Calculation of the Spin-Axis Orientation of the *Telstar* Satellites from Optical Data

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The orientation of a satellite can be calculated if the aspect of two or more known reference vectors can be measured with respect to a body-fixed coordinate frame. On the Telstar satellites one such line may be determined by solar aspect cells, but the other is determined by ground observation of flashes of sunlight reflected from three mirrors mounted on the spinning satellite. The simple geometric considerations are discussed, as are the optimum placement of the mirrors, the amount of data to be used, and an error analysis. Spin-axis orientation data are used as the basis for steering the spin axis by ground command of the current in a coil of wire around the equator of the satellite.

I. INTRODUCTION

A novel method was devised to determine the orientation in space of the Telstar satellites. The method used grew out of limitations on weight, power, and telemetry facilities, and consists of three highly reflective mirrors and six solar aspect cells attached to the outside of the satellites. The mirrors reflect sunlight to a ground observation station,* and the solar aspect cells measure the direction of the sun with respect to reference coordinates in the satellite. The method has proved successful in determining the orientation of the Telstar satellites with sufficient accuracy both to deduce the residual magnetic moments of the satellites and to correct their precession by means of an equatorial torque coil. The remainder of the discussion concerns the details of the attitude determination scheme. See Ref. 1 for a description of the photoelectric equipment used. We will restrict the discussion to a spin-stabilized

* The use of mirrors for this purpose was first suggested by D. Gibble of Bell Laboratories.

satellite and further assume that the precessional motion occurs at a slow rate.

The orientation of a satellite can be calculated if the directions of two known lines in inertial space can be measured with respect to a body-fixed reference frame. One such reference line can be measured by solar aspect cells, and, in the case of a spin-stabilized satellite, another can be determined by observing flashes of sunlight reflected from a mirror attached to the satellite. Because of the indeterminancy of the angular displacement about each reference being observed from the satellite, we know only that any given body axis must lie on a cone about the reference line. The semivertex angle of the cone is the angle between the body axis and the reference line, and is the quantity determined by the attitude sensing scheme. Consequently, if two reference lines are known with respect to the satellite, the orientation of any desired body axis must lie along one of the two lines of intersection of the two cones which have been determined. Since the inertial coordinates of the reference lines are known, the orientation of the body axis can be calculated. Any ambiguity between the two possible orientations can usually be resolved by the general knowledge of the observations, and in fact, no difficulty was experienced with the Telstar satellites in resolving the ambiguity.

1.1 *Mirrors Attached to Satellite*

There are three mirrors attached to the satellite, one being a plane mirror approximately 4 × 6 inches in size mounted so the normal to the mirror makes an angle of 68° with the symmetry or spin axis of the satellite; see Fig. 1. The other two mirrors are each approximately half the size of the first and both are mounted so the normal is inclined 95° to the spin axis. The two 95° mirrors are 120° apart in aximuth around the satellite and provide a coding of the flash train to enable the ground station to determine which mirror is flashing.

If a train of flashes of reflected sunlight is observed at the earth and the mirror which is flashing can be identified, we know that the spin axis must lie on a cone of either 68° or 95° semiangle about the line normal to the mirror at the instant of a flash. This line can be calculated, since it lies in the plane of the sun, satellite, and ground station and bisects the angle between the incident and reflected rays.

The angles 68° and 95° were chosen because mirrors placed at those angles would afford nearly the maximum number of flash observations for the nominal orbit and attitude. Calculations showed that these

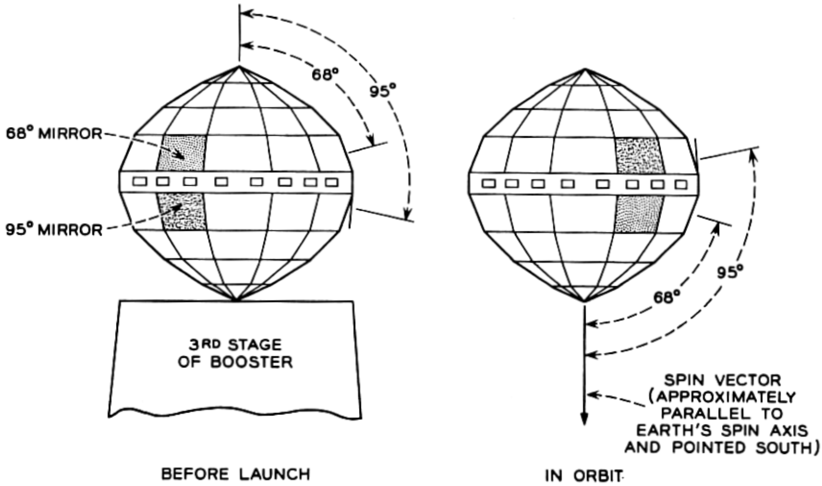

Fig. 1 — Placement of mirrors.

two mirrors would provide an average of more than one flash observation per night, weather permitting, for the first 55 nights, after which there would be no night passes for about one month due to precession of the satellite orbit and movement of the sun in the geocentric frame of reference.

