A Technique for Measuring Small Optical Loss Using an Oscillating Spherical Mirror Interferometer

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The measurement of very small optical losses (the order of a few per cent) by conventional methods becomes very difficult because of the extreme accuracy required. This article shows that both high mirror reflectances and low transmission losses can be readily measured using an oscillating mirror interferometer as a frequency spectrum analyzer. The theory developed shows that when this type of interferometer is excited by a continuous gaseous laser, the total optical loss is proportional to the frequency resolution or the finesse. The theory also shows that the first-order velocity effect produced by having the mirror move at a velocity of one foot per hour can be large if the total optical loss is about 0.25 per cent. For the velocities and optical losses we have encountered so far in our measurement system, the first-order mirror velocity effect can be neglected. The range of reflectance of mirrors we have measured is from 94 to 99.5 per cent, and the measurements for the optical transmission loss range from 0.2 to 3 per cent. The accuracy to which a 1 per cent loss can be repeated is 1.0 ± 0.1 per cent. It was found that the transmission loss through an optical grade of fused quartz (Homosil) at 6328 Å is about 1 db per meter, and that for Plexiglas II is about 2 db per meter.

I. INTRODUCTION

In developing a long-distance optical communication system employing a large number of components, it is essential to be able to measure accurately the optical transmission loss of each component. These components may include mirrors with reflectances in the order of 99 per cent, Brewster angle output windows, various lenses, and other passive elements with optical transmission losses of 1 per cent or less. For such small losses, the conventional measuring techniques become increasingly difficult because of the extreme accuracy required.

In conventional measuring systems, the loss (or reflectance) is calculated by comparing the electrical output of a photodetector for two different optical conditions, first with the unknown in the system, and then with the unknown removed. As the magnitude of the optical loss decreases, this comparison becomes more and more inaccurate, as it requires the measurement of a small difference between two relatively large photodetector outputs. In these measuring systems, a number of methods have been developed to minimize these errors, which are principally optical.[1] However, it is believed that measuring techniques to be described below will more readily measure very small optical losses.

The proposed method uses a frequency spectrum analyzer at optical frequencies. Such an analyzer can be obtained by using a Fabry-Perot type of interferometer as a transmission element between an optical source and a photodetector.[2,3] When the mirror separation of the interferometer is varied periodically by moving one of the mirrors linearly, a large photocell output will be obtained whenever the optical cavity is in resonance at any frequency that may be present in the optical source. If the photodetector output is observed on an oscilloscope whose sweep is synchronized with the mirror drive, the scope will display the energy distribution of an optical source as a function of frequency. If this type of Fabry-Perot interferometer is illuminated by a continuous laser with its extremely narrow line output (one or two cycles wide), the linewidth of the pattern displayed on the scope is determined by the optical losses in the cavity itself, and by the velocity of the moving mirror. If the excursion of the moving mirror is several optical wavelengths, the scope pattern will repeat several times during a single sweep trace. That is, as the mirror separation increases, the mth harmonic of the cavity becomes resonant with the source; a short time later, the $(m + 1)$th harmonic is resonant with the same optical frequency, then the $(m + 2)$th, and so on. At each resonant point, there will be several output scope pulses, since the laser usually has an output at more than one frequency. The ability of any interferometer to separate or resolve two adjacent optical frequencies is determined by the finesse of the system. In the moving-mirror interferometer, the finesse is equal to the ratio of the fundamental pulse group spacing to the half-power-height width of any pulse. It will be shown below that the total power loss of the Fabry-Perot cavity, expressed as a ratio, is equal to π divided by the finesse.

This interferometer method of measuring small optical losses has been suggested on several occasions,[4] but it is believed that this is the first time such a system has been so fully developed. Since this system of measurements requires an optical source with an output linewidth very narrow compared to that of the optical cavity, loss measurements cannot be made over a continuous range of wavelengths, but only at the discrete wavelengths at which cw lasers have been developed. Since some light must be transmitted through the cavity, the reflectance can be measured only for those mirrors not coated with an opaque reflecting surface, such as the multiple-layer dielectric coated mirror.

