Evaluation of the Net Radiant Heat Transfer between Specularly Reflecting Plates

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(Manuscript received July 16, 1962)

The radiant heat transfer between two parallel infinite plates was determined. The plates were assumed to be specular, anisotropic reflectors and emitters as characterized by the electromagnetic theory for highly polished electrical conductors.

Numerical results are given for specific metals from 4.2 to 1500° K. Also, the results are expressed in generalized form for obtaining the net radiant heat transfer between any two parallel, infinite metal plates given only the temperatures and electrical resistivities.

Total hemispherical and normal emissivities were determined using the same methods. The results were in very good agreement with empirical equations given in the literature. For a contrasting comparison, Christiansen's equation for the net radiant heat transfer between two parallel, diffuse, gray surfaces of infinite extent was evaluated using these emissivities. The values obtained were less than those computed for the net radiant heat transfer between specular plates.

I. INTRODUCTION

Radiative exchange often becomes the controlling mode of heat transfer when systems are found to exist at either low or high temperature levels. In the former case, for instance, it often becomes one of the major heat leaks to cryogenic fluids stored in dewars.

In the following, the net radiant heat transfer between two parallel infinite surfaces is calculated for behavior in all respects as predicted by the electromagnetic theory of radiation for polished electrical conductors.

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II. FORMULATION AND SOLUTION

The monochromatic emissivity, absorptivity, and reflectivity are known to be azimuthally symmetric about the normal to a conducting surface. Electromagnetic theory[1,2] indicates that the monochromatic directional emissivity of low-emissivity metals is given by:

$$\epsilon(\theta,\lambda) = \alpha(\theta,\lambda) = 1 - \frac{1}{2}\left[\frac{\frac{60\lambda}{re} - 2\sqrt{\frac{30\lambda}{re}}\cos\theta + \cos^2\theta}{\frac{60\lambda}{re} + 2\sqrt{\frac{30\lambda}{re}}\cos\theta + \cos^2\theta} + \frac{\frac{60\lambda\cos^2\theta}{re} - 2\sqrt{\frac{30\lambda}{re}}\cos\theta + 1}{\frac{60\lambda\cos^2\theta}{re} + 2\sqrt{\frac{30\lambda}{re}}\cos\theta + 1}\right]. \quad (1)$$

Simplified expressions based on electromagnetic theory have been shown to be in agreement with experiment for temperatures as high as 1800°K.[2] However, deviation from experiment is presumed for higher temperatures.

2.1 *Radiant Heat Transfer*

For the arrangement shown in Figs. 1 and 2, the monochromatic radiation emitted from a unit area of surface 1 into a solid angle $d\omega$ inclined at an angle θ from the normal is

$$\epsilon_1(\theta,\lambda)I(bb,\lambda,T_1)\cos\theta\, d\omega. \quad (2)$$

A fraction $\alpha_2(\theta,\lambda)$ of this is absorbed at surface 2 for the first reflection. For specular reflection, a fraction $\alpha_2(\theta,\lambda)[1 - \epsilon_1(\theta,\lambda)][1 - \alpha_2(\theta,\lambda)]$ is absorbed at surface 2 for the second reflection. Ultimately, the monochromatic radiation emitted from surface 1 in a direction θ that is absorbed by surface 2 is

$$\begin{aligned}\epsilon_1(\theta,\lambda)\alpha_2(\theta,\lambda)\{1 &+ [1 - \epsilon_1(\theta,\lambda)][1 - \alpha_2(\theta,\lambda)] \\ &+ [1 - \epsilon_1(\theta,\lambda)]^2[1 - \alpha_2(\theta,\lambda)]^2 \\ &+ \cdots\}\, I(bb,\lambda,T_1)\cos\theta\, d\omega \qquad (3) \\ &= \frac{\epsilon_1(\theta,\lambda)\alpha_2(\theta,\lambda)I(bb,\lambda,T_1)\cos\theta\, d\omega}{1 - [1 - \epsilon_1(\theta,\lambda)][1 - \alpha_2(\theta,\lambda)]}.\end{aligned}$$

