B.S.T.J. BRIEFS

On the Spectrum of Optical Waves Propagated through the Atmosphere

By D. C. HOGG

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It is well known from our day-to-day experience that distant objects appear to shimmer, especially on still, hot days when relatively large temperature and humidity gradients may exist. It is true, however, that refractive index gradients are in the air to some degree at all times; that is, they may be present at night and also when the air is disturbed by the winds. It is not surprising therefore that the power received at some distance from an optical source fluctuates randomly and possesses a characteristic low-frequency power spectrum.

The purpose of this note is to discuss typical low-frequency spectra resulting from propagation of 0.63-micron radiation over a 2.6-km path. A vertically polarized helium-neon maser[1] of power output 10 mw and a reflecting telescope of 9-cm diameter comprise the source. With all modes of the maser propagating, the beam spreads to an ill-defined and ever-changing disk of about 25-cm diameter.* The receiver is a refracting telescope of 5-cm diameter with associated filters, polarizers, and attenuators which feed a photomultiplier; it is located in the dense central region of the transmitted beam. The beamwidth of the receiver is large compared with that of the transmitter. For measurement of the power spectrum, the output of the photomultiplier is taken to a wave analyzer whose bandwidth is 4 cps.

A typical power spectrum is shown in Fig. 1, the measured points being indicated by open circles.† The abscissa, $F = f - f_o$, is sideband frequency, and the ordinate indicates relative received power. On this scale the direct-current level is 133 units; thus the power in the lower frequency components is about 13 db below the dc level. The curve has been extrapolated from 10 cps to 1 cps. These data were obtained with the full aperture of the source telescope illuminated.

At this point it is well to note that when the source is brought within

* Dependent upon weather conditions.

† Similar data, using an incoherent source, were reported by V. I. Tatarski in Ref. 2.

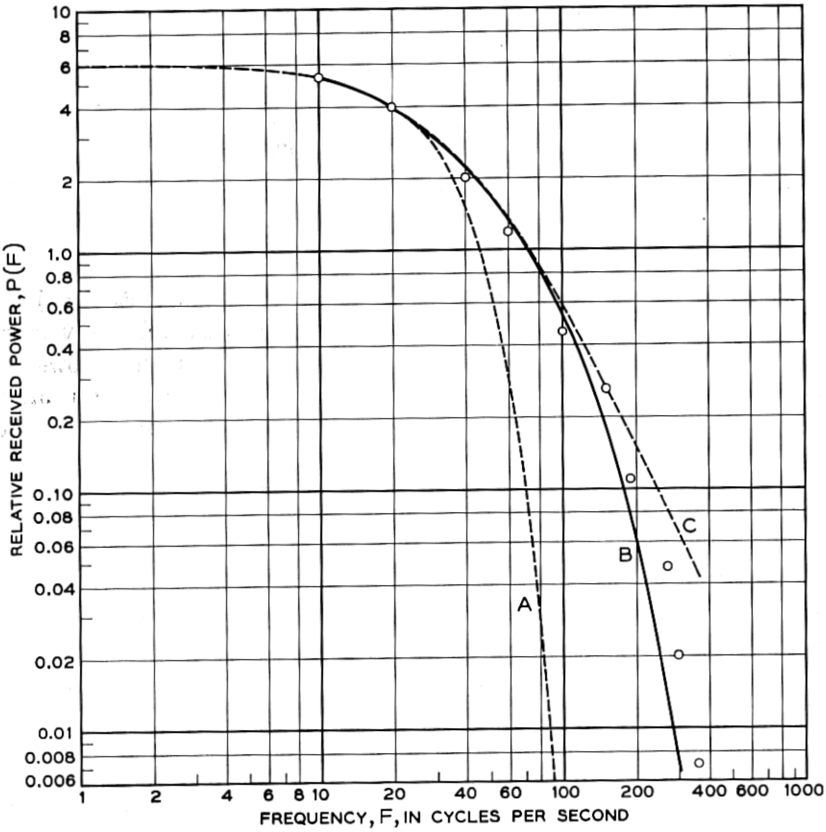

Fig. 1 — Low-frequency spectrum: $\lambda = 0.63\ \mu$ for a 2.6-km path length.

the laboratory housing the receiver and the same measurement procedure is followed, no such spectrum is observed.

Three theoretical distributions, $P(F)$, shown as curves in Fig. 1, accompany the experimental data. Curve A is a Gaussian, B an exponential; in C, $P(F)$ is of the form $(1 + KF^2)^{-1}$. In all cases, the theoretical curves have been fitted to the experimental data at $F = 30$ cps, where the spectrum has fallen to one-half its very-low-frequency value. The exponential appears to best represent the data.