1.2 *Solar Aspect Determination*

The determination of the sun's aspect is made on the satellite by six solar cells placed on the nearly spherical surface of the satellite. These solar cells are placed symmetrically and oppositely pointed on each of three orthogonal axes, Fig. 2. Each cell responds to sunlight over one hemisphere — i.e., 2π steradians — giving a current response approximately proportional to the cosine between the direction of the sun and the axis of the solar cell. Thus no more than three cells are illuminated at any one time, and the direction of the sun with respect to the orthogonal triad through the solar cells is found by relatively simple calculations using the currents generated by the three illuminated cells.

The orthogonal triad through the solar cells is located symmetrically with respect to the spin axis of the satellite, three axes piercing the upper hemisphere of the satellite and their opposite extensions piercing the lower hemisphere, the solar cells being attached at the intersections of the axes and the shell of the satellite. Thus the three axes in either

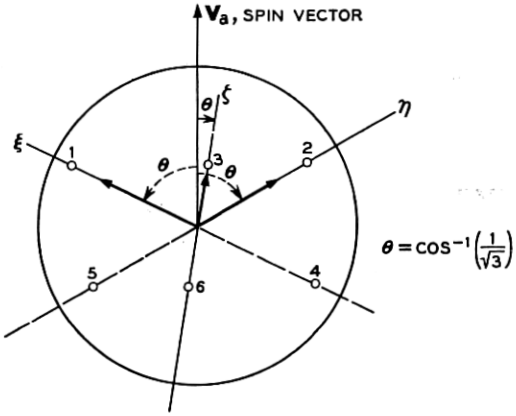

Fig. 2 — Solar aspect cell geometry.

hemisphere each have a direction cosine of $\sqrt{1/3}$ with the extension of the spin axis in that hemisphere.

Once the aspect of the sun is determined with respect to the orthogonal triad through the solar aspect cells, the angle between the satellite-sun line and the spin axis is thus easily computed.

In principle this solar aspect scheme is satisfactory, but it does suffer from an inherent defect. Fundamentally, the method is an analog method depending on absolute values of solar cell current for solar aspect determination. Thus any drift, either in the properties of the cells or in the data amplifying and transmitting system, will cause inaccuracies in the aspect determination. In order to minimize these effects, the cells have been preirradiated to lessen the effect of radiation in the satellite environment. Further, thermistors are used to measure the temperature of the cells and a temperature correction is applied to the solar cell current readings. Finally, two methods of computing the solar aspect are used to try to isolate any discrepancies that arise. However, there is no way to arrive at a correct aspect determination if the solar cells give erroneous readings. The best that can be achieved is to obtain some average result by distributing errors in some rational manner, such as distributing them in proportion to the currents measured in the three illuminated cells.

Consideration was given to correcting for angular rotation of the satellite during the time required to read the solar aspect cell currents. The six cell currents are measured a nominal 100 microseconds apart, so at 3 revolutions/second the satellite rotates 0.54° from first to last reading. However, calculations showed that the solar aspect cell data would need an accuracy in excess of the capability of the solar aspect cells and data encoder to show these effects. Therefore, no spin correction was included in the analysis.

II. DETERMINATION OF SPIN-AXIS ORIENTATION

In this section we want to consider the problem of determining spin-axis orientation from observational data which give the directions with respect to the spin axis of two or more known reference lines. One of these reference lines will be the line normal to a mirror attached to the satellite at the instant a flash of reflected sunlight is observed at the ground station, and the other will be either a second such reference line or the line from the satellite to the sun. Since the change in the satellite-sun line is very slow, two solar aspect observations are not very useful for determining spin-axis orientation, as the two cones would nearly coincide and hence yield results of questionable accuracy. Clearly, any pair of observations could be used for spin-axis determination, but for useful information on the spin-axis precession the observations should be made as close together in time as possible. There would be no advantage in studying the use of three or more observations simultaneously, as the three or more cones would in general not have a common intersection because of observational errors. Thus the observed data should be used in pairs for determination of spin-axis orientation and any averaging should be done by operating on the resulting direction vectors.

2.1 *Reference Line Obtained from Flash Observations*

Let the inertial reference frame used be the x, y, z frame, and let the associated unit vectors be $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$, respectively. At the instant a flash of sunlight reflected from the mirror attached to the satellite is seen at the ground station, the sun, satellite, and ground station coordinates in the x, y, z frame can be obtained by well-known methods.

Let the vector $\bar{R}_s$ from the earth mass center to the sun be (see Fig. 3)

