Evaluation of Solar Cells by Means of Spectral Analysis

By H. K. GUMMEL and F. M. SMITS*

(Manuscript received February 20, 1964)

An approach toward testing of solar cells is outlined, and a test set and evaluation procedure of test results are described. Outer space short-circuit current is calculated from spectral response measurements performed on the cells. From this and additional measurements that determine the forward diode characteristic, the maximum obtainable power and the voltage at which maximum power is delivered are computed. The accuracy of outer space short-circuit current predictions is ±2 to 3 per cent when suitable standards are employed.

I. INTRODUCTION

The solar cell is a device which converts light energy into electrical energy. The electrical output of the cell, or response, depends on the spectral composition of the incident light. For space applications, determination of the response of solar cells to sunlight not filtered by the atmosphere is of great importance.

In characterization of the performance of a solar cell under outer space illumination, the most important parameter is the short-circuit current under such illumination. Once this current is known, the current-voltage output characteristic can be measured under a light source of arbitrary spectral distribution but of an intensity adjusted to produce the outer space short-circuit current.

This paper shows that it is practical to obtain the outer space short-circuit current from measurements of the spectral response of solar cells. Of the alternative approaches, one method relies on measurements under terrestrial sunlight while another method attempts to simulate the spectrum of the sun. Each of these two methods has practical difficulties that offset its inherent simplicity.

The direct sunlight measurements must be made outdoors unless

* Sandia Corporation, Albuquerque, New Mexico.

special buildings or sun tracking facilities are available and should be made at high mountain altitudes. Only a few hours around noon on clear days can be used for precision work. These requirements constitute a considerable difficulty, especially for laboratories located on the East Coast. Even if measurements are made at high altitudes, corrections must still be applied for the alteration of the spectrum and the reduction of over-all intensity by atmospheric absorption and scattering.

Direct solar simulation requires a light source which is constant in time and which has a spectral composition equivalent to that of the sun. Absolute spectral measurements of high accuracy are required for calibration and maintenance. This problem is eased, however, if calibrated solar cells are available against which the calibration of the simulator can be checked.

The only information available from a solar simulator, as well as from direct sunlight measurements, is the total current of a solar cell; detailed spectral response information is not obtained.

In the method which is described here, spectral response measurements are performed on the solar cells themselves. The spectral response at a given wavelength, multiplied by the sun's intensity at the same wavelength, gives the contribution to the short-circuit current at this particular wavelength. The total short-circuit current is obtained by integration of these contributions over all wavelengths. The spectrum of the sun is thus introduced only in calculations; the problem of building and maintaining a sun simulator is avoided. The accuracies with which the spectral response of the cells can be measured are, in principle, comparable to the accuracies with which the spectral distribution of the output of a solar simulator can be measured. However, if appropriately calibrated standard solar cells[1] are used, the accuracy of the outer space currents as determined by the test equipment can be considerably higher than that of the spectral measurements themselves.

An automatic testing facility based on such principles has been developed which, in addition to the spectral measurements, evaluates the current-voltage characteristic of the solar cell under test. The measurements are recorded on IBM cards to facilitate data handling. Evaluation of the measurements and any statistical analysis of the results is then easily done on an electronic computer.

This paper gives a description of the test equipment and the testing procedure. First, a description of the theory is given that relates spectral information and short-circuit current. There follows a detailed account of the design of the test set. The procedure for evaluation of the test results is described in the last section.

II. THEORY

In the derivation of the relation between the spectral response of solar cells and short-circuit current it is convenient to represent spectral response in terms of quantum efficiency, defined here as the number of electrons delivered into a short circuit per photon incident on the solar cell. This represents an over-all efficiency and includes the effects of light reflection at the surface, internal carrier-pair generation efficiency, and loss of carriers due to recombination.

If the quantum efficiency is known, the outer space short-circuit current, I_{scos}, can be calculated by integration over all wavelengths of the product of the incident outer space photon flux density[2] $\varphi(\lambda)$ and the quantum efficiency $Q(\lambda)$ of the cell

$$I_{scos} = Aq \int Q(\lambda)\varphi(\lambda)\, d\lambda \tag{1}$$

where A is the cell area and q the electronic charge. The quantum efficiency of a solar cell is a smoothly varying function of wavelength. For this reason it is adequate to sample the quantum efficiency only at a small number of discrete wavelengths. The integral can be approximated by a sum

$$I_{scos} = Aq \sum_i Q(\lambda_i)\varphi(\lambda_i)\Delta_i \tag{2}$$

where the photon flux entering the sum must be smoothed according to the intervals at which the quantum efficiency is sampled.

The λ_i are the wavelengths at which the quantum efficiency is measured, and the Δ_i are determined by the wavelength intervals between these points and the integration scheme used, e.g., trapezoidal approximation, Simpson's rule, Gaussian quadrature, etc.