The interferometer loss measuring technique is very sensitive to building and floor vibrations, and to air currents. The reason for this is as follows: if the separation between the two mirrors in the cavity is changed by one half wavelength, the resonant frequency of the interferometer is changed by the natural frequency of the cavity, which in our system is about 1 kmc, whereas the bandwidth of the cavity for 1 per cent optical loss is only about 3.0 mc. Thus, even very small random variations in the mirror spacing in both the laser and in the cavity, produced by either vibrations or air currents, will cause the output pulse pattern displayed on the scope to have large random time position variation.

II. THEORY

The relation for which the mirror reflectance and other optical losses can be calculated from measurable quantities is derived in the Appendix, and will be given briefly here. The effects of the velocity of the moving mirror were included in the Appendix.

Let

R_1, R_2 = power reflectance of the two mirrors expressed as a ratio
g = power loss per single pass through any material within the cavity, also expressed as a ratio
T_c = fundamental pulse group spacing
T_p = half-power output pulse width
v = velocity of moving mirror
d_0 = mirror spacing
t = time
λ = optical wavelength in free space
c = velocity of light in free space

$$\beta_0 = \frac{4\pi d_0}{\lambda}\frac{v}{c}$$

$$x = g(R_1R_2)^{\frac{1}{2}}$$

$$\alpha \equiv -\frac{4\pi v t}{\lambda}$$

$$g_0 \equiv [g(1 - R_1)(1 - R_2)]^{\frac{1}{2}}.$$

An exceedingly good approximation for the time response of the interferometer appropriate for computer calculation is given by (5) of the Appendix:

$$I_T \approx \left[\sum_{n=0}^{\infty} g_0 x^n \cos (\alpha n + \beta_0 n^2) \right]^2 + \left[\sum_{n=0}^{\infty} g_0 x^n \sin (\alpha n + \beta_0 n^2) \right]^2 . \quad (5)$$

Since x is very nearly unity, a large number of terms must be taken in the series. For a maximum error of ϵ in stopping the series after the first $N - 1$ terms, we have

$$N \approx \frac{\log (2/\epsilon)}{1 - x}. \quad (6)$$

For example

$$\epsilon = 1/1000 \quad \text{and} \quad x = 0.9975, \qquad N = 3040.$$

The values of I_T shown in Fig. 1 were calculated for the following three

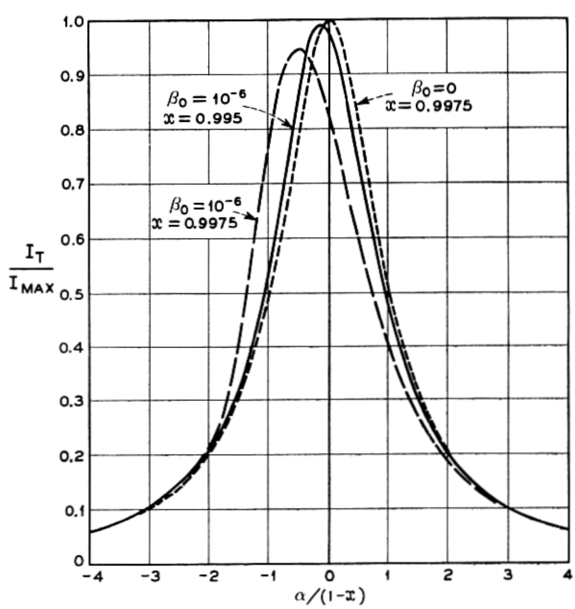

FIG. 1 — Interferometer time response as affected by mirror velocity.

conditions: (a) $\beta_0 = 10^{-6}$, and $x = 0.9975$, (b) $\beta_0 = 0$, $x = 0.9975$, and (c) $\beta_0 = 10^{-6}$ and $x = 0.995$. The curves given in Fig. 1 show that the first-order mirror velocity effect is to decrease the maximum response, increase the half-power-height pulse width, and make the response unsymmetrical about the maximum response.