$$\bar{R}_s = x_s\hat{\imath} + y_s\hat{\jmath} + z_s\hat{k}.$$

The vector $\bar{R}_g$ from the earth mass center to the ground station is

$$\bar{R}_g = x_g\hat{\imath} + y_g\hat{\imath} + z_g\hat{k}$$

and the vector $\bar{R}_b$ from the earth mass center to the satellite is

$$\bar{R}_b = x_b\hat{\imath} + y_b\hat{\jmath} + z_b\hat{k}.$$

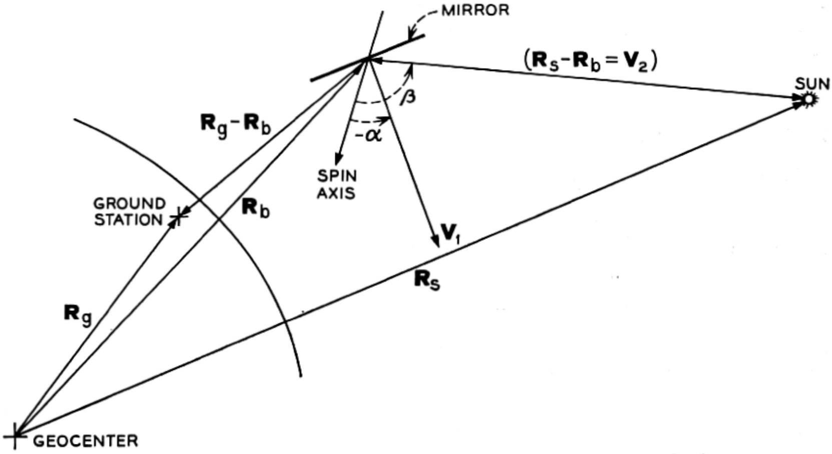

Fig. 3 — Reference frame for calculations from mirror flashes.

The vector $\bar{R}_i$ in the direction of the light ray incident to the mirror is

$$\bar{R}_i = \bar{R}_b - \bar{R}_s .$$

Since the solar parallax is only a few seconds of arc even from the satellite, we could approximate $\bar{R}_i$ by $-\bar{R}_s$, but this approximation will not be made, as very little simplification results.

The vector $\bar{R}_r$ in the direction of the reflected ray is

$$\bar{R}_r = \bar{R}_g - \bar{R}_b .$$

Let us adopt the symbol $\hat{R}$ to denote a unit vector, and the symbol R to denote the absolute value of a vector. The incident and reflected unit vectors are

$$\hat{R}_i = x_i\hat{\imath} + y_i\hat{\jmath} + z_i\hat{k}$$
$$\hat{R}_r = x_r\hat{\imath} + y_r\hat{\jmath} + z_r\hat{k}$$

where

$$x_i = \frac{x_b - x_s}{R_i}, \qquad y_i = \frac{y_b - y_s}{R_i}, \qquad z_i = \frac{z_b - z_s}{R_i}$$

$$x_r = \frac{x_g - x_b}{R_r}, \qquad y_r = \frac{y_g - y_b}{R_r}, \qquad z_r = \frac{z_g - z_b}{R_r}$$

$$\frac{1}{R_i} = [(x_b - x_s)^2 + (y_b - y_s)^2 + (z_b - z_s)^2]^{-\frac{1}{2}}$$

$$\approx \frac{1}{R_s}\left[1 + \frac{1}{R_s^2}(x_sx_b + y_sy_b + z_sz_b)\right]$$

$$R_r = [(x_g - x_b)^2 + (y_g - y_b)^2 + (z_g - z_b)^2]^{\frac{1}{2}}$$
$$= [R_g^2 + R_b^2 - 2(x_g x_b + y_g y_b + z_g z_b)]^{\frac{1}{2}}$$
$$R_s = [x_s^2 + y_s^2 + z_s^2]^{\frac{1}{2}}, \text{ distance from geocenter to sun}$$
$$R_g = \text{earth radius at ground station}$$
$$R_b = [x_b^2 + y_b^2 + z_b^2]^{\frac{1}{2}}, \text{ distance from geocenter to satellite.}$$

The vector $\bar{V}_1$ normal to the mirror is thus given by the expression

$$\bar{V}_1 = \hat{R}_r - \hat{R}_i$$

since $\bar{V}_1$ must bisect the angle between the incident and reflected rays and be normal to the mirror.

If we let x_1, y_1, z_1 be the direction cosines of the unit vector $\hat{V}_1$, we can write

$$\hat{V}_1 = x_1\hat{\imath} + y_1\hat{\jmath} + z_1\hat{k}$$

where

$$x_1 = \frac{x_r - x_i}{V_1}$$

$$y_1 = \frac{y_r - y_i}{V_1}$$

$$z_1 = \frac{z_r - z_i}{V_1}$$

$$V_1 = [(x_r - x_i)^2 + (y_r - y_i)^2 + (z_r - z_i)^2]^{\frac{1}{2}}$$
$$= [2 - 2(x_r x_i + y_r y_i + z_r z_i)]^{\frac{1}{2}}.$$

The vector $\hat{V}_1$ is our desired reference vector and can be computed explicitly by the above formulas for any time at which a flash is observed.

2.2 *Second Reference Line*

We will let our second reference line be represented by the unit vector $\hat{V}_2$

$$\hat{V}_2 = x_2\hat{\imath} + y_2\hat{\jmath} + z_2\hat{k}.$$

The vector $\hat{V}_2$ could represent the satellite-sun line in the case of a solar aspect determination, it could represent a second reference line determined from a flash observation, or it could be any other reference line whose orientation is measured with respect to the satellite body axes as well as in the x, y, z system.

2.3 *Spin-Axis Orientation*

First we let $\hat{V}_3$ be the unit vector in the direction of the spin axis. If we observe a flash from the satellite and can identify the mirror which reflected the flash, then we know the angle between the normal to the mirror and the spin axis, say α. Similarly, if we determine the sun's aspect with respect to the spin axis or if we observe a flash from a second mirror, we know that the spin axis makes an angle, say β, with our second reference line. These two conditions give us sufficient information to determine the direction of the spin axis. If we know the spin axis makes an angle α with $\hat{V}_1$ and an angle β with $\hat{V}_2$, the spin axis must therefore lie along one of the two intersections of a cone of semiangle α about $\hat{V}_1$ and a cone of semi-angle β about $\hat{V}_2$. In general we do not know which intersection is the correct location of the spin axis, but we should be able to resolve the ambiguity by the general knowledge of the situation.