The quantum efficiencies are measured as shown schematically in Fig. 1. Light from a tungsten light source is passed through interference filters which are mounted on a turntable. To eliminate the influence of fluctuations and drift in the light level, the ratio of the response of the cell to be measured to that of a monitor cell is taken. In terms of such ratios, R_i, the quantum efficiency of the sample cell can be expressed as

$$Q(\lambda) = R_i Q_{\mathrm{ref}}(\lambda_i) \tag{3}$$

where $Q_{\mathrm{ref}}(\lambda_i)$ is the quantum efficiency of the monitor cell. The sum (2) can thus be written:

$$I_{scos} = \sum_i R_i[AqQ_{\mathrm{ref}}(\lambda_i)\varphi(\lambda_i)\Delta_i]. \tag{4}$$

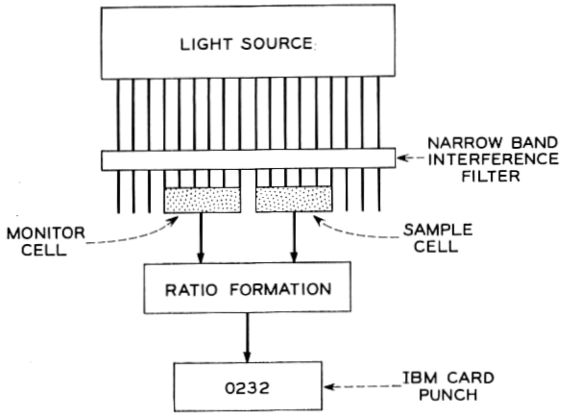

Fig. 1 — Method of measuring ratio of quantum efficiencies.

Defining the quantities in the brackets as "weighting factors"

$$W_i = AqQ_{\text{ref}}(\lambda_i)\varphi(\lambda_i)\Delta_i\,, \tag{5}$$

one obtains for the outer space short-circuit current

$$I_{scos} = \sum_i R_i W_i\,. \tag{6}$$

The weighting factors include the spectral composition of outer space sunlight and the quantum efficiency of the monitor cell. In the derivation, outer space sunlight served only as an example. Clearly, the method is not restricted to outer space illumination but can be applied, with appropriate weighting factors, for any illumination of fixed spectral composition and intensity. Practical schemes for obtaining the weighting factors in connection with the calibration of a set of solar cells will be discussed below.

As a preliminary step in obtaining weighting factors for outer space current prediction, a set of weighting factors referred to as terrestrial weights was determined for the short-circuit current of solar cells measured on a clear day near noon at a high altitude. The set of solar cells which were to be calibrated as standards had a wide range in spectral response, as shown in Fig. 2. It included cells bombarded with various fluxes — up to $2 \times 10^{16}/\text{cm}^2$ — of 1-Mev electrons and virgin cells.

To find the terrestrial weighting factors, first the quantum efficiency of the monitor cell at the various wavelengths is measured on a relative scale by comparison of the monitor cell output with that of a spectrally flat detector, such as a thermocouple. Combining this information with relative spectral intensities of the sun under which the observation was taken, one obtains the short-circuit current to within a common constant factor for all cells. This factor is readily determined from the actual measurements of the short-circuit currents. It is significant that only

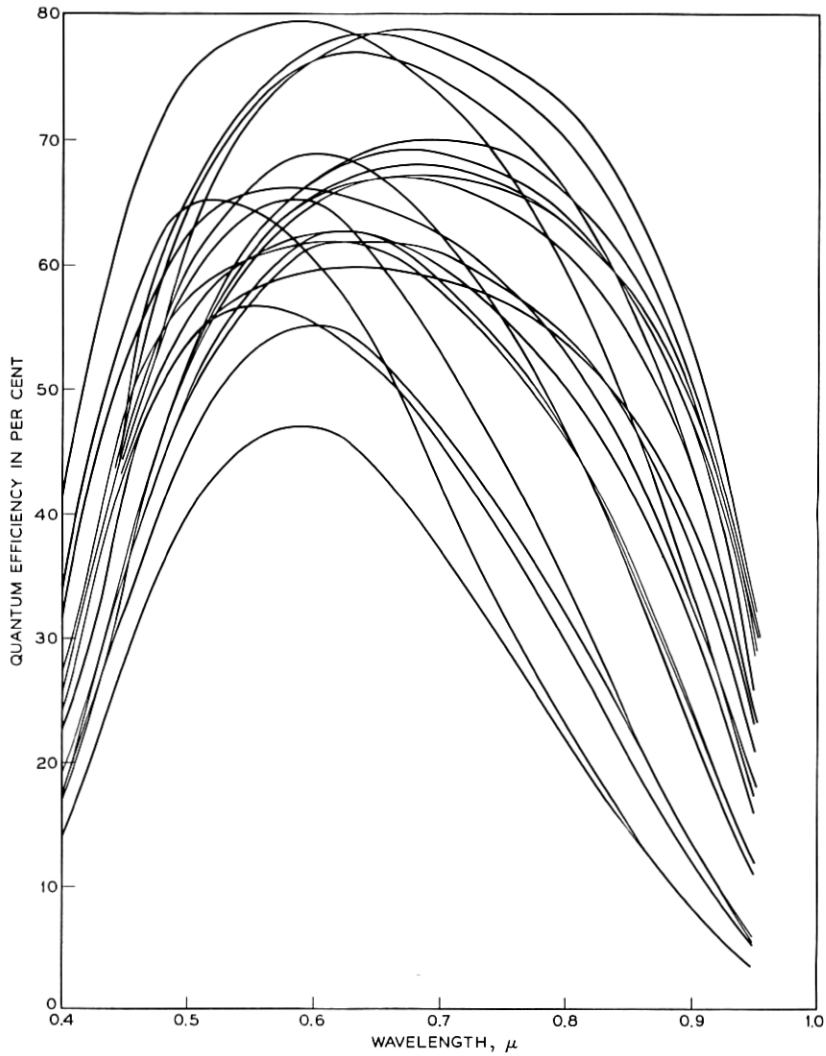

Fig. 2 — Quantum efficiencies of set of cells used as standards.

the current measurements determine the absolute scale, while all other measurements need to be performed on a relative scale.

