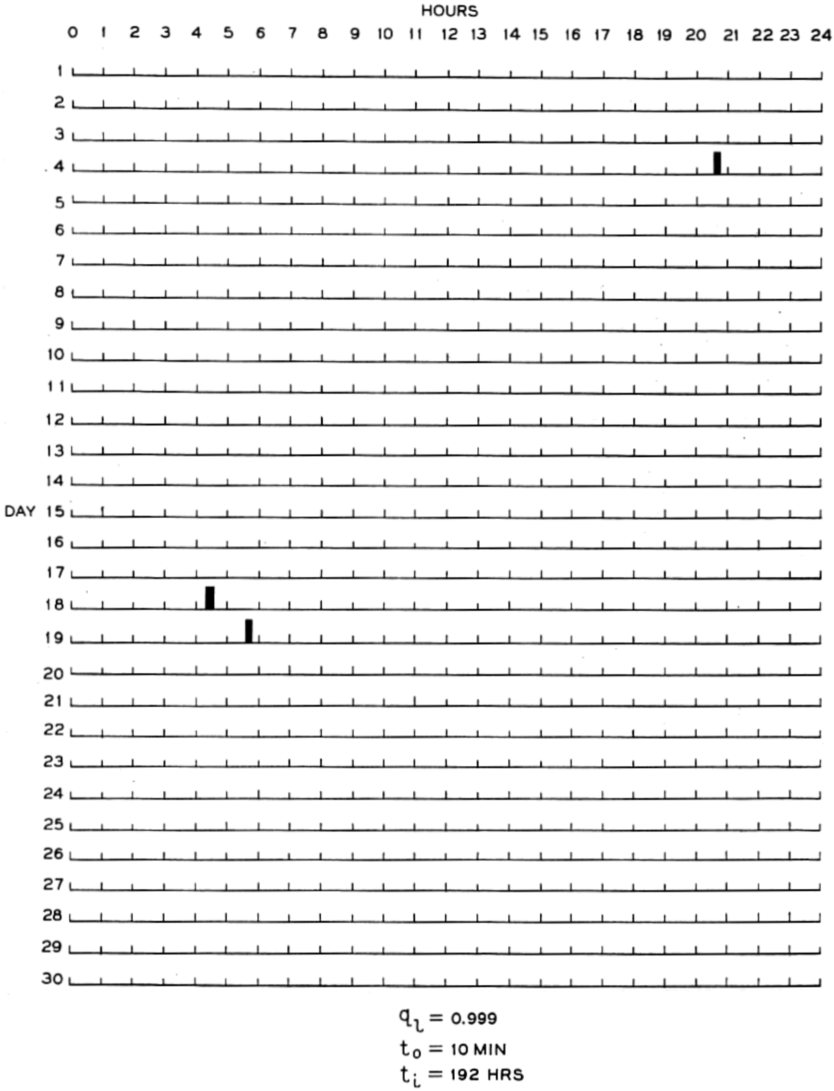



Fig. 13 — Maine–London service from 27 satellites in 6000-mile polar orbits.

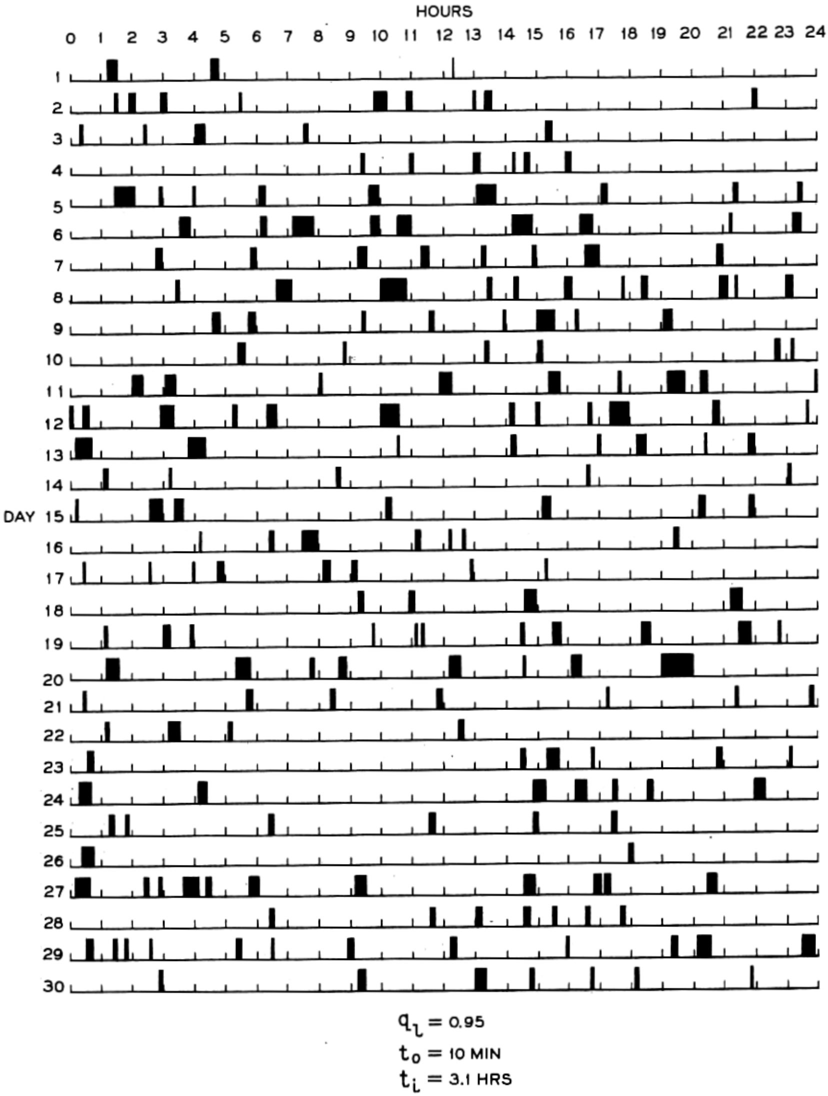



Fig. 14 — Maine–London service from 18 satellites in 3000-mile polar orbits.

the delays occur less frequently, but they also become longer. Eventually a point would be reached where the delays in service would be excessive and would approach the severity of catastrophic failures.

### 4.3 *Analytical Description of Periods of Service and No-Service*

A number of persons have made analytical studies of the lengths of service delays (outages) and the time between these delays. Some of these expressions are included here for completeness. Additional results are given in a companion paper by S. O. Rice[5] who makes use of a traffic model. The results predicted by the model compare favorably with simulated results described in the previous section.

The average length of the service delays is given by

$$t_o = \frac{(1 - p)T}{n} \tag{7}$$

$n$ and $p$ have been defined previously, and $T$ is the average time between successive appearances of a particular satellite. If the visibility region were circular and centered on the north pole, $T$ for polar orbits would be equal to the orbital period $P$, which is given in Fig. 2. For polar orbits, $P$ is a good approximation for $T$ for regions which include one of the earth's poles such as the Maine–London region in Fig. 5. For other regions, such as Seattle–Hawaii, only a fraction of all possible orientations of the orbit plane with the earth result in visibility, so there will be some times longer than $P$ between successive appearances. An expression for $T$ for these cases has not been determined. For equatorial orbits, $T$ would equal $P$ if the earth were not rotating. For the kind of equatorial orbits assumed here, the satellite moves in the direction of the earth's rotation, so the time between successive appearances above a point on the earth is longer than $P$. In this case, the expression for $T$ is

$$T = \frac{P}{1 - \dfrac{P}{24}} \, . \tag{8}$$

The average time between service delays is given by

$$t_i = \frac{q_1}{1 - q_1} t_o \, . \tag{9}$$

Some useful approximations for the distribution of the lengths of delays and times between delays have also been given by Ref. 5. The fraction of delays which would be longer than $t$ is approximately $\exp(-t/t_o)$, and the fraction of delays which are separated by a time longer than $t$ is approximately $\exp(-t/t_i)$.

Equations (7) and (9) and the exponential approximations for the distributions have been compared with many simulated results, and the agreement testifies favorably for the usefulness of these expressions in describing random-orbit service analytically.

## V. AN EXAMPLE OF A WORLD-WIDE SYSTEM

The material given in the previous sections discusses techniques which can be used to describe the service provided by a set of satellites. The application of these techniques is illustrated in this section by describing the world-wide service which would be provided by a particular system of satellites in 6000-mile orbits. This example system provides essentially uniform service throughout the world with a modest number of satellites.

### 5.1 *Description of the Orbits*

This system would consist of 24 satellites in 6000-mile circular orbits. It is assumed that four satellites would be placed into orbit with a single rocket, so each plane would contain four satellites. Three of these planes would be polar, and three would have an inclination of 28°.* The three planes in each group would be uniformly spaced (their ascending nodes would be 120° apart) as shown in Fig. 15. The lines on the figure represent the intersections of the orbit planes with the surface of the earth at an arbitrary time. A discussion of the motion of the orbit planes and its effect on service is given in Section VI. A possible scheme for effecting the separation of the multiply-launched satellites is also discussed in Section VI. For the present, it is assumed that the 24 satellites are randomly phased.