Thus, the total radiation emitted from a unit area of surface 1 and

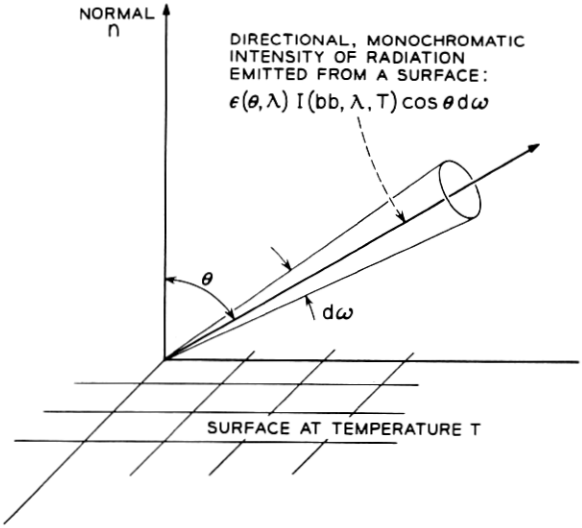


Fig. 1 — Coordinate system.

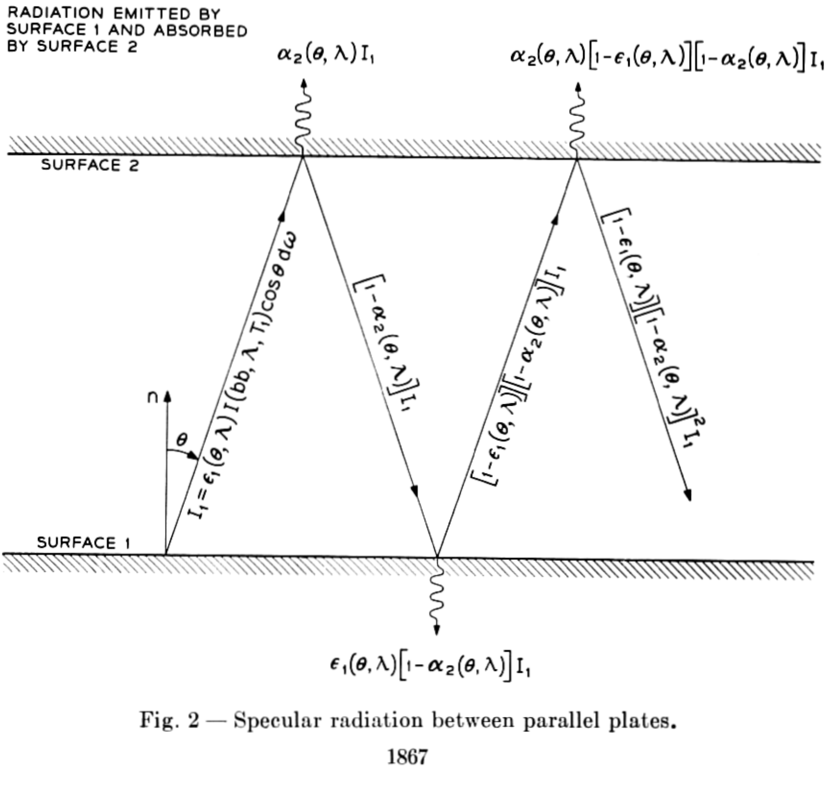


Fig. 2 — Specular radiation between parallel plates.

absorbed by surface 2 is

$$q_{1\to 2} = \int_0^\infty \int_0^{\pi/2} \frac{\epsilon_1(\theta,\lambda)\alpha_2(\theta,\lambda)}{1 - [1 - \epsilon_1(\theta,\lambda)][1 - \alpha_2(\theta,\lambda)]} \cdot E(bb,\lambda,T_1) \sin 2\theta \, d\theta \, d\lambda. \tag{4}$$

The net radiation from surface 1 to surface 2 is

$$q_{\text{net}} = q_{1\to 2} - q_{2\to 1}$$

where $q_{2\to 1}$ is evaluated from (4) with the subscripts reversed.