However, if the set of solar cells used in such measurements has a sufficiently wide spread in spectral response characteristics, it is possible to find the weighting factors even without any prior information on the quantum efficiency calibration and on the spectral composition of the sun. The appropriate weights are simply those which reproduce the current of all cells with the least error.

In the present work, the terrestrial weighting factors were determined by a judicious combination of the two methods. Fig. 3 shows the resulting comparison of the measured short-circuit current and the calculated current. In this figure the most heavily prebombarded cells appear at the lower left-hand corner.

To convert the terrestrial weighting factors into outer space weighting factors, it is necessary to increase the weighting factors at each wavelength by the ratio of outer space solar intensity to terrestrial solar intensity at this particular wavelength. Such information is readily available from solar spectral recordings performed by the Smithsonian Institution.[1] For the set of solar cells shown in Fig. 2 an outer space

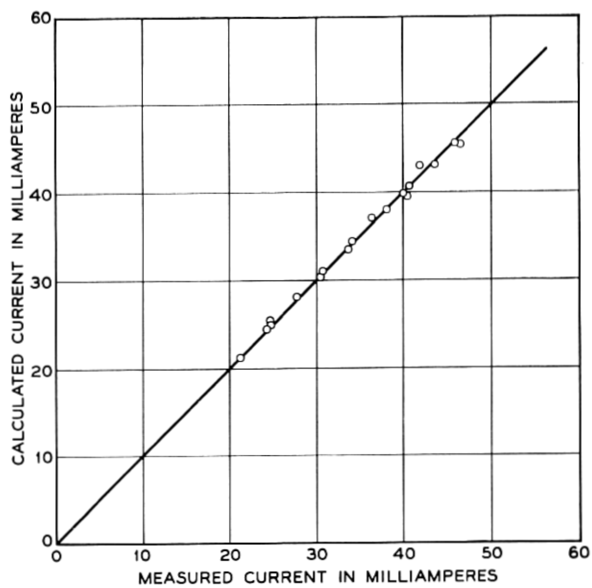

Fig. 3 — Comparison of directly measured current with current calculated from spectral information.

extrapolation was performed by this procedure. Thus, weighting factors were determined for the prediction of outer space short-circuit currents from the spectral measurements performed on the test set.

The calibrated cells serve as standards for the weighting factors. Thus, the monitor cell in the test equipment need not provide the long-time standard. It serves only as a short-time standard that can be compared readily with the calibrated cells.

For further characterization of solar cells, additional measurements under tungsten light filtered through heat-absorbing glasses, later referred to as "white light," are performed. These are the short-circuit current, the open-circuit voltage, the current delivered into a 10-ohm load, and the current delivered into 0.45 volts. The reverse leakage and the forward voltage with 50 ma passing through the solar cell are measured in the dark.

In a special mode of operation the test equipment can form the weighted sum, thus giving the short-circuit current directly. In general, however, it is preferred to evaluate all the test results on a digital computer.

The obtained outer space short-circuit current is then used in conjunction with the measurements under the white light to calculate the over-all output characteristic for outer space illumination, in particular the maximum-power point and the voltage at maximum power.

In addition, the short-circuit current under the white light source is also calculated from the spectral response with an appropriate set of weighting factors. The percentage difference between the calculated and the measured short-circuit current is determined and recorded. Generally, this difference is small, indicating that the outer space short-circuit current calculation can be trusted. Occasionally, solar cells are observed in which the calculated current deviates substantially from the measured current. This indicates that the particular solar cell has a nonlinear response, e.g., its quantum efficiency is light level dependent. The results for the outer space short-circuit current obtained on such solar cells are correspondingly in error. Alternative methods of calculation that are applicable to such nonlinear cells are described in Section IV.

III. TEST EQUIPMENT

As mentioned, two groups of measurements are performed in the test set. One group of eight measurements evaluates the spectral response of the cells, while the measurements in the other group evaluate the current-voltage characteristic of the solar cell.

For those tests in which the output is porportional to the incident light intensity, the ratio of the response of the sample cell to that of a reference cell is measured. This eliminates the effect of drift and fluctuations in the light source and the interference filters.

The set consists of the following major components:

1. optical system comprising light source and interference filters,
2. mechanical system providing for optical filter transport and actuation of control switches,
3. electronics and ratio-formation,
4. control system, giving control commands for operation of counters and punch-out, and providing for the routing of the signals through appropriate amplifiers and attenuators, etc., and
5. output channels: printer or translator and card punch.