We can express the above statements analytically by the following set of equations

$$\hat{V}_1 \cdot \hat{V}_3 = \cos \alpha = x_1 x_3 + y_1 y_3 + z_1 z_3 \tag{1}$$

$$\hat{V}_2 \cdot \hat{V}_3 = \cos \beta = x_2 x_3 + y_2 y_3 + z_2 z_3 \tag{2}$$

$$\hat{V}_3 \cdot \hat{V}_3 = 1 = x_3^2 + y_3^2 + z_3^2. \tag{3}$$

The solution of (1)–(3) is found to be

$$x_3 = \frac{A + B y_3}{C} \tag{4}$$

$$y_3 = \frac{-(AB + A'B') \pm [(AB + A'B')^2 - (A^2 - C^2 + A'^2)(B^2 + C^2 + B'^2)]^{\frac{1}{2}}}{B^2 + C^2 + B'^2} \tag{5}$$

$$z_3 = \frac{-(A' + B' y_3)}{C} \tag{6}$$

where

$$A = z_2 \cos \alpha - z_1 \cos \beta$$

$$B = z_1 y_2 - z_2 y_1$$

$$A' = x_2 \cos \alpha - x_1 \cos \beta$$

$$B' = x_1y_2 - x_2y_1$$
$$C = z_2x_1 - z_1x_2 .$$

Given the direction cosines of the two reference vectors $\hat{V}_1$ and $\hat{V}_2$ plus α and β, we can use (4)–(6) to calculate the direction cosines of the spin axis. The plus and minus square root term in (5) is due to the two possible locations of the spin axis.

III. CONSIDERATION OF ERRORS

It is of interest to know the effect of observational errors on the calculated orientation of the spin axis. In the following the significant sources of error are discussed and the effects of these errors on the orientation of the spin axis are calculated.

Since we know the positions of the sun, satellite, and ground station to within a small fraction of a degree of arc, we can reasonably assume that the direction of V_2 is known with negligible error and that the uncertainty in V_1 is a fraction of a degree that does not exceed the errors in the observations themselves. Our main sources of observational error are in the determination of the aspect angles α and β, so we will consider the effect of small errors in α and β on the orientation of the spin axis. The principal errors we can expect in α and β are

1. Error in measuring the angle α between the spin axis and the normal to the mirrors. This is a mechanical error of approximately $\pm 1/2°$. There is also the possibility of some wobble in the spin motion, although the nutation damper should minimize wobbling.

2. Error in determining the direction cosines of the sun with respect to body axes. This gives an error in the other cone semi-angle, β. This error has been about $\pm 1/4°$ on most passes, but has been considerably worse on others, possibly due to interference from earth-shine. Since the sun subtends an angle of $1/2°$ at the satellite, not much increase in accuracy can be expected with the solar aspect system.

3. Error in determining the center of the sun's image on the ground, i.e., ascertaining the time of the center of the train of light flashes. This error has the effect of changing the direction of the reference vector $\bar{V}_1$ by $\pm \approx 1/4°$, since the semi-angle the sun subtends at the ground station is $1/2°$ to $3/4°$, depending on the flatness of the mirrors. We will simplify our error analysis by considering the effect of this error to be an additional error in α.

4. The angular errors of $\bar{V}_1$ resulting from the position ephemeris for the satellite would enter into the error analysis in a fashion similar to item 3.

3.1 *Analysis*

First let α_0 and β_0 be the values of the cone semiangles that determine the true orientation of the spin axis, and let $\hat{V}_0$ be the unit vector in the direction of the true spin axis. Similarly, let α and β be the observed values of the cone semi-angles that determine the orientation of an estimated unit vector $\hat{V}$. We will assume that α and β differ only slightly from α_0 and β_0, respectively, so that $\hat{V}$ and $\hat{V}_0$ differ only slightly. Once we have determined the reference vectors $\hat{V}_1$ and $\hat{V}_2$ and the cone semi-angles α and β, the orientation of the spin-axis unit vector is calculated from (4)–(6). We are assuming that $\hat{V}_1$ and $\hat{V}_2$ are known with negligible error and are considering only the effect on orientation of errors in α and β, so x_1, y_1, z_1 and x_2, y_2, z_2 are fixed quantities. The two unit vectors corresponding to the true and observed spin-axis orientation are thus

$$\begin{aligned} \hat{V}_0(\alpha_0, \beta_0) &= x_0\hat{\imath} + y_0\hat{\jmath} + z_0\hat{k} \\ \hat{V}(\alpha, \beta) &= x\hat{\imath} + y\hat{\jmath} + z\hat{k}. \end{aligned}$$

We let $\delta\bar{V}$ be the small vector difference between $\hat{V}$ and $\hat{V}_0$, so we can write

$$\hat{V} = \hat{V}_0 + \delta\bar{V}.$$

For small changes of α and β we can approximate $\delta\bar{V}$ by the first- and second-order terms of a Taylor series.