Using the IBM 7090 computer, this equation was evaluated for specific metals and also for various resistances and temperatures. Only a fraction of a minute computation time was required per case. The required resistivity values were obtained from Refs. 3–10.

Specific examples of solutions are given in Table I. In Fig. 3, the radiation $q_{1\to 2}$ is graphically expressed in general form in terms of only T_1,

Table I — Net Radiation Between Parallel Plates — Specific Examples

T_1° K	T_2° K	Computed Values — Figure 3			Christiansen's Equation (2)
		$q_{1\to 2}$	$q_{2\to 1}$	q_{net}	$q_{\text{net}} = \frac{\sigma[T_1^4 - T_2^4]}{1/\epsilon_{h_1} + 1/\epsilon_{h_2} - 1}$
		BOTH SURFACES GOLD			
77	4.2	1.65×10^{-7}	3.42×10^{-13}	1.65×10^{-7} watts/sq cm	0.442×10^{-7} watts/sq cm
290	77	2.58×10^{-4}	6.65×10^{-7}	2.58×10^{-4}	1.52×10^{-4}
1000	290	0.129	4.97×10^{-4}	0.129	0.0812
		BOTH SURFACES IRON			
500	290	1.4×10^{-2}	1×10^{-3}	13×10^{-3}	8.82×10^{-3}
1000	290	0.4	0.0013	0.4	0.195
1000	500	0.44	0.016	0.424	0.322
		SURFACE 1 IRON — SURFACE 2 GOLD			
290	77	3.147×10^{-4}	1.2×10^{-6}	3.14×10^{-4}	1.68×10^{-4}
1000	290	0.1733	6.74×10^{-4}	0.172	0.1
		SURFACE 1 GOLD — SURFACE 2 STAINLESS 18-8			
290	77	6.35×10^{-4}	10^{-6}	6.34×10^{-4}	5.5×10^{-4}
1000	290	0.258	10^{-3}	0.257	0.22

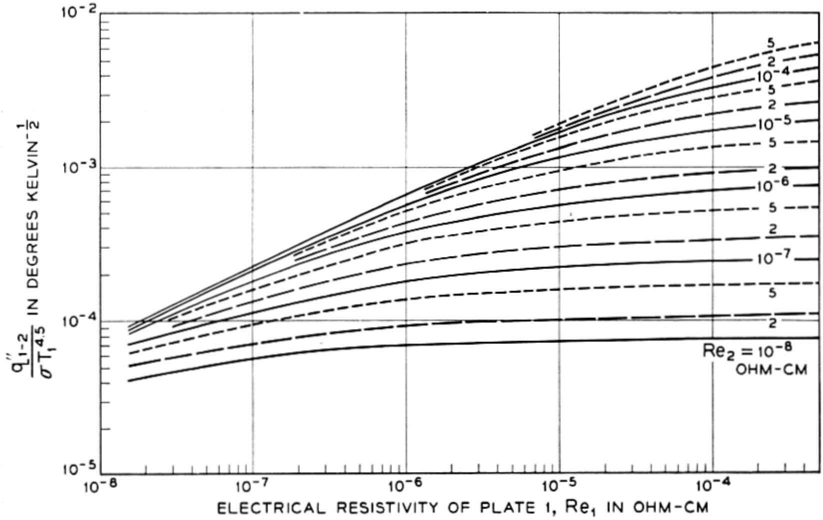


Fig. 3 — Radiation emitted by plate 1 that is absorbed by plate 2.

re_1, and re_2. As is indicated by the examples in Table I, Fig. 3 can be used to determine the net radiation, $q_{1\to2} - q_{2\to1}$, between any two similar or dissimilar parallel infinite metal plates. The choice of the ordinate in Fig. 3 was arbitrary. The choice of $\sigma T^{4.5}$ in the denominator removed part of the dependence of emissivity on temperature and resulted in an ordered family of curves.