### 5.2 *Service from the United States*

Fig. 16 shows the qualities of service which would be provided by the system of 24 satellites if stations were placed in Maine, Seattle, Puerto Rico, and Guam. A point within the 0.99 contour would be able

* Equatorial orbits would be more useful in this system than 28° orbits; however, a rocket launched from Cape Canaveral must use difficult, power-consuming maneuvers to achieve an equatorial orbit. It is shown in Ref. 6 that the Atlas Agena can place about 1300 pounds into a 6000-mile, 28° orbit but can place only about 200 pounds into an equatorial orbit of the same altitude. About 1000 pounds can be placed into a 6000-mile polar orbit from the Pacific Missile Range.

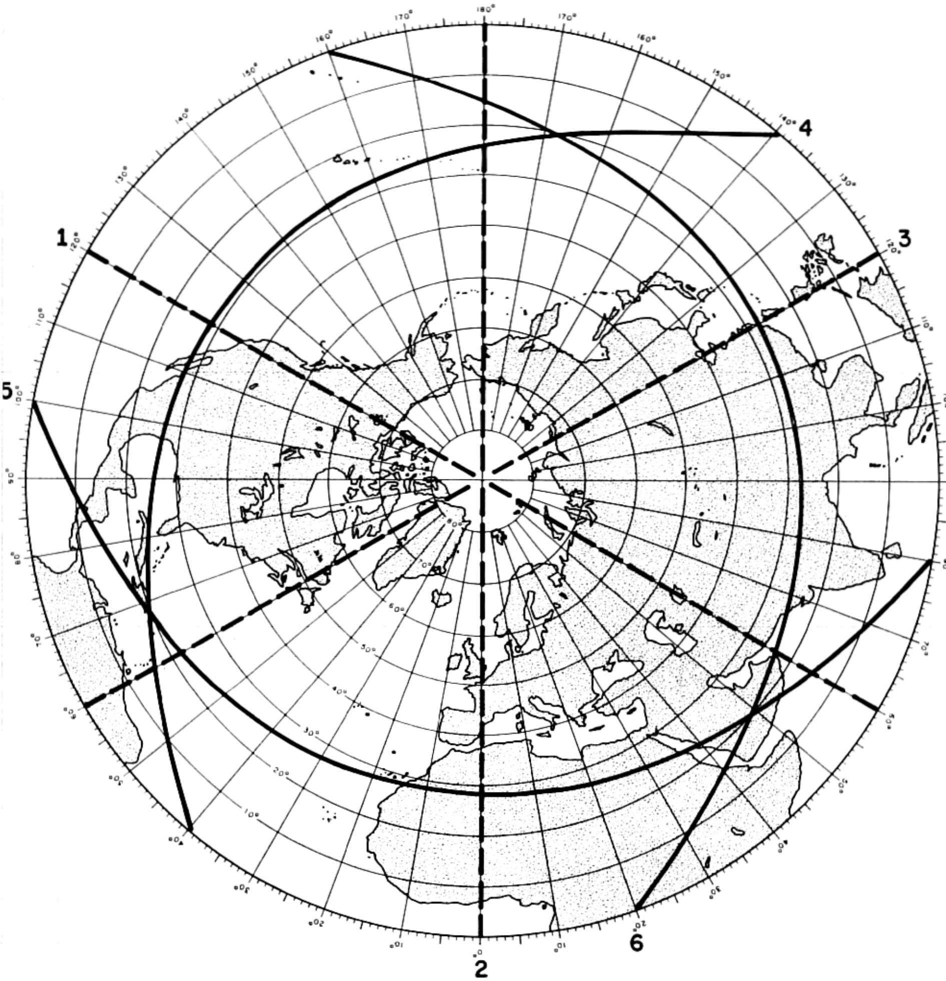
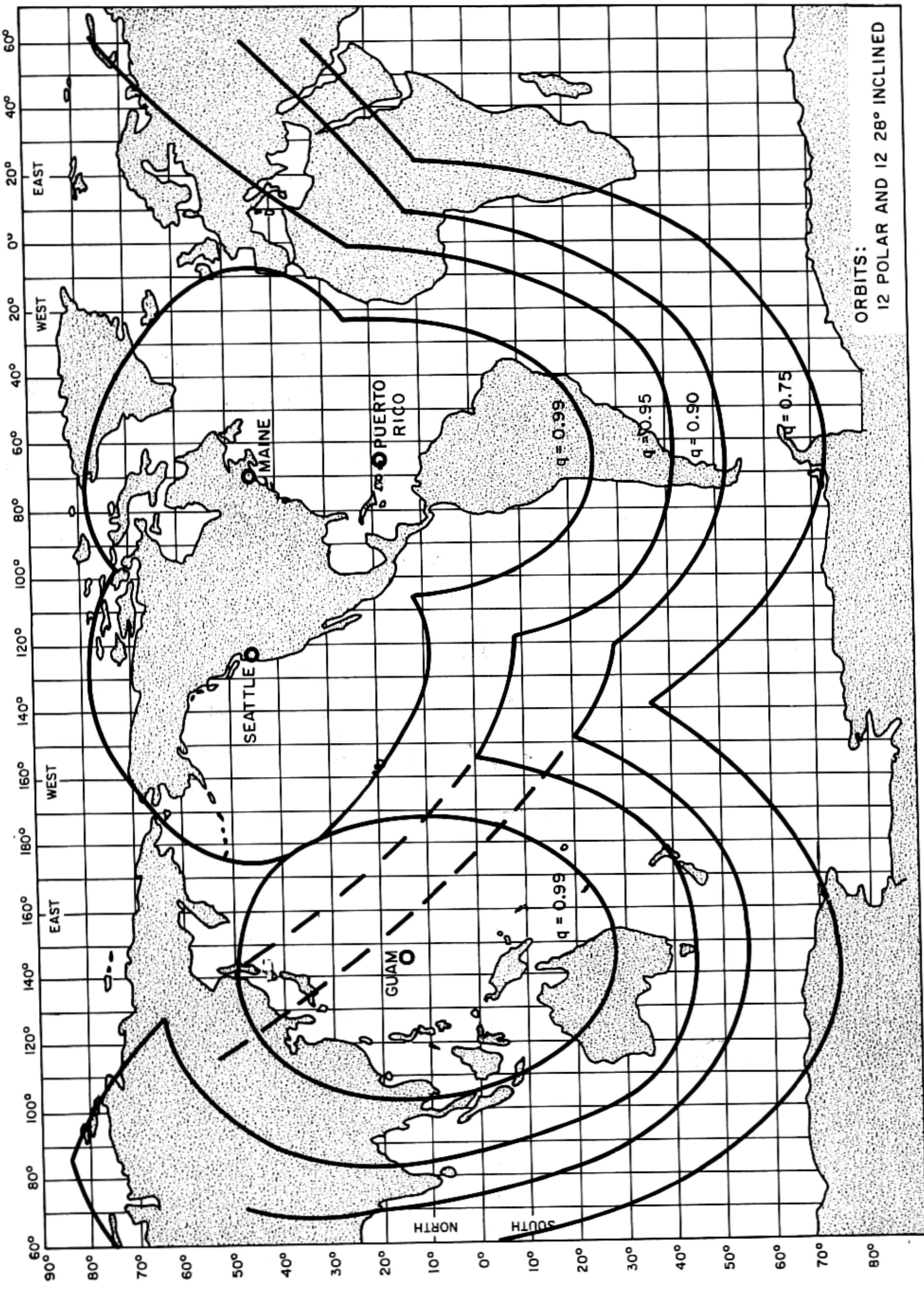



Fig. 15 — Orientation of the orbit planes of the 24-satellite system. The dotted lines are the intersections of the polar planes with the surface of the earth, and the solid lines are the intersections of the 28° planes with the earth's surface. The planes are numbered at the ascending nodes of the orbits.

to communicate with one of these stations more than 0.99 of the time. A point north of the 0.95 contour would have service with one of the stations more than 0.95 of the time, etc. These contours were determined by using (6) in Section 4.1.2 for values of $p$ from the table in the Appendix. The location of the ground stations in the figure were chosen




[illegible] satellites in 6000-mile orbits (12 polar and 12 28°-inclined).

in an attempt to reach as much of the world as possible from points in the United States and its outlying territories.