$$\begin{aligned} \hat{V} &= \hat{V}_0 + \delta\bar{V} \\ &\approx \hat{V}_0 + \left.\frac{\partial\hat{V}}{\partial\alpha}\right|_{\substack{\alpha=\alpha_0\\ \beta=\beta_0}} \delta\alpha + \left.\frac{\partial\hat{V}}{\partial\beta}\right|_{\substack{\alpha=\alpha_0\\ \beta=\beta_0}} \delta\beta + \left.\frac{1}{2}\frac{\partial^2\hat{V}}{\partial\alpha^2}\right|_{\substack{\alpha=\alpha_0\\ \beta=\beta_0}} \delta\alpha^2 \\ &\quad + \left.\frac{1}{2}\frac{\partial^2\hat{V}}{\partial\beta^2}\right|_{\substack{\alpha=\alpha_0\\ \beta=\beta_0}} \delta\beta^2 + \left.\frac{\partial^2\hat{V}}{\partial\alpha\,\partial\beta}\right|_{\substack{\alpha=\alpha_0\\ \beta=\beta_0}} \delta\alpha\,\delta\beta. \end{aligned}$$

The small angle between $\hat{V}$ and $\hat{V}_0$, denoted ϵ, is found from

$$\hat{V}\cdot\hat{V}_0 = \cos\epsilon$$

so

$$1 + \hat{V}_0\cdot\left(\left.\frac{\partial\hat{V}}{\partial\alpha}\right|_{\substack{\alpha=\alpha_0\\ \beta=\beta_0}} \delta\alpha + \left.\frac{\partial\hat{V}}{\partial\beta}\right|_{\substack{\alpha=\alpha_0\\ \beta=\beta_0}} \delta\beta\right)$$

$$+\frac{1}{2}\hat{V}_0\cdot\left(\frac{\partial^2\hat{V}}{\partial\alpha^2}\bigg|_{\substack{\alpha=\alpha_0\\ \beta=\beta_0}}\delta\alpha^2+\frac{\partial^2\hat{V}}{\partial\beta^2}\bigg|_{\substack{\alpha=\alpha_0\\ \beta=\beta_0}}\delta\beta^2+2\frac{\partial^2\hat{V}}{\partial\alpha\,\partial\beta}\bigg|_{\substack{\alpha=\alpha_0\\ \beta=\beta_0}}\delta\alpha\,\delta\beta\right) \quad (7)$$

$$+\ 0(\delta\alpha^3, \delta\beta^3)$$

$$= \cos\epsilon$$

$$= 1-\frac{\epsilon^2}{2}+0\ (\epsilon^4).$$

The effect of variations in $\delta\alpha$ and $\delta\beta$ on $\cos\epsilon$ is of second order, so the term containing first derivatives of $\hat{V}$ in (7) vanishes, as can be readily verified. Thus for ϵ we find

$$\epsilon^2 = -\hat{V}_0\cdot\left(\frac{\partial^2\hat{V}}{\partial\alpha^2}\bigg|_{\substack{\alpha=\alpha_0\\ \beta=\beta_0}}\delta\alpha^2+\frac{\partial^2\hat{V}}{\partial\beta^2}\bigg|_{\substack{\alpha=\alpha_0\\ \beta=\beta_0}}\delta\beta^2+2\frac{\partial^2\hat{V}}{\partial\alpha\,\partial\beta}\bigg|_{\substack{\alpha=\alpha_0\\ \beta=\beta_0}}\delta\alpha\,\delta\beta. \quad (8)$$

The reference vectors $\hat{V}_1$ and $\hat{V}_2$ define a plane, so there is no loss of generality if we choose a ξ axis in the $\hat{V}_2$ direction and let $\hat{V}_1$ lie in the ξ, η plane where we transformed from the x, y, z axes to ξ, η, ζ axes. The details of the transformation are unnecessary, since we seek only magnitudes of error in orientation and not direction. If $\hat{e}_1$, $\hat{e}_2$, and $\hat{e}_3$ are the unit vectors in the ξ, η, ζ directions, respectively, then

$$\hat{V}_1 = \cos\gamma\,\hat{e}_1+\sin\gamma\,\hat{e}_2$$

$$\hat{V}_2 = \hat{e}_1$$

where

$$\hat{V}_1\cdot\hat{V}_2 = \cos\gamma$$

i.e., γ is the angle between the reference vectors $\bar{V}_1$ and $\bar{V}_2$. We infer from (1)–(3) that the unit vector $\hat{V}$ in the direction of the spin axis has the direction cosines

$$\xi = \cos\beta$$

$$\eta = \frac{\cos\alpha-\cos\gamma\cos\beta}{\sin\gamma}$$

$$\zeta = \pm\left[1-\cos^2\beta-\left(\frac{\cos\alpha-\cos\gamma\cos\beta}{\sin\gamma}\right)^2\right]^{\frac{1}{2}}$$

where α and β are the angles between the spin axis and $\hat{V}_1$ and $\hat{V}_2$, respectively.