Fig. 2 shows the effect of change in source beamwidth on the width (and shape) of the spectrum. For curve 2, the source beamwidth is about ten times that for curve 1. One notes that the half width increases from 40 to 120 cps with that increase in beamwidth. From this measure-

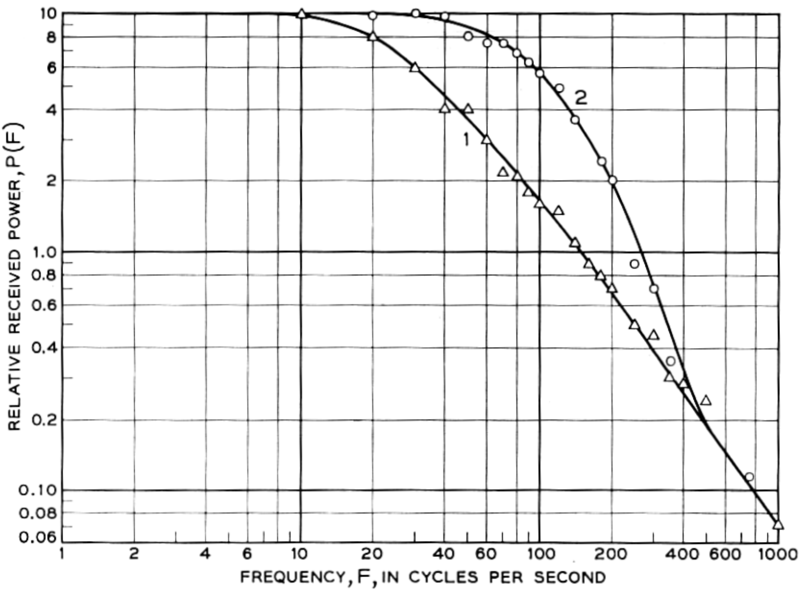

Fig. 2 — Low-frequency spectrum: $\lambda = 0.63\ \mu$ for a 2.6-km path length.

ment we deduce that the fluctuation rate is roughly proportional to the square root of the beamwidth. It is interesting that the two spectra appear to coalesce at the higher frequencies; however, in interpreting these data, one must recall that the near field of the source extends well toward the receiver in the case of curve 1.

Of course, when one listens to the detected envelope of the optical wave, it is heard as low-frequency audio noise.

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A Relation Between the Basis Functions of Periodically Varying Nondissipative Circuits

By SIDNEY DARLINGTON

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This note relates to circuits of linear, periodically time-varying, positive capacitors and inductors. More specifically, it notes a property of the basis functions, or natural modes which describe the transient behavior of the circuits. Some other properties of the basis functions are described in Ref. 1 in this issue.

In accordance with Ref. 1

$$I = (S + pCp)\Phi, \qquad E = p\Phi \tag{1}$$

where I and E are column matrices, or vectors representing the excitation currents into and the response voltages at the various nodes, and Φ is an auxiliary vector variable. The matrices C and S are the node matrices of the capacitances and of the reciprocals of the inductances. They are symmetric and at least positive semidefinite. We will assume that C is positive definite. If it is not so originally, a similar equation with a positive definite C can be derived from (1), for example by the transformations described in the Appendix of Ref. 1. The symbol p represents differentiation (*not* frequency) and operates on all quantities following it.

Setting $I = 0$ in (1) makes it a homogeneous, second-order, vector differential equation in, say n dimensions, with periodically varying coefficients. Thus the well-known Floquet-Poincaré theorem requires the solution to be as follows

$$\Phi = \sum_{\sigma=1}^{2n} k_\sigma \varphi_\sigma\,, \qquad \varphi_\sigma = H_\sigma(t)e^{s_\sigma t}. \tag{2}$$

Here the k_σ's are arbitrary scalar constants and the φ_σ's are the basis functions, or natural modes. The characteristic exponents s_σ are scalar constants. The coefficients H_σ are time-varying vectors. When the s_σ's are all different, the H_σ's vary periodically. Otherwise, they are at most polynomials in t with periodically varying vector coefficients.

The real parts of the s_σ's indicate the damping of the basis functions. When the inductors and capacitors are fixed, all basis functions are undamped. However, when the components vary periodically some of the basis functions may have nonzero damping (as is well known). Some bounds on the damping are derived in Ref. 1. It is also known that the sum of all the characteristic exponents must be zero (when the circuit is nondissipative).