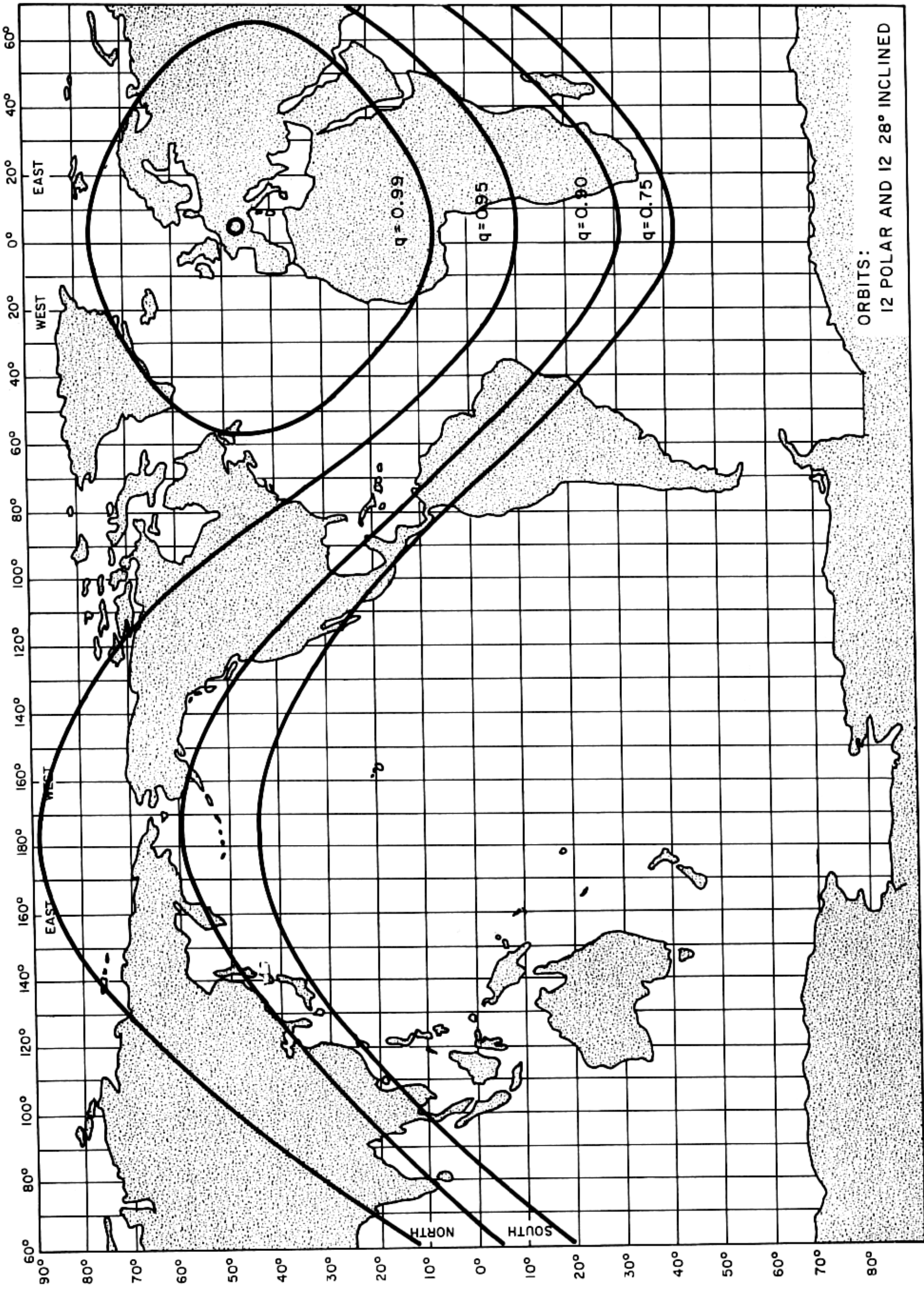
In order to be useful, the stations in Puerto Rico and Guam would have to be connected to circuits in the United States. For Puerto Rico, this connection could be made with Maine by way of a satellite or with Florida by submarine cable. The length of the path through a single satellite link, i.e., the distance from one ground station through the satellite to another ground station, can be as much as 17,000 miles for certain positions of a satellite at a 6000-mile altitude. Thus a circuit from South America to Maine consisting of two satellite links in tandem could be as long as 34,000 miles. It is well known that the performance of telephone circuits is adversely affected by the time delay associated with long path lengths, and ideally the path should be as short as possible. Thus for telephone communication with South America and Africa by way of Puerto Rico, the cable connection from Puerto Rico to Florida would probably be more desirable than a satellite connection with Maine; however, both alternatives may be desirable to provide diversity.

Guam could be connected to the United States by using a satellite link from Guam to Hawaii and another satellite link from Hawaii to Seattle. Circuits from Asia to the United States would then consist of three satellite links in tandem which could be as much as 50,000 miles long. A submarine cable which will connect Guam to the United States is presently planned for service in 1964. This cable would probably be a more attractive way of relaying telephone circuits to the United States than would the two satellite links, but again it may be desirable to have both means available.

The plan of Fig. 16 resulted from an attempt to reach as much of the world as possible through facilities under control of the United States. Most of the world could be reached with reasonably good qualities of service, but the service to Africa is probably not adequate. A plan which results in better service to this area will be given in Section 5.4.

### 5.3 *Service from Europe*

Fig. 17 shows contours of constant $q_1$ from an arbitrary point in western Europe. Much of the Pacific area cannot be reached from Europe by a single satellite link because the two regions are on opposite sides of the world. Africa could be served from Europe about as well as South America could be served from the U. S., and Europe–South American service would be limited to about the same extent as U. S.–African serv-


ORBITS:
12 POLAR AND 12 28° INCLINED
q=0.99
q=0.95
q=0.90
q=0.75
EAST
WEST
NORTH
SOUTH

ice. Service to all of these areas could, of course, be improved by establishing additional ground stations at appropriate points and using multiple satellite links in tandem.

Other contours could be drawn with respect to other parts of the world; however, the contours in Figs. 16 and 17 are probably adequate to describe world-wide service. These contours can be moved in longitude or they can be inverted and centered at the corresponding latitude in the southern hemisphere. The next section presents a plan in which consolidation of traffic could be used to provide more efficient communication between all parts of the world.

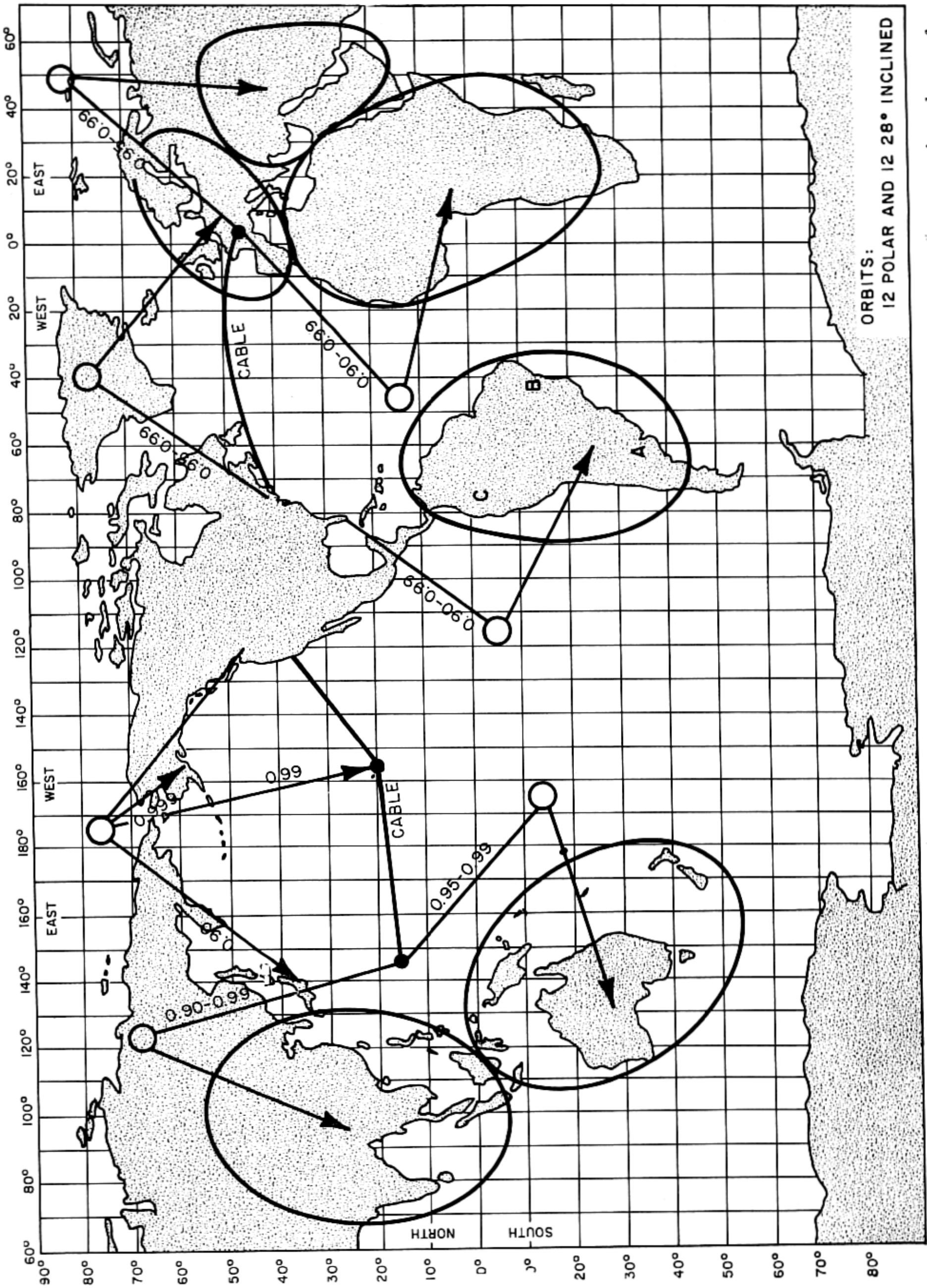
### 5.4 *A Possible Method of Operation for World-Wide Service*

North America has an extensive land-line network which would enable all points to communicate with overseas points through one of the assumed ground stations in Fig. 16. Europe also has an extensive network, but land lines between countries on other continents are not as well developed. Each of these countries would need a direct connection, by way of a satellite, with a network through which other countries of interest could be reached.

