Engineering of T1 Carrier System Repeatered Lines

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The repeatered lines for the T1 carrier system (24 voice channels, PCM) are cable pairs equipped with transistorized regenerative amplifiers. The line signal is a train of 1,544,000 bipolar pulse positions per second. The line engineering methods, which tell how to select cable facilities and where to place repeaters, are based on the theory and measurements reported here. Using typical cable data, specific examples are given of the design limits imposed by the principal types of interference: crosstalk from other T1 systems and noise originating in central offices.

I. INTRODUCTION AND PROBLEM STATEMENT

1.1 *System Description*

The T1 carrier system transmits 24 voice channels over two cable pairs, one for each direction of transmission. The channels are time-division multiplexed and encoded by PCM, so that the line signal is a train of 1,544,000 bipolar pulse positions per second. Transistorized regenerative amplifiers, powered directly over the transmission pairs, detect the pulses and retransmit them along the line. The amplifiers are placed in manholes or on telephone poles at a nominal spacing of 6000 feet, for 22-gauge paper-insulated cable. This conforms to the common "H" loading spacing. General equipment features have been described for an experimental system,[1] and further aspects of the T1 system are described in another paper.[2]

In this paper we give the underlying theory for engineering the repeatered lines. The emphasis is on the principles that affect the system layout. Although some specific objectives are developed, we have not tried to list all of the factors that may affect a specific layout.

1.2 *Repeatered Line Engineering*

1.2.1 *The Span as a Line Engineering Unit*

A major departure in T1 line engineering, compared to previous short-haul carrier systems, is the design of routes in units called *spans,* as com-

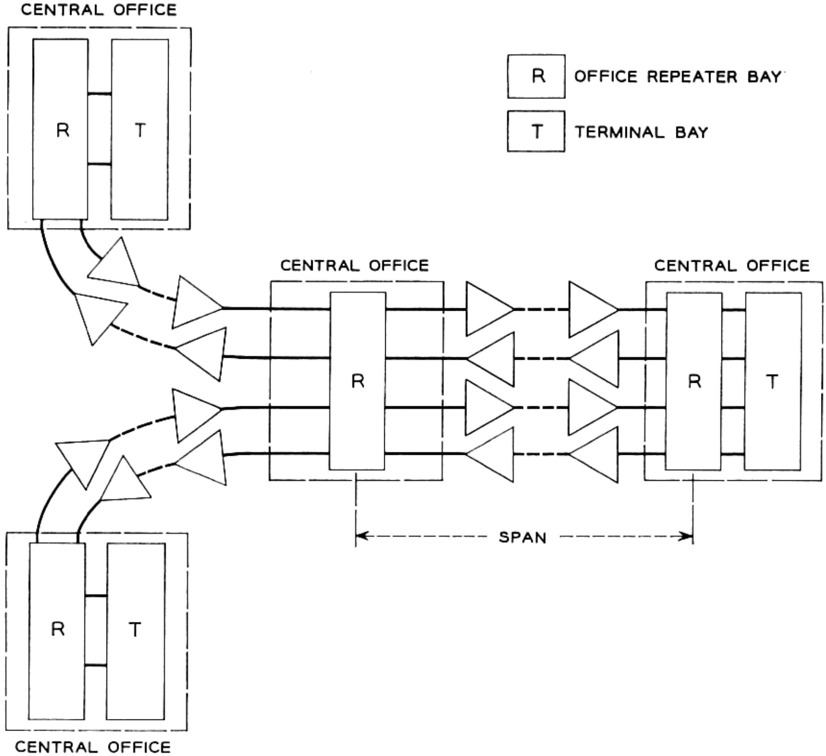

Fig. 1 — Spans.

pared to complete systems from terminal to terminal, or from one office served by the voice circuits to another. Referring to Fig. 1, a span is a collection of repeatered lines* between office repeater bays. A repeatered line for a particular system (terminal to terminal) is made up by interconnecting span lines at the offices along the route. It is simpler to administer, operate, and maintain the large numbers of telephone circuits that are needed in typical central office areas if the T1 lines are thought of as composing the spans to which they belong, rather than identifying the lines by their ultimate terminals. For example, any line in a span may serve as a spare for another, irrespective of the terminals served by the trunks using the spans. By building up the network on the span basis,

* Two regenerative amplifiers, with common power circuitry, make up a plug-in package called a repeater. Each repeatered line consists of two pairs of wires, one for each transmission direction, and the necessary amplifiers. Our "amplifier" is the same functional unit called a "repeater" by Mayo[3] and a "reconstructive repeater" by Aaron.[4]

it is also easier to provide order wires for communication by maintenance personnel along the lines, to plan the fault-location requirements,[2] and to supply dc power for the repeaters. Our point of view, then, is to assure satisfactory pulse transmission in the lines of each span.

The prime markets for the T1 system are areas of high telephone development, such as the large networks in metropolitan areas. The majority of systems will provide trunks, e.g., local, tandem, and toll connecting trunks, between central office buildings in these areas. Typical cable lengths between buildings, corresponding to span lengths, are 2 to 10 miles. Terminal-to-terminal system lengths may be as great as 50 miles.

Message channels are needed in large numbers along T1 routes. Repeaters will be installed in a case suitable for location in manholes or on telephone poles. Each case will mount 25 repeaters, that is, 50 regenerative amplifiers as packaged. If the case is spliced into the main cable as in Fig. 2(a), 25 amplifiers serve as many pairs in one direction of transmission, and 25 the other; all 50 pairs are in the same cable, and this is called *one-cable operation*. In Fig. 2(b), all 50 amplifiers serve as many pairs for only one direction of transmission, and 50 more pairs in a second cable sheath, spliced to a second case, are needed for the opposite direction; this is *two-cable operation*.* Each fully-equipped repeater case thus serves the lines in both directions of transmission for 600 message channels in one-cable operation, or in one direction for 1200 channels in two-cable operation. To allow for flexibility in pair assignment and to accommodate growth, pairs not immediately needed as T1 lines may be used for voice-frequency circuits by plugging into the repeater case special load coil cases or through connectors, which are interchangeable with the repeaters.

1.2.2 *Objective*

In T1 lines fidelity of transmission is measured by the error rate, which is the fraction of received pulse positions in which a pulse is present when none was transmitted, or vice versa. The error rate objective for a terminal-to-terminal line is based on two desiderata. First, the noise power in a speech channel caused by line errors must be negligible compared to noise and distortion from other sources. The performance of the terminal then is controlling.[2] Second, the distinctive, low-level noise pulses in a speech channel which result from line errors must occur infrequently enough to be unobjectionable to the user. The second is more

* A repeater case is never spliced into more than one cable.

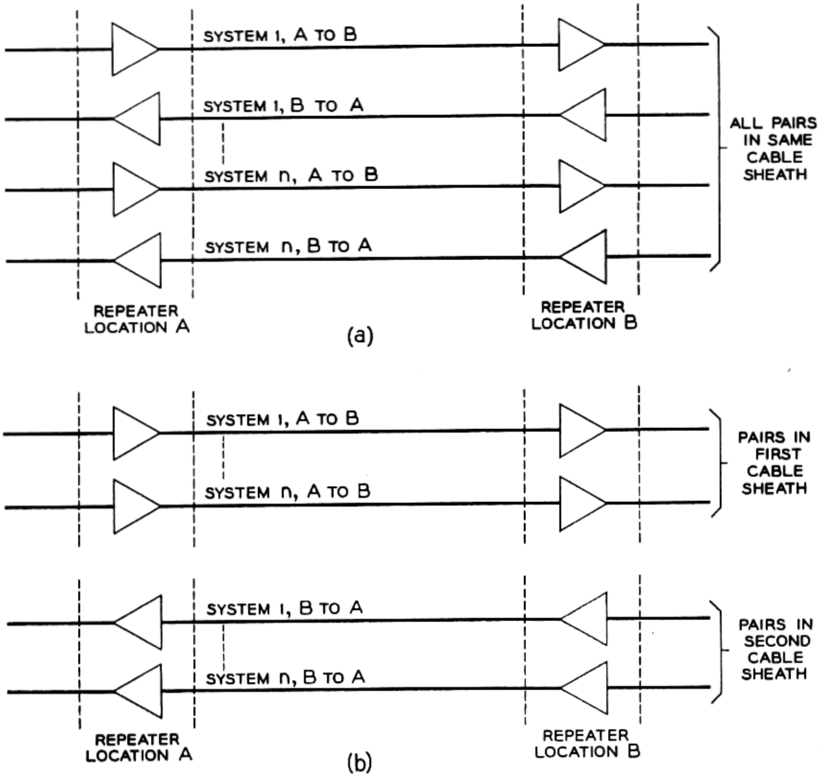

Fig. 2 — One- and two-cable operation.

difficult to satisfy than the first. Some tests conducted by W. L. Ross indicate that errors occurring at a rate of 10^{-6} are very difficult to detect by listening, and this rate has been made the system objective.[1]

The goal of line engineering is satisfactory transmission over the lines of each span. The span objective is an error rate of 3×10^{-7}; this allows three span lines in a system to have the maximum error rate. Since most span lines will have substantially lower error rates, the system objective will almost always be met, even in long systems. The system error rate of 10^{-6} is a conservative figure; an error rate of 10^{-5} will not seriously impair speech quality.

1.2.3 *Further Requirements*

Many aspects of the T1 system are novel; for example, the application of PCM. We wish to explore the theory that is relevant to line engineering, and then to reduce its results to manageable procedures for the layout of lines. Studies of the existing plant and trunk forecasts will show the numbers of voice channels needed between particular terminals and the available routes and cables. Starting with these, the procedures must tell how to lay out the T1 lines, or more specifically, how to choose the one- or two-cable mode (Fig. 2), how to choose the cable and pair assignments, and where to place the repeaters.

System layouts must come reasonably close to an economic optimum in the installed cost of repeaters. Measurements of cable properties, such as noise levels or crosstalk coupling losses, should not be necessary to proceed with the design. It should be possible to arrive at a workable layout without extensive calculation. In specific cases, the procedures may allow a number of alternatives, and the selection can be assumed to rest on the economics of the complete trunk plan.

1.3 *Types of Cables*

T1 carrier continues a trend in carrier telephone transmission toward the greater utilization, in terms of bandwidth, of cable originally installed in the outside plant for the transmission of voice frequencies. Prior examples are the N and ON systems.[5,6] Each new system places its own demands on the cable plant. A natural development is that additions to this plant will be more and more influenced by potential carrier-frequency applications.

The present system is specifically intended for use on paired cables with paper strip, pulp or plastic conductor insulation. Load coils, building-out capacitors, and bridged taps must be disconnected from the pairs intended for carrier use. Cables of layer or unit construction with staggered pair twists will have satisfactory crosstalk properties, as will be seen below, but cables with nonstaggered twists are not acceptable. The preponderance of T1 applications will be on cables of larger sizes, say 200 pairs or more, although smaller cables will occasionally be used. A majority of installations, particularly in metropolitan centers, will utilize underground cable in ducts. In the outlying sections of cities, aerial cable is encountered; a field trial is presently under way to test the system performance in such cables, especially as regards the greater exposure to lightning and the wider temperature variations.

A considerable amount of 22-gauge cable is in place along the trunk routes involved; this gauge is therefore the design center for T1. Sections of gauges 19, 24, and 26 are also possible, and of mixed gauges where necessary.

More detailed cable characteristics that are desirable for T1 purposes are mentioned in the following sections.

II. AMPLIFIER PROPERTIES AFFECTING LINE ENGINEERING

2.1 *Required Signal-to-Noise Ratio*

An ideal binary pulse detection system[7,8] can operate with noise amplitudes as great as half the received pulse amplitude. An error is made only when the noise value at the sampling instant is more than this amount and occurs with appropriate sign. A practical regenerative amplifier[3] as built for the T1 system cannot operate with such a low signal-to-noise ratio. We assume that the probability of an error, that is, of retransmitting a pulse when none is received or vice versa, is the probability with which the noise amplitude exceeds one-fourth of the received pulse height. Production amplifiers are, of course, not all identical, and the required signal-to-noise ratio is taken as 12 db with a standard deviation of 1 db, based on observations of a number of samples.

2.2 *Operating Signal Range*

In discussing signal levels and line lengths, a convenient short cut is to replace the random pulse train, Fig. 3, with the periodic train of Fig 4.

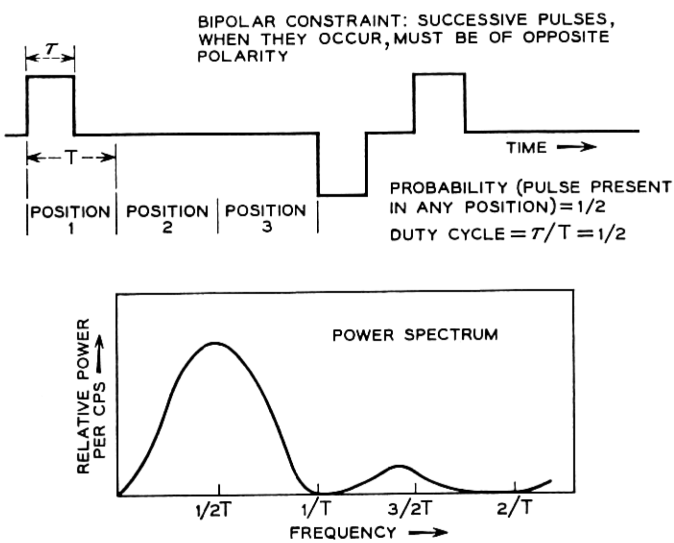

Fig. 3 — Random bipolar pulse train.

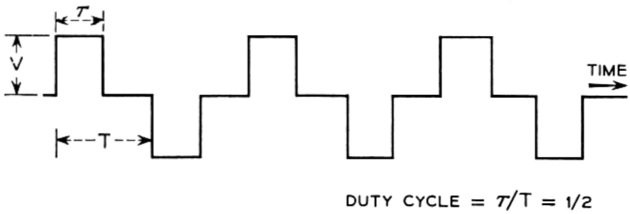

Fig. 4 — Continuous bipolar pulse train.

The random train, assuming balanced pulses, has no discrete frequency components.[4] The maximum of its power spectrum is at about half the pulse repetition frequency, or 772 kc.* The continuous train has 81 per cent of its power at this frequency and the balance at its odd harmonics. The peak amplitude of the 772-kc component for the wave of Fig. 4 is 0.90 V, or 2.7 volts for $V = 3$ volts at the amplifier output. For engineering purposes, the received pulse height may be found by calculating the peak amplitude of the received 772-kc component. Suppose, for example, that the cable pair between the output of one amplifier and the input of the next has 20 db attenuation at 772 kc. Then the pulse height at the sampling point, viz., at the equalized preamplifier output,[3] is 0.27 volts, increased by the preamplifier gain at 772 kc. *We shall refer always to the attenuation at 772 kc and 55 F (degrees fahrenheit), when the term "loss" is used.*

The amplifier operates most effectively over a range of losses centered at 31 db, the loss of 6000 feet of 22-gauge paper insulated cable. In Fig. 5, the relative signal-to-noise ratio, at a constant error rate of about 10^{-6}, is plotted against the loss, for the particular case of near-end crosstalk interference. If we set a goal of 1 db departure from optimum, the figure shows that the loss may be 27 to 35 db. A wider range of line losses is made possible by building out shorter sections.[3] Each amplifier has a buildout network at its input, selected on the basis of pair loss measurements at the time the repeater case is installed. The networks simulate the attenuation-vs-frequency characteristic of 22-gauge cable pairs, and are identified by their losses at 772 kc. For practical reasons, networks are supplied only in multiples of 2.4 db, as explained below.

The size of the loss steps for the networks is dictated by the loss range allowable for line and buildout together, 27 to 35 db, and by the variations due to temperature and to pair-to-pair differences. The increase in attenuation from 55 F to 100 F, the maximum temperature expected for

* The maximum is at 711 kc; the spectrum level at 772 kc is 98 per cent of this maximum.