The purpose of this note is to point out the following: *Corresponding to any nondissipative circuit of periodically varying positive inductors and capacitors, any characteristic exponents which have nonzero real parts occur in equal positive and negative pairs.* Since complex exponents must occur in conjugate pairs, it follows that all the s_σ's must fit into pairs or quadruplets of the following sorts:

$$\begin{matrix} +i\omega_k & +\alpha_j & +\alpha_h + i\omega_h \\ -i\omega_k\,, & -\alpha_j\,, & +\alpha_h - i\omega_h \\ & & -\alpha_h - i\omega_h \\ & & -\alpha_h + i\omega_h\,. \end{matrix} \tag{3}$$

The theorem follows at once from certain general properties of adjointly related differential equations, which are stated below without derivation. The solution of the nonhomogeneous equation (1) may be expressed in terms of functions of the excitation time, τ, and the response time, t. The adjoint equation is the equation whose solution corresponds to interchanging the two times, τ and t. It can be shown that the characteristic exponents of the adjoint equation are the negatives of those of the original equation. It can also be shown that equation (1) is self-adjoint. (The equation is its own adjoint.) Thus the negatives of the characteristic exponents of the original equation are also the characteristic exponents of the original equation.

When the components are fixed, vectors I and E are related by a set of odd rational functions of a frequency variable. Then Φ, whose derivative is E, is related to I by even rational functions. It appears that, more generally, self-adjoint differential equations are useful counterparts, for time-varying circuits, of even rational functions in the theory of fixed circuits. (See also Ref. 2.)

Recall Foster's canonical fixed nondissipative one-ports, comprising series- or parallel-connected subcircuits of one or two components each. The grouping (3) suggests that there *may* be a time-varying counterpart, in which the most complicated subcircuits correspond to quadruplets of $\pm$ complex characteristic exponents. However, although the existence of the grouping (3) is necessary for such a configuration, it does not in itself prove, or even support a strong conjecture, that the configuration is, in fact, a canonical periodically varying nondissipative one-port.

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A 5-Gigacycle Tunnel Diode Oscillator with 9-Milliwatt Output from a Single Diode

By M. V. SCHNEIDER

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This brief paper describes a tunnel diode oscillator which has been built by using a single diode in a low-impedance strip transmission line.

Tunnel diodes were made from p-type zinc-doped gallium arsenide with a doping level of 7×10^{19} carriers per cubic centimeter. Peak currents in the range of 150–300 ma are obtained by alloying tin pellets with a diameter of approximately 1 mil to the gallium arsenide wafer.

A new type of mount has been used in order to reduce the package reactance as well as the contact resistance of the diode in the transmission

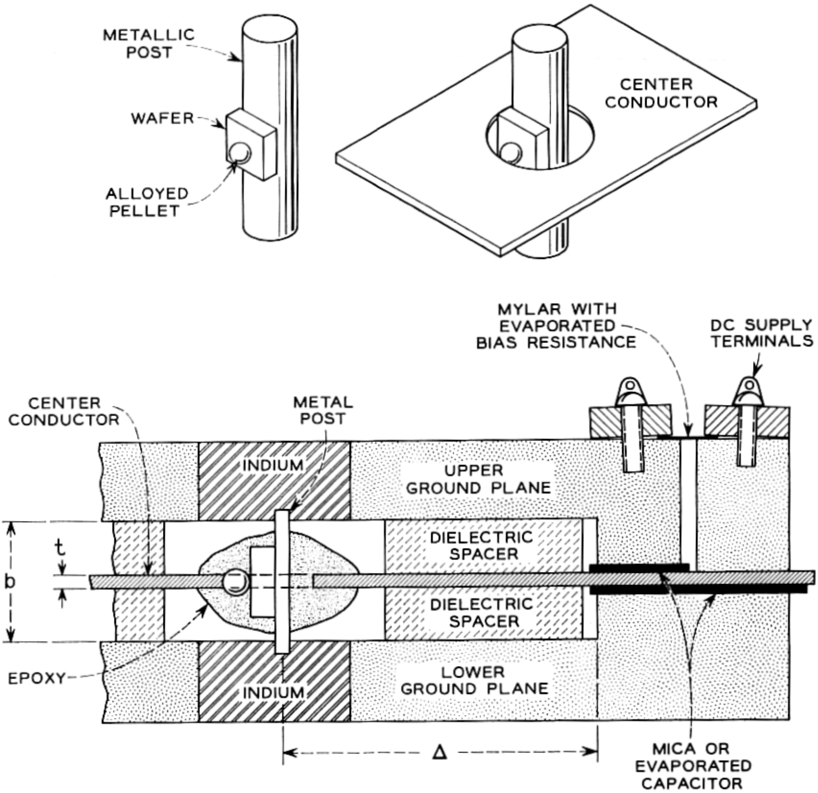

Fig. 1 — Tunnel diode mounted in balanced strip transmission line.