A possible scheme for world-wide communication is shown in Fig. 18. In this plan, countries within the encircled regions would communicate with stations in western Europe, Guam, Seattle, Maine, or Miami.* Since these stations would be interconnected by submarine cables and land lines, any of the encircled regions could be reached from any of these stations by using cable facilities and one satellite link, e.g., U. S.–African circuits could go by cable to Europe and then by satellite to Africa. Any two encircled regions could communicate by using two satellite links which would be connected by cable facilities, e.g., circuits from Asia to South America could go by satellite to Guam, then by cable and land lines to Miami, and then by satellite to South America.

The ranges in quality of service shown for each path in the figure assume that all countries within the encircled region could use the same satellite. The South American service will be discussed to point out some implications of this kind of operation. In Fig. 18, stations A, B, and C in South America would always use the satellite which is mutually visible to Miami and station A. If no satellite satisfied this condition, station A would be out of service, and a satellite which was

* This plan does not require direct communication by satellite between the U. S. and Africa, and the service to South America from a station in Puerto Rico would not be greatly different than the service from Miami would be. For this reason, Miami was chosen as the U. S. station for this part of the world.




...ldwide network which uses satellites and submarine cables. The ranges of qualities of service shown for mutually visible to Miami and B would be used, etc. Thus stations close to Miami would have high qualities of service and those further from Miami would have lower qualities of service. As shown on the figure, the range of $q$ from Miami to countries in South America is 0.90–0.99.

Communication between countries in South America could also be accomplished by way of the satellite which is used to reach Miami. The quality of service between two countries would be approximately the product of the qualities which the individual countries would have with Miami. The service between these countries could be improved if they also used mutually visible satellites which were not visible to Miami, but this would require each country to have another ground antenna.

According to Fig. 18, South America and Africa would communicate over a long network of land lines, cables, and two satellite links, but it is apparent that direct communication by way of one satellite link between these regions would be possible. However, this direct communication would require all South American countries and all African countries to have another ground antenna specifically for this purpose. This would not be true if each of these areas had a land-line network which interconnected all countries. Direct communication between Europe and South America could also be achieved if each South American country had still another ground antenna. The discussion of South American service can be extended to the other regions which are encircled in Fig. 18.

The method of operation illustrated in Fig. 18 minimizes the number of ground antennas needed by individual countries by consolidating traffic through central points which are interconnected by terrestrial facilities. With this method, routes which are frequently used could be served by single-satellite links, and other routes which are used less frequently could be served by two satellite links in tandem.

### 5.5 *Alternate Routing by Way of Two Satellite Links*

During the times when no satellites are mutually visible to a pair of stations, each station will often be able to see a satellite which is out of view of the other station. If there were a relay point which could see both of these satellites, a path with two satellite links could be established by way of the relay station. For example, Anchorage, Alaska might be a useful relay point for Maine and Europe. Figs. 16 and 17 give qualities of service of about 0.95 and 0.90 for Maine–Anchorage and Anchorage–Europe respectively. Another location which is well-situated for relays between Maine and Europe is the Cape Verde Islands off the west coast of Africa. This relay point could also provide an alternate route for Miami–South America and Europe–Africa.

The 24-satellite system was simulated for a four-month period in order to obtain an appreciation for the usefulness of the alternate-routing concept. In this simulation the Maine–Europe path had 98 periods during which a satellite was not mutually visible. During 53 of these periods, satellites were mutually visible between Maine and Anchorage and between Anchorage and Europe, so an alternate route could have been established. A relay in the Cape Verde Islands would have been useful during 31 of the remaining 45 periods. Thus about 86 per cent of the service delays during this simulated period could have been avoided by using these two relay points. Of course some penalty would remain, during these periods, in the form of degraded communication performance, but it is possible that this degradation could be tolerated for the short and infrequent intervals which would be involved.

### 5.6 *System Growth and Service Improvement*

It is expected that over a period of years an increase in demand and a desire to improve qualities of service would require the expansion of an initial system. The orbits in which satellites are added to the system would depend on the global distribution of the increase in demand for service. It is likely that polar satellites would be added at a faster rate than would 28° satellites, and if this were the case, the shapes of the contour lines would change from those given in Figs. 16 and 17. The service improvement offered by increases in the number of satellites will be illustrated by a simple example in which the satellites in each orbit are increased by equal numbers.

As noted in Section 5.2, the contours of Figs. 16 and 17 represent qualities of service for the 24-satellite system determined by the equation

$$1 - q_1 = (1 - p_1)^{12} (1 - p_2)^{12}.$$

If the number of satellites in each of the two orbit inclinations is changed to $n$, it follows that

$$(1 - q_1') = (1 - q_1)^{n/12},$$

where $q_1'$ is the new quality of service for a contour line. Therefore, these contours remain valid, with the appropriate changes in $q_1$, for different numbers of satellites in this type of system. The following table presents the values of $q_1$ for each of the contour lines for several $n$.

| $n$ | = | *12* | *16* | *20* | *24* |
|---|---|---|---|---|---|
| Inner contour | = | 0.99 | 0.998 | 0.9995 | 0.9999 |
| Second contour | = | 0.95 | 0.982 | 0.9934 | 0.9975 |
| Third contour | = | 0.90 | 0.954 | 0.978 | 0.99 |
| Outer contour | = | 0.75 | 0.842 | 0.901 | 0.937 |

An increase in $q_1$ which results from an expanded system may be considered solely as an improvement in service quality. However, this increase also affects the number of paths which can be made available between two points. For example, from the above table, for $n = 20$, $q_1 = 0.9995$ for the inner contour. Inspection of Fig. 9 shows that 2 paths could be established with ordered priority with $q_2 = 0.995$; similarly, for three paths, $q_3 = 0.978$ etc. From Fig. 10 similar information may be obtained for paths with equal priority. For the above example $\bar{q}_2 = 0.9973$, $\bar{q}_3 = 0.9915$, etc.

The communication scheme of Fig. 18 is also valid for an expanded system of an equal number of satellites in 28°-inclined and polar orbits. The qualities of service indicated would be changed by replacing them by their corresponding values in the above table.

## VI. SOME NONRANDOM CHARACTERISTICS AND THEIR EFFECT ON SERVICE

The system described in the previous section would be nonrandom in two respects. In the first place, the four satellites in each plane are not controlled in phase, so these satellites could bunch together. If this should occur, the four satellites would be of little more use than a single satellite. Secondly, the orbit planes will rotate slowly about the earth's axis at different rates, so the three orbit planes in each inclination could coincide after a long period of time. Certain parts of the world could then see the satellites only during parts of the day. This section discusses these two nonrandom characteristics and presents simulation results which show their effect on service.

### 6.1 *Relative Satellite Velocities and Phasing of Successive Launches*

If several satellites were placed into the same orbit plane, the most uniform coverage would result if the satellites were uniformly spaced. However, even if uniform spacing could be achieved initially, it is not possible to give all satellites exactly the same velocity. Thus the satellites would drift from the desired relative positions, and to preserve the uniform spacing, a propulsion system would be needed on each of the satellites. This would probably represent a severe requirement on a system which must have an appreciable lifetime.