After rewriting (8) with the help of ξ, η, ζ we obtain for ϵ

$$\epsilon^2 = \frac{\delta\alpha^2 - 2\left(\dfrac{\cos\gamma - \cos\alpha_0\cos\beta_0}{\sin\alpha_0\sin\beta_0}\right)\delta\alpha\,\delta\beta + \delta\beta^2}{1 - \left(\dfrac{\cos\gamma - \cos\alpha_0\cos\beta_0}{\sin\alpha_0\sin\beta_0}\right)^2} \tag{9}$$

provided

$$\frac{\cos\gamma - \cos\alpha_0\cos\beta_0}{\sin\alpha_0\sin\beta_0} \neq 1. \tag{10}$$

The restriction (10) merely assures that the two cones have two intersections, i.e.,

$$\gamma > |\alpha_0 - \beta_0|$$

$$\gamma < \alpha_0 + \beta_0 .$$

Vanishing of the denominator in (9) occurs for the singular instances when the two cones intersect in one line only or when they coincide. In these situations no solution exists if either α_0 or β_0 is varied, and so the expression for ϵ^2 is not defined.

There are four examples which are of interest, and these are examined next.

3.2 *Solar Aspect and Mirror Observation*

Usually the two reference vectors determined will be the satellite-sun line and the normal to the mirror, angles α and β being the angles between these reference lines and the spin axis.

Let $\hat{V}_2$ be directed toward the sun. The satellite is intended to be aligned nearly perpendicular to the ecliptic plane, so we will set $\beta_0 = 90°$. Equation (9) becomes

$$\epsilon^2 = \frac{\delta\alpha^2 - 2\dfrac{\cos\gamma}{\sin\alpha_0}\delta\alpha\,\delta\beta + \delta\beta^2}{1 - \left(\dfrac{\cos\gamma}{\sin\alpha_0}\right)^2}$$

and since there are two mirrors on the satellite having normals which are inclined 68° and 95° to the spin axis, α_0 can take on only the values 68° and 95°. The variation in ϵ with changes in γ is displayed in Fig. 4.

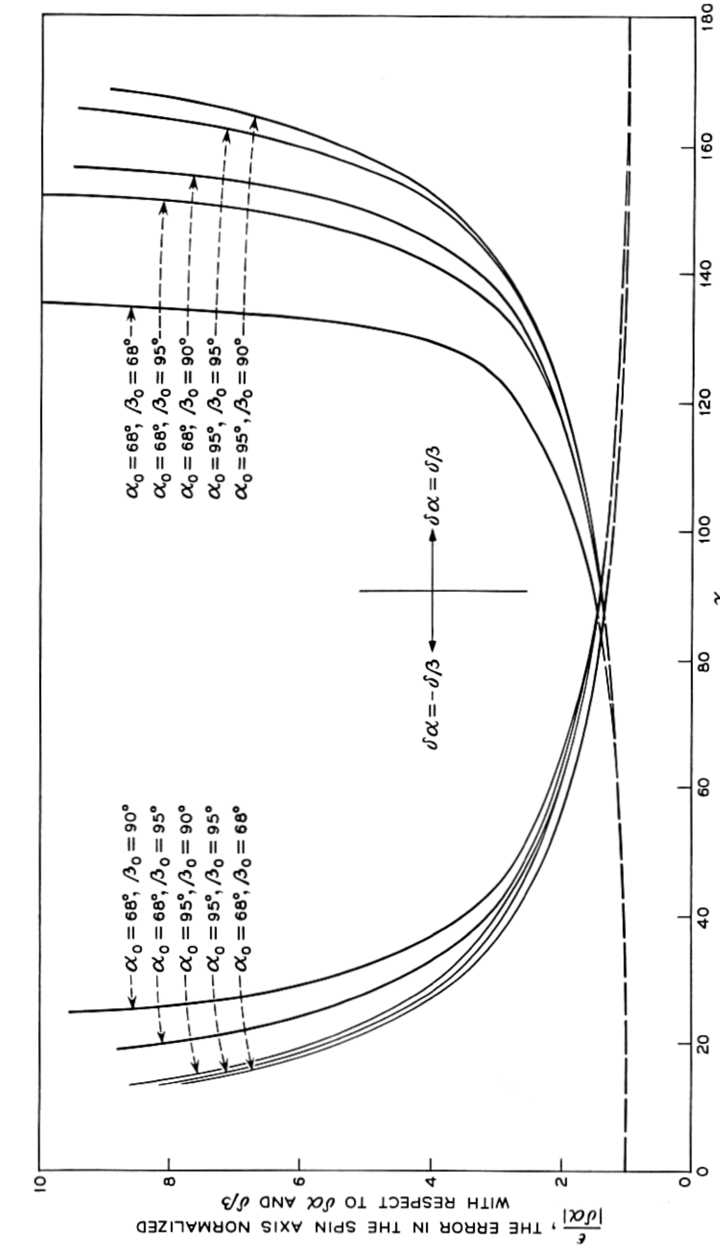

Fig. 4 — Variation of geometric errors as a function of γ.

3.3 *Two-Mirror Observations*

The three possible combinations of two mirror observations are

$$\alpha_0 = 68°, \qquad \beta_0 = 68°$$

$$\alpha_0 = 68°, \qquad \beta_0 = 95° \text{ (or vice versa)}$$

$$\alpha_0 = 95°, \qquad \beta_0 = 95°.$$

The results of varying γ for these three possible combinations, among others, are also shown in Fig. 4.