3.1 *Optical and Mechanical System*

The optical system is shown schematically in Fig. 4. A 1-kw incandescent projection lamp (General Electric type PH/1M/T20MP) serves as the light source. The light is focused by a spherical mirror (diameter 12 inches, focal length $5\frac{3}{4}$ inches) and directed by a plane mirror towards the sample and reference cells, which are placed adjacent to each other in a light-tight box. To insure uniform illumination of both cells, diffusing glass is interposed into the light beam. Long-wavelength light is attenuated by heat-absorbing glass and nearly monochromatic light at various wavelengths is selected by narrow-band interference filters. These filters are mounted on a disk that is driven intermittently by a Geneva motion and interposes the filters sequentially into the optical path. The filters have transmission bandwidths of the order 0.01 μ. For the evaluation of silicon cells, filters at the following wavelengths are used: 0.4, 0.45, 0.5, 0.6, 0.7, 0.8, 0.9, and 0.95 μ. For measurements on the solar cells in the dark, four positions are blocked; the remaining four positions admit white light for measurements of additional solar cell characteristics. The white light intensity is adjusted to generate in a typical solar cell a short-circuit current that is comparable to the short-circuit current expected in outer space.

The important factors of the mechanical system are also shown in Fig. 4. A 1-rps motor drives the Geneva motion, which advances the filter disk. Thus the time available for any individual test is 1 sec. The motor axle contains a set of adjustable cams that actuate microswitches for the timing of various functions within a single test cycle. The wipers of a set of 16-position rotary switch decks are driven by the filter disk axle to permit the selection of the different types of tests to be performed.

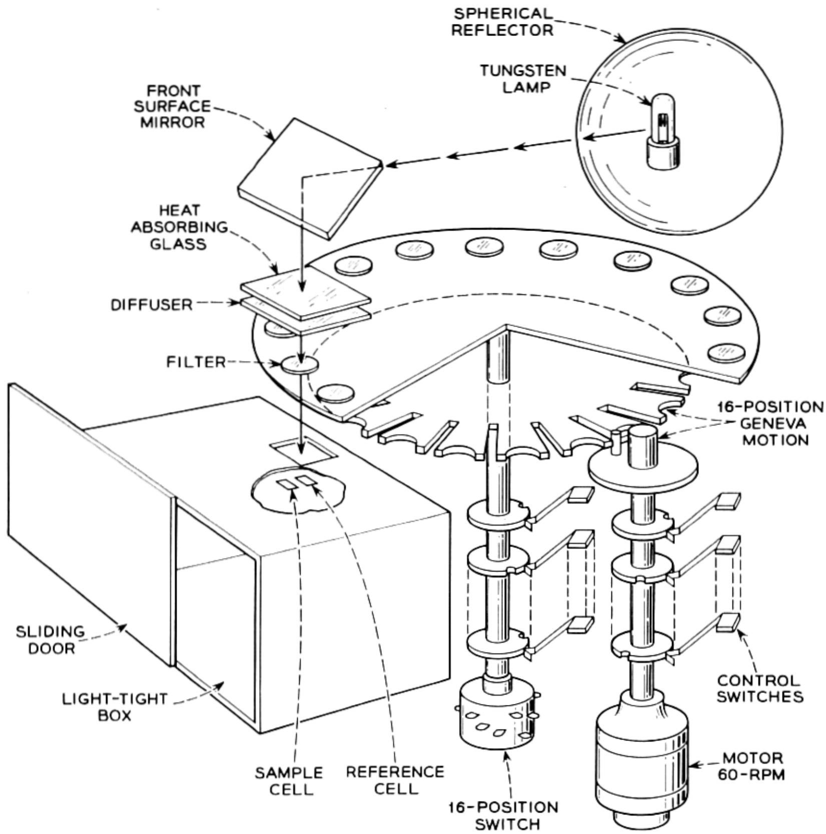

Fig. 4 — Optical and mechanical system.

3.2 *Electronics and Ratio Formation*

The various signals encountered are measured by a combination of voltage-to-frequency converter (Dymec 1122B) and counter. This combination, when used in both the sample and reference channel, makes fast and accurate ratio formation possible.

If the input voltage at the sample channel is V_s volts (see Fig. 5), and if the voltage-to-frequency converter (vfc) gives A cycles per second per volt, the total count N_s obtained in t seconds is

$$N_s = V_s A t. \tag{7}$$

Similarly one obtains in the reference channel

$$N_r = V_r A t. \tag{8}$$

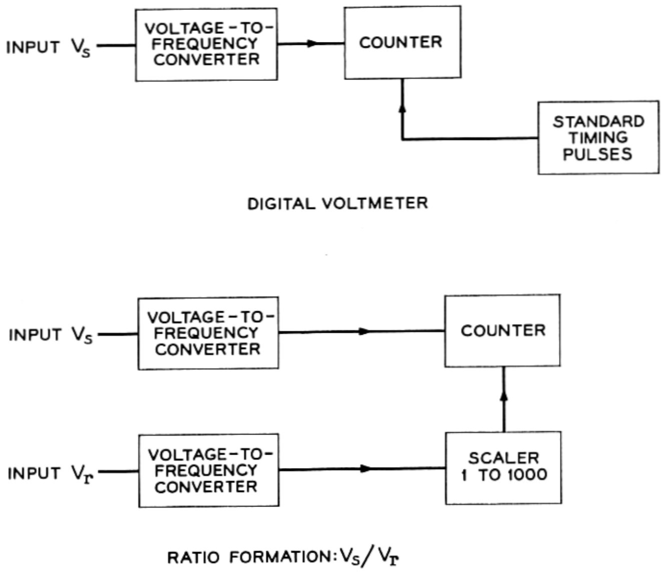

Fig. 5 — Method of ratio formation.