Fig. 19 illustrates one method of separating four satellites in such a

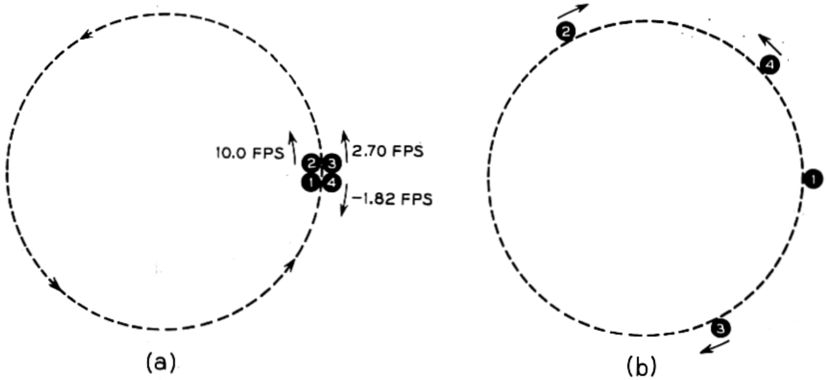



Fig. 19 — Geometry of satellite separation: (a) Incremental velocities relative to satellite #1. (b) Relative positions after 3 months in orbit.

way that the effect of future bunching of the satellites would be small. In Fig. 19(a) the satellites initially would be moving counter-clockwise with the same velocities (they would be placed into orbit as one package). Then the satellites would be separated by springs in order to give three of them the indicated velocities relative to satellite number 1. Figure 19(b) shows the relative directions of motion and the relative positions after three months in orbit. A satellite which is given an increase in velocity goes into a slightly higher elliptical orbit with a slightly longer period, and its angular velocity therefore decreases. This is the reason satellites 2 and 3 fall behind satellite 1 even though they have higher velocities. The reverse is true of satellite 4.

The relative velocities were determined by a computer simulation. Many combinations of velocities (all small enough to be within the capability of springs and large enough to effect early separation) were tried, and the relative positions of the satellites were followed for a period of ten years. The set shown in Fig. 19 is the one which resulted in the most uniform coverage over the study period; however, errors of $\pm 3$ per cent could be tolerated in each of these velocities without significantly changing the coverage.

The three planes of satellites in each group would no doubt be launched a month or so apart because of preparation time and availability of launch facilities. There is also a coverage advantage to be gained from phasing the launchings. Some pairs of stations could see all polar orbits at all times, and other pairs could see all 28° orbits at all times. If the satellites in each of the planes were given the set of velocities shown in Fig. 19 at different times (different launch dates), the satellites in one plane could be made to be well-spaced at times when the satellites in another plane were bunched.

The desired intervals between launchings were also determined by computer simulation. The first set of satellites was allowed to disperse for one month, and then many combinations of second and third launching times over a six-month period were tried. For each combination, the coverage was examined for a ten-year period. The best combination which was found specified launching the second plane 1.5 months after the first, and the third plane 4.5 months after the first. If either attempt were to fail, there would be other times several weeks later which would serve equally well. The two groups of planes would be handled separately from different launching points, and no phasing between groups would be attempted.

The above procedures are not necessarily optimum, but they do represent a way of achieving orbits which do not result in severe bunching of the satellites within the planes.

### 6.2 *Motion of the Orbit Planes*

The plane of the orbit of an earth satellite will not in general maintain a fixed orientation in inertial space. Instead, there will be a slow rotation of the plane about the earth's axis. This rotation is caused by the non-spherical shape of the earth, and for circular orbits the rate of rotation, in radians per orbit, is given by[7]

$$\Delta\Omega = \frac{2\pi J R^2 \cos i}{(R + H)^2} \tag{4}$$

where

$R$ = radius of the earth

$H$ = altitude of the satellite above the earth

$J$ = $1.625 \times 10^{-3}$, the first oblateness coefficient[8]

$i$ = inclination of the orbit plane.

The direction of the rotation of the plane is opposite to that of the earth's rotation for inclinations less than 90°, and is in the same direction as the earth's rotation for inclinations greater than 90°. The rotation vanishes for true polar orbits ($\cos i = 0$), and a 6000-mile, 28° orbit rotates at a rate of about 100° per year.

In the system shown in Fig. 15, the three orbit planes in each group would have no relative motion if each plane has the same inclination.

Small errors in inclination can be expected, however, and the resultant relative motions could cause the planes to bunch together after a number of years. The magnitudes and directions of the inclination errors which are made will determine the extent of the bunching and the time of its occurrence.

If it is assumed that the accuracy of the inclination angle will be within 5°, which is probably a pessimistic assumption, the most severe bunching of the near-polar planes would result for inclinations of 85°, 90°, and 95° respectively for planes 1, 2, and 3 in Fig. 15. With these inclinations, plane 2 would not rotate, plane 1 would rotate clockwise at a rate of about 10° per year, and plane 3 would rotate counter-clockwise, also at a rate of 10° per year. Thus the three planes would be nearly coincident* after about 6 years.

The worst combination of inclination angles for planes 4, 5, and 6 would be 33°, 28°, and 23° respectively. All of these orbits would rotate in a clockwise direction, but plane 4 would rotate about 5° per year less than plane 5, and plane 6 would rotate about 5° per year more than plane 5. The planes would be closest to coincidence when their ascending nodes were at the same position. Since the ascending nodes would be 120° apart initially, nearest-coincidence of the three planes would occur after 24 years.

The bunched system which will be discussed here is shown in Fig. 20. The orbits have the inclinations discussed in the preceding paragraphs, and the figure represents the orientation of the planes six years after the satellites are placed into orbit. Note that the assumed inclinations represent a rather pessimistic example since 5° errors would need to be made in exactly the right combination. The maximum error may also be pessimistic, and a maximum error of one-half this value would delay this bunching until twelve years after the system was started.

Even if the original system bunched together in this way after six years, it is quite likely the effect would not be severe if satellite failures and system growth are considered. The logical way to replace satellites which fail would be to establish a new plane with four satellites in it rather than try to replace individuals in the original planes.† The new plane could be placed in a gap which was caused by bunching. It is quite likely that more than 24 satellites would be needed by this time to improve the quality of service and to establish multiple paths between some stations. These additional satellites would provide another way to fill gaps caused by bunching of planes. If one considers such

* The planes would not be exactly coincident because they would have different inclinations.

† Assuming one rocket will place four satellites into orbit.

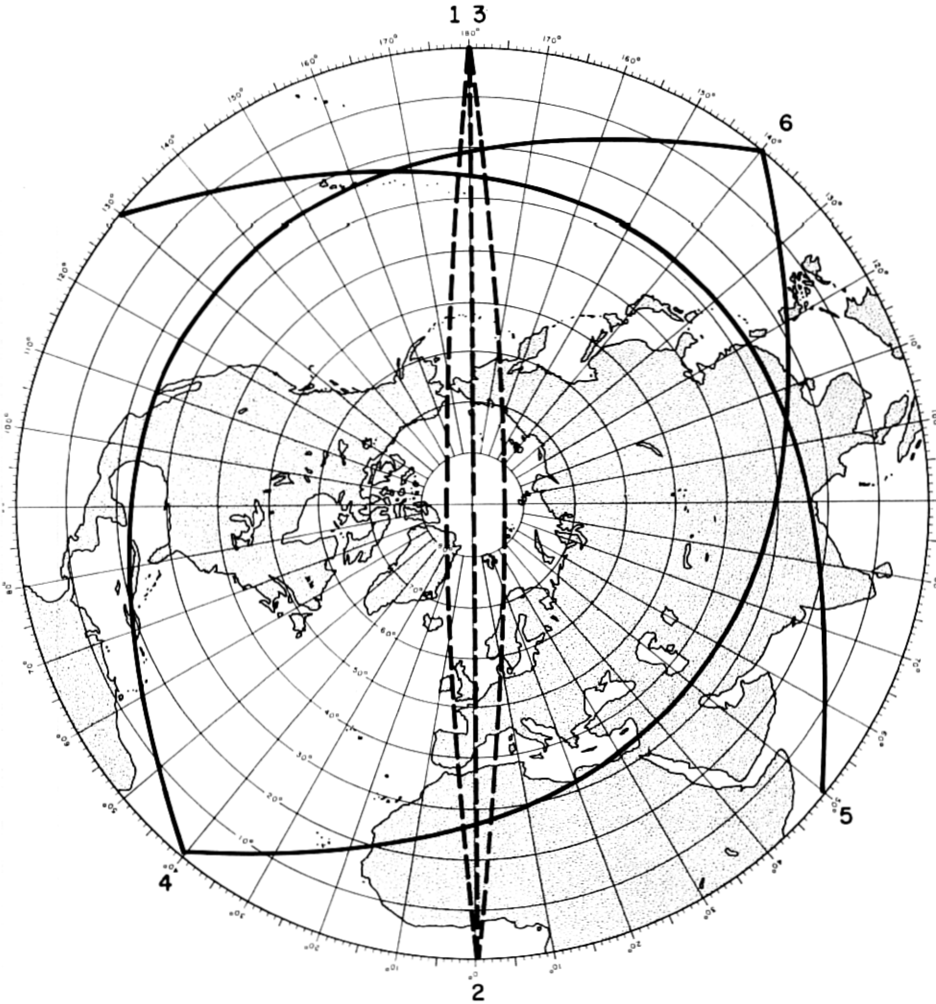



Fig. 20 — Orientation of the orbit planes of the 24-satellite system for a severe case of plane bunching. This bunching would occur after six years if a certain combination of 5° errors were made in the inclinations of four of the planes.

things as satellites failing at random, new satellites being added with random errors in inclination, and motions of orbit planes in different directions and at different rates, it is not difficult to postulate the evolution of a random system after a number of years.