The curves of Fig. 4 show that if the angle between $\hat{V}_1$ and $\hat{V}_2$ is between 50° and 130° then

$$|\epsilon| < +2\sqrt{\delta\alpha^2 + \delta\beta^2}.$$

If $\delta\alpha = \delta\beta$ and $50° < \gamma < 130°$, then

$$\epsilon < 2.828\,\delta\alpha.$$

Thus if the observational errors in α and β are 1°, the error in determining spin-axis orientation would be less than 2.828° provided γ, the angle between $\bar{V}_1$ and $\bar{V}_2$, satisfies the above inequality.

IV. RESULTS

The motion of the spin axis of the Telstar satellites has been derived from observations of flashes of sunlight reflected from the mirrors attached to the satellite and from use of telemetry data from the solar aspect cells mounted on the satellite. Figs. 5 and 6 and Table I sum-

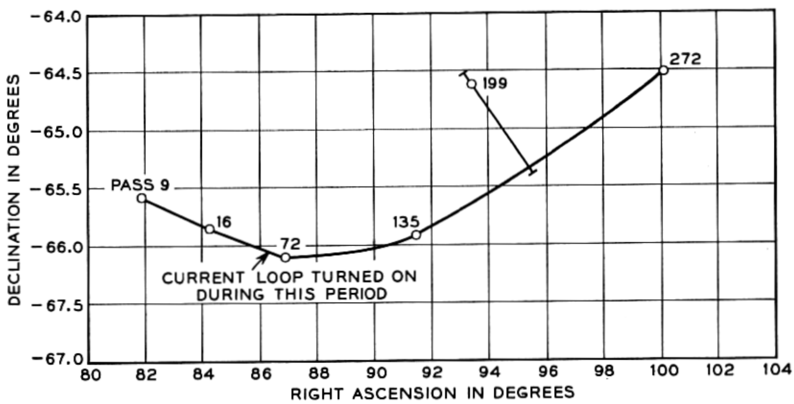

Fig. 5 — Motion of spin axis in geocentric coordinates.

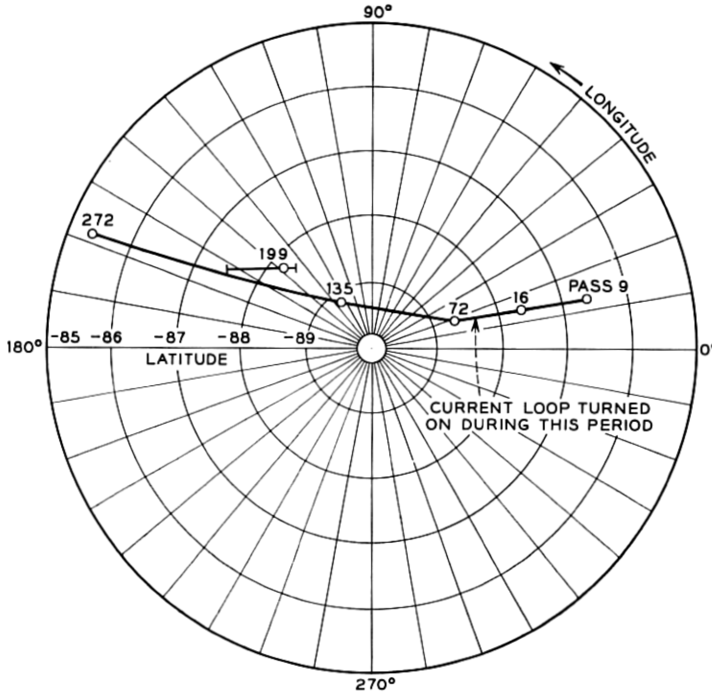

Fig. 6 — Motion of spin axis in polar coordinates referred to ecliptic plane.

marize the early precession of the Telstar I satellite.* The portion between passes 16 and 72 covers the period of time the torque coil on the satellite is known to have been activated (presumably from orbits 65 thru 70).

The accuracy obtained to date appears to be much higher than was originally anticipated. It is estimated that the angular error in determining the spin-axis orientation is less than 1/2° for all observations except pass 199. The orientation determined for pass 199 is questionable due both to poor geometric relations and to many inaccurate frames of telemetry data. Some more detailed information is given below for two of the fixes reported.

No high-frequency precession or "coning" has been detected so far, so apparently the nutation damper is operating satisfactorily.

* Right ascension is measured eastward from vernal equinox in the earth's equatorial plane; declination is measured plus or minus from earth's equatorial plane; latitude is measured plus or minus from ecliptic plane; longitude is measured eastward from vernal equinox in the ecliptic plane.

TABLE I — SPIN-AXIS ORIENTATION

Pass No.	Date	Universal Time	Right Ascension°	Declination°	Latitude°	Longitude°
7, 8, 9	7/10	0849	81.96	−65.57	−86.60	13.22
16	7/12	0214	84.39	−65.81	−87.62	15.52
72	7/18	0553	86.86	−66.10	−88.67	18.55
135	7/25	0328	91.22	−65.86	−89.19	125.95
199	8/1	0345	93.41	−64.58	−88.18	139.87
272	8/9	0341	100.08	−64.51	−85.36	158.43

Sensitivity checks have shown that timing mirror flashes to the nearest second is accurate enough.