From a number of computed values for (5), of which only a few are given in Fig. 1, it can be shown that the first-order effect of the moving mirror's velocity is to increase the half-power pulse width according to the relation (see Appendix)

$$\frac{Tm}{Tp} \approx \left[1 + 13.8 \frac{\beta_0^2}{(1-x)^4}\right]^{\frac{1}{2}}. \tag{7}$$

This relation holds only when T_m is within a few per cent of T_p. For larger values, the computed results are less than those given in (7). Thus, in order to have the width increase by less than 1 per cent,

$$\frac{\beta_0}{(1-x)^2} \leqq \frac{1}{25}.$$

This relation can readily be satisfied unless the total loss becomes extremely small. In our measurements, the mirror spacing is varied about one micron at a 20-cycle frequency. The mirror spacing is about 15 cm and the wavelength is 6328 Å. Then, if $x = 0.995$ (the largest we have measured),

$$v = \tfrac{1}{250} \text{ cm/sec} \approx 0.4 \text{ ft/hour}$$

and

$$\frac{\beta_0}{(1-x)^2} = \frac{0.4 \times 10^{-6}}{25 \times 10^{-6}} = \frac{1}{62.5}.$$

Hence, for optical losses of 0.5 per cent or greater, we can neglect the first-order mirror velocity effects.

If mirror velocity effects can be neglected, then the system response given by (10) and (11) of the Appendix can be used. The relation between the optical loss and the finesse of the system is

$$g(R_1R_2)^{\frac{1}{2}} = 1 - (\pi T_p/T_c) + \tfrac{1}{2}(\pi T_p/T_c)^2 + \cdots \tag{11}$$

where T_c/T_p is defined as the finesse. This equation points out the potential accuracy with which small losses can be measured. The interferometer method actually measures how much the combined loss, $g(R_1R_2)^{\frac{1}{2}}$, differs from unity. Obviously, the smaller this difference, the greater can be the experimental errors to obtain a fixed accuracy in

$g(R_1R_2)^{\frac{1}{2}}$. For a value of $g(R_1R_2)^{\frac{1}{2}}$ near 0.99, a 10 per cent error made in the measurement of T_p/T_c would give an error of only one part in a thousand in the value of $g(R_1R_2)^{\frac{1}{2}}$.

To obtain a good signal-to-noise ratio in the electrical output of the photodetector, it is important to obtain a maximum amount of light transmission through the interferometer. From (10), the fraction of the incident light transmitted through the cavity is given by (see Appendix)

$$I_m = \frac{g(1 - R_1)(1 - R_2)}{[1 - g(R_1R_2)^{\frac{1}{2}}]^2} . \tag{10}$$

Equation (10) shows that for $g = 1$ and $R_1 = R_2$, I_m is unity for any value of reflectance, but if $R_1 \neq R_2$, I_m will be less than unity. For example, if $R_1 = 0.995$, $R_2 = 0.97$ and $g = 1$, then $I_m = 0.49$. The remaining 51 per cent of the light is reflected back towards the source. Therefore the two mirror reflectances should be identical for maximum transmission. For measurements of transmission losses in the cavity, there is some advantage to be obtained by not having the mirror reflectances too great. As an example, for $g = 0.97$ and $R_1 = R_2 = 0.99$, $I_m = 0.062$. For the same loss and $R_1 = R_2 = 0.97$, $I_m = 0.25$. In the second case, however, there is a greater chance of making an error in measuring g since it is a smaller fraction of the total loss.

The shape of the output pulse expressed as a function of time is given by (12) of the Appendix

$$I(t) \approx \frac{I_m}{1 + 4t^2/T_p^2} . \tag{12}$$

The frequency spectrum of this time pulse is

$$F(\omega) = \frac{\pi}{2} T_p I_m \exp\left(-\frac{T_p}{2}|\omega|\right).$$

Now, the value of T_p is a function of the velocity of the moving mirror and hence, subject to the limitations on velocity discussed above, may be made as large or small as is desired. In our laboratory, the motion of the moving mirror was so selected that for a 1 per cent total cavity loss, the pulse width is about 30 μsec, and the frequency spectrum is down to 1 per cent of its low-frequency value at about 50 kc. The required bandwidth of the photodetector and its associated electrical circuits, including the viewing oscilloscope, is increased by a decrease in the total optical loss, as this decreases the pulse width, T_p. In order that the system be capable of measuring optical losses as small as 0.25 per cent, the over-all bandwidth of the electrical components should be at least 200 kc.