6.3 *A Study of the Effect of Plane Bunching*

This section presents simulation results for three different systems of satellites. The first of these is the uniformly spaced system represented by Fig. 15, the second is the bunched system of Fig. 20, and the third is this same bunched system with four satellites added in another 28° plane (the ascending node of the plane is 180° from that of plane 5). The services each of these systems would furnish to Maine–London, Seattle–Hawaii, and London–Johannesburg were simulated for six-month periods.

In the uniform system, the phases of the satellites were specified by using the relative velocities and phasings of successive launches discussed previously. The bunched system was described by advancing these phases through six years and arranging the ascending nodes and inclinations to correspond to those of Fig. 20.

#### 6.3.1 *Maine–London Service*

Fig. 21 shows ten-day portions of the simulated services of the three systems for the Maine–London path. Fig. 21(a) is a representative ten-day period for the uniform system. The statistics $q_1$, $t_o$, and $t_i$ (see Section IV) are given for the ten days and for the six months to provide a means of comparing the services quantitatively. It can be seen from these statistics that the ten-day period shown for the uniform system has longer periods of no service and longer times between these periods than the averages for the six months.

Fig. 21(b) shows the poorest service which resulted for the bunched system. The poorest ten days were chosen so that the effect of bunching of satellites within the planes would also be included. Although this group of days cannot be compared directly with the first group, the six-month statistics can be compared. For the six-month periods, $q_1$ is 0.984 for both systems, and the periods of no service are longer but less frequent for the bunched system than for the uniform system. The ten-day statistics for the two systems differ by similar amounts from the six-month statistics of the uniform system. From these comparisons, it is difficult to say that one system is better than the other for this path.

It is not surprising that this path would be only slightly affected by the bunching because most of the service is furnished by the near-polar satellites and Maine and London always have mutual visibility of these three orbits as shown in Fig. 5. Because the earth rotates under the orbits, there would be two times (12 hours apart) each day when the portions of these orbits which could be seen by Maine and London would be a minimum. The periods of no service would most likely occur during these times, and a 12-hour pattern in these periods would result. This pattern is evident in Fig. 21(b).

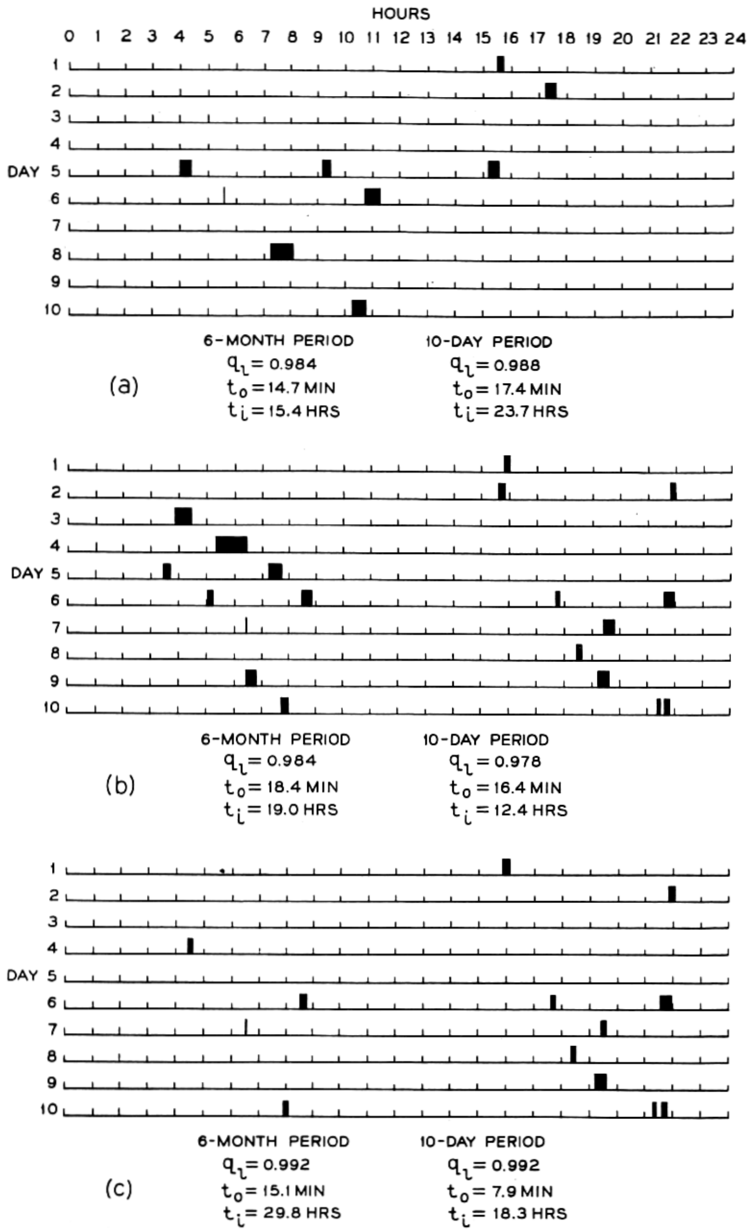



Fig. 21 — Maine–London service: (a) Representative service furnished by the uniformly spaced system shown in Fig. 15. (b) Service during the poorest 10-day period from the 6-month simulation of the bunched system shown in Fig. 20. (c) Service during the period shown in Fig. 21(b) if four satellites in a 28° plane were added to the bunched system.

It will be shown later that the other two paths in the study would be more seriously affected by the bunching, and the third system with 28 satellites will be studied as a means of relieving the effect of the bunching. This system was also simulated for the Maine–London path for completeness, and the results are given in Fig. 21(c). The same ten-day period is shown for Figs. 21(b) and 21(c), so a direct comparison can be made. It is clear from this comparison that the four additional satellites more than compensate for the possible, small effect of bunching on this path.

### 6.3.2 *Seattle–Hawaii Service*

Fig. 22 gives similar ten-day displays for the Seattle–Hawaii path. Comparison of the six-month statistics for the first two systems shows the service to be affected considerably by the plane bunching. In Figure 22(b), the periods of no service tend to occur during the same part of each day, and this again is caused by the rotation of the earth under the orbits. Consideration of Fig. 20 and the visibility region of Fig. 5 shows there would be a part of each day for which only orbit plane 6 would be over the visibility region and it is during this time that the no-service periods would most likely occur. Twelve hours later orbits 4 and 5 are over the visibility region but orbits 1, 2, and 3 are not; since the service at this time is still rather good, it is apparent that the effect of the bunching of the polar orbits is not large.

These characteristics suggest the addition of another 28° plane with an ascending node 180° from that of plane 5, and the service which would result from this system of 28 satellites is shown in Fig. 22(c). In this figure the no-service periods are considerably reduced from those of Fig. 22(b), and they occur during the two parts of the day when the polar orbits are not visible. Comparison of the six-month statistics for the 28-satellite system with those of the uniform 24-satellite system suggests that both systems would provide about the same service to the Seattle–Hawaii path. It might be said, then, that the four added satellites represent the penalty of the bunching for this path.

### 6.3.3 *London–Johannesburg Service*

The service to the London–Johannesburg path is represented in Fig. 23. Comparison of the first two groups shows that a severe pattern in the periods of no service is caused by the bunching of the planes, and comparison of the six-month statistics shows a definite degradation in the service to this path. The nature of the reduction in service is the