4.1 *Attitude Calculations*

The times at which mirror flashes have been observed are the basis for attitude calculations; early observations for the Telstar I satellite are listed in Table II.

4.1.1 *Passes 7, 8, 9*

Two mirror flashes were observed on each of passes 7, 8, and 9, thus providing a fix for each of these passes. The accuracy of these three fixes was not sufficiently great to permit determining the motion between passes, so all three were averaged to obtain a single fix. The motion between successive passes is about 0.03°, probably below the precision of the present methods to resolve, although additional efforts were made on the data for passes 7, 8, and 9 to try to obtain such precision.

The results of various combinations of mirror flashes are given in Table III to show the scatter obtained. This exercise demonstrates that

TABLE II — MIRROR FLASH OBSERVATIONS

Observation No.	Pass No.	Date	Universal Time, h.m.s.	Mirror Observed
M1	7	7/11	0235 + 26	68°
M2	7	7/11	0250 + 47	68°
M3	8	7/11	0518 + 08	68°
M4	8	7/11	0535 + 36	68°
M5	9	7/11	0817 + 22	68°
M6	9	7/11	0821 + 30	95°
M7	16	7/12	0214 + 19	68°
M8	72	7/18	0553 + 26	68°
M9	135	7/25	0328 + 07	68°
M10	136	7/25	0609 + 31	68°
M11	199	8/1	0345 + 37	68°
M12	272	8/9	0340 + 58	68°

TABLE III — RESULTS FOR PASSES 7, 8, AND 9

Mirror Observations	Right Ascension°	Declination°
M1 & M2	82.09	−65.73
M3 & M4	82.59	−65.70
M5 & M6	81.00	−65.23
M1 & M5	82.97	−65.74
M2 & M5	81.27	−65.25
M3 & M6	81.77	−65.77

adequate attitude determination is possible from mirror data alone when solar aspect readings are unavailable or considered unreliable.

4.1.2 *Pass 135*

Mirror flashes were obtained for each of passes 135 and 136 and were used to obtain fix number 1 given in Table IV. Additionally, all the solar aspect data for pass 135 was combined with the mirror flash for pass 135, and 17 selected aspect readings averaged to obtain fix number 2. The average of these two was used for the pass 135 attitude fix.

4.1.3 *Remaining Observations*

All other attitude fixes reported here were obtained by combining one mirror observation with all the solar aspect telemetry data for the pass during which the mirror was observed.

4.2 *Residual Magnetic Moment*

The average residual magnetic moment has been determined approximately by comparing the observed spin axis precession against theoretical predictions. The value obtained is 0.5 ampere-turn-meter^2, which is of the same order of magnitude as the pre-launch estimates.

V. CONCLUSION

The conclusion drawn from these results is that it clearly is possible to track the motion of the spin axis with mirrors and solar aspect cells,

TABLE IV — RESULTS FOR PASS 135

No.	Right Ascension°	Declination°
1	91.10	−65.80
2	91.33	−65.97

and that controlling the spin-axis orientation slowly by means of a torque coil is feasible. Much more extensive tests of the procedure have been performed than described here, which cover the remaining life of the Telstar I satellite and the Telstar II satellite. The computational procedure has been modified so that no data from the solar aspect cells are required. In general, the resulting attitude angles fall well within the error estimates given in Section 3.3. A report on this work is being prepared by L. C. Thomas.

REFERENCE

1. Courtney-Pratt, J. S., Hett, J. H., and McLaughlin, J. W., Optical Measurements on *Telstar* to Determine the Orientation of the Spin Axis and the Spin Rate, J. Soc. Motion Picture and Television Engineers, **72,** June, 1963, pp. 462–484; the abstract of this paper is reproduced here to briefly introduce the design and operation of the equipment used to detect the flashes of light from the mirrors:

"One plane mirror and two faceted mirrors were fitted to Telstar. A photoelectric telescope was mounted on a radar antenna pedestal. Appropriate recording gear was built. The 12-in. aperture Cassegrain photoelectric telescope could be pointed at the satellite directly from prediction tapes, and the pointing accuracy could on occasion be improved by homing on the microwave beacon on the satellite or by applying corrections from visual observation through other telescopes with apertures of up to 6 in. It has been possible to detect the flashes of sunlight reflected from the mirrors on Telstar at slant ranges up to 3,700 miles. Directly from pairs of these observations, or by combining this information with telemetry data from the solar sensors on the satellite, it has been possible to determine the orientation of the spin axis of Telstar and to examine how this is moving in time. This normal precession of the spin axis is due mainly to the interaction of the residual magnetic moment of the satellite with the magnetic field of the earth. It was also possible to study how the spin axis moved when the torque coil was operated.

Measurements have been made of the spin rate of the satellite and of its rate of decay. The spin rate will drop to $\frac{1}{10}$ of its initial value in about 750 days.

Photographs of the glints have been taken, and a study is now being made of the feasibility of combining precise photoelectric observation of the time of the flashes with high-precision photographic observation of the instantaneous position of the satellite relative to the fixed star background."

For a more complete description, the reader will wish to consult the full article.