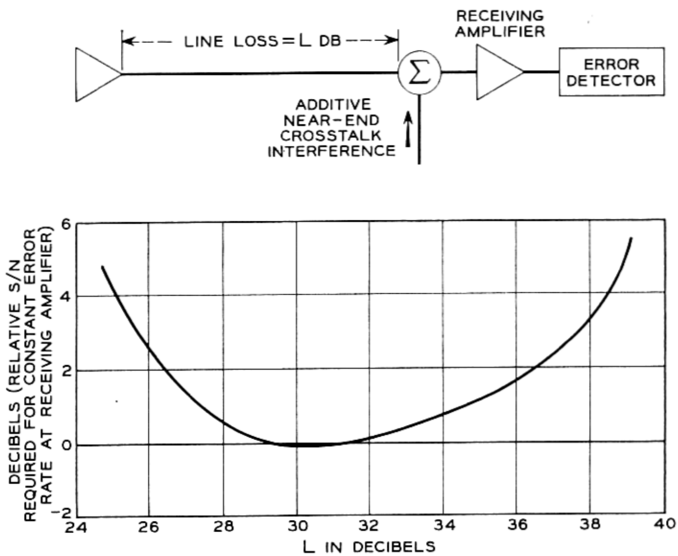

Fig. 5 — Effect of line loss on amplifier performance.

underground cable, is 1.3 db for 6300 feet* of 22-gauge cable. A comparable decrease in attenuation occurs when the temperature drops from 55 F to zero F, the minimum expected for underground cable. The standard deviation of pair losses is 0.6 db for this kind of section; therefore, the one per cent highest loss pair has an additional 1.5-db loss. So far we have accumulated a variation of ±2.8 db. If the buildout networks are made in steps small enough to have the line plus network loss 31 ± 1.2 db, i.e., steps of 2.4 db, the desired range of 31 ± 4 db is achieved. Thus, networks of loss 0, 2.4, 4.8, . . . db are needed. In the experimental system,[3] coarser steps of buildout loss were used. The list of networks for the commercial system has been filled in with the intermediate steps, to a maximum of 26.4 db. Line sections shorter than 32.2 db are built out to 31 ± 1.2 db. Generally all amplifiers in one direction in the same repeater section are equipped with the same buildout.

In making the preliminary layout, section losses are calculated from plant records showing the cable lengths and types. The average losses of common exchange cables are listed in Table I.[9,10,11] For T1 calculations, we use the "engineering loss," which is the average loss increased by a small amount to account for month-to-month variations in cable produc-

* Maximum "H" loading spacing.

Table I — Average Cable Losses, 772 kc, 55 F

Gauge	Insulation	Capacitance, microfarads/mile	Loss, db/mile
19	polyethylene	0.066	11.8
19	polyethylene	0.083	15.1
19	paper strip	0.066	15.2
19	paper strip	0.084	19.1
22	polyethylene	0.083	21.9
22	paper pulp	0.082	26.5
24	paper strip	0.072	29.5
26	paper pulp	0.079	39.2

tion. For 22 gauge cable with average loss of 26.5 db per mile, as an example, the engineering loss is 27.0 db per mile, or 32.2 db per 6300 feet. When a section is composed of mixed gauges of cable, or of high- and low-capacitance types of the same gauge, a "junction loss" is added to the engineering loss. The junction loss is based on the reflection loss at the junction, suitably reduced if only a short section of different gauge is present. It is not more than 0.3 db for common cable types.

2.3 *Minimum Section Loss*

The amplifier output and input impedances are not perfectly matched to the characteristic impedance of the cable pairs over the band of frequencies needed for pulse transmission. As a result, reflections of pulses may travel back from an amplifier input to the output circuit of the previous amplifier and interfere with the regeneration of pulses there, increasing the error rate. Experience has shown that this is serious only for amplifiers separated by less than 9 db of cable loss, a situation that may arise adjacent to central offices (cf. Section 4.5). For this reason, central office amplifiers are equipped with fixed pads of 100 ohms impedance and 3 db loss, at both input and output. These pads also reduce reflections from cable plugs and terminating cable discontinuities at the cable vault. Further, a minimum line loss of 6 db is required between the central office main distributing frame and the nearest outside repeater point. Sections not adjacent to central offices must have a minimum line loss of 9 db; in practice, these sections are normally long enough to meet this limit without special consideration. Operation with these minimum section losses* also helps in fault locating,[2] during which the amplifier is especially sensitive to interference from reflections.

* The two highest buildouts (Section 2.2) of 24 and 26.4 db were originally provided to complete the series. With a minimum line loss of 9 db, these buildouts are not used.

III. SOURCES OF INTERFERENCE

3.1 *General Remarks*

In the experimental system,[4] a choice of fundamental importance was made: the transmission of bipolar pulses (see Fig. 3) with timing wave extraction at half the pulse repetition rate. A simplified model of crosstalk was assumed in Ref. 4 to estimate the relative effects of crosstalk on the timing wave that is derived at a disturbed amplifier, for various pulse transmission schemes. Experience with the experimental and prototype systems has confirmed the value of the earlier choice. In this paper, we deal with the effects of interference on the pulse detection process. In practical installations, errors due to additive interference at the decision point of the amplifier, rather than those due to timing wave jitter, limit the accuracy of transmission of the pulse train.

We have stated the performance criterion in Section 1.2.2 in terms of amplifier error rate. For ideal pulse detection, this rate may be found if we know the amplitude of the received pulses and the amplitude distribution of the interfering voltage at the point where the regeneration decision is made. As an experimental observation, the T1 amplifier operates closely enough to this ideal amplitude-sensitive fashion that we may base error rate calculations on this model (cf. Section 2.1). In this section the major sources of interference are identified, and in Section IV the limits placed on repeatered line layout by the controlling sources are derived.

Internal sources of noise are principally thermal noise and disturbances introduced by the power supply. It is apparent that voltages from these sources are too small to enter into line engineering.[4] For example, the rms thermal noise voltage[12] in a band from zero to two megacyles per second, at a temperature of 300 degrees Kelvin and resistance 100 ohms (the characteristic impedance of 22-gauge cable at 772 kc) is 2 microvolts. The signal level (see Section 2.2), or received pulse height as equalized, at the end of a line section of 32.2 db loss is 0.066 volts, more than 90 db above the rms noise.

External sources induce voltages directly into the repeatered lines and indirectly via crosstalk coupling paths.* For example, central office noise affects T1 lines principally via crosstalk paths from noisy non-T1 pairs. Also, atmospheric static, lightning and radio transmitters induce voltages in exposed parts of the wire plant such as open wire extensions or sub-

* The term "crosstalk" refers to voltages induced in T1 pairs from nearby conductors. The resulting interference is not in general audible speech or even "babble," as in voice-frequency usage.

scriber drops, which, in turn, may join pairs in T1 cables. A further interference problem is the coordination of T1 with other transmission systems on cable pairs or other conductors in the same sheath. The possibility of *mutual* interference exists between T1 and other systems with similar frequency bands, particularly type N carrier (48 to 256 kc), type L carrier (308 to 8320 kc) and type A2A television (baseband to 4500 kc).

3.2 *Controlling Types of Interference*

To proceed in an orderly fashion, we must know the relative importance of the several sources of interference and then develop line engineering procedures which assure us of satisfactory operation under the most adverse amounts to be expected, of the controlling types of noise, in any combination. At this point, we draw heavily on experience in field trials of the experimental and prototype T1 systems, the former between Summit and South Orange, New Jersey,[1] the latter between Newark and Passaic, New Jersey, as well as on data gathered at Bell Telephone Laboratories and in operating telephone company areas. The conclusion, in brief, is that *crosstalk from other T1 systems* in the same cable and *central office noise* are the forms of interference that require specific engineering measures to combat.

Other sources do not seem at present to cause serious levels of interference. If future applications warrant it, these other sources may be taken into account in line engineering by methods similar to those given here for office noise and intersystem crosstalk. The effects of atmospheric electricity will be better known quantitatively after the aerial cable trial, mentioned in Section 1.3. To date, no instance of interference from radio transmitters has occurred. With regard to other transmission systems, coordination with type N carrier is excluded in existing plans for T1, primarily to avoid crosstalk to type N. Studies are now being made of T1 interference to or from type L carrier or type A2A video systems in the same cable sheath. The latter two systems operate on coaxials and shielded pairs, respectively, in the cores of some paired cables.

3.3 *Crosstalk from Other T1 Systems*

Near-end, far-end, and interaction crosstalk must be considered. Preliminary measurements[13] by the Outside Plant Systems Engineering Department at Bell Telephone Laboratories showed that interaction crosstalk coupling losses are extremely high in multipair cables. For this reason, this kind of crosstalk has not been dealt with further.

3.3.1 *Near-End Crosstalk*

Near-end crosstalk was recognized as a significant problem in one-cable systems during the development and testing of the experimental system.[4] Referring to Fig. 2(a), the high-level outputs of one set of amplifiers are coupled to the inputs of another set via near-end crosstalk paths, the most damaging of which are in the cable near the repeater case. It was primarily to lessen the effects of near-end crosstalk on amplifier timing that the bipolar pulse train of Fig. 3 was introduced. Its power spectrum peaks at about 772 kc (see Section 2.2), whereas a unipolar pulse train contains relatively more energy at higher frequencies, as well as certain discrete components. A secondary advantage is that the absence of a dc component in the bipolar case eases low-frequency transmission requirements.

The nature of near-end crosstalk interference has been studied with the laboratory setup[14,15] of Fig. 6, in which crosstalk from random bipolar word generators is added to the input of a test amplifier to determine its effect on error rate. The disturbing word generators have independent random pulse train outputs; a typical one is shown in ideal form in Fig. 3. Each generator is timed by its own crystal oscillator. The signal word generator of Fig. 6 can be set to give a fixed repetitive pattern or a pseudo-random pattern in which every eighth position and the position following it are, respectively, a forced mark and a forced space. Patterns

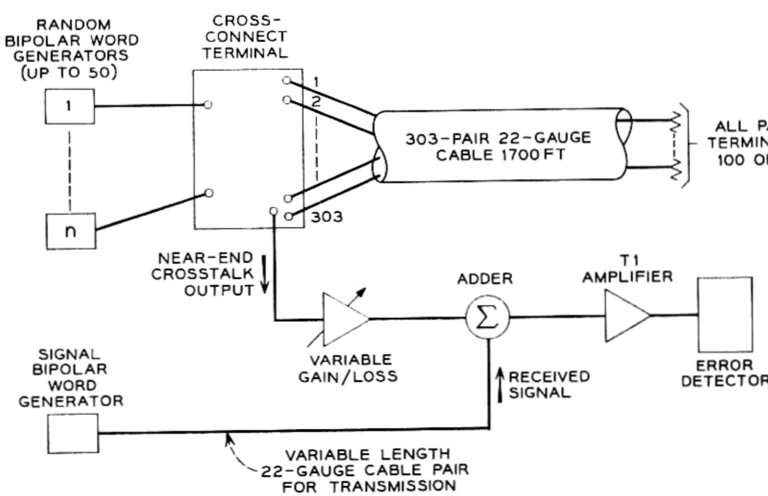

Fig. 6 — Laboratory setup for near-end crosstalk tests.

with very long runs of spaces must be avoided, as they may lead to self-oscillation in the receiving amplifier.[3] In the present paper, our conclusions are not significantly affected by the choice of pattern for the received signal.

In Appendix A, a theory is developed for predicting the interfering power that arises when n systems operate in a cable. A knowledge of the mean value of this power, and of its standard deviation as we select combinations of n pairs, allows us to develop line engineering procedures, as is done in Section IV. As further pointed out in Appendix A, the laboratory tests have shown that the near-end crosstalk interfering voltage has a Gaussian amplitude distribution, out to several standard deviations.[15] This is the reason we may calculate error rates in Section IV using only the rms value of the interference.

It is largely a result of experience with frequency-division carrier systems that many measurements of crosstalk losses in multipair cables have been made at selected frequencies in the band of interest. The near-end crosstalk coupling loss between typical cable pairs varies irregularly with frequency, as illustrated in Fig. 7. In fact, measurements across the frequency band on one pair combination are almost valueless; the statistical properties must be found by measuring a large number of pair combinations.[9] In Table II, we have summarized data available on coupling losses at 772 kc in cables with staggered pair twists. Losses are given for pairs in 100-pair splicing groups, to correspond to pair selections in actual cables planned for T1 use. Using these data with the theory of Appendix A, as explained in Section 4.2, line engineering tables have been compiled to aid in the layout of one-cable systems.

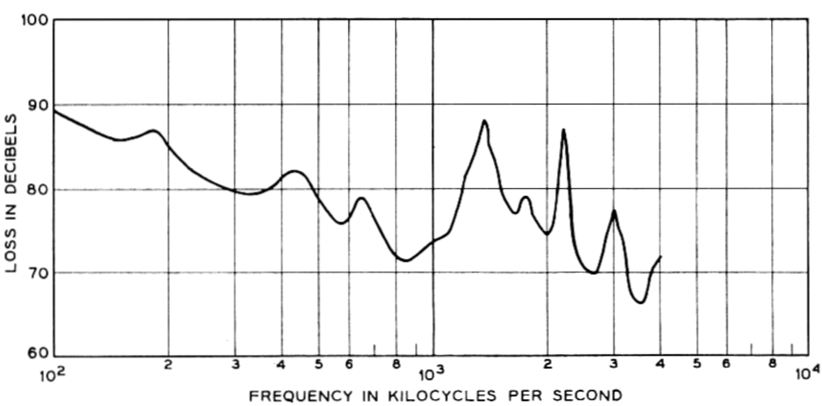

Fig. 7 — Near-end crosstalk loss vs frequency.

TABLE II — NEAR-END CROSSTALK COUPLING LOSSES AT 772 KC FOR 22-GAUGE CABLES WITH STAGGERED PAIR TWISTS, 100-PAIR SPLICING GROUPS

Cable Construction	Mean, db	Standard Deviation, db
Unit		
Pairs in same group	82	11
Pairs in adjacent groups	90	9
Pairs in alternate groups	103	7
Layer		
Pairs in same group	75	9
Pairs in adjacent groups	83	8

These data apply, without correction for length, to sections at least 1000 feet long. The data were taken on sections about 6000 feet long.

3.3.2 *Far-End Crosstalk*

Far-end crosstalk coupling paths are always present among cable pairs in the same transmission direction. Three situations, illustrated in Fig. 8, are critical in T1 engineering. In Fig. 8(a), there are lines for the same direction of transmission in which the amplifiers are slightly misaligned. This may happen in one- or two-cable operation when repeater cases are spliced into the main cable at different points along the main cable. In Fig. 8(b), a greater misalignment is shown. This can occur when systems are added along a two-cable route, and there is not enough space at existing repeater locations to accommodate the additional repeater cases. It may be necessary, for example, to use the next manhole along the line for an underground installation. In parts (a) and (b) of the figure, the most damaging far-end crosstalk exposures, shown by arrows, are in the cable between an amplifier output and the nearest amplifier input. In Fig. 8(c), we have a junction, where lines from different systems enter the same cable; the figure shows only one direction of transmission. If L_A and L_B are unequal, say $L_B > L_A$, signals from the "A" amplifier output interfere more seriously with the "B" system than vice versa, by way of the coupling path shown.

The theory of far-end crosstalk is developed in Appendix B. As in the near-end case, we find the statistical properties of the interfering power when n systems are present in a cable. The assumption that the crosstalk voltage has a Gaussian amplitude distribution again allows us to estimate error rates based on interference power. The theory is applied to the layout of two-cable systems in Section 4.3, and to junctions in Section 4.4.

3.4 *Central Office Noise*

In a telephone switching office, a variety of devices and circuits create transient and repetitive currents with energy spread over a wide frequency band. The more common sources are relays and switches that interrupt direct or alternating currents, rectifier power supplies and ac power wiring, and lines carrying ringing signals and other plant and test tones. Carrier wiring inside the office, e.g., from main distributing frame to carrier equipment bays, can be isolated from these disturbances by shielding and by physical separation. Office noise can also find its way into carrier lines by secondary induction. That is, voice-frequency pairs within the office are exposed to office noise, and a number of these pairs may enter cables which contain carrier pairs in the outside plant. The noise voltages are coupled from the voice-frequency pairs into the carrier pairs by crosstalk, chiefly near-end and far-end.