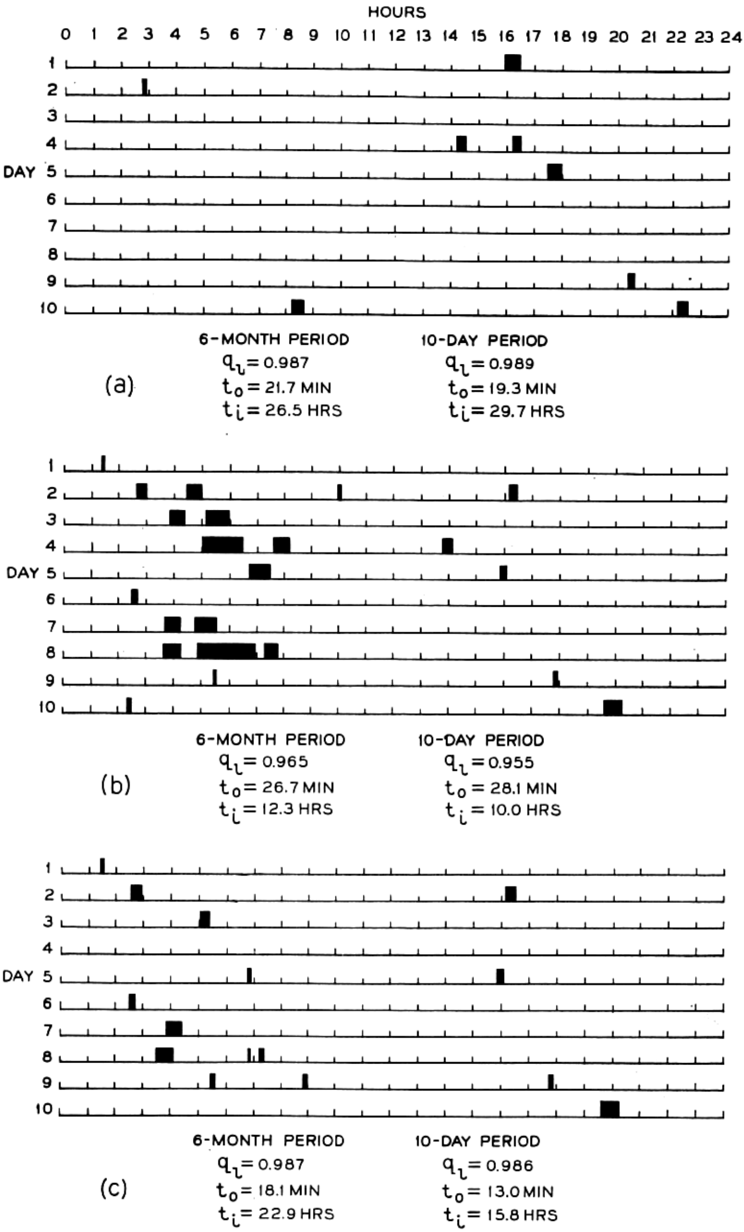



Fig. 22 — Seattle–Hawaii service: (a) Representative service furnished by the uniformly spaced system shown in Fig. 15. (b) Service during the poorest 10-day period from the 6-month simulation of the bunched system shown in Fig. 20. (c) Service during the period shown in Fig. 22(b) if four satellites in a 28° plane were added to the bunched system.

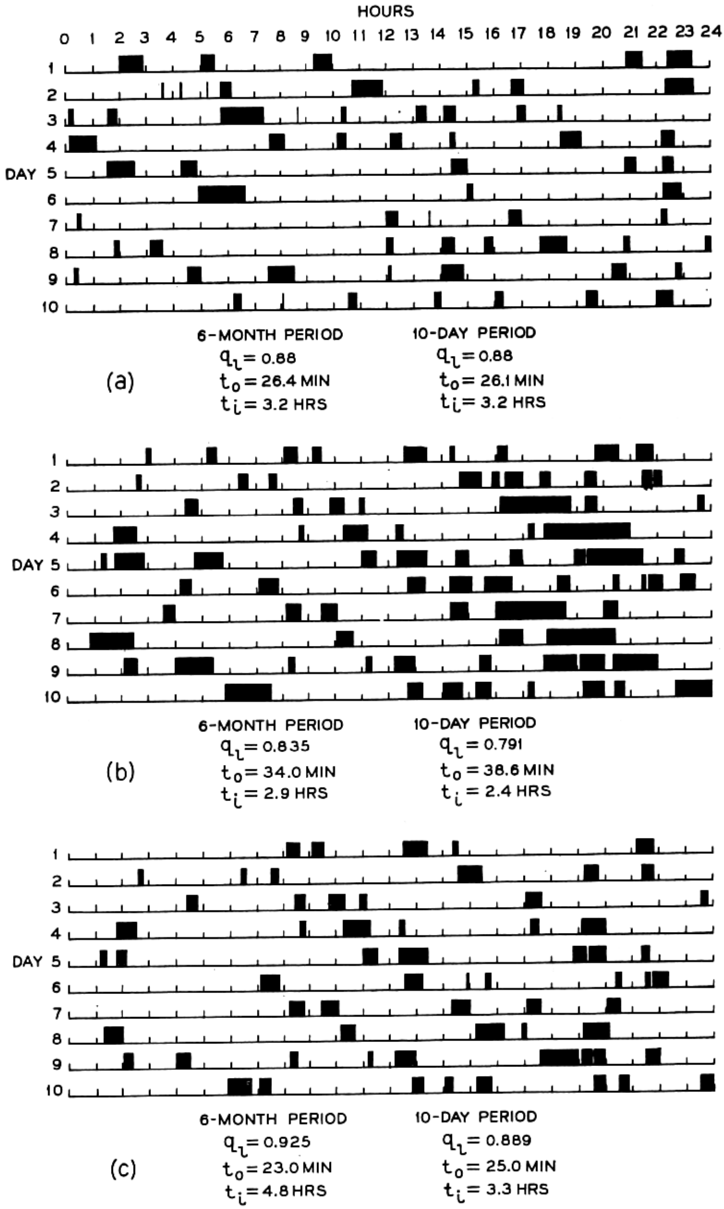



Fig. 23 — London–Johannesburg service: (a) Representative service furnished by the uniformly spaced system shown in Fig. 15. (b) Service during the poorest 10-day period from the 6-month simulation of the bunched system shown in Fig. 20. (c) Service during the period shown in Fig. 23(b) if four satellites in a 28° plane were added to the bunched system.

same as it was for the Seattle–Hawaii path, and Fig. 23(c) shows that the four additional satellites would be more than adequate to overcome the effect. This statement is best supported by comparing the six-month statistics for the three systems.

#### 6.3.4 *Summary*

The service furnished to Maine–London by the bunched system of Fig. 20 would be essentially the same as the service furnished by the uniform system. The services furnished to the other two paths would be degraded by the assumed bunching of planes, but this degradation could be compensated by adding four satellites in a 28° orbit. Two of the paths would have better service from this bunched system of 28 satellites than they would have from the uniform system of 24 satellites. Thus the penalty of the bunching is less than four satellites.

The assumed bunching would result after six years if large (5°) errors were made in the inclinations in exactly the right combination. A reduction of the error by a factor of two would double the time required for bunching. Also, if errors in the range of 0°–5° were made on a random basis, the bunching after six years would not be as severe as that which was assumed.

Satellite failures and system growth were not included in the study, but they represent important factors in the control of bunching. If four satellites were to fail from each group of twelve during the six years, their replacements could easily fill in the gaps caused by the bunching. These gaps could also be filled by satellites which might be added to increase the qualities of service from their initial values. Because the bunching is slow, the constant redistribution resulting from replacements and additions would be expected to make the bunching of planes of minor significance, and through these processes a random system would no doubt evolve.

## VII. CONCLUSION

The service which would be provided by satellites in uncontrolled orbits has been described analytically and by simulation. Section IV provides equations and graphical and tabular material which can be used to describe the service of systems which are random, and the same material can be used to estimate the service which would be provided by systems which have certain nonrandom characteristics. Results of computer simulations of several systems have been presented to develop an intuitive feeling for various qualities of service.

The example system which was studied shows that rather good worldwide service could be started with a modest number of satellites. The usefulness of existing facilities in extending this service and in providing for efficient consolidation of traffic has also been illustrated. A method of improving the continuity of service by using alternate relay points has been included.

The nonrandom characteristics of the example system were investigated by computer simulation to determine their effect on service. A method of phasing multiply-launched satellites was presented, and the effect of bunching of orbit planes was studied. A severe case of bunching was shown to degrade the service to some representative pairs of stations, but this degradation was shown to be more than compensated by a small increase in the number of satellites. Since the bunching of planes in ordered systems would occur at a slow rate, it is felt that replacements of satellites which fail and the additions of satellites for growth would provide an adequate means of controlling the effect of this bunching. It is also felt that these replacements and the additions would result in the eventual evolution of a random system in which significant bunching would be less probable.

## VIII. ACKNOWLEDGMENTS

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## APPENDIX

Table II presents values of $p \times 10^3$ for numerous ground station locations, orbit altitudes and inclinations. The first three columns give the co-latitudes of the ground stations and their longitude separation in degrees. Because of symmetry with respect to the equator, two co-latitudes may be measured from either the south pole or the north pole. Some ground stations of interest are named next to their corresponding locations. Where one station is paired with several others, the first name is not repeated and the second is indented.

The other columns are headed by $H$, the circular orbit altitude and $i$, the inclination. Linear interpolation may be used between successive altitudes and inclinations with reasonable accuracy.