To form the ratio V_s/V_r the reference counter is used as a scaler, in such a way that it gives an output pulse for every 1000 input pulses. The time between output pulses is then

$$t = \frac{1000}{AV_r}. \tag{9}$$

The output pulses from the scaler gate the counter in the sample channel in such a way that one pulse turns it on and the next one turns it off. Thus t in (7) is identical to that in (9). This leads to

$$N_s = 1000\ (V_s/V_r) \tag{10}$$

i.e., the count in the sample channel counter gives the desired ratio of the voltages with adequate resolution.

The voltage-to-frequency converter and counter combination, when used as a voltmeter, gives an output that represents the input voltage averaged over the time of measurement. Thus fluctuations are smoothed out. The effective noise bandwidth is therefore the reciprocal of the time of measurements. In spite of the small noise bandwidth, however, this combination has a fast transient response. Thus, high-resolution, low-noise measurements during short time intervals are possible. With a wideband amplifier (Kintel type 112) preceding the voltage-to-frequency converter, the equivalent noise at the input that actually has been observed is of the order of 1 μv for 0.1-second measurement intervals even after an estimated 750 hours of operation of the system. The chopper at the input of the preamplifier deteriorates with time; for a new chopper the noise may be considerably below 1 μv.

The signals to be measured are in the range 0.5 mv to 10 mv except for the spectral response measurement at the shortest wavelength, 0.4 μ, where the output of the light source is low and the signal is about 100 μv. For all other wavelengths the amplifier noise thus contributes less than 0.2 per cent to the measurement error. Since the contribution of the 0.4 μ spectral region to the total short-circuit current is small, the error contributed by the amplifier to the total short-circuit current is also below 0.2 per cent.

The short-circuit current is obtained from the measured voltage drop across a shunting resistor. The resistor has to be of sufficiently low value that the voltage developed across it would result in a forward current in the dark that is negligible compared to the short-circuit current. For the spectral measurements, a resistor of 10 ohms is adequate, while for the white light measurements a 0.1-ohm resistor is used. The switching of resistors at the low signal levels has to be done carefully to prevent introduction of spurious thermal emf's. Mercury relays are employed in an arrangement shown in Fig. 6. For the measurement of the open-circuit voltage ($\approx\frac{1}{2}$ volt) and other voltages of comparable magnitude, the preamplifier is bypassed and the signal is fed directly into the voltage-to-frequency converter.

3.3 *The Control System*

The programming of the various tests is accomplished through the 16-position switch coupled to the axis of the drive motor. Microswitches control reset commands at the beginning of each test; approximately in the middle of each test, (after the amplifier transients have died down) a new counting cycle is initiated. The last operation within each test is the print command to the printer or IBM punch.

The switching of the signal paths for the various tests is accomplished by relays which are activated from the 16-position switch through a diode matrix as shown in Fig. 7.

The following quantities are measured in the tests: in the eight spectral readings, the ratio of the short-circuit currents of sample cell and reference cell at eight wavelengths,

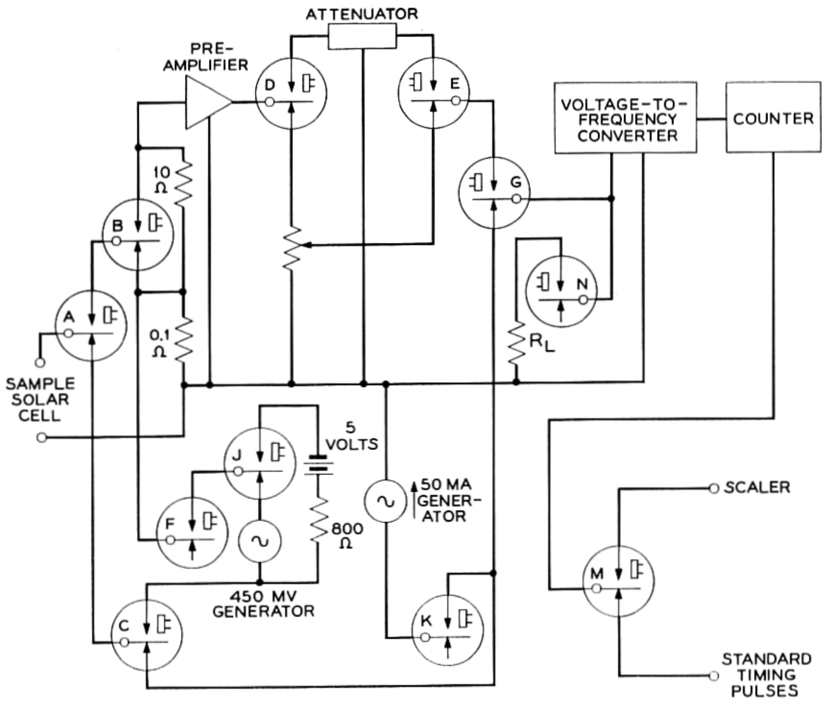

Fig. 6 — Switching of signal paths.