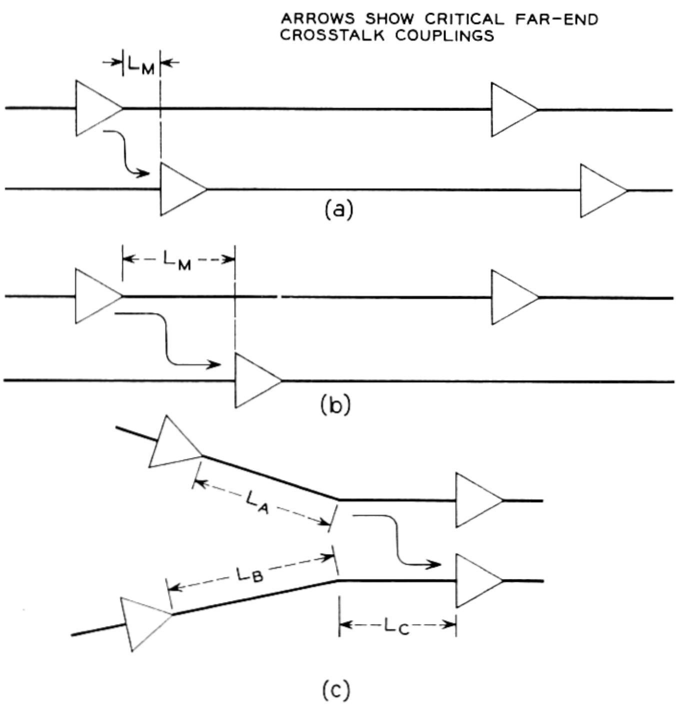

Fig. 8 — Far-end crosstalk situations: (a) sections with repeaters almost aligned, L_M small; (b) sections with greater misalignment of repeaters, L_M large; (c) junction.

Such secondary couplings are important in T1 lines only in the vicinity of central offices, as the noise on the noncarrier pairs, which is effective in the T1 band, is rapidly attenuated further out. In Appendix C some of the properties of office noise are described, and in Section IV a simple application of these facts is made to arrive at line engineering methods.

IV. DETAILED ENGINEERING OF REPEATER SECTIONS

4.1 *Error Rate Objectives for Repeater Sections*

In engineering a repeater section one has to take account of errors due to crosstalk and office noise so as to produce a sufficiently small contribution to the span error rate objective (Section 1.2.2) of 3×10^{-7}, while achieving nearly maximum economy in the use of repeaters.

The error rate caused by a combination of crosstalk and office noise is very nearly the sum of the error rates caused by the two kinds of interference separately (Appendix C). It is therefore possible to set an error rate objective for noise and one for crosstalk, and to find the separate design limits (cf. Section 4.5) that will meet these objectives. We have set the objective for error rate due to office noise in one section at 10^{-7}; assuming two sections in a span to be fully exposed to noise, this assigns two-thirds of the span error rate. The remainder is assigned to crosstalk. Because of the extreme sensitivity of error rate to small differences in crosstalk loss or section loss, error rates due to crosstalk will vary radically from one section to the next. The section objective is an error rate of 10^{-7} for the worst one per cent amplifier. This means that in a five-section span about 95 per cent of the lines will meet the span objective. This is a reasonable performance level. Marginal lines may be set aside for use as spares, and the investment in still poorer lines may be salvaged by reusing the repeaters elsewhere and returning the cable pairs to voice-frequency use.

4.2 *One-Cable Systems*

4.2.1 *Design Section Loss*

We have mentioned in Section 3.3.1 that one-cable installations are subject to near-end crosstalk effects. We must limit the number of systems installed in a given cable, or the repeater section loss, or both, to stay within the error rate objective of Section 4.1. The quantitative limits are obtained as follows.

Suppose a repeater section has cable loss L db. Then the received pulse height at the preamplifier output is equivalent to $S + G - L$ dbm,* where S is the peak value of the 772 kc signal component at the transmitting amplifier output (cf. Section 2.2) in dbm, and G is the preamplifier gain at 772 kc. Let P_ϵ be the instantaneous noise power in dbm which is exceeded with probability ϵ. Then, for a required signal-to-noise ratio of A db (cf. Section 2.1), the error rate will be ϵ when $S + G - L - A - P_\epsilon = 0$. For near-end crosstalk, the interfering voltage has a Gaussian amplitude distribution (cf. Section 3.3.1). If its rms value for n interfering systems is equivalent to P_n dbm, then

$$P_\epsilon = P_n + 20 \log y(\epsilon) \text{ dbm} \tag{1}$$

where $y(\epsilon)$ is the (positive) quantity such that

$$\epsilon = (2\pi)^{-1/2} \int_y^\infty \exp\ (-t^2/2)\ dt. \tag{2}$$

Actually, S, L, A, and P_ϵ are statistical quantities. The mean value of S is $\bar{S} = 10 \log (2.7)^2/100 + 30 = 18.6$ dbm, and its standard deviation is $\sigma_S = 0.5$ db, owing to variations in the amplifier output circuits. The mean pair loss, $\bar{L}$, is a design parameter to be found here; we use a standard deviation $\sigma_L = 0.6$ db for $\bar{L}$ about 30 db. The mean amplifier signal-to-noise ratio $\bar{A}$, as mentioned in Section 2.1, is 12 db, with $\sigma_A = 1.0$ db. As to P_ϵ, its mean value, introducing the mean $\bar{P}_n$ as in (34), is

$$\begin{aligned} \bar{P}_\epsilon &= \bar{P}_n + 20 \log y(\epsilon) \\ &= 48 - m + \sigma + 10 \log (n/25) + 20 \log y(\epsilon) \end{aligned} \tag{3}$$

where m and σ are the mean and standard deviation, respectively, of the near-end crosstalk coupling losses at 772 kc, in db. The standard deviation of P_ϵ, which is the same as that of P_n, is underestimated by the theory of Appendix A. A value of $\sigma_{P_n} = \sigma_{P_\epsilon} = 3.2$ db has been determined experimentally.[15]

Accordingly, we now define another statistical quantity, the *margin at error rate* ϵ,

$$M_\epsilon = S + G - L - A - P_\epsilon . \tag{4}$$

Substituting in (4) the values given, with $G = 23.7$ db, and $y = 5.2$ for $\epsilon = 10^{-7}$ as in (2), we find

* dbm = decibels relative to one milliwatt.

$$\begin{aligned}\overline{M}_\epsilon &= 18.6 + 23.7 - \bar{L} - 12 - 48 + m - \sigma - 10 \log (n/25) \\ &\qquad - 20 \log 5.2 \\ &= m - \sigma - \bar{L} - 10 \log (n/25) - 32.\end{aligned} \tag{5}$$

Also,

$$\sigma_{M_\epsilon}^2 = \sigma_S^2 + \sigma_L^2 + \sigma_A^2 + \sigma_{P_\epsilon}^2 = (0.5)^2 + (0.6)^2 + (1.0)^2 + (3.2)^2$$

or

$$\sigma_{M_\epsilon} = 3.4 \text{ db}, \tag{6}$$

the standard deviation of M_ϵ. The random variable M_ϵ is a measure of system performance which depends upon the statistical variations of signal level, pair loss, required amplifier signal-to-noise ratio, and near-end crosstalk interference. If, for a particular choice of the variables on the right side of (4), M_ϵ is positive, the amplifier operates at an error rate less than 10^{-7}; if $M_\epsilon = 0$, the rate is 10^{-7}; and, if M_ϵ is negative, the rate exceeds 10^{-7}. Assuming that M_ϵ has a normal distribution, 99 per cent of the amplifiers will operate with error rate less than $\epsilon (= 10^{-7}$ here) provided that

$$\overline{M}_\epsilon - 2.33\sigma_{M_\epsilon} = \overline{M}_\epsilon - 8.0 \geqq 0 \tag{7}$$

using (6). If we have equality in (7), i.e., if we place the one per cent point of the cumulative distribution of M_ϵ just at zero db, we will meet the error rate objective. Then from (5) and (7), rearranging, we have

$$\bar{L} + 10 \log (n/25) = m - \sigma - 40. \tag{8}$$

This equation gives upper bounds on both $\bar{L}$ and n. That is, the mean section loss, $\bar{L}$, and the number of systems, n, must both be kept low enough so that the left side of (8) does not exceed the right side. If it does, we violate the error rate objective. Conversely, we may regard (8) as placing a lower bound on $m - \sigma$, derived from the properties of the cable, for a given $\bar{L}$ and n.

As an example of the use of (8), assume $n = 200$ and 22-gauge unit cable, with pairs for the two transmission directions in adjacent splicing groups. Although Table II gives $m = 90$ db and $\sigma = 9$ db, we shall use the more conservative value $m = 84$ db. With this 6-db allowance we recognize that crosstalk measurements at 772 kc have been made on only a few of the many sizes and types of cable in the plant, and that some cables may have lower crosstalk losses than given in the table. For $n = 200$ systems, (8) shows that $\bar{L}$ must not exceed 26 db. If we accept this value of loss at 100 F, the loss at 55 F is 25 db. The latter figure, then, is the *design section loss* for this type of 22-gauge cable. That is, it is the maximum loss allowable for one-cable operation with 200 systems. For an engineering loss (Section 2.2) of 27 db per mile, a length of 4900 feet has 25-db loss, so this is the maximum distance allowable between repeater points for such an installation.*

We do not have data comparable to Table II for cable gauges other than 22. To estimate the near-end crosstalk losses for other gauges, we note that the near-end crosstalk coupling ratio varies, for the lengths and frequencies involved here, directly with the attenuation in db (Appendix A). For example, to estimate m for 19-gauge cable with average loss 19.1 db per mile, as in Table I, we use the corresponding m in Table II, and add $10 \log (19.1/26.5) = -1.4$ db. For 19-gauge layer cable, adjacent splicing groups, then, $m = 83 - 1.4 = 81.6$ db. Lacking more specific information, we may assume the standard deviation figures in Table II to hold for other gauges.

A value of $\bar{L}$ from (8) which leads to a design section loss greater than 32.2 db simply means that the section loss may be 32.2 db, as we cannot exceed this value in any circumstances (Section 2.2). Using (8), with this limitation and with the correction for gauge described above, engineering tables have been prepared that give the design section losses for the various types of cable to be found in the field, for any number of systems.

4.2.2 *Selection of Cables and Pairs*

We would like to be free to assign systems to cable pairs as they are available along spans, without selecting individual pairs within the cable sections. Such assignment is possible with voice-frequency circuits, but T1 systems need more careful consideration. For one-cable operation, the two directions of transmission must be assigned to groups of pairs with a satisfactory range of near-end crosstalk coupling losses. More exactly, we wish to be sure that the mean loss and the standard deviation of losses are in accordance with Table II, or better,† in designing sections according to (8).

In large cables of unit construction, pairs are stranded into units of

* Amplifiers in one-cable sections are also exposed to far-end crosstalk from coupling paths among the pairs in the same transmission direction. To simplify the exposition in this section, the far-end crosstalk interference power, which adds to that due to near-end crosstalk, has been neglected. The far-end power may be calculated using the results of Section 4.3. When this is done, the result is equivalent to reducing the 6-db allowance slightly.

† I.e., a greater mean loss, or smaller standard deviation.

25 or 26 pairs for 19-gauge, 50 or 51 pairs for 22-gauge, and 100 or 101 pairs for 24- and 26-gauge cables. The units are identified by position and color coding. Measurements[13] on a 900-pair, 22-gauge cable with paper pulp insulation, as in Fig. 9, show that the mean near-end loss at 750 kc for pairs in adjacent units, e.g., one pair in unit 1, the other in unit 2 of Fig. 9, is 13 db greater than for pairs within the same unit. The mean loss for pairs in alternate units, such as units 1 and 3 of Fig. 9, is 30 db greater than the within-unit value, and even greater differences are possible for pairs in, say, diametrically opposite units of a cable of this size. The standard deviations of the losses are not so dramatically changed by pair location, but are usually smaller for the more widely separated units. Similar remarks apply to cables of layer construction.

The common splicing group in large multipair cables is 100 pairs, i.e., two 50-pair units or four 25-pair units. For orderly administration of the outside plant, T1 pairs are assigned by splicing groups. There are three principal ways in which pairs for the two directions of transmission may be chosen, as given in Table III. The table also contains examples of the three configurations that apply to the unit cable of Fig. 9. If pairs for both directions are in the same splicing group, the design section losses from (8) are undesirably low. Placing the two transmission directions in adjacent groups increases the near-end crosstalk loss, and a reasonable number of systems can be operated in the same sheath with full repeater spacing. Going over to alternate-group separation,

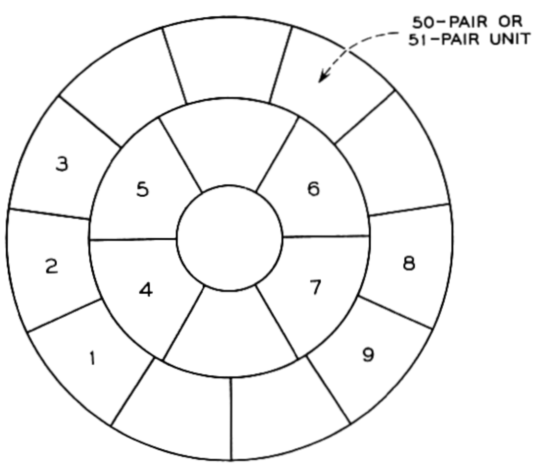

Fig. 9 — 900-pair, 22-gauge unit cable.

TABLE III — ASSIGNMENT OF SPLICING GROUPS IN ONE-CABLE OPERATION

Short Description	Explanation	Example for Unit Cable, Fig. 9		Design Section Loss in db, 22-gauge Unit Cable		
		E-W Pairs in Units	W-E Pairs in Units	25 Systems	50 Systems	100 Systems
Same group	Pairs for both directions are in the same 100–pair splicing group.	1 & 2	1 & 2	24.0	21.1	18.2
Adjacent group	Pairs for the two directions are in separate groups that are next to one another.	1 & 2	4 & 5	32.2	30.7	27.9
Alternate group	Pairs for the two directions are in separate groups that have no units or layers touching.	1 & 2	6 & 7 or 8 & 9 etc.	32.2	32.2	32.2

there is a further increase in near-end crosstalk loss, and in the allowable number of systems. Therefore, to accommodate more systems in a given sheath, we may reduce repeater spacings or assign the systems to groups of pairs with greater separation.

It is not always possible, merely by examination of the usual outside plant records, to be sure that pair separation is maintained over a span length. In specific instances, especially where older cables are involved, it may be necessary to open some splices to determine the splicing pattern and to make rearrangements. If enough measurements for statistical validity can be made, it may be helpful to measure the near-end losses at or near 772 kc. In placing new cables where it is likely that T1 systems will be installed, modern splicing practices will be followed, including complete color-coded pair-to-pair splicing, as is possible with polyethylene-insulated cables.[11] This will generally make rearrangements or measurements unnecessary.

4.2.3 *Staggered-Repeater Operation*

Situations arise in planning T1 installations where one-cable operation is needed, but the crosstalk properties of the cable are not up to the requirements outlined above. For example, it may be doubtful if the pairs are in separate 100-pair splicing groups and inconvenient to open up enough splices to find out. In such cases, *staggered-repeater* *operation*, as shown in Fig. 10, may be possible. Pairs for both transmission directions are in one cable, but the repeater cases are installed at staggered points so as to reduce near-end crosstalk interference. For example, the outputs of the amplifiers at repeater location A no longer face the inputs of those at B directly through the near-end crosstalk paths, but are first attenuated by L_1 db, the loss between the two locations. In effect, the mean near-end crosstalk coupling loss is increased by L_1, and we are able to meet the interference objective, even though the same cable is unsuitable for one-cable operation as in Fig. 2(a).

The optimum layout for staggered-repeater operation is that in which all losses L_1, L_2, L_3, etc. are exactly the same and are equal to half the design section loss. In a practical layout, it may not be possible to locate repeater cases in this way. Suppose the design section loss for one-cable use, without repeater staggering, is D db. If we require that each loss L_1, L_2, etc. (Fig. 10) be at least 10 db, then we have a minimum near-end loss improvement of 10 db. The design loss D_s, between amplifiers in the same direction, is thus $D + 10$, or, if this number exceeds 32.2, $D_s = 32.2$. Of course, L_1, L_2, etc. must each be less than $D_s - 10$.

At a constant value of design section loss, more systems may be installed in a given cable with staggered-repeater operation than in the usual one-cable mode.