2.2 *Emissivities*

An expression similar to (4) can be developed for the total hemispherical emissivity of a perfectly smooth, clean metal surface:

$$\epsilon_h = \frac{1}{E(bb,T)} \int_0^\infty \int_0^{\pi/2} \epsilon(\theta,\lambda) E(bb,\lambda,T) \sin 2\theta \, d\theta \, d\lambda. \tag{5}$$

This equation was also evaluated for seven metals at various temperatures; the results, depicted in Fig. 4, agree very well with empirical equations (based on electromagnetic theory predictions) that have been applied over specific ranges of reT.[1,2]

Available experimental emittance values for polished metal surfaces are generally from one to three times the computed values. Slight imperfections and oxides on the actual surfaces could account for the variances. Most of the available copious data were taken at pressures above 10^{-6}

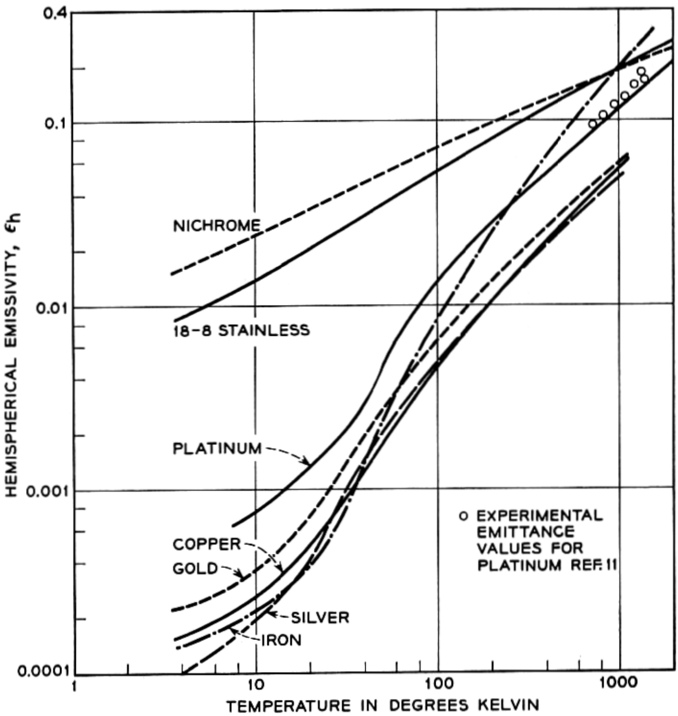


Fig. 4 — Computed total hemispherical emissivities.

Torr, and the samples were not prepared in high vacuum. The results of a recent intensive experimental investigation[11] are in good agreement with the computed values for platinum as shown in Fig. 4. Apparently these investigators chose platinum for their tests because of good surface stability compared with other metals.

The "emittance" of a surface is taken to be equal to the "emissivity" when the surface is opaque, optically flat, and identical to the interior of the material.[11]

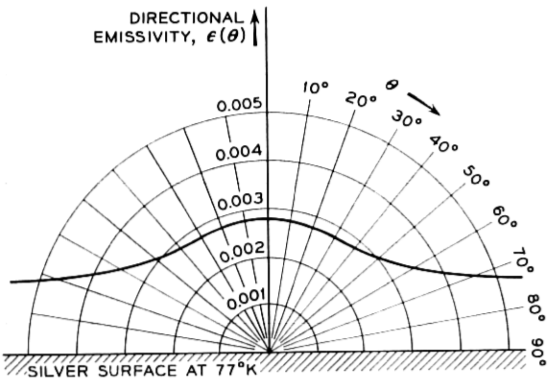
Below about 20 degrees K, the resistivity of the materials considered approaches a residual value. This is more noticeable with pure metals than in alloys, and is reflected in the emissivity values in Fig. 4.