I_{rev}, the current flowing through the cell when a reverse voltage of 5 v is applied through an 800-ohm resistor,

V_L, the voltage developed across a load resistor R_L ($\approx$10 ohms) when the cell is illuminated by white light (see optical system),

V_f, the voltage across the cell when a forward current I_f, normally 50 ma, is passed through the cell in the dark,

I_{sc}, the short-circuit current measured directly under white light.

I_{scR}, the ratio of the short-circuit current of the sample cell under white light to that of the reference cell (In this measurement the amplifier gains are set such that for proper adjustment of the light source the numerical value of I_{scR} is the same as that of I_{sc}.),

I_{45}, the current delivered by the cell under white light illumination into a 0.45-volt voltage source, and

V_{oc}, the open-circuit voltage of the cell under white light illumination.

The short-circuit current is measured both directly (I_{sc}) and normalized (I_{scR}). In the linearity test, the normalized current is to be used, as it is compared with the normalized spectral readings. In the evaluation of the output characteristics, the unnormalized short circuit current is used in conjunction with the open-circuit voltage and the voltage across a load resistor.

IV. EVALUATION OF THE SHORT-CIRCUIT CURRENT

The short-circuit current is evaluated from the raw data according to (6). This evaluation can be performed either directly by the test equipment concurrently with the measurement of the R_i or by a separate calculation, preferably on an electronic computer.

For the evaluation in the test set the counter is not reset between the spectral response measurements R_i. The multiplication by W_i is carried out by interposing attenuators between the preamplifier and voltage-to-frequency converter of the sample channel. Such attenuators are switched by the 16-position switch, which is coupled with the axle of the filter disk. The number added into the counter at each filter position equals the current contributed by the wavelength band represented by the filter, and the final number on the counter gives the total short-circuit current.

The direct test set evaluation of the short-circuit current is of importance if no computing facilities are available or if results are needed

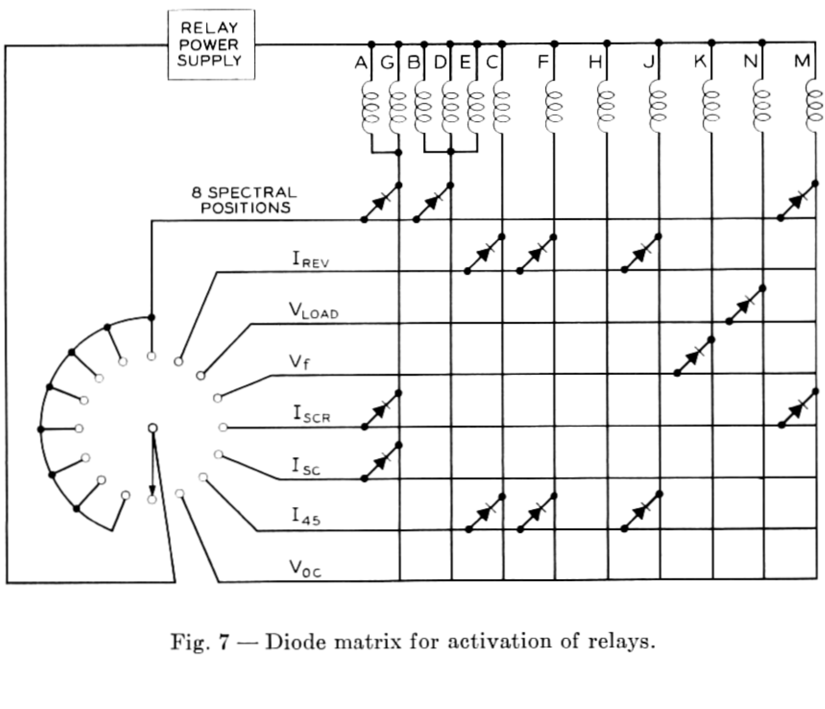

Fig. 7 — Diode matrix for activation of relays.

immediately. On most other occasions, it is preferable to have the computer process the raw R_i data, since one set of data can then be analyzed for cell response under various light sources.

In addition to providing short-circuit current under outer space illumination, the computer program, as used for routine evaluation of solar cells at Bell Telephone Laboratories, evaluates the short-circuit current for the white light that is used in some of the tests of the output characteristic, as discussed in Section II. For a check of consistency, the program then computes the percentage deviation of this current from the short-circuit current measured directly under white light. This check is important for the detection of cells whose short-circuit current changes nonlinearly with light intensity.

As an additional feature, the program computes the ratio of the computed outer space current to the computed white light current. This quantity, CC, is of interest for a number of reasons. If the maximum intensity of the white light falls at a different wavelength than the maximum intensity of outer space sunlight, then CC provides a "color index" for the cells. In the test set described here the white light is deficient in infrared and red light due to filtering with heat-absorbing glass. Therefore, red-sensitive cells have large CC numbers while blue-sensitive ones have smaller CC numbers.