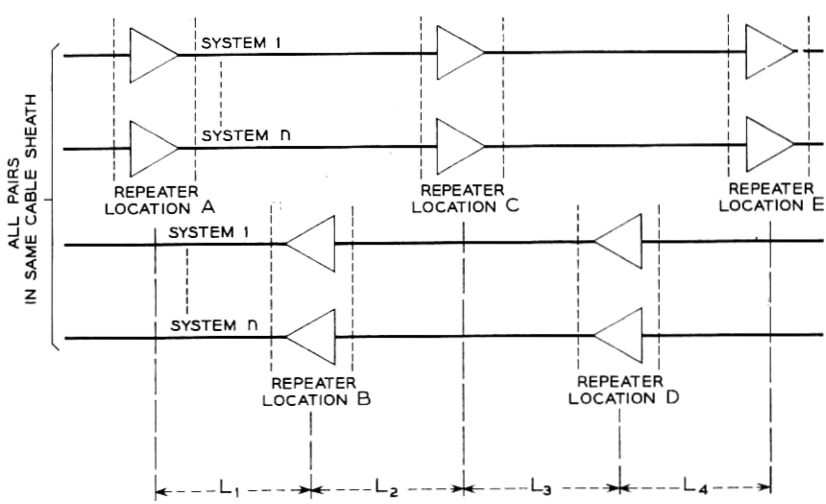

Fig. 10 — Staggered-repeater operation.

4.3 *Two-Cable Systems*

The general far-end crosstalk exposure of Fig. 11 is analyzed in Appendix B. The mean interfering power, $\bar{Q}_n$, at the preamplifier output of amplifier 2, due to crosstalk from amplifier 1, is given by (47), and the standard deviation of this power, $\sigma_{Q_n} = \sigma_{j,n}$ by (44). By selecting the values of L, L_1 and L_2, we may determine limiting conditions for various layouts.

For example, let $L = L_1 = 32.2$ db, and $L_2 = 0$. This is the far-end coupling arrangement in an ordinary repeater section of maximum loss, with repeater locations aligned. Let us further assume that we have paper-insulated, 22-gauge cable with the properties listed in Table IV (Appendix B). As a worst case, suppose that there are 50 systems in the same cable unit, all but one of which may be regarded as interfering with the remaining one, or, closely, $n = 50$. The mean interfering power $\bar{Q}_{50}$, and its standard deviation, $\sigma_{Q_{50}}$, are calculated in Appendix B as -33.2 dbm and 5.2 dbm respectively.

Our calculation of error rate due to this interfering power is quite similar to that for near-end crosstalk, Section 4.2.1. As in (1), the power level exceeded by the interfering voltage with probability $\epsilon = 10^{-7}$, assuming again that the interference has a Gaussian amplitude distribution, is $P_\epsilon = Q_n + 20 \log 5.2 = Q_n + 14.3$. The mean over various selections of n crosstalk couplings is thus $\bar{P}_\epsilon = -33.2 + 14.3 = -18.9$ dbm. We may find the mean margin $\overline{M}_\epsilon$ by substituting in (4) the values $\bar{S} = 18.6$, $G = 23.7$, $L = 32.2$, $\bar{A} = 12$, and $\bar{P}_\epsilon = -18.9$; the result is $\overline{M}_\epsilon = 17.0$ db. For the standard deviation of M_ϵ, we take only that due to P_ϵ, or 5.2 db, as contributions from the variances of S and A are small. Thus 99 per cent of the amplifiers will have a margin at least

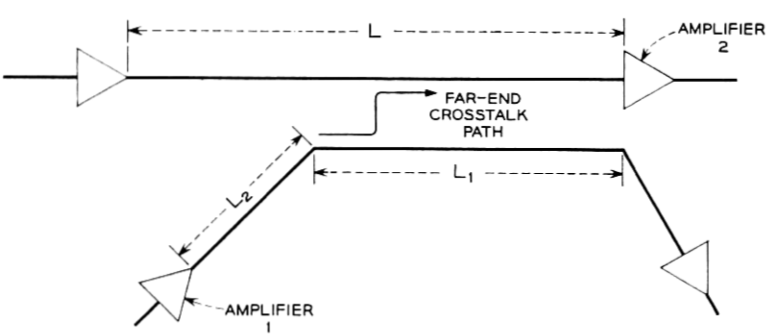

Fig. 11 — General far-end crosstalk exposure.

$\overline{M}_\epsilon - 2.33 \times 5.2 = 4.9$ db, again assuming a Gaussian distribution for M_ϵ.

The effect of far-end crosstalk is more severe for an equivalent exposure in 19-gauge cable of lower loss per mile. Specifically, if we assume that the cable loss is 15.9 db per mile, and also that its crosstalk properties are the same as those used for the 22-gauge cable, the 99 per cent margin is 2.9 db for a 32.2-db section.

Neither of these values is quite as favorable as the 6-db reserve margin called for in our near-end crosstalk calculations (Section 4.2.1). As the 32.2-db section of 19-gauge cable, more than two miles in length, is rather an extreme case, we conclude that the line loss of ordinary repeater sections for one-or two-cable operation is not seriously limited by far-end crosstalk.

The calculation above relates to far-end interference within a single group of 50 systems in one cable unit. A different situation is shown in Figs. 8(a) and (b). Here we have far-end crosstalk between two groups of systems in two-cable operation, when the repeater locations for the two groups are different. This may come about, for example, when the number of repeater cases at each location is limited by manhole space, for underground systems. In Figs. 8(a) and (b), we have far-end paths between one set of amplifier outputs, and the inputs of another set, separated by a line section of loss L_M. To apply the theory, we allow L_1 of Fig. 11, which corresponds to L_M of Figs. 8(a) and (b), to vary from small values to 32.2 db, and again find the 99 per cent margin, $M_{99} = \overline{M}_\epsilon - 2.33\, \sigma_{j,n}$, for $L_2 = 0$ and $L = 32.2$ db. For purposes of illustration, we select the cable properties as follows. A 600-pair, 22-gauge unit cable may have 400 systems in outside units surrounding 200 systems in interior units, in two-cable operation. We therefore use $n = 400$ interfering systems. The other cable parameters are taken from Table IV, choosing the adjacent-unit value, $m_{j_0} = 77$ db. This is pessimistic, since many of the far-end crosstalk coupling paths are between more widely separated pairs, in this instance. When we calculate M_{99} on this basis, we find a variation with L_1 as shown in Fig. 12.

For low values of L_1, Fig. 11, the output of amplifier 1 is coupled to the input of amplifier 2 by a short far-end crosstalk path. This corresponds to the situation of Fig. 8(a), with L_M small. We see from Fig. 12 that L_1 must be less than 0.05 db in order to have M_{99} at least 6 db. We conclude that splices of apparatus cases to the main cable should be within 10 feet. Greater distances are tolerable for numbers of interfering systems less than 400.

The curve of Fig. 12 clearly shows that higher values of L_1 less than

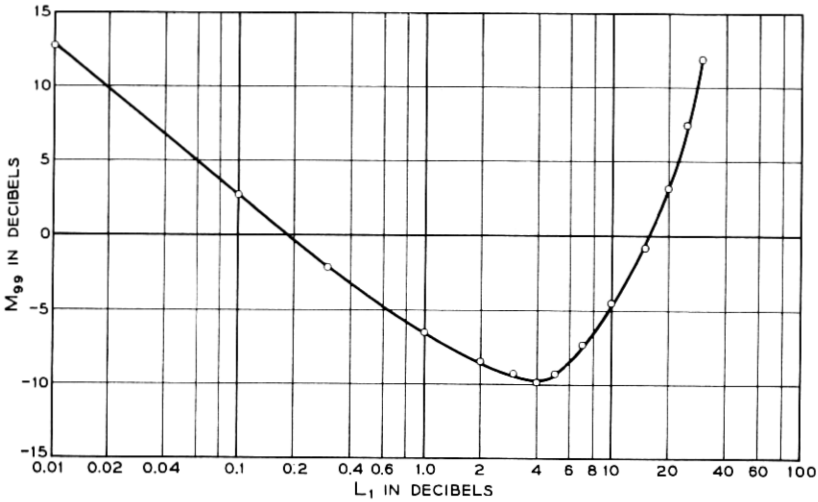

Fig. 12 — M_{99} versus L_1.

24 db will not provide the necessary margin. For sections spaced as in Fig. 8(b), then, we find that the interfering amplifier output must be at least 24 db from the disturbed amplifier input. This requirement cannot be met for sections limited to a maximum loss of 32.2 db. The reason is that interference in the opposite direction, i.e. from the 200-system group into the 400-system group in our example, also requires a minimum spacing, which we can estimate to be about 3 db less, or 21 db. This leads to a 45-db section loss. We conclude that misalignment of sections as in Fig. 8(b) is ruled out. More lenient spacing requirements result for cables of higher loss per mile, but the amount by which sections may be offset is still controlled by far-end crosstalk.

4.4 *System Junctions*

In an extensive area with T1 systems connecting several central offices, junctions such as those in Fig. 13 will occur. In the figure, systems connect office A with office C, and C with B, but for simplicity no through systems from A to B are included. The controlling far-end crosstalk exposures are in the cable sections incoming to amplifiers in office C, i.e., in cable section 3 for the two-cable case, Fig. 13(a), and in the incoming pairs of cable section 9 for the one-cable case, Fig. 13(b).

Assume in Fig. 13(c) that $L_B > L_A$, and that there are n interfering systems in the A branch which join systems from the B branch at the

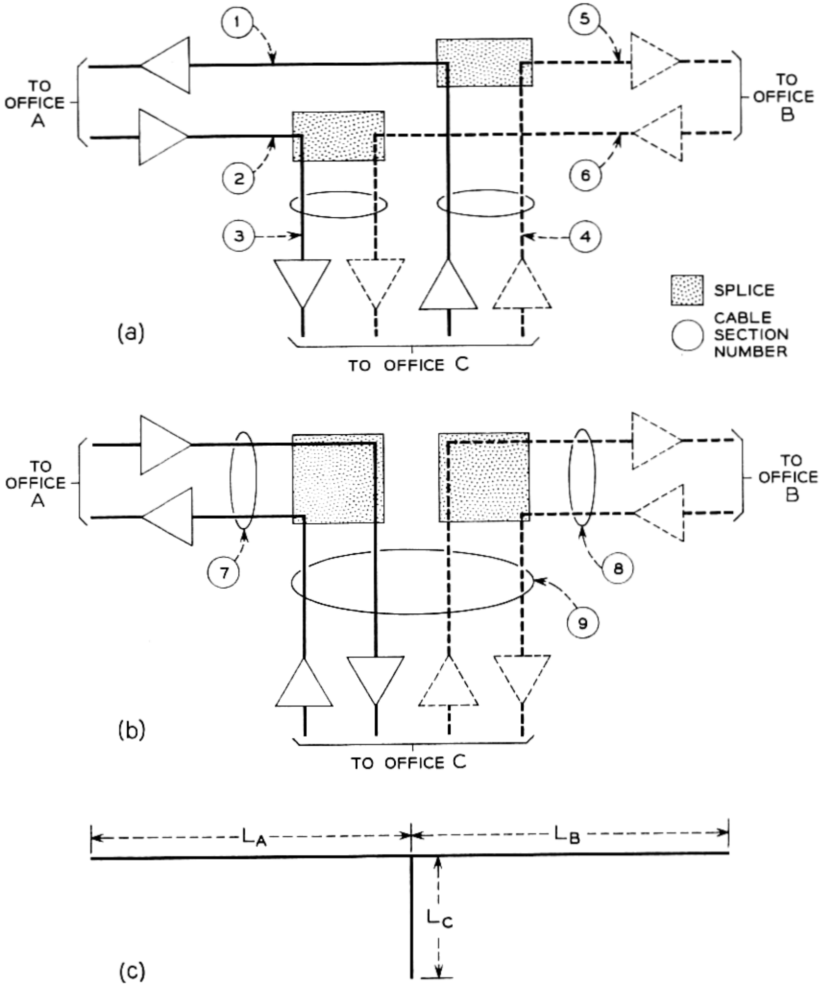

Fig. 13 — System junctions: (a) junction of two-cable systems; (b) junction of one-cable systems; (c) schematic showing losses.

junction. The far-end exposure has loss L_C, which corresponds to L_1, Fig. 11. For L in Fig. 11 we have $L_B + L_C$, the loss in the signal path of the disturbed systems. For L_2 of Fig. 11 we have L_A, the loss in the disturbing pairs between the disturbing amplifiers and the junction.

The 99 per cent margin M_{99} is again given by

$$M_{99} = \overline{M}_\epsilon - 2.33\, \sigma_{M_\epsilon} = \overline{M}_\epsilon - 2.33\, \sigma_{j,\,n}$$

where j is the number of cable reels in the exposure, or C, section. We may find $\overline{M}_\epsilon$ as in (4), with $\bar{S} = 18.6$, $G = 23.7$, $L = L_B + L_C$, and $\bar{P}_\epsilon = \bar{Q}_n + 14.3$ as before. The necessary value of $\bar{Q}_n$, the mean far-end disturbing power in dbm, is obtained from (47) for $j = 1, 2, 3, \ldots$ reels, or from (48) when the exposure section is shorter than one reel in length. When the required substitutions are made, again using the data of Table IV, and defining Δ as the maximum loss difference $L_B - L_A$ which will keep M_{99} at least 6 db, we find that Δ varies with L_C, the exposure section loss, as in Fig. 14. The number of systems, n, is a parameter in this figure. The curves are plotted on the assumption that systems entering the junction from the A side have at least adjacent-unit separation in the exposure section from systems entering from the B side, i.e. a mean equal-level far-end crosstalk loss at 772 kc, m_{j_0}, of 77 db is used.

The value of Δ from Fig. 14 is the maximum permitted when far-end crosstalk is controlling, e.g., for an exposure as in cable section 3, Fig. 13(a). Other limits must also be taken into account. For example, suppose $n = 200$ systems are in the A branch, and the exposure length is $L_C = 10$ db. The value of Δ from Fig. 14 is 13.4 db. If $L_A = 12$ db it would seem that L_B might be as much as $12 + 13.4 = 25.4$ db. But the

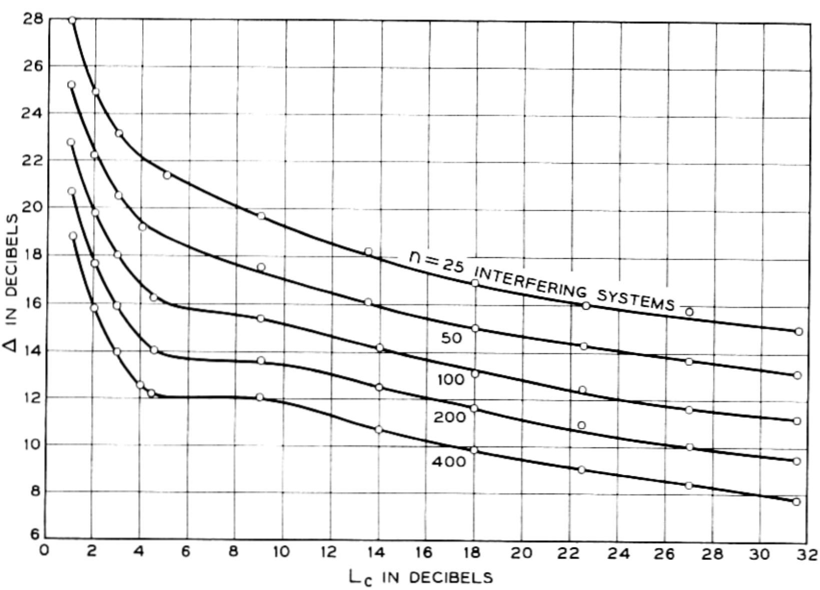

Fig. 14 — Δ versus L_C, 22-gauge cable.

total loss of the disturbed section, $L_B + L_C$, would then be $25.4 + 10 = 35.4$ db, more than the maximum possible loss of 32.2 db. For one-cable operation with design section loss less than 32.2 db, the restriction is based on this loss rather than 32.2 db; further, the margin M_{99} is affected by both near-end and far-end crosstalk interference power. Similarly, if the junction forms an entrance into a central office, the section losses $L_A + L_C$ and $L_B + L_C$ are limited as described in Section 4.5.

The curves of Fig. 14 may be used for mean far-end losses other than 77 db by adding $m_{j_0} - 77$ db to the indicated values of Δ, since Δ varies directly with this loss. For example, if there are 25 systems entering a junction of exposure length $L_C = 10$ db, and they are spliced into the same cable unit as the disturbed systems, we may estimate $m_{j_0} = 63$, so that $\Delta = 19.2 + 63 - 77 = 5.2$ db.