The values in Fig. 4 should be applied with qualification to surfaces at very low temperatures, because of the increased importance of the anomalous skin effect which was not considered here. The anomalous skin effect theory indicates that the skin resistance, instead of the bulk resistivity, becomes important in absorption and reflection for the longer wavelengths (approaching those of diffusely behaving microwaves) that are effective in thermal radiation at extremely low temperatures. At the present time, there is considerable uncertainty concerning the nature of thermal radiation between surfaces below 100°K. This is primarily due to difficulties in taking measurements and to conflicting results. Also, recent evidence suggests that the diffuse or specular nature of a metallic surface at low temperatures may be strongly dependent upon both the temperature of the surface and the wavelength of the incident radiation.[12, 13]

The resistivity of nichrome is nearly constant over the range of temperature considered; thus the slope of the curve for nichrome in Fig. 4 is due only to the temperature dependence of the emissivity in the evaluation that was made. A converse example would be the large change in resistance at a certain temperature during the quantum transition of a superconductor to the superconducting state. The classical expressions solved here predict perfect reflection for the superconducting state, but they would not necessarily be expected to be applicable. In the visible region, no change in reflection has been reported during the superconducting transition; however, an increase in reflection has been reported for frequencies less than the superconducting energy gap frequency (on the order of 3×10^{11} cycles per second).

Equations 4 and 5 can also be readily solved for a particular wavelength or for a particular radiant energy distribution other than the Planckian spectral distribution used here.

Directional and normal emissivity values were also obtained by evaluating (5) for specific values of θ. An exemplary case is given in Fig.



COMPUTED DATA NOT PLOTTED

θ	$\epsilon(\theta)$
80°	0.0081
85°	0.0157
87°	0.0256
88°	0.0380
88.5°	0.0500
89°	0.0725
89.5°	0.1340
89.9°	0.3640
89.9999°	0.0005
90°	0

Fig. 5 — An example of the directional emissivities computed.

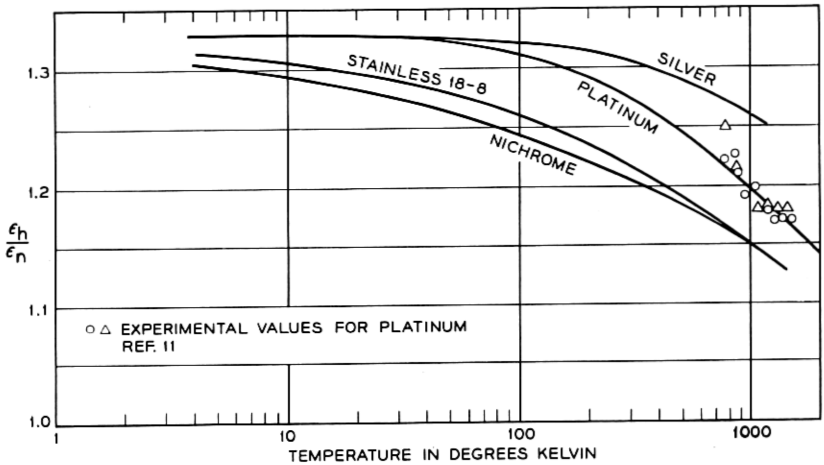


Fig. 6 — Ratio of total hemispherical to total normal emissivity.

5. The ratio ϵ_h/ϵ_n was found to approach the theoretical limiting value of $\frac{4}{3}$ (Ref. 2) using the values computed for the low resistivity metals at very low temperatures, as shown in Fig. 6.