# TABLE II

| | Colat 1 | Colat 2 | Longitude Difference | H = 2500 N. Mi | | | | | H = 4000 N. Mi | | | | | | H = 6000 N. Mi | | | | | | H = 8000 N. Mi | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | i = 0° | 35° | 50° | 70° | 90° | i = 0° | 28° | 35° | 50° | 70° | 90° | i = 0° | 28° | 35° | 50° | 70° | 90° | i = 0° | 28° | 35° | 50° | 70° | 90° |
| | 10 | 45 | 0 | | | | | | | | | | | | | 088 | | | | 252 | | | | | | |
| | 20 | 45 | 100 | | | | | | | | | | | | | 037 | | | | 221 | | | | | | |
| | 20 | 45 | 160 | | | | | | | | | | | | | 000 | | | | 199 | | | | | | |
| Anchorage, Alaska– | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Seattle | 28 | 42 | 28 | | 075 | 110 | 149 | 184 | | | 115 | 152 | 198 | 239 | | | 150 | 186 | 242 | 266 | | | 177 | 217 | 283 | 304 |
| Tokyo | 28 | 55 | 70 | | 033 | 059 | 088 | 075 | | | 063 | 093 | 130 | 142 | | | 090 | 123 | 164 | 190 | | | 114 | 149 | 192 | 230 |
| | 30 | 45 | 130 | | | | | | | | | | | | | 014 | | | | 189 | | | | | | |
| | 30 | 45 | 140 | | | | | | | | | | | | | 007 | | | | 184 | | | | | | |
| | 30 | 45 | 180 | | | | | | | | | | | | | 000 | | | | 176 | | | | | | |
| | 34 | 45 | 108 | | | | | | | | | | | 154 | | | | | | 196 | | | | | | 222 |
| Lands End, England– | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Maine | 39 | 45 | 65 | 000 | 047 | 069 | 094 | 124 | 000 | | 078 | 102 | 137 | 187 | 030 | 060 | 107 | 133 | 185 | 230 | 080 | | 127 | 154 | 220 | 256 |
| New Delhi | 39 | 62 | 83 | | 026 | 040 | 051 | 040 | | | 052 | 069 | 090 | 082 | | | 076 | 096 | 123 | 132 | | | 095 | 115 | 152 | 180 |
| Leopoldville | 39 | 94 | 20 | | | | | | | 123 | | | | | | 161 | | | | | | 187 | | | | |
| Johannesburg | 39 | 115 | 30 | | 016 | 011 | 009 | 009 | | 077 | 053 | 036 | 029 | 027 | 81 | 122 | 094 | 062 | 050 | 046 | 188 | 151 | 132 | 085 | 066 | 062 |
| | | | | | | | | | | | | | | | | | | | | | | | | | | — |
| | 40 | 76 | 0 | | | | | | | | | | | | | | | | | 141 | | | | | | |
| Seattle– | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 42 | 48 | 76 | 000 | | | | 086 | 000 | | | | | 160 | | | | | | 200 | 085 | | | | | 231 |
| Tokyo | 42 | 55 | 98 | 000 | 013 | 027 | 041 | 038 | 000 | | 036 | 053 | 076 | 102 | 000 | | 058 | 075 | 111 | 150 | 033 | | 074 | 094 | 143 | 188 |
| Hawaii | 42 | 70 | 33 | 000 | 089 | 100 | 081 | 070 | 122 | | 127 | 137 | 134 | 110 | 175 | | 158 | 166 | 171 | 148 | 219 | | 184 | 193 | 196 | 179 |
| | 42 | 71 | 71 | | | | | | | | | | | | | | | | | 086 | | | | | | |
| | 42 | 76 | 93 | | | | | | | | | | | | | | | | | 060 | | | | | | |
| Maine– | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 45 | 45 | 47 | | | | | | 060 | | | | | 196 | 116 | | | | | 242 | 152 | | | | | 270 |
| Rome | 45 | 48 | 83 | | | | | | | 045 | | | | 142 | | 080 | | 101 | 134 | 165 | | 098 | | | | 170 |
| Madrid | 45 | 49 | 65 | | | | | | | 077 | | | | 143 | | 108 | | | | 153 | | 122 | | | | 182 |
| | 45 | 55 | 100 | | | | | | | | | | | | | 045 | | | | 150 | | | | | | |

TABLE II—*Continued*

| | Colat 1 | Colat 2 | Longitude Difference | H = 2500 N. Mi | | | | | H = 4000 N. Mi | | | | | | H = 6000 N. Mi | | | | | | H = 8000 N. Mi | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | i = 0° | 35° | 50° | 70° | 90° | i = 0° | 28° | 35° | 50° | 70° | 90° | i = 0° | 28° | 35° | 50° | 70° | 90° | i = 0° | 28° | 35° | 50° | 70° | 90° |
| Dakar | 45 | 76 | 52 | | | | | | | | | | | | | 128 | | | | 112 | | | | | | |
| | 45 | 80 | 100 | | | | | | | | | | | | | 041 | | | | 046 | | | | | | |
| | 45 | 90 | 0 | | | | | | | | | | | | | 191 | | | | | 107 | | | | | |
| | 45 | 90 | 0 | | | | | | 200 | | | | | 080 | 263 | | | | | 111 | 305 | | | | | 131 |
| Rio de Janeiro | 45 | 112 | 27 | 077 | 033 | 023 | 018 | 017 | 138 | | 077 | 052 | 041 | 038 | 188 | | 119 | 082 | 063 | 057 | 225 | | 159 | 105 | 082 | 077 |
| | 45 | 120 | 0 | | | | | | | | | | | | | 138 | | | | 054 | | | | | | |
| | 48 | 51 | 4 | 136 | | | | 155 | 218 | | | | | 237 | 263 | | | | | 281 | 302 | | | | | 317 |
| | 48 | 58 | 21 | 113 | | | | 112 | 184 | | | | | 171 | 228 | | | | | 224 | 282 | | | | | 295 |
| | 51 | 56 | 40 | 068 | | | | 093 | 136 | | | | | 154 | 178 | | | | | 207 | 233 | | | | | 272 |
| Tokyo, Japan–Guam | 55 | 76 | 5 | | | | | | | 193 | | | | | | 235 | | | | 165 | | 277 | | | | |
| | 56 | 58 | 23 | 137 | | | | 109 | 197 | | | | | 166 | 238 | | | | | 223 | 286 | | | | | 291 |
| | 56 | 69 | 41 | 105 | | | | 068 | 160 | | | | | 109 | 239 | | | | | 153 | 227 | | | | | 180 |
| | 58 | 80 | 27 | 158 | | | | 069 | 209 | | | | | 105 | 242 | | | | | 138 | 266 | | | | | 164 |
| Cairo, Egypt–Puerto Rico | 60 | 71 | 98 | | | | | | | 033 | | | | | | 067 | | | | 052 | | 084 | | | | |
| New Delhi, India–Sydney, Australia | 62 | 123 | 74 | | 001 | 001 | 001 | | 068 | | 023 | 017 | 014 | 013 | 100 | | 060 | 040 | 032 | 029 | 126 | | 091 | 059 | 046 | 043 |
| Miami, Florida– | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 64 | 91 | 2 | | | | | | | 219 | | | | | | 232 | | | | 140 | | 298 | | | | |
| Rio de Janeiro | 64 | 112 | 37 | 135 | 055 | 039 | 031 | 029 | 181 | | 108 | 073 | 058 | 054 | 214 | | 153 | 105 | 084 | 078 | 241 | | 198 | 134 | 103 | 097 |
| Santiago, Chile | 64 | 123 | 9 | | | | | | | 133 | | | | | | 191 | | | | 077 | | 234 | | | | |
| | 68 | 76 | 57 | | | | | | | 118 | | | | | | 155 | | | | 103 | | 178 | | | | |
| Hawaii– | | | | | | | | | | | | | | | | | | | | | | | | | | |
| San Francisco | 70 | 53 | 32 | 129 | 104 | 107 | 089 | 076 | 181 | | 146 | 146 | 143 | 118 | 216 | | 182 | 178 | 178 | 164 | 240 | | 209 | 203 | 206 | 189 |
| Manila | 70 | 75 | 84 | | | | | | | | | | | | | | | 077 | 068 | | | | | | | |
| Sydney | 70 | 123 | 53 | 081 | 028 | 020 | 016 | 016 | 128 | | 077 | 048 | 039 | 036 | 166 | | 121 | 079 | 061 | 056 | 200 | | 154 | 105 | 080 | 074 |
| Puerto Rico–Dakar | 71 | 76 | 48 | | | | | | | 140 | | | | | | 180 | | | | 116 | | 202 | | | | |
| | 71 | 89 | 63 | | | | | | | | | | | | | | | | | 077 | | | | | | |

| | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Leopoldville | 71 | 94 | 81 | | | | | | | 070 | | | | | | 102 | | | | 045 | | 120 | | | | |
| | 71 | 100 | 56 | | | | | | | | | | | | | | | | | 078 | | | | | | |
| | 71 | 120 | 36 | | | | | | | | | | | | | | | | | 079 | | | | | | |
| | 71 | 130 | 0 | | | | | | | | | | | | | | | | | 079 | | | | | | |
| | 71 | 150 | 0 | | | | | | | | | | | | | | | | | 050 | | | | | | |
| Guam– | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 76 | 25 | 0 | | | | | | | | | | | | | | | | | 119 | | | | | | |
| Bangkok | 76 | 76 | 44 | | | | | | | 150 | | | | | | 195 | | | | 121 | | 214 | | | | |
| | 76 | 76 | 50 | | | | | | | | | | | | | | | | | 109 | | | | | | |
| | 76 | 100 | 55 | | | | | | | | | | | | | | | | | 082 | | | | | | |
| Sidney | 76 | 123 | 7 | | | | | | | 169 | | | | | | 223 | | | | 099 | | 259 | | | | |
| | 76 | 130 | 0 | | | | | | | | | | | | | | | | | 088 | | | | | | |
| | 90 | 90 | 33 | | | | | | 205 | | | | | 098 | 233 | | | | | 129 | 256 | | | | | 154 |

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