4.5 *Sections Near Central Offices*

Repeater sections that terminate in central offices, and others in which switched telephone pairs share a cable sheath with the T1 lines, are subject to office noise interference (Section 3.4). The fundamental information for engineering these sections is in Appendix C. As shown there, the signal-to-noise ratio, and hence the error rate, of an amplifier is determined by the difference between two losses. The first of these is the loss in the path by which the noise reaches the amplifier input. It is convenient to imagine that the noise originates at the office termination of the switched voice-frequency pairs. Thus, the noise path loss is a sum, in general, of a crosstalk coupling loss and a direct transmission loss. For the latter, it is adequate to substitute the loss at 772 kc of the pairs over which the noise travels. The second loss we need is that in the signal path, exclusive of the buildout network loss (Section 2.2), office pad loss (Section 2.3), and office cable loss. This is simply the cable loss incoming to the amplifier. In Fig. 20, error rate is plotted as a function of the difference in noise and signal path losses. An error rate of 10^{-7} is attained if this difference is at least 52 db. As explained in Appendix C, the error rates of Fig. 20 prevail under very severe noise conditions, so that we adopt this objective.

Fig. 15(a) shows the incoming T1 pair in a simple entrance section. For the office amplifier, the signal path loss is L_1, the pair loss between the nearest outside amplifier and the main frame. The noise path loss, for the near-end crosstalk coupling path A, may be taken as 75 db (see Appendix C). We therefore must have $75 - L_1 \geqq 52$, or $L_1 \leqq 23$ db. For the first outside amplifier, the signal path loss is L_2, and the noise

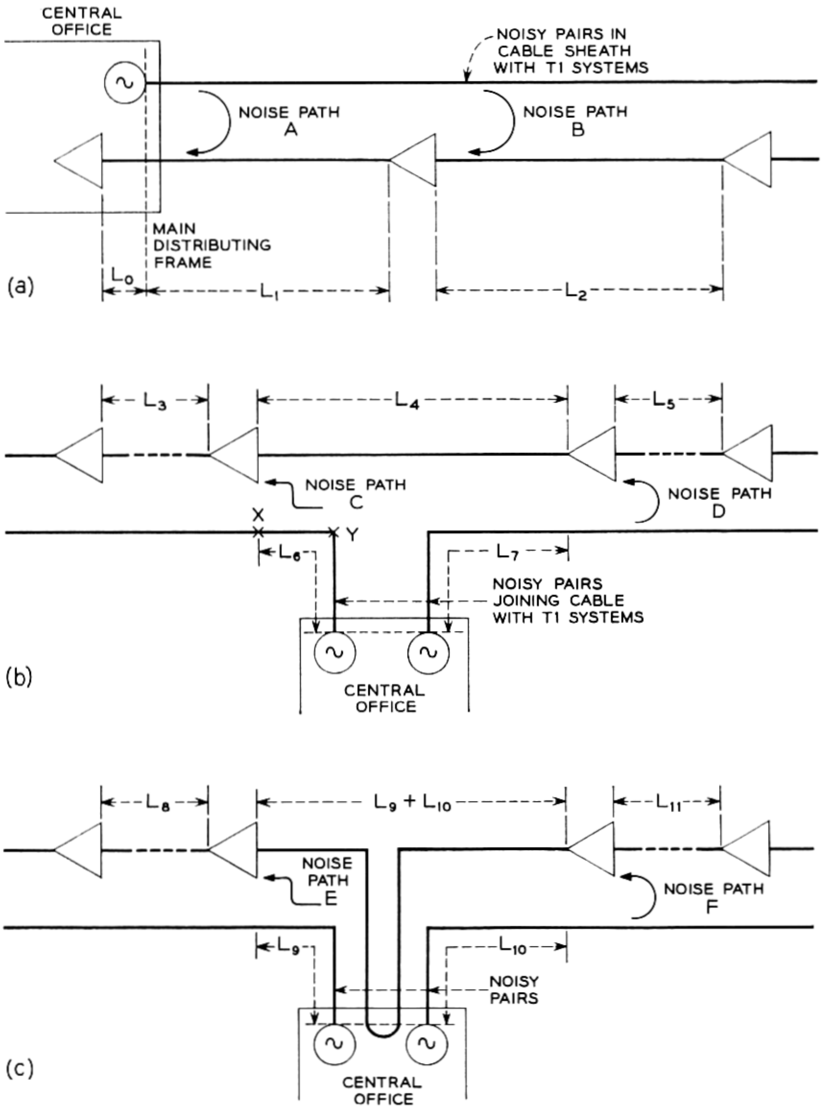

Fig. 15 — Repeater sections near central office: (a) T1 systems entering office; (b) T1 systems not entering office; (c) T1 pairs cross-connected at office main distributing frame.

path loss is $L_1 + 75$, for the near-end path B. We thus have $L_1 + 75 - L_2 \geqq 52$, or $L_2 \leqq L_1 + 23$. Notice that L_1 must be less than 9.2 db before any reduction in L_2 is needed, as L_2 cannot exceed 32.2 db. Sections farther from the office are not affected, owing to the increasing loss in the noise path.

In Fig. 15(b), one direction of T1 transmission is shown in an arrangement where the T1 lines do not enter the office, but are exposed to disturbing pairs which do enter it. The amplifier operating with signal loss L_4 has a total noise path loss, for path C, of $L_6 + \bar{L}_f$, where $\bar{L}_f$ is the mean equal-level far-end crosstalk coupling loss at 772 kc for the cable section between points X and Y in the figure. [The distance from X to Y in feet may be substituted for l in (49), to approximate $\bar{L}_f$.] The losses L_4 and L_6 must satisfy the relation $L_6 + \bar{L}_f - L_4 \geqq 52$. For near-end path D, we have a noise path loss of $L_7 + 75$, for the amplifier that operates with signal loss L_5, so we require $L_7 + 75 - L_5 \geqq 52$, or $L_5 \leqq L_7 + 23$. For one-cable operation, with oppositely-directed amplifiers at the same points as those in Fig. 15(b), a similar set of inequalities applies, relating to noise paths not shown in the figure.

A third arrangement is that of Fig. 15(c), in which the T1 lines are cross-connected at the main frame, for flexibility in installing a terminal at the office later. For noise path E, we have $L_9 + \bar{L}_f - (L_9 + L_{10}) \geqq 52$, or $\bar{L}_f - L_{10} \geqq 52$; here $\bar{L}_f$ is the far-end coupling loss for the entire L_9 section. For near-end path F, we find $L_{10} + 75 - L_{11} \geqq 52$, or $L_{11} \leqq 23 + L_{10}$. For one-cable operation, a comment similar to that above applies.

In arriving at the section limits due to office noise, we have not included a 6-db safety margin, as was done for near-end crosstalk in Section 4.2.1. The reason is apparent from Fig. 16, in which curves of error rate vs section loss are shown for (a) an end section, as in Fig. 15(a), designed with $L_1 = 23$ db, and (b) a one-cable section for which the design loss is 23 db. In the latter case, the error rate on the curve is that of the one per cent poorest amplifier; notice that this amplifier will not have an error rate greater than 10^{-7} until the section loss is 6 db more than the design section loss, or 29 db. The error rate due to crosstalk increases catastrophically beyond the 6 db margin. Since a section error rate of 10^{-6} is probably tolerable, while an error rate of 10^{-5} is not, there is as great a margin of safety for noise as for crosstalk. The steep slope of the curve of crosstalk error rate vs section loss in the critical error rate region necessitates considerable care in design.

In one-cable systems, the sections near offices are subject to interfer-

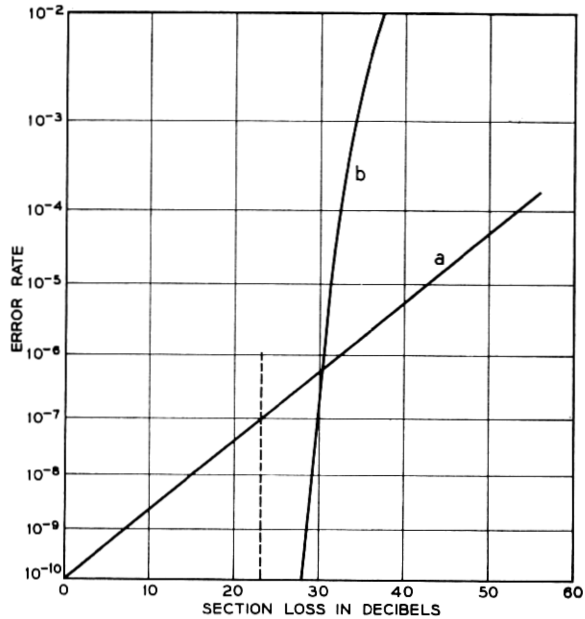

Fig. 16 — Error rates (a) in an office repeater due to switching noise and (b) in a one-cable section with design loss 23 db (poorest one per cent).

ence from near-end crosstalk, as discussed in Section 4.2, and also to office noise. For any such section, two losses may be found: the design section loss for one-cable operation (Section 4.2.1), and the maximum loss permitted by operation in the presence of central office noise. If we always choose the lesser of these two losses, we are assured of meeting the span error rate objective of Section 4.1. This simplifies the design, and we incur only a very small cost penalty in the number of repeaters needed.

The assignment of the T1 lines to cable pairs for either one- or two-cable operation may effectively isolate them from office noise disturbances, and the restrictions above may be relaxed. For example, in Fig. 15(a) the carrier pairs may be part of a splicing group in which the non-carrier pairs do not enter the office, but are cross-connected to outgoing pairs at the main frame. The prime source of office noise in the carrier systems is then coupling from noisy pairs in the same sheath which do enter the office, but are found in a different splicing group, with correspondingly greater crosstalk coupling losses to the carrier pairs. A similar situation occurs when an entire splicing group, say a 50-pair unit, is initially equipped for T1, and the integrity of the group is maintained by unit-to-unit splicing, at least in the repeater sections near the office. Still another favorable case, for a layout like Fig. 15(b), is that in which the T1 pairs are in a group containing pairs spliced to an entrance cable, but to a complement of the entrance cable that does not enter the office. As described in Appendix C, noise measurements in these more favorable situations have shown that the effective crosstalk coupling losses in the noise paths are increased by 10 db. For example, in Fig. 15(a), we would use a loss of 85 db for near-end path A, instead of 75 db. As a result, there is no restriction arising from office noise, since the calculated maximum section loss exceeds 32.2 db.

Recalling the minimum line loss requirement for end sections of 6 db (Section 2.3), we find that these sections will lie in the range of 6 to 23 db, or, for 22-gauge cable, 1200 to 4500 feet. The advantage against office noise is great enough that the carrier wiring inside the office need not be physically separated from other wiring, e.g., by special cable racks or hangers. In most installations, the noise exposure inside will not be as great as that for which we have made allowance, by shortening the sections, in the outside plant.

V. CONCLUSION

In Sections II, III, and IV, we have given in piecemeal fashion the limitations on line layout that arise from the amplifier properties, and those that are needed to meet the error rate objective (Section 1.2.2) in the presence of interference. We conclude with a general description of how a system layout is made, taking into account these limitations.

Let us assume that the local trunk forecasts have shown the numbers of circuits needed and their allotment among various interoffice routes. These routes define the T1 spans. As far as possible, forecasts are made for a long growth period; each span must be designed for the ultimate number of T1 lines it will contain, rather than for the initial number of systems installed. A survey of the cable maps and circuit assignment records will show what cables and pairs are available for carrier use. With the present 25-repeater case, counts of 50 pairs must be cleared in each cable, for a minimum of 50 systems in two-cable operation or 25 systems in one-cable operation.* At this point, the decision as to the type of operation for each span may be made. When cables with assured

* As mentioned in Section 1.2.1, pairs may be equipped with through connectors or load coils instead of repeaters, when they are not needed immediately for carrier systems.

splicing characteristics (Section 4.2.2) are available, one-cable operation is attractive. Otherwise, and especially when very large numbers of systems are to be placed in a span, two-cable operation may be used. Splicing rearrangements and installation of new cable particularly suitable for carrier may also be considered at this stage.

A preliminary layout of each span is now made. Tentative repeater locations are selected so that repeater section lengths fall within the basic limits of 32.2 db, for two-cable operation, or the design section loss (Section 4.2.1) for one-cable. For simple spans directly between offices, section losses near the offices may readily be kept within the minimum and maximum limits of Section 4.5. Some estimates of office cabling losses are made at this point, as the office equipment bays are not yet installed. In two-cable operation, the outgoing section near an office is not limited in the same way as the incoming section, and they may be designed independently. It is quite possible to have the two directions in two-cable systems in different types of cable, or even along different routes.

When junctions occur, some revision of the preliminary layout may be called for, to meet the limits of Section 4.4. A common situation is a junction of two systems near an office, so that the section carrying inputs from both systems is subject to the end-section limits (Section 4.5) as well. Usually there is enough latitude to find a satisfactory layout without changing the fundamental plan. In exceptional cases, staggered-repeater operation (Section 4.2.3) may provide an alternative.

During the early stages of planning, a detailed examination of the cable routes, including manholes (for underground cable) and pole lines (for aerial cable) is essential. The results will show if some repeater locations must be shifted for traffic or other practical reasons. Further, it may be necessary to enlarge some manholes, or build auxiliary manholes, when many repeater cases are to be installed. Again, the necessary layout changes can most often be made well within the design limits.

For the convenience of operating telephone company engineers, the theoretical material developed here has been summarized in tables, e.g. for design section loss (Section 4.2.1). Along with the tables, general procedures for line layout, similar to those above, are given. Experience so far has shown that this information enables these engineers to plan line layouts in a simple and straightforward way, and that the resulting lines meet the transmission objective of Section 1.2.2. Unusual situations will arise in the field, as with any new system, that are not covered by the engineering information already supplied. As more experience is had with such situations, the theory here can be applied to them, and additional experimental data collected, with the goal of making the tables and recommendations more complete.

VI. ACKNOWLEDGMENTS

The experimental arrangements for the study of the effects of multi-system crosstalk on T1 repeaters were set up by W. L. Ross with the assistance of H. S. Piper, Jr., B. W. Boxx, and of D. B. Robinson, Jr., who designed the pulse generators. The experimental study itself was conducted largely by Robinson and R. G. DeWitt. An investigation of office noise which continued over several years involved many persons, but it is appropriate to mention the work of Robinson and of R. A. Gustafson, Jr. In numerous difficulties, we have had the assistance of K. E. Fultz, under whose direction all of this work was done, and to whom several basic concepts are attributable.

Much essential information about the existing telephone cable plant could only have been obtained through discussions with C. S. Thaeler and J. Mallett and others of the Laboratories and with engineers of the American Telephone and Telegraph Company. Finally, it is a pleasure to recall the discussions we have had of analytical methods with S. O. Rice.

APPENDIX A

Near-End Crosstalk

Consider two pairs in a cable section of length l and propagation constant γ, terminated in the characteristic impedance Z_0 at each end. A current I_0 is applied to one pair at one end. Let the mutual impedance unbalance at a distance x from this end be $Z(x)$ per unit length and the admittance unbalance be $Y(x)$. Then it can be shown[16] that the incremental crosstalk current dI on the disturbed pair at the transmitting end, due to an incremental length of cable dx at distance x is given by

$$\frac{dI}{I_0} = -\left[\frac{Z_0 Y}{16} + \frac{Z}{4Z_0}\right] e^{-2\gamma x}\, dx. \tag{9}$$

The mutual impedance unbalance Z is due to inductance unbalance. The admittance unbalance Y is due primarily to capacitance unbalance. To a first-order approximation these effects are independent of frequency. At the frequencies of interest here Z_0 may be assumed to be a constant resistance. Equation (9) may therefore be written

$$\frac{dI}{I_0} = i\omega C(x)\, e^{-2\gamma x}\, dx \tag{10}$$

in which $C(x)$ is a real function of x, independent of frequency, which will be called here the unbalance function.[17] The statistical properties to be ascribed to $C(x)$ are as follows. Consider a large number of similar cables of length l. Each has, among others, two pairs, numbered 1 and 2, which are always similarly located with respect to each other. For each cable there is a $C(x)$ for these two pairs. The following assumptions are made for $C(x)$:

(*i*) When the length l is large enough, any kind of average for $C(x)$ over all values of x for one cable is equivalent to the same average taken at one value of x over all cables.