In the standard procedure for calculating the outer space short-circuit current, the measured R_i values are multiplied by proper weighting factors W_i and summed as in (6). Fluctuations in intensity and spectral composition of the light source cause no first-order error, since the R_i are ratios. However, if the cell current varies nonlinearly with light intensity, the current at full solar illumination, as calculated from spectral measurements at low light intensity, is in error. From the comparison of measured white light response and calculated white light response one knows whether or not a cell is linear. Nonlinear cells are encountered rarely enough so that they present no serious problem. Nevertheless, the program can be used — if required — in such a way that errors due to cell nonlinearity are minimized; however, errors will be introduced if the spectral content of the white light changes. The procedure is to multiply the measured white light current I_{scR} by the factor CC. Since nonlinearities cause errors in both numerator and denominator, the influence is reduced to second-order effects.

V. THE OUTPUT CHARACTERISTIC

As discussed before in connection with the short-circuit current for outer space illumination, the output characteristic of the solar cell could be established under a light source of arbitrary spectral composition but adjusted in intensity to produce the known outer space short-circuit current. To do this, however, would be a very tedious and time-consuming procedure. As an alternative, one measures the output characteristic under a light source that generates a short-circuit current close to, but not necessarily equal to, the outer space short-circuit current. It then is possible to calculate the outer space response from these measurements and the known outer space short-circuit current.

For such calculations of the output characteristic the following model of a solar cell is used:

$$I = I_{sc} - I_o \left\{ \exp\left[\frac{q(V + IR_s)}{nkT}\right] - 1 \right\}. \tag{11}$$

Here I is the output current at voltage V, I_{sc} is the light induced current (under the given illumination) and corresponds to a current generator in parallel with the solar cell diode. R_s is a lumped resistance approximating the contact resistance and sheet resistance in the front layer of the cell. I_o, the diode forward-current constant, is orders of magnitude smaller than I_{sc}; therefore the term (-1) in (11) can be neglected. With this simplification one can express I_o in terms of the open-circuit voltage, V_{oc}, i.e. that voltage for which the left side of (11) is zero, which leads to

$$I = I_{sc}\left[1 - \exp\frac{q(V + R_s I - V_{oc})}{nkT}\right]. \tag{12}$$

An ideal diode would have a value of 1 for the coefficient n in the exponent. A number of effects* cause deviations from this simple behavior, and frequently the output curve cannot be fitted by a constant value of n over the entire voltage range and for different light intensities. However, one may fit the output characteristic over a limited range near the maximum power point with a constant n for a particular light intensity. Only small errors are introduced if one uses the same value of n near the maximum power point under slightly different illumination.

Similarly the value of the resistance R_s in (12) is not strictly a constant, since it involves a spreading resistance in the thin diffused layer. Nevertheless, in general R_s will be a very weak function of current and light level so that, for this calculation, it can be treated as a constant.

With these assumptions the output characteristic under outer space illumination is determined by the four quantities I_{sc}, V_{oc}, n, and R_s, which must be established by measurement. The first two of these de-

* Among these effects are space-charge recombination and sheet resistance. See also Ref. 3.

pend on the particular light level. From the measurements of I_{sc}, V_{oc}, and V_L under the white light, and from the measurement of V_f in the dark, all four quantities can be extracted. By replacing I_{sc} in (12) by the known outer space short-circuit current I_{scos} and setting $I = 0$, one obtains the open-circuit voltage V_{ocos} for outer space illumination. The parameters I_{scos}, V_{ocos}, n, and R_s are now used to characterize the output characteristic of the cell under outer space illumination. This permits a calculation of all quantities of interest according to (12), in particular the maximum power point. An approach for performing these calculations on an electronic computer is given in the Appendix.

Most of the measurements on the test are reproducible over periods of hours with an rms deviation from the mean of about 0.5 per cent. If calibrations on the test set are to be made to an accuracy approaching 0.5 per cent or less, the fine adjustments become very tedious, as the criterion has to be the result of a statistical analysis of several measurements.

A preferable procedure is to make adjustments on the test set to a moderate accuracy and to have the computer apply fine corrections. Two modes of operation are used. In one mode these corrections are applied manually as parameters that are entered with the data. In the other mode, a set of standard cells is measured along with the cells to be evaluated. The data cards of the standard cells also contain their calibrated outer space short-circuit current value. The computer can then determine the percentage deviation of the calculated value of the outer space short-circuit current from the calibration value for each of the standard cells. The calculated values of I_{scos} for the cells under test are then corrected on the basis of the average of these deviations. A similar correction is applied to the white light current.

With the latter mode of operation, the long time standard is provided by a group of standard cells, and the built-in monitor cell serves only as a short-time reference. If there are no uncertainties introduced by long-time drifts of the standards, absolute accuracies of outer space short-circuit current predictions of 2 to 3 per cent should be realizable. Comparison of preflight predictions with flight data on the Telstar satellites and the Anna 1B satellite[4] confirm that an accuracy of 3 per cent or better has been achieved.

