leading to red (7000 Å) electroluminescent junctions. The diffusion was done in an evacuated and sealed-off quartz tube using as a source[2] a Zn + GaP mixture. The efficiency of the emission was determined with an integrating sphere and a photomultiplier with S-1 response calibrated in absolute units, and was found at room temperature to be about 1.0×10^{-3} photons per electron for the best samples. Red electroluminescence in GaP was previously reported to have efficiencies of about 10^{-4} (see Ref. 3) and $10^{-4} - 10^{-3}$ (see Ref. 4).

If silver contacts are alloyed onto the rough side of the solution-regrown GaP crystals, green electroluminescence can frequently be observed at the contact area. The efficiency of the green emission was found to be 4×10^{-5} photons (5550 Å) at 300°K observed outside the crystal per recombining electron-hole pair for the best samples. This compares with efficiencies of 3×10^{-5} measured by Gershenzon et al.[5] and efficiencies smaller than 10^{-4} as indicated by Allen et al.[3]

The figure shows one of the red electroluminescent crystals with a Zn-diffused junction photographed in its own electroluminescent light.

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Matching of Optical Modes

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In experiments with coherent laser light it is frequently necessary to transform a given Gaussian beam[1,2] into a Gaussian beam with certain desired parameters. It is required, for example, to transform the light beam emerging from a laser oscillating in a fundamental mode in order to provide for optimum injection into a light transmission line[2,3] (consisting of a sequence of lenses), or for optimum coupling into a spherical mirror interferometer.[4] In these cases one has to "match" the incoming beam to the natural mode of the system in question. Lenses inserted in the beam perform the matching transformation. The design of a matching configuration has to take full account of the laws[1,2,3] that govern optical modes. This leads to a somewhat complex analysis.[5] The results, however, are quite simple matching formulae which are presented in this brief. A matching experiment is described for illustration.

The given beam is characterized by its minimum beam radius[1,6] (spot size) w_1 and by the location of the beam waist. The problem is to transform this beam into another with a minimum radius w_2. The quantities w_1 and w_2 determine a characteristic "matching length" f_0 given by

$$f_0 = \pi \frac{w_1 w_2}{\lambda} \tag{1}$$

where λ is the wavelength. One beam is transformed into the other if a lens with a focal length f larger than f_0 is spaced between the two beam minima as shown in Fig. 1. The distances d_1 and d_2 between the lens and the beam minima have to satisfy the following matching conditions

$$\frac{d_1}{f} = 1 \pm \frac{w_1}{w_2} \sqrt{1 - \frac{f_0^2}{f^2}} \tag{2}$$

$$\frac{d_2}{f} = 1 \pm \frac{w_2}{w_1} \sqrt{1 - \frac{f_0^2}{f^2}} \tag{3}$$

where the same sign should be used in both equations. From (2) and (3) it follows that matching is not possible if $f < f_0$. If one chooses $f = f_0$ then $d_1 = f_0$ and $d_2 = f_0$; the beam minima are located in the two focal planes of the lens.

When one uses more than one lens to achieve the desired beam transformation, the above matching formulae are still applicable. Then f is the focal length of the lens combination, and d_1 and d_2 are measured from the principal planes. If the modes of two given optical systems are to be matched, one need not evaluate the beam parameters w_1 and w_2, which are functions[1,6] of λ and the system parameters: the matching parame-

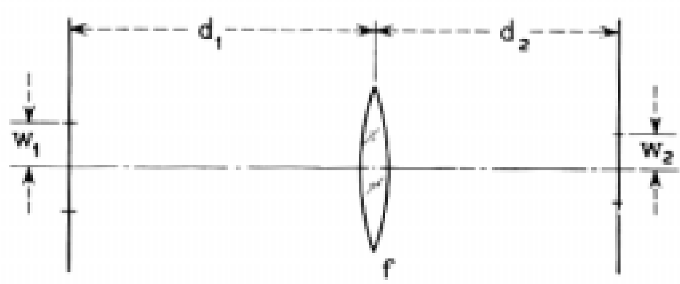

Fig. 1 — Matching configuration.

ters f_0, $\frac{w_1}{w_2}$, d_1, and d_2 are independent of λ and can be expressed in terms of the system parameters alone.

In our experimental study the light beam was taken from a He-Ne gas laser oscillating in a fundamental mode at $\lambda = 0.63$ micron. The laser cavity consisted of a concave mirror of 1 meter focal length and a flat output mirror. The mirror spacing was 1.7 meters. The (minimum) beam radius at the flat is computed[1] as $w_1 = 0.37$ mm. This beam was passed through a matching lens and then injected through a slit into a mirror system formed by two concave mirrors of 12.5 meters focal length

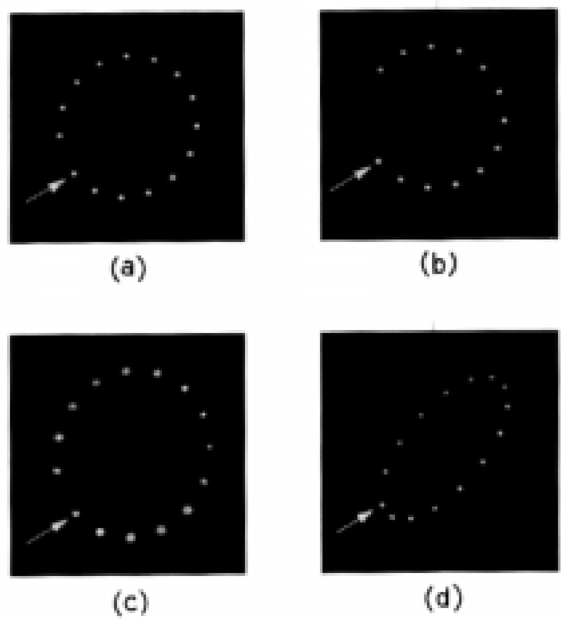

Fig. 2 — Photographs of beam spots on mirror.

spaced 50 centimeters apart. The injection angle was so chosen that the beam was reflected back and forth between the mirrors many times before it was finally intercepted, with the points of beam impact on each mirror forming a circular pattern. Such a beam configuration was described and analyzed in Ref. 7. As the beam passes back and forth between the mirrors its radius is changed in the same way as for transmission through a sequence of lenses[2,3,8] with corresponding parameters. The minimum beam radius of a fundamental mode of this sequence is computed as $w_2 = 0.7$ mm.

From the above data one obtains a matching length of $f_0 = 1.3$ meters. A lens of a focal length of $f = 1.3$ meters was available and was used as matching lens. Therefore, spacings $d_1 = d_2 = f_0 = f = 1.3$ meters were required for matching.

A mirror of the multiple-pass system was slightly transparent and Fig. 2 shows photographs of the beam-impact points taken through this mirror. In Fig. 2(a) the arrow marks the point where the injected beam strikes the mirror first. After one return trip the point of impact is the neighboring point to the right. Subsequent impact points after a corresponding number of return trips appear counterclockwise on a circle. The beam was intercepted after 14 return trips. For illustration we show Fig. 2(b), where the beam was intercepted after 12 return trips. In both cases mode-matching conditions were fulfilled and all beam radii at impact are seen to be the same. In Fig. 2(c) one can see how the beam radii at the mirror vary periodically[9] if some mismatch is introduced: the spacing d_1 was misadjusted by about 25 cm. Fig. 2(d) shows the elliptical pattern obtained for another injection angle. Here the modes were matched again and all beam spots are of equal size.

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