(*ii*) $C(x)$ is normally distributed in amplitude, with mean zero. Most of the following discussion does not depend on assumption (*ii*). In view of the first assumption, the autocorrelation function $S(r)$ and the power spectrum $G(s)$ may be defined as follows:

$$S(r) = \text{ave}\,[C(x)C(x + r)] \tag{11}$$

$$G(s) = 4 \int_0^\infty S(r) \cos 2\pi sr\, dr. \tag{12}$$

The response $R(\omega)$ is defined as the ratio of the total near-end crosstalk current to the current I_0, and is obtained by integrating (10):

$$R(\omega) = i\omega \int_0^l C(x)\, e^{-2\gamma x}\, dx$$

or

$$R(\omega) = i\omega \int_0^l C(x)\, e^{-2\alpha x}\, (\cos 2\beta x + i \sin 2\beta x)\, dx \tag{13}$$

in which α and β are the attenuation and phase constants respectively. Since β is nearly proportional to frequency at video frequencies, $R(\omega)$ will fluctuate rapidly in a random way with frequency. The in-phase and quadrature components of $R(\omega)$ tend strongly to be normally distributed with mean zero regardless of the distribution of C because they are sums of many random components.

The "average" behavior of $R(\omega)$ is most easily examined by computing the average value of $|R|^2$. Denote this function by $p(\omega)$. Then

$$p(\omega) = \text{ave}\left\{\omega^2 \int_0^l \int_0^l C(x)C(y) \exp\,[-2(\alpha + i\beta)x - 2(\alpha - i\beta)y]\, dy\, dx\right\}.$$

The averaging may be done inside the integral and (11) used to give

$$p(\omega) = \omega^2 \int_0^l \int_0^l S(x-y) \exp\left[-2\alpha(x+y) - 2i\beta(x-y)\right] dy\, dx. \qquad (14)$$

Let $x + y = u$, $x - y = v$. Equation (14) becomes

$$p(\omega) = \frac{\omega^2}{2} \int_0^l \int_{-u}^{u} S(v)\, e^{-2\alpha u - 2i\beta v}\, dv\, du + \frac{\omega^2}{2} \int_l^{2l} \int_{-2l+u}^{2l-u} S(v)\, e^{-2\alpha u - 2i\beta v}\, dv\, du.$$

Making use of the fact that $S(v)$ is even and changing variables in the second integral leads to

$$p(\omega) = \omega^2 \int_0^l (e^{-2\alpha u} + e^{2\alpha u - 4\alpha l}) \int_0^u S(v) \cos 2\beta v\, dv\, du.$$

This can be integrated by parts to give

$$p(\omega) = \frac{\omega^2}{2\alpha} \int_0^l (e^{-2\alpha u} - e^{2\alpha u - 4\alpha l}) S(u) \cos 2\beta u\, du.$$

It will be seen shortly that at video frequencies $S(u)$ is extremely small for values of u large enough to make $e^{-2\alpha u}$ substantially different from unity. Using this fact and (12), we have[17] for l greater than a few feet,

$$p(\omega) = \frac{\omega^2}{8\alpha} (1 - e^{-4\alpha l})\, G(\beta/\pi). \qquad (15)$$

Over the video range β is approximately proportional to frequency and may be replaced by $2\pi f/c$ where c is the propagation velocity in miles per second. The length factor $1 - e^{-4\alpha l}$ is unity for frequencies and spacings prevalent in T1 applications so (15) becomes

$$p(\omega) = \frac{\omega^2}{8\alpha} G(2f/v). \qquad (16)$$

The function $p(\omega)$ has been found empirically by averaging many crosstalk measurements to increase 15 db per decade over the range 100 kc to 10 mc (above the latter frequency its behavior is unknown). Therefore, since α is approximately proportional to the square root of ω, $G(2f/c)$ is nearly constant over this range. This yields some information concerning $S(r)$. This function is required to take its maximum value at the origin.

If we assume the function

$$S(r) = S(0) e^{-k_2 |r|} \qquad (17)$$

then G is given by

$$G(x) = 4S(0) \int_0^\infty e^{-k_2 r} \cos 2\pi x r \, dr$$

$$= \frac{4S(0)k_2}{4\pi^2 x^2 + k_2^2}.$$

Consequently $G(2f/c)$ is given by

$$G(2f/c) = \frac{4S(0)k_2 c^2}{16\pi^2 f^2 + k_2 c^2}. \tag{18}$$

The departure of $p(\omega)$ from the 15-db per decade line is of the order of the measurement inaccuracy, about 1 db. The value of k_2 can be estimated by assuming $G(2f/c)$ falls off 1 db at 10 mc from its value at the origin. Assuming $c = 1.2 \times 10^5$ miles per second, this leads to the result

$$1/k_2 = 5 \times 10^{-4} \text{ miles}. \tag{19}$$

Consequently the correlation function $S(r)$ has decreased to e^{-1} times its value at the origin when r is about 2.6 feet; in other words, the crosstalk unbalances at points further apart than this are essentially uncorrelated. The object of staggering pair twists is to make this correlation range small, i.e., to increase k_2, since as (18) shows, the crosstalk power is inversely proportional to k_2 in the frequency range of interest. It appears that further improvement in near-end crosstalk by this means will be difficult to achieve.

Having found the average behavior of the crosstalk function $|R(\omega)|$ with frequency, it is worth while to look at its amplitude distribution. For the ensemble of cables pictured earlier, the average value of $|R(\omega)|^2$ for a particular pair combination is given by (16), but a given cable will generally have a different value, and the distribution of these values enters into crosstalk calculations. The distribution of $|R(\omega)|$ may be investigated by dealing with the real and imaginary parts of $R(\omega)$, denoted by $X(\omega)$ and $Y(\omega)$, respectively. Let

$$\overline{X^2} = m_{11}, \ \overline{Y^2} = m_{22}, \ \overline{XY} = m_{12}.$$

Since X and Y are normally distributed, the joint probability distribution $p(x,y)$ of X and Y is given by

$$p(x,y) = (2\pi)^{-1}(m_{11}m_{22} - m_{12}^2)^{-\frac{1}{2}} \cdot \exp\left\{\frac{-m_{22}x^2 - m_{11}y^2 + 2m_{12}xy}{2(m_{11}m_{22} - m_{12}^2)}\right\}. \tag{20}$$

From this the desired distribution of $|R| = (X^2 + Y^2)^{1/2}$ can be obtained. From (13), for a long cable section,

$$X = -\omega \int_0^\infty e^{-2\alpha x} C(x) \sin 2\beta x \, dx$$

$$Y = \omega \int_0^\infty e^{-2\alpha x} C(x) \cos 2\beta x \, dx.$$

Multiplication and averaging leads to the results

$$m_{11} = \frac{\omega^2}{2} \int_0^\infty e^{-2\alpha u} \cos 2\beta u \int_0^u S(v) \, dv \, du + (4\alpha)^{-1}\omega^2 \int_0^\infty e^{-2\alpha u} S(u) \cos 2\beta u \, du$$

$$m_{22} = -\frac{\omega^2}{2} \int_0^\infty e^{-2\alpha u} \cos 2\beta u \int_0^u S(v) \, dv \, du + (4\alpha)^{-1}\omega^2 \int_0^\infty e^{-2\alpha u} S(u) \cos 2\beta u \, du$$

$$m_{12} = \frac{\omega^2}{2} \int_0^\infty e^{-\alpha u} \sin 2\beta u \int_0^u S(v) \, dv \, du \, .$$

Substitution of an estimate of $S(r)$ such as is given by (17) and (19) shows that m_{11} is approximately equal to m_{22} and m_{12} may be neglected over the range 100 kc to 10 mc. Equation (20) then becomes

$$p(x,y) = (2\pi m_{11})^{-1} \exp\left[-\frac{x^2 + y^2}{2m_{11}}\right].$$

The probability that the point (X,Y) lies in the differential area $dx\,dy$ is given by $p(x,y)\,dx\,dy$. By making the change of variable $x = r\cos\theta$, $y = r\sin\theta$ the probability density of $|R|$ may be found to be

$$p_1(r) = \frac{r}{m_{11}} \exp\left[-r^2/2m_{11}\right], \qquad (r \geqq 0).$$

The phase angle θ is uniformly distributed over $(0, 2\pi)$.

The discussion of near-end crosstalk may be summarized as follows. For a given pair combination in a reel of cable which is long enough so that $e^{-4\alpha l}$ is small in comparison to unity, the magnitude of the near-end crosstalk response $R(\omega)$ to a one-volt sinusoid of frequency $\omega/2\pi$ has the probability density

$$\text{prob}\ (r < |R| < r + dr) = p_1(r)\,dr = \frac{2r}{p(\omega)} e^{-r^2/p(\omega)}\,dr. \qquad (21)$$

Over the frequency range 100 kc to 10 mc, $p(\omega)$ for multipair cables is given quite accurately by

$$p(\omega) = k\omega^{\frac{3}{2}} \tag{22}$$

in which k is a constant which varies from one combination of two pairs to another.

The statistical properties of the values of k corresponding to the many possible combinations of two pairs in a cable may not be predicted from this theory but must be estimated from measurements of crosstalk loss. The analysis up to this point permits only the prediction of the statistical properties of measurements made at different frequencies on a single combination of two pairs. Let $L(\omega)$ be the crosstalk loss in db for a particular pair combination at a frequency $\omega/2\pi$. Then $L(\omega)$ is a random function of frequency, with an amplitude distribution and therefore a mean and standard deviation at each frequency. It will be convenient in what follows to use the probability density of $|R|^2$ rather than of $|R|$; this is found from (21) to be

$$\text{prob } (u < |R|^2 < u + du) = p_2(u) = \frac{1}{p(\omega)} e^{-u/p(\omega)}.$$

For a given k corresponding to a particular choice of disturbing and disturbed pairs, the expected value of the crosstalk loss $L(\omega)$ in db as a function of frequency is given by

$$\begin{aligned} \langle L(\omega) \rangle &= -\langle 10 \log |R|^2 \rangle \\ &= -10 \log e \, \langle \ln |R|^2 \rangle. \end{aligned} \tag{23}$$

From (21),

$$\begin{aligned} \langle \ln |R|^2 \rangle &= \frac{1}{p(\omega)} \int_0^\infty \ln u \, e^{-u/p(\omega)} \, du \\ &= \psi(1) + \ln p(\omega) \end{aligned}$$

in which $\psi(x)$ is the derivative of the logarithm of the gamma function,[18,19] and $\psi(1) = -0.577$. Using (22), (23) becomes

$$\langle L(\omega) \rangle = -10\psi(1) \log e - 10 \log k - 15 \log \omega. \tag{24}$$

The variance of $L(\omega)$ is given by

$$\langle L^2(\omega) \rangle - \langle L(\omega) \rangle^2 = 10^2 \psi'(1) \log^2 e$$

from which the standard deviation σ is found to be

$$\sigma = 10 \log e \sqrt{\psi'(1)} = 5.56 \text{ db}. \tag{25}$$

Equation (25) shows that the standard deviation of the loss in db is independent of frequency. If the same pair combination in many different cables is tested at a given frequency, or in other words if the loss in db is measured at one frequency on many pair combinations having the same value of k, the standard deviation of all the readings will be 5.56 db at any frequency in the range of 100 kc to 10 mc. The average value at any frequency ω will be given by (24).

In practice, crosstalk loss measurements have been made at relatively few frequencies, often at only one frequency which is of particular interest in a given application, but on many pair combinations. The mean value of the losses in db of all the pair combinations decreases 15 db per decade in the band from 100 kc to 10 mc. The standard deviation is always greater than 5.56 db, due to the contribution of the variation in the value of k from one pair combination to another. Measurements[13] of near-end crosstalk loss in a 22-gauge cable of unit construction yielded standard deviations of about 9 db for pair combinations in which both pairs were in the same unit, 7.5 db for combinations in which the two pairs were in adjacent units, and 6.5 db for combinations in which the two pairs were separated by one unit. The distribution of the loss in db at any frequency fits the normal law quite well and the normal distribution is usually assumed in making calculations.

In order to complete a picture of near-end crosstalk which will be adequate for repeatered line engineering, it is only necessary to find the distribution of the k's which together with the distribution

$$p_2(u) = \frac{1}{k\omega^{\frac{3}{2}}} \exp -(u/k\omega^{\frac{3}{2}})$$

will give the empirically obtained normal distribution of crosstalk losses. The latter distribution is assumed to have a mean loss of m db at a reference frequency ω_0 and a standard deviation of σ db at all frequencies. At an arbitrary frequency ω, the probability density of loss $L(\omega)$ in db is given by

$$\text{prob } (x < L(\omega) < x + dx) = \frac{1}{\sqrt{2\pi}\sigma} \exp -\left[\left(x - m - 15 \log \frac{\omega}{\omega_0}\right)^2 \bigg/ 2\sigma^2\right] dx.$$

Since $L(\omega) = -10 \log |R(\omega)|^2$, the density of $|R|^2$ is given by

$$\text{prob } (x < |R|^2 < x + dx) = \frac{10 \log e}{\sqrt{2\pi}\,\sigma x} \exp -\left[\left(10 \log x + m - 15 \log \frac{\omega}{\omega_0}\right)^2 \bigg/ 2\sigma^2\right] dx.$$

If we set $C = (10 \log e)^{-1}$ the density becomes

$$\frac{1}{\sqrt{2\pi}C\sigma x} \exp -\left[\left(\ln x + mC - \frac{3}{2}\ln\frac{\omega}{\omega_0}\right)^2 \Big/ 2C^2\sigma^2\right] dx.$$

It is convenient to redefine k in (22) so that

$$p(\omega) = k\left(\frac{\omega}{\omega_0}\right)^{\frac{3}{2}}. \tag{26}$$

Now what is needed is a probability density $p_k(x)$ for k such that

$$\frac{1}{\sqrt{2\pi}C\sigma x} \exp -\left[\left(\ln x + mC - \frac{3}{2}\ln\frac{\omega}{\omega_0}\right)^2 \Big/ 2C^2\sigma^2\right]$$
$$= \int_0^\infty p_k(u)\,\frac{1}{u(\omega/\omega_0)^{\frac{3}{2}}} \exp -\left[\left(\frac{x}{u}\right) \Big/ \left(\frac{\omega}{\omega_0}\right)^{\frac{3}{2}}\right] du.$$

However, an accurate solution of this equation is not necessary. The lognormal distribution itself is only a convenient approximation to the actual distribution. For p_k we require a function which is a probability density, whose convolution may be expressed in elementary functions, which possesses at least two arbitrary parameters, and which, applied to the right side of (24), permits the calculation of the first and second moments of $\langle L(\omega)\rangle$. A satisfactory function is

$$p_k(x) = \frac{\lambda^\nu}{\Gamma(\nu)}\, x^{\nu-1} e^{-\lambda x} (\nu > 0, \lambda \geqq 0).$$

If we now redefine k in (24) to agree with (26), and denote expectations relative to $p_k(x)$ by E, then

$$E\langle L(\omega)\rangle = m - 15 \log\left(\frac{\omega}{\omega_0}\right)$$
$$= -10 \log e\,[\psi(1) + \psi(\nu)] + 10 \log \lambda - 15 \log\left(\frac{\omega}{\omega_0}\right).$$

Similarly the variance in db may be calculated for each distribution to yield

$$E(\langle L(\omega)\rangle)^2 - (E\langle L(\omega)\rangle)^2 = (10 \log e)^2\,[\psi'(1) + \psi'(\nu)]\,\sigma^2.$$

Consequently we set

$$\begin{aligned} \psi'(\nu) &= (10 \log e)^{-2}\sigma^2 - \psi'(1) \\ 10 \log \lambda &= m + 10 \log e\,[\psi(1) + \psi(\nu)]. \end{aligned} \tag{27}$$

The results so far may be summarized as follows. Suppose the distribution of near-end crosstalk losses between two groups of pairs in a given type of exchange cable is found empirically to possess a mean of m db and a standard deviation of σ db at a frequency ω_0. Then for each combination consisting of a pair in each group, the probability density of $|R(\omega)|^2$, the squared amplitude ratio of crosstalk voltage to transmitted voltage, is

$$\text{prob}\,(x < |R(\omega)|^2 < x + dx) = k^{-1}\left(\frac{\omega}{\omega_0}\right)^{-\frac{3}{2}} \exp\left[-xk^{-1}\left(\frac{\omega}{\omega_0}\right)^{-\frac{3}{2}}\right] dx. \quad (28)$$

The distribution of the k's for all the pair combinations is given by

$$\text{prob}\,(x < k < x + dx) = p_k(x)\,dx = \frac{\lambda^\nu}{\Gamma(\nu)}\, x^{\nu-1} e^{-\lambda x}\, dx \quad (29)$$

in which λ and ν are given by (27). In (28), the average value of $|R(\omega)|^2$ is $k(\omega/\omega_0)^{\frac{3}{2}}$.