TABLE I — DIMENSIONS OF BALANCED STRIP TRANSMISSION LINE

Ground plane spacing	b = 6 mils
Width of center conductor	w = 100 mils
Thickness of center conductor	t = 1 mil
Distance diode to RF short	Δ = 100 mils
Impedance of strip transmission line	Z = 2.8 ohms

line. The mount is shown in Fig. 1. A hole with an 8-mil diameter is made in the 1-mil center conductor of the strip transmission line. The gallium arsenide wafer is alloyed to a 3-mil gold wire containing 3 per cent zinc. The diode is inserted into the hole, and the pellet is soldered

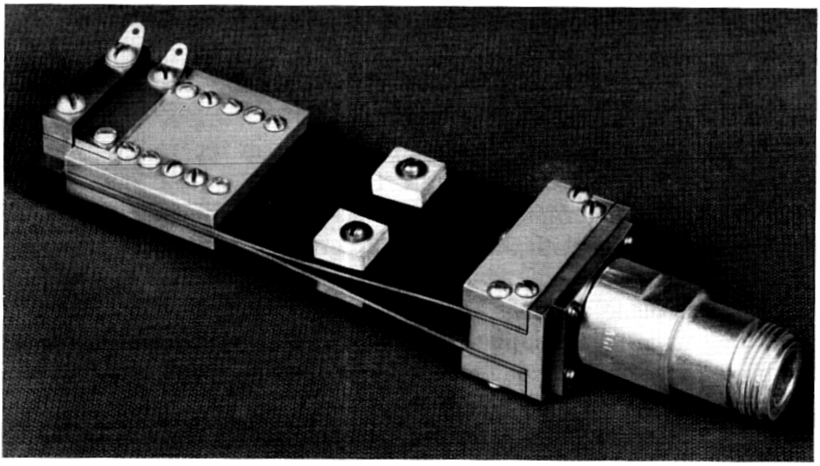

Fig. 2 — Complete tunnel diode oscillator.

to the tinned edge of the center conductor. A small drop of epoxy is used to lock the diode into the hole. The gold wire connects the upper and lower parts of the ground plane after the line is assembled.

It is advisable to have a radio-frequency short close to the diode in order to prevent the oscillator from multimoding. The distance between the diode and the RF short determines the frequency of oscillation. The short shown in Fig. 1 consists of a symmetrical step in the ground plane. The center conductor is insulated from the ground planes by an evaporated oxide film or a thin sheet of mica. This section serves at the same time as a bypass capacitor for the dc supply. The bias resistance consists of a thin nichrome film evaporated on Mylar.

The dimensions of a particular assembly are given in Table I.

A special line transformer is used for matching the low-impedance

TABLE II—ELECTRICAL PARAMETERS OF GALLIUM ARSENIDE TUNNEL DIODE

Peak current	I_p = 210 ma
Valley current	I_v = 12 ma
Peak voltage	E_p = 225 mv
Valley voltage	E_v = 560 mv
Junction capacitance	C = 12 pf

structure into a 50-ohm coaxial line. The impedance transformation is obtained by increasing the ground plane spacing b from 6 mils to approximately 300 mils. This type of transformation leads to a structure with relatively low losses. A broadband launcher is used for the transition from balanced strip transmission line to coaxial line.

The complete oscillator is shown in Fig. 2. An output power of 9.1 mw at 4.85 gc was obtained from a single diode. The electrical parameters of the diode are listed in Table II.

The junction capacitance has been measured in the valley by using the techniques developed by D. E. Thomas.[1] Several attempts were made in order to determine the series resistance of high-current diodes in a transmission line with the dimensions given in Table I. It was found that the DeLoach method[2] gives very satisfactory results for relatively large ground plane spacings (10 mils and above); however, measurements at smaller ground plane spacings are at the present time not possible because of increasing line losses and because of launching problems.

Several oscillators were built with diodes having peak currents in the range from 150 ma to 250 ma. The output power ranged from 6 mw to 10 mw and the frequency of oscillation from 3.5 gc to 5.5 gc. No degradation effects have been observed. One particular oscillator has been running continuously for two months with a power stability of better than ±2 per cent, measured with a temperature stabilized power meter.

The author expresses his thanks to R. Neeld, who has developed the necessary techniques for assembling and mounting devices with an over-all size of several mils.

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