All of the computed values for all of the metals considered are normalized to one curve in Fig. 7. An approximation to the theoretical expression evaluated here and some recent experimental results are included in Fig. 7 for comparison. The departure of a real surface from the

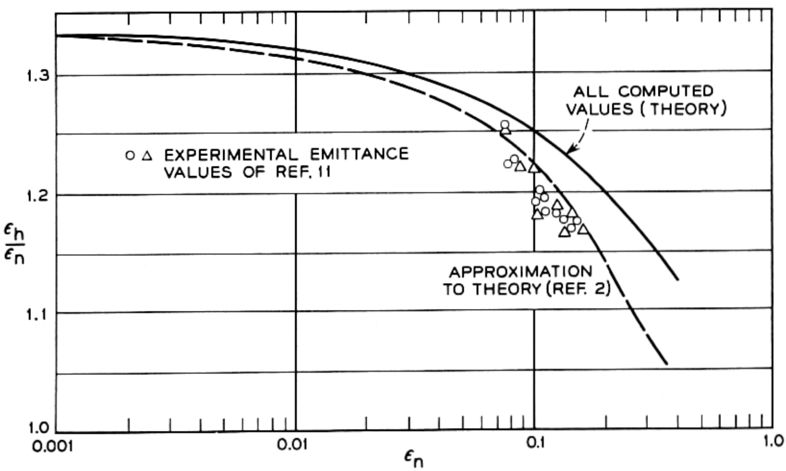


Fig. 7 — Ratio of total hemispherical to total normal emissivity.

ideal optical flat surface and the change in actual surface resistivity from the bulk value would be expected to make the experimentally measured emissivity ratio ϵ_h/ϵ_n lower than would be theoretically predicted.[11]

The computed values of hemispherical emissivity were used in evaluating Christiansen's equation for the net radiant heat transfer between parallel, diffuse, gray surfaces of infinite extent. Christiansen's equation may be written as:[2]

$$q_{\text{net}} = \frac{\sigma[T_1^4 - T_2^4]}{1/\epsilon_{h_1} + 1/\epsilon_{h_2} - 1} .$$

The results for several cases are tabulated in Table I for comparison with the computed examples for net specular radiation. The corresponding values computed for the net specular radiant heat transfer are larger. This may be attributed to the angular and spectral effects. Goodman[14] compared Christiansen's equation with values computed from experimental spectral emissivities for aluminum and Inconel at temperatures of 400°C to 1000°C. The spectral results were 2 per cent to 29 per cent greater than the values predicted by Christiansen's equation.

III. CONCLUSIONS

The radiant heat transfer between any two parallel, specular, infinite, uniform, metal plates was determined and is expressed in terms of only the temperatures and electrical resistivities in Fig. 3. The results exceed the predictions of Christiansen's equation for diffuse radiation between parallel gray plates of infinite extent. Christiansen's equation was evaluated using the computed emissivity values.

Both the radiant heat transfer and the emissivity values presented represent limiting values that can be expected for perfectly clean, smooth metallic surfaces. The results should be very useful in interpreting data and in estimating values where adequate data are lacking.

The solutions presented can be readily evaluated with the aid of a computer to obtain any additional radiant heat transfer and emissivity values that might be of particular interest.

APPENDIX

Nomenclature

$\alpha(\theta,\lambda)$ monochromatic directional absorptivity
$\epsilon(\theta,\lambda)$ monochromatic directional emissivity

ϵ_h total hemispherical emissivity

ϵ_n total normal emissivity

θ direction angle with respect to a normal to the surface

λ wavelength in centimeters

$d\omega$ element of solid angle: $\sin\theta \, d\theta \, d\varphi$ where φ is the azimuth angle

$E(bb,\lambda,T)$ monochromatic emissive power of a black body at temperature T:

$$\frac{3.7404 \times 10^{-12}}{\lambda^5 [\exp(1.4387/\lambda T) - 1]} \text{ watts/cm}^2$$

$E(bb,T)$ emissive power of a black body: σT^4 where $\sigma = 5.6699 \times 10^{-12}$ watts/cm^2 °K^4

$I(bb,\lambda,T)$ monochromatic areal radiant intensity of a black body at temperature T:

$$\frac{E(bb,\lambda,T)}{\pi}$$

re electrical resistivity ohm-cm.

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