We may now calculate the mean and standard deviation of the interference power at the preamplifier output when there are n systems in operation. Let $r(f)$ be the Fourier transform of the preamplifier impulse response, and $W(f)$ the average power spectrum of the pulse train at the amplifier output in watts per cycle. Consider a hypothetical interferer coupled to the disturbed amplifier by a near-end crosstalk path whose power ratio at frequency f is just $k(f/f_0)^{\frac{3}{2}}$. If the expected value of the interference power due to this interferer is denoted by $w_c(k)$, then

$$w_c(k) = k \int_0^\infty |r(f)|^2\, W(f) \left(\frac{f}{f_0}\right)^{\frac{3}{2}} df.$$

The actual interference power will be close to this value because in summing $|R\,(\omega)|^2$ over a range of frequencies the variation tends to be averaged out. The standard deviation of w_c for a given value of k is taken to be zero.

The total interference power is the sum of many contributions $w_c(k)$ with k's chosen at random from the distribution given by (29), in which λ and ν are determined from the mean m and standard deviation σ in db of the crosstalk loss distribution. The distribution of interference power due to n interferers depends upon the distribution of the random variable which is the sum of n values of k chosen at random, say k_n. This is the nth convolution $p_k^{(n)}(x)$ of $p_k(x)$ and is given by

$$p_k^{(n)}(x) = \frac{\lambda^{n\nu}}{\Gamma(n\nu)}\, x^{n\nu-1} e^{-\lambda x}. \quad (30)$$

The mean and standard deviation of the interference power w_c in dbm at the preamplifier output may now be determined. From (30) one finds

$$\text{ave }(10 \log k_n) = 10 \log e\,[\psi(n\nu) - \ln \lambda]$$

$$\text{ave }(100 \log^2 k_n) - [\text{ave }(10 \log k_n)]^2 = (10 \log e)^2 \psi'(n\nu).$$

The mean interference power in dbm at the preamplifier output is therefore given by

$$\begin{aligned} \bar{P}_n &= \text{ave }[10 \log w_c(k_n)] + 30 \\ &= 30 + 10 \log e\,[\psi(n\nu) - \psi(\nu) - \psi(1)] \\ &\qquad - m + 10 \log \left\{ \int_0^\infty |r(f)|^2\, W(f) \left(\frac{f}{f_0}\right)^{\frac{3}{2}} df \right\}. \end{aligned}$$

The standard deviation is

$$\sigma_{P_n} = 10 \log e\,[\psi'(n\nu)]^{1/2}. \tag{31}$$

The value of ν must be determined from (27).

The quantity

$$30 + 10 \log \left\{ \int_0^\infty |r(f)|^2\, W(f) \left(\frac{f}{f_0}\right)^{\frac{3}{2}} df \right\} - 10 \log e\, \psi(1) \tag{32}$$

is the interference in dbm at the preamplifier output due to one amplifier when $m = 0$ at frequency $f_0 = \omega_0/2\pi$. This frequency will be taken to be half the repetition frequency of the pulse train, or 772 kc. The preamplifier response in db is given in Fig. 17. The expression for $W(f)$ may

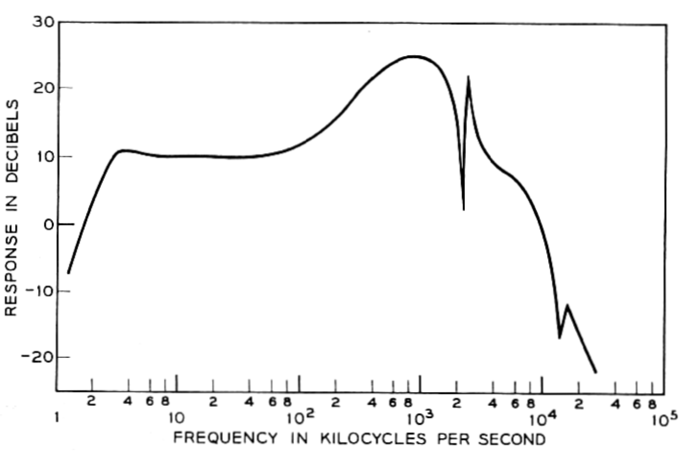

Fig. 17 — Preamplifier response.

be written[4] as

$$W(f) = \frac{2V^2 f_0}{100\pi^2 f^2} \sin^2 (\pi f/4f_0) \left(1 - \cos \frac{\pi f}{f_0}\right). \tag{33}$$

This assumes rectangular pulses of height V volts base-to-peak, $1/4f_0$ seconds long at an impedance level of 100 ohms, with probability $\frac{1}{2}$ for pulse and $\frac{1}{2}$ for space, each time slot being independent of preceding ones. It also takes into account the polarity constraint that each pulse is of opposite polarity from the preceding one; see Fig. 3. When $V = 3$ volts is substituted in (33), and (32) is integrated numerically, the result is 39.4 dbm. The mean interference power in dbm at the preamplifier output due to n interfering amplifiers is therefore

$$\bar{P}_n = 39.4 + 10 \log e \, [\psi(n\nu) - \psi(\nu)] - m$$

in which m is the mean crosstalk loss in db at 772 kc and ν is to be determined from the standard deviation σ of crosstalk loss in db by

$$\begin{aligned} \psi'(\nu) &= \sigma^2(10 \log e)^{-2} - \psi'(1) \\ &= 0.0530\sigma^2 - 1.645. \end{aligned}$$

For engineering calculations, a convenient approximation to $\bar{P}_n$ is

$$\bar{P}_n = 48 - m + \sigma + 10 \log (n/25). \tag{34}$$

This equation, based on well-known properties of the ψ-function,[18,19] is valid within 0.2 db for σ between 6 and 14 db.

These results for $\bar{P}_n$ are in good agreement with experiment, but (31) has been found to yield too small a value for σ_{P_n}. This may be traced to the neglect of the variance of $w_c(k)$ and the use of its average value only. The exact analysis has not been attempted, but an experimental value[15] of σ_{P_n} is used in engineering (Section 4.2.1).

A crucial point in the application in Section 4.2 of these results on near-end crosstalk is the Gaussian amplitude distribution of the interfering voltage. This was established experimentally[15] with the setup of Fig. 6, as follows. A fixed crosstalk output was obtained by choice of the connections at the cross-connect terminal. The variable gain in the crosstalk path (see Fig. 6) was adjusted to cause an error rate of 1.2×10^{-6}, a convenient value for measurement, in the receiving T1 amplifier. The interfering power for this condition was taken as a reference value. Next, the interfering power was varied from the reference level, by changing the gain setting, and the variation of error rate observed. The resulting points, for 3 and 25 interferers, are plotted in Fig. 18. Quite similar results were found for 1, 6, 12, and 50 interferers.

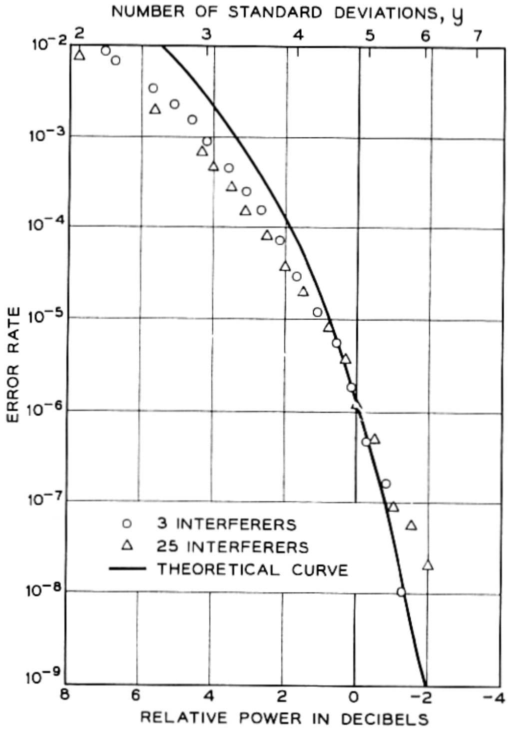

Fig. 18 — Error rate versus relative power.

The solid curve of Fig. 18 is the theoretical result that would be obtained if the crosstalk had a Gaussian amplitude distribution and a power level such as to cause an error rate of 1.2×10^{-6} in the reference condition. As a Gaussian signal exceeds an amplitude of 4.85 times its standard deviation (rms value) with probability* 1.2×10^{-6}, we have plotted $y = 4.85$ on the upper horizontal scale of Fig. 18, to coincide with 0 db on the lower scale. For a value of power G db greater than the reference, the value of y is $4.85 \times 10^{-G/20}$. For example, if the gain is set to give 2 db less power than in the reference condition, or $G = -2$, the Gaussian interference must exceed $4.85 \times 10^{2/20} = 6.1$ times its rms value to cause an error, an event with probability 1.1×10^{-9}.

Measurements of the near-end crosstalk amplitude distribution out to

* This is the "double-tail" probability, or twice the value ϵ in (2), for a given number of standard deviations, y, because the data were taken with a one-out-of-eight pulse pattern as the transmitted signal, and most errors are insertions of pulses. The probability of such errors is 2ϵ.

two or three standard deviations have been made with a conventional amplitude distribution analyzer. These measurements confirm the Gaussian distribution to a probability level of 10^{-3} or so. The confirmation we have described for the lower probabilities that occur for five or six standard deviations is essential in line engineering calculations with error rates as low as 10^{-7}.

APPENDIX B

Far-End Crosstalk

Consider again two pairs of propagation constant $\gamma(f)$ in a cable section of length l, which are terminated in their characteristic impedance Z_0 at each end. A current I_0 of frequency f is applied to one pair at one end. Let the mutual impedance unbalance at a distance x from this end be $Z(x)$ per unit length and the admittance unbalance be $Y(x)$. Then it can be shown[16] that the incremental crosstalk current dI on the disturbed pair at the receiving end, due to an incremental length of cable dx at distance x, is given by

$$\frac{dI}{I_0} = \left(\frac{Z_0 Y}{16} - \frac{Z}{4Z_0}\right) e^{-\gamma l}\, dx. \tag{35}$$

At frequencies of interest we may make the approximation that Z, Z_0, and Y are independent of frequency. Equation (35) may then be written

$$dI/I_0 = i\omega C_r(x) e^{-\gamma l} dx$$

in which $C_r(x)$ may be interpreted as the unbalance per unit length. It is convenient to deal with "equal-level" crosstalk loss, that is, with the ratio $dI/I_r = i\omega C_r(x) dx$ where I_r is the current on the disturbing pair at its receiving end. If we define $F(f)$ as the response of the disturbed pair at the receiving end of the cable to a sinusoid impressed on the disturbing pair at the sending end which is large enough to produce unit amplitude at the receiving end, then

$$F(f) = i2\pi f \int_0^l C_r(x)\, dx. \tag{36}$$

The quantity of immediate interest is the expected value of $|F|^2$. From (36), and setting $\langle C_r(x) C_r(y) \rangle = S_r(x - y)$ we get

$$\langle |F(f)|^2 \rangle = 2\omega^2 \int_0^l (l - x) S_r(x)\, dx.$$

If l is very much larger than the range over which $S_r(x)$ is non-negligible, the above equation becomes

$$\langle |F(f)|^2 \rangle = 2\omega^2 l \int_0^\infty S_r(x)\, dx. \tag{37}$$

This result shows the well-known 6-db per octave average slope of the response with frequency and the proportionality to cable length.[17]

From (36) we expect the function $i^{-1}F(f)$ to be normally distributed with mean zero since $C_r(x)$ has mean zero. This would be true even if C_r were not normally distributed, provided that l is large compared to the range over which C_r is self-correlated. Thus for a particular pair combination, $i^{-1}F(f)$ will have a value in the range $(x, x + dx)$ with probability

$$(2\pi)^{-\frac{1}{2}}\mu^{-1}(f) \exp \{ -[x^2/2\mu^2(f)]\} dx,$$

where μ is the standard deviation. From this the probability density of $|F(f)|^2$ is found to be

$$(2\pi x)^{-\frac{1}{2}}\mu^{-1} \exp - (x/2\mu^2).$$

The first moment of this distribution is $\mu^2(f)$, which may therefore be equated to the right-hand side of (37). It is convenient to set

$$2 \int_0^\infty S_r(x)\, dx = \frac{k}{{\omega_0}^2}.$$

Then $\mu^2(f) = kl\omega^2/{\omega_0}^2$ and kl is the expected value of the power on the disturbed pair at the receiving end, per watt of received power in the disturbing pair, at a frequency ω_0. As in the near-end case, k is a random variable, a particular value being associated with each pair combination. We assume that the value of k possessed by a pair A in combination with pair B is independent of its value in combination with any other pair. For the probability that k for a pair combination chosen at random lies in the range $(u, u + du)$ we assume

$$P_k(u) = \frac{\lambda^\nu}{\Gamma(\nu)}\, u^{\nu-1} e^{-\lambda u} \qquad (\nu > 0, \lambda \geqq 0). \tag{38}$$

The probability that the equal-level far-end crosstalk response $|F(f)|^2$ of a pair combination chosen at random at a frequency f lies in the range $(x, x + dx)$ is therefore given by

$$P(x)\, dx = dx \frac{\lambda^\nu}{\Gamma(\nu)} \int_0^\infty u^{\nu-1} \frac{\omega_0}{\omega\sqrt{2\pi lux}} \exp\left(-\lambda u - {\omega_0}^2 x/2\omega^2 ul\right) du.$$

The equal-level far-end crosstalk loss in db is given by $-10 \log |F|^2$. For the above distribution we have the mean loss

$$m_1 = -(10 \log e)\left[\psi(\nu) + \psi\left(\frac{1}{2}\right)\right] + 10 \log \lambda + 10 \log \frac{{\omega_0}^2}{2\omega^2} - 10 \log l,$$

and the standard deviation in db

$$\sigma_1 = (10 \log e)\left[\psi'(\nu) + \psi'\left(\frac{1}{2}\right)\right]^{\frac{1}{2}}.$$

A repeater section is usually made up of several reels spliced together. For a given pair in a cable section of j reels the far-end coupling is therefore made up of j couplings in parallel, each having a value of k chosen at random from the distribution (38). Each coupling may be thought of as a lumped capacitance which may be positive or negative and is equal to the integral of $C_r(x)$. The currents therefore add linearly. The density of the resultant current, relative to the received current is, therefore, the j-fold convolution of the density function of $i^{-1}F(f)$, i.e.

$$\lambda^\nu[\Gamma(\nu)]^{-1} \int_0^\infty u^{\nu-1} (\omega_0/\omega\sqrt{2\pi lu}) \exp(-\lambda u - {\omega_0}^2x^2/2\omega^2ul)\, du.$$

Multiplying by exp izx and integrating over x first, we get for the characteristic function $\Phi(z)$

$$\Phi(z) = \{\lambda[\lambda + (\omega^2z^2/2{\omega_0}^2)]^{-1}\}^\nu.$$

Taking the jth power of this function merely replaces ν by $j\nu$ and we see that the convolution is given by

$$P_j(x) = \frac{\lambda^{j\nu}}{\Gamma(j\nu)} \int_0^\infty u^{j\nu-1} \frac{\omega_0}{\omega\sqrt{2\pi lu}} \exp(-\lambda u - {\omega_0}^2x^2/2\omega^2ul)\, du. \quad (39)$$

For this distribution we have for the mean loss in db and the standard deviation in db

$$m_j = -(10 \log e)\left[\psi(j\nu) + \psi\left(\frac{1}{2}\right)\right] + 10 \log \lambda + 10 \log \frac{{\omega_0}^2}{2\omega^2} - 10 \log l \quad (40)$$

$$\sigma_j = (10 \log e)\left[\psi'(j\nu) + \psi'\left(\frac{1}{2}\right)\right]^{\frac{1}{2}}. \quad (41)$$

We may now apply these results to the general far-end crosstalk situation of Fig. 11. Suppose that the cable section in which the far-end exposure takes place has loss L_1 and length L_1/K miles, where K is the loss in db per mile, and that it consists of j reels of length l miles, i.e., $jl = L_1/k$. The ratio of the power at the input of amplifier 2 to that arriving at the exposure section on the disturbing pair at frequency f, for a pair combination giving the mean dbm power, is

$$(2l/\lambda)(f/f_0)^2 \exp\left[\psi(j\nu) + \psi\left(\frac{1}{2}\right) - 2\alpha(f)(L_1/K)\right]$$

where $\alpha(f)$ is the cable loss in nepers per mile. Taking into account the preamplifier characteristic $r(f)$, the power spectrum $W(f)$ of the amplifier output signal as in (33), and the loss, L_2, of the section of cable from amplifier 1 to the exposure, the total interference power at the preamplifier output for one disturber is

$$(2L_1/\lambda jK) \exp\left[\psi(j\nu) + \psi\left(\frac{1}{2}\right)\right] \int_0^\infty (f/f_0)^2 W(f)\, |r(f)|^2 \cdot \exp\left[-2\alpha(f)(L_1 + L_2)/K\right] df.$$

The interference power in dbm for this mean interferer is

$$\begin{aligned}\bar{Q}_1 = {} & 10 \log (2L_1/\lambda jK) + 10 \log e\left[\psi(j\nu) + \psi\left(\frac{1}{2}\right)\right] \\ & + 30 + 10 \log \int_0^\infty (f/f_0)^2 W(f)\, |r(f)|^2 \\ & \cdot \exp\left[-2\alpha(f)(L_1 + L_2)/K\right] df.\end{aligned} \qquad (42)$$

The standard deviation of Q_1 is given by (41).