VI. ACKNOWLEDGMENTS

The authors acknowledge the contributions of P. J. Kamps, who did the mechanical design, and of W. G. Ansley, who provided the digital recording system.

APPENDIX

A.1 *Method of Calculation of n and R_s*

The quantity n is determined by fitting the output characteristic between V_L and V_{oc}. As one has to make allowances for R_s, the series resistance, an iterative scheme is convenient. Assume for a moment that the series resistance R_s is zero. One can then compute the value of n or nkT/q from (12)

$$\frac{nkT}{q} = \frac{V_{oc} - V_L}{\ln\left(1 - \frac{V_L}{R_L I_{sc}}\right)}. \tag{13}$$

Using this value of nkT/q one can compute the value of the voltage across the cell for a current $I_{sc} - I_f$ through the illuminated cell

$$V_c = V_{oc} + \frac{nkT}{q} \ln\left(\frac{I_f}{I_{sc}}\right). \tag{14}$$

If there were no series resistance, then the measured voltage V_f should be equal to V_c. Because of the series resistance, the voltage V_f is higher by $R_s \cdot I_f$. One thus obtains an initial estimate of the resistance

$$R_s = \frac{V_f - V_c}{I_f}. \tag{15}$$

Now this value of R_s is used for an improvement of (13). During the measurement of V_L, current is flowing out of the cell, and the junction voltage is the measured voltage V_L increased by the voltage drop $(V_L/R_L)/R_s$. Thus one obtains

$$\frac{nkT}{q} = \frac{V_{oc} - V_L\left(1 + \frac{R_s}{R_L}\right)}{\ln\left(1 - \frac{V_L}{R_L I_S}\right)}. \tag{16}$$

The new value of nkT/q thus obtained can be used to compute an improved value of V_c, and in turn, of R_s, and so on. The convergence of this procedure is quite rapid, since nkT/q in (14) is multiplied by $\ln(I_f/I_{sc})$ and I_f was chosen to be near I_{sc}.

A.2 *Method of Calculation of Outer Space Quantities*

Using the quantities I_{sc}, V_{oc}, (nkT/q) and R_s, as determined in the previous section, the outer space quantities are evaluated as follows.

Outer space open-circuit voltage:

$$V_{ocos} = V_{oc} + \frac{nkT}{q} \ln \frac{I_{scos}}{I_{sc}} . \qquad (17)$$

For further evaluation of the characteristics it is convenient to introduce the following normalizations:

$$y = \frac{qV}{nkT} \qquad (18)$$

$$u = \frac{I}{I_{scos}}$$

$$x = \frac{I_{scos}}{I_{scos} - I} = \frac{1}{1 - u} \qquad (19)$$

$$z = \frac{qV_{ocos}}{nkT} \qquad (20)$$

$$\beta = \frac{qR_s I_{scos}}{nkT} . \qquad (21)$$

The normalized current u at the normalized voltage y is obtained from a solution of the equation

$$u = 1 - e^{y+\beta u-z}. \qquad (22)$$

In the computer program, this equation is again solved by iteration. The power at the voltage corresponding to y is now given by

$$P = (nkT/q)I_{scos}yu. \qquad (23)$$

To obtain the maximum power, (22) is rewritten so that the normalized voltage appears as a function of the normalized current

$$y = z - \beta u + \ln (1 - u). \qquad (24)$$

The normalized power π is obtained by multiplying (24) by u

$$\pi = u[z - \beta u + \ln (1 - u)]. \qquad (25)$$

The maximum is obtained by equating the derivative to zero. This yields the condition

$$z = \frac{u}{1 - u} + 2\beta u - \ln (1 - u). \qquad (26)$$

In (25), u is required as a function of z and β. Again, on the computer a solution is conveniently found by iteration. To get (25) into a form suitable for this, a switch is made to the variable x, defined in equation (19). Then

$$x = z + 1 - 2\beta\left(1 - \frac{1}{x}\right) - \ln x \qquad (27)$$

and in this form the equation can be solved by iteration, starting with $x = 1$ on the right-hand side. One now obtains for the maximum power

$$P_{\max} = I_{scos} V_{ocos} \frac{(x - 1)^2}{x}\left(1 + \frac{\beta}{x}\right), \qquad (28)$$

and for the voltage at which maximum power is delivered

$$V_{mp} = \frac{nkT}{q}(x - 1)\left(1 + \frac{\beta}{x}\right). \qquad (29)$$

REFERENCES

1. Gummel, H. K., Smits, F. M., and Froiland, A. R., A Method for Terrestrial Determination of Solar Cell Short Circuit Current under Outer Space Solar Illumination, Wescon 1961, Paper 7/3.
2. In this work the spectral solar irradiance data of Johnson, F. S., J. Meteorology, **11**, 1954, p. 431, were used.
3. Queisser, H. J., Forward Characteristics and Efficiencies of Silicon Solar Cells, Solid State Electronics, **5**, No. 1, 1962, and Gettering Effects on the Forward Characteristics of Silicon Solar Cells, Proc. I.R.E., **50**, 1962, p. 486.
4. The authors are indebted to Dr. R. E. Fischell of the Johns Hopkins University, Applied Physics Laboratory, for flight performance data on experimental cells on which preflight measurements were made on the test set described here.