To determine the interfering power for n interferers, we can obtain from (39) the density for the square of the current and find its n-fold convolution. To obtain an approximate result we may use the Edson-Alford formula,[20] which gives an approximation to the distribution of the power sum expressed in db, of n quantities whose distribution in db is assumed to be Gaussian. The increase in the mean power in dbm for n interferers as compared to one interferer is

$$a_{j,n} = 5[\log (n^3 \exp C^2\sigma_j^2) - \log (\exp C^2\sigma_j^2 + n - 1)] \qquad (43)$$

and the standard deviation is

$$\sigma_{j,n} = 6.593 [\log (\exp C^2\sigma_j^2 + n - 1) - \log n]^{\frac{1}{2}} \qquad (44)$$

where $C = 1/(10 \log e)$. Performing the numerical integration in (42) with the assumption that $\alpha(f) = \alpha(f_0)(f/f_0)^{\frac{1}{2}}$, we find that the integral is very closely given by $7.0 - (L_1 + L_2)$. Thus the mean interfering power in dbm for n systems is

$$\bar{Q}_n = 10 \log (2L_1/\lambda jK) + 10 \log e \, [\psi(j\nu) + \psi(\tfrac{1}{2})] + 37.0 - (L_1 + L_2) + a_{j,n} \tag{45}$$

with standard deviation $\sigma_{j,n}$ as in (44).

The values of λ and ν in (45) must be determined by substituting in (40) and (41) the measured mean, m_{j_0}, and standard deviation σ_{j_0}, of the equal-level far-end crosstalk losses at a single frequency (f_0 is convenient) for a cable made up of j_0 reels of length l_0 miles each. If the loss m_{j_0} is measured at f_0, we find that

$$10 \log \lambda = m_{j_0} + 10 \log e \, [\psi(j_0\nu) + \psi(\tfrac{1}{2})] + 10 \log 2l_0 . \tag{46}$$

Substituting (46) in (45),

$$\bar{Q}_n = 10 \log (L_1/jKl_0) - m_{j_0} + 10 \log e \, [\psi(j\nu) - \psi(j_0\nu)] + 37.0 - (L_1 + L_2) + a_{j,n} . \tag{47}$$

Instead of L_1/jKl_0, the equivalent expression l/l_0 may be used.

Equation (47) gives the mean interfering power, $\bar{Q}_n$, strictly speaking, only when the exposure section is made up of j reels, for $j = 1, 2, 3, \cdots$. For exposure sections of arbitrary loss L_1, and an assumed reel length of l miles, we may use $j = L_1/Kl$, which will not in general be an integer. The results for $\bar{Q}_n$ and $\sigma_{j,n}$ will be interpolations between their values for the nearest integer j's. For small values of L_1 when we would have $j < 1$, we cannot apply the convolution results (40) and (41); instead, the one-reel distribution applies. Thus, for $L_1/Kl < 1$, we have, putting $j = 1$ in (47),

$$\bar{Q}_n = 10 \log (L_1/Kl_0) - m_{j_0} + 10 \log e \, [\psi(\nu) - \psi(j_0\nu)] + 37.0 - (L_1 + L_2) + a_{1,n} . \tag{48}$$

The standard deviation of Q_n is $\sigma_{1,n}$.

To illustrate (47), suppose we have $n = 50$ systems in 22-gauge paper cable with $L = L_1 = 32.2$ db and $L_2 = 0$ in Fig. 11. If we take 900 feet as the reel length, the 32.2 db section will contain $j = 7$ reels. For the cable crosstalk parameters, we use the data of Table IV, choosing $m_{j_0} = 63$ db for pairs in the same unit. Other measurements[21] on a single reel of cable gave a standard deviation σ_1 of 11 db. Substituting $\sigma_1 = 11$ in (41) shows that $\nu \approx 1$, and we adopt the value $\nu = 1$ for convenience ($\sigma_1 = 11.2$ db).

Table IV — Far-End Crosstalk Data[13]

Type of cable:	22-gauge, paper-insulated, 900 pairs
Length:	6106 feet
Number of reels, j_0 :	9
Reel length, l_0 :	$6106/(9 \times 5280) = 0.13$ miles
Loss per mile, K:	26.5 db per mile (772 kc, 55 F)
Mean equal-level far-end crosstalk coupling loss, at 772 kc, m_{j_0} :	(a) for pairs in same unit, 63 db (b) for pairs in adjacent units, 77 db
Estimated parameter ν (see text)	1

From (41), we find that $\sigma_7 = 9.8$ db; putting this in (43), $a_{7,50} =$ 25.0 db. The mean interference power $\bar{Q}_{50}$ is then $=33.2$ dbm, from (47). Further, its standard deviation $\sigma_{Q_{50}} = 5.2$ dbm, using (44).

APPENDIX C

Central Office Noise

The operation of electromagnetic relays in telephone offices produces electrical transients which propagate down the pairs.[22] Through crosstalk in the cable, these disturbances may appear on pairs not connected to switches. The energy distribution of this noise is very broad, reaching frequencies well in excess of two megacycles. The transients are of several types and are quite complex. If T1 lines were operated at a lower signal level they would cause bursts of errors, each burst lasting less than a millisecond and containing from one or two to several hundred errors. Since the noise is primarily due to switching, the error rate is strongly dependent upon office activity, being highest during the busy hours and disappearing in the early hours of the morning. Since a crosstalk path is involved, the levels of the noise transients reflect the approximately log-normal distribution of crosstalk loss and vary widely.

Because of the complexity of the office noise phenomenon, it is necessary to lean heavily on experimental results. The feature of the noise which is of most interest in T1 carrier work is the error rate which it produces in the office amplifier, as a function of the length of the repeater section. The error rate is predictable from the amplitude distribution of the noise at the output of the linear preamplifier, that is, at the point in the amplifier at which the decision is made whether a pulse has been received or not. The amplifier may be assumed to make an error if the noise amplitude is greater than one-fourth the pulse height (Section 2.1). The error rate to be expected on a typical office repeater section has therefore been studied by means of a sampling and comparison device which is connected to an otherwise idle pair at the main frame. A noise sample increases the count on a register by one if the amplitude is greater than a preset level.

Since the repeater section is always built out to full length (Section 2.2), the signal pulse shape and amplitude at the preamplifier output are always the same, within the allowable ±4 db limits. The noise, on the other hand, is attenuated by whatever buildout is used. Since the buildout loss is a function of frequency and the noise transients are quite variable in structure, the relation between section loss and error rate must also be found experimentally. The noise may also be attenuated by pair loss* in the disturbing pair if the amplifier is not in the office but near it. Since the buildouts are designed to imitate pair loss, the effect on the error rate of adding pair loss in the noise path is the same as the effect of shortening the repeater section.

In some cases the noise exposure occurs via far-end rather than near-end crosstalk. The dependence on the crosstalk mode may be estimated from the relative near-end and far-end crosstalk losses. Again, if the disturbing pairs are in a different unit or layer of the cable than the T1 pairs, the loss in the crosstalk path is greater than if both are in the same unit, and this may be taken into account in engineering end sections.

Since both crosstalk interference and impulse noise are generally present in sections near offices, the error rate due to any combination of the two must be determined.

To summarize, the following questions must be answered in order to estimate the restrictions which should be placed on repeater sections exposed to office noise:

(a) What is the worst noise level to be expected in cable pairs entering an office?

(b) How does noise error rate depend upon section loss, distance of the repeater from the office, crosstalk mode, and segregation in the sheath?

(c) If office noise and crosstalk separately produce error rates R_1 and R_2, what error rate will both together produce?

Considering the first of these questions, amplitude distributions for positive noise voltages were measured *at the output of a T1 preamplifier* on several pairs at their main frame terminations in one office. Curve 1 of Fig. 19 is for the median pair. The ordinate is the normal probability scale, while the abscissa gives the instantaneous level at the output of the preamplifier in dbm, with no buildout in the amplifier. Thus a level dis-

* Noise in the disturbing pair is found in both the longitudinal and metallic modes. At T1 frequencies the losses in the two modes may be taken to be the same for the present purpose.

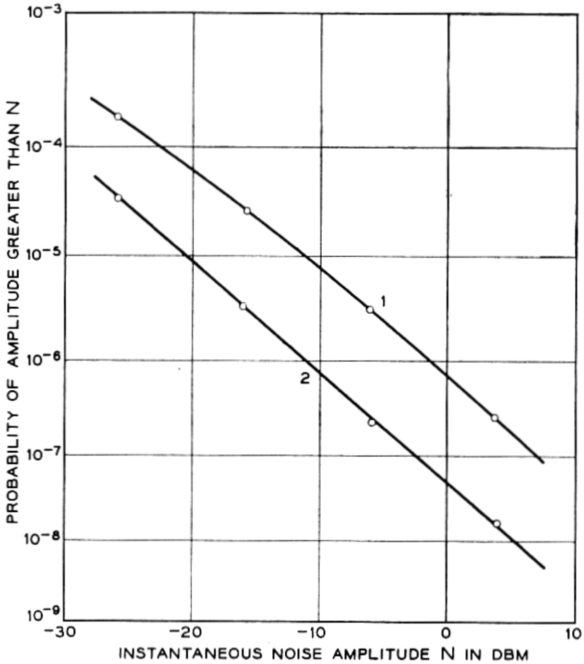

Fig. 19 — Noise amplitude distribution at preamplifier output with 9.6-db buildout (curve 2) and without (curve 1).

tribution which was normal in db would appear as a straight line. Of several offices involving step-by-step, panel, and crossbar switching machines, this office gave as high a noise level as any. We have, therefore, taken curve 1 of Fig. 19 to be representative of noise levels in noisy offices.

Considering the second question, if section loss is decreased, the noise level is reduced by the presence of the buildout. Curve 2 of Fig. 19 shows the effect of the 9.6 db buildout on the noise distribution of curve 1. The effect is quite closely the same as the introduction of the same amount of flat loss. Thus reducing the section loss or moving the repeater away from the office affects the noise level by the number of db represented by the change, measured at 772 kc.

For calculations based on Fig. 19, we assume that the mean near-end coupling loss is 75 db at 772 kc, a figure that is typical for pairs in the same unit or layer of large cables.[13] In the case of far-end coupling, the approximation

$$\bar{L}_f \approx 63 - 10 \log (l/6106) \tag{49}$$

may be used; $\bar{L}_f$ is the mean equal-level far-end crosstalk coupling loss at 772 kc for a cable of length l feet (cf. Appendix B). To this loss is added the direct loss of the pair between the office and the disturbed amplifier.

The additional loss in the crosstalk path due to segregation of T1 pairs in a unit or layer of the cable containing no switched pairs can be estimated from the relative mean crosstalk losses. Near-end crosstalk loss in unit cables is about 8 to 13 db greater in mean value for pairs in adjacent units than for pairs in the same unit. Far-end crosstalk loss is about 10 to 15 db greater for pairs in adjacent units than for pairs in the same unit. These are metallic-to-metallic losses, whereas longitudinal-to-metallic couplings may be dominant. Some noise data is available which indicates about a 10-db reduction of office noise for pairs in an idle unit in the sheath as compared with pairs in working units. This estimate has been used for both near-end and far-end noise coupling.

Curve 1 of Fig. 19 may now be translated into a curve of error rate as a function of the difference between loss in the noise path and loss in the signal path. For example, the curve shows that a noise level of -1.5 dbm at the preamplifier output is exceeded with probability 10^{-6}. Therefore, the amplifier error rate will be 10^{-6} when the signal-to-noise ratio at this point is 12 db (Section 2.1), i.e. when the signal level is $-1.5 + 12 = 10.5$ dbm. The signal level (Section 2.2) at the preamplifier output is $\bar{S} + G - \bar{L}$, where these symbols have the same meaning as in Section 4.2.1. Substituting $\bar{S} = 18.6$ dbm and $G = 23.7$, we find that the mean pair loss, i.e. the loss in the signal path, is $\bar{L} = 31.8$ db for a signal level of 10.5 dbm. We have assumed a mean loss in the noise path of 75 db in Fig. 19. The difference between noise path loss and signal path loss is thus $75 - 31.8 = 43.2$ db. In Fig. 19, error rate 10^{-6} is plotted above the abscissa 43.2 db. Correspondingly, each abscissa of Fig. 19 is increased by $43.2 + 1.5 = 44.7$ db to obtain a point of Fig. 20. When the losses in the noise and signal paths for a particular amplifier are known, the error rate in the presence of severe office noise may be read from Fig. 20.

Finally, the error rate resulting from a combination of crosstalk and noise must be examined. This requires choosing a mathematical representation of the office noise amplitude probability density p_n and finding its convolution with the normal distribution, which applies to crosstalk voltage amplitudes.

A convenient representation for this purpose is:

$$p_n(x) = \frac{\sqrt{2}\alpha^3}{\pi} (\alpha^4 + x^4)^{-1}. \tag{50}$$

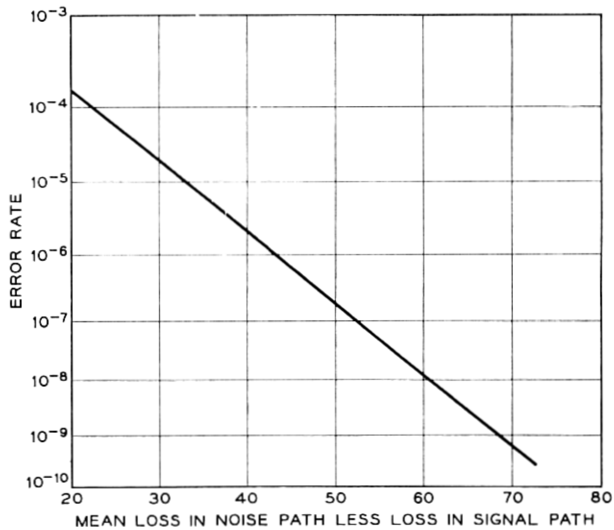

Fig. 20 — Error rate as a function of loss difference.

This function fits typical noise curves quite well. The fit is not as good for the rather extreme case of curve 1 of Fig. 19, which shows a relatively higher probability of very large amplitudes, but it is satisfactory for the present purpose. For large values of the ratio of the random amplitude variable X to the standard deviation α we have

$$\text{Prob}\left(\frac{X}{\alpha} \geqq u\right) = \frac{\sqrt{2}}{\pi}\left\{\frac{1}{3u^3} - \frac{1}{7u^7} + \frac{1}{11u^{11}} + \cdots\right\}.$$

For an error rate of 10^{-7} all terms except the first can be ignored and we find $u = 114.5$. The probability density of the amplitude of the crosstalk interference at the preamplifier output is given by

$$p_c(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-x^2/2\sigma^2}. \tag{51}$$

If Y is the crosstalk interference amplitude variable, solving the equation

$$\text{Prob}\left(\frac{Y}{\sigma} \geqq u\right) = 10^{-7}$$

yields $u = 5.2$. Thus an error rate of 10^{-7} will occur if the error threshold (one-fourth the pulse height) is 5.2 times the rms crosstalk interference or if it is 114.5 times the rms office noise. We are interested in the error rate if these amounts of noise and crosstalk interference occur in the same amplifier. This may be calculated by finding the distribution of the sum $Y + X$ of the two variables. Let

$$\alpha = \frac{5.2}{114.5}\sigma.$$

By working with the characteristic functions corresponding to the probability densities of (50) and (51) it may be shown that

$$\text{Prob}\ (X + Y \geqq 5.2\sigma) = 2.31 \times 10^{-7}.$$

Thus the error rate due to the sum of the two types of interference is approximately the sum of the error rates due to the two interferences separately. For practical purposes, design requirements for office noise and crosstalk may therefore be considered separately.

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