The above theory was developed for the assumptions that the incident beam of monochromatic light was collimated and that the plane surface mirrors were infinite in size. For finite-size mirrors with plane or spherical surfaces, and for a finite-size input light beam diameter, the electromagnetic energy inside the interferometer can be described by the familiar TEM modes.[5,6,7] For a given input light beam condition and for a given set of mirrors, the energy within the cavity can be characterized by selecting the appropriate amplitudes of the TEM modes. If the input beam spot size is too large or too small, a large portion of the input energy will be found in higher transverse modes of quite large order. If the laser is adjusted to operate in only the TEM_{00q} mode, then it has been shown[8] that the light energy will be principally in the fundamental TEM_{00q} mode in the cavity, if the spot size and the surfaces of constant phases of the input beam are both equal to those for the TEM_{00q} resonant cavity mode. As is usual in matching problems, these conditions are not too critical.

When the measuring interferometer has spherical mirrors at nonconfocal spacing, an incident light ray at a small angle off the system axis will produce repeated reflections, which in general will trace an ellipse on the mirrors.[9] Under special conditions, the points of reflection lie on a circle and are displaced by some angle after every round trip. When this angle is a multiple of 2π, the rays will exactly retrace their paths, and the trace of reflections on a mirror will break up in a number of separate and equally spaced dots. Under these reentrant conditions, the cavity will become resonant not only at a multiple of the fundamental cavity frequency, but also at multiples of a much lower frequency which is not quite a subharmonic of the fundamental cavity frequency. For these conditions, the oscilloscope pattern of the photocell output will show, in addition to the main response, a number of smaller equally spaced pulses. Since the response of one of the "off-axis" modes can coincide (or nearly coincide) with the main response, the measurement of the loss under these conditions can be considerably in error unless these off-axis mode responses are made very small.

In general, the system should be designed to be nondegenerate. That is, the length of the interferometer must be so selected as to avoid any possible overlapping of the cavity resonant response of the different orders of the TEM modes to any of the several optical frequencies present in the laser source. Usually, this is not difficult to accomplish.

III. MEASUREMENT SYSTEM

A block diagram of the components in the interferometer measuring system is given in Fig. 2. The He-Ne laser has external spherical mirrors

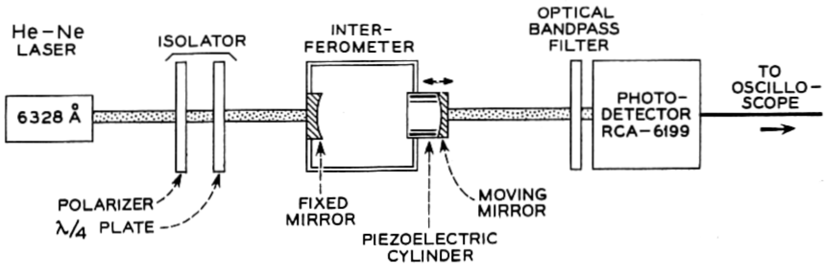

FIG. 2 — Schematic diagram of the interferometer measuring system.

and Brewster angle output windows, and operates at 6328 Å. To reduce the effects of vibrations, the laser and the cavity structures were made very rigid by using a construction similar to that used by Bennett in his magnetostrictively tuned laser.[10] The interferometer mirrors are mounted with suitable tilt controls in 6-inch square steel end blocks, 1 inch thick. The two blocks are tied rigidly together at each of the four corners by 1-inch diameter Invar rods 16 cm long. The interferometer cavity, shown in Fig. 3, uses a piezoelectric transducer to vary the mirror spacing in the cavity. It is a ceramic cylinder $1\frac{1}{4}$ inches ID, $1\frac{1}{2}$ inches OD, and $1\frac{1}{2}$ inches long.[11] The mirror holder is epoxied to one end of the cylinder. The other end is epoxied to a mounting plate which is fastened through suitable tilt controls to the steel end block. The laser, interferometer, and other optical components were fastened rigidly to a heavy steel optical table. This 4 by 8-foot table is supported on six small airplane inner tubes encased in heavy canvas covers. For this type of support, the natural frequency of the table is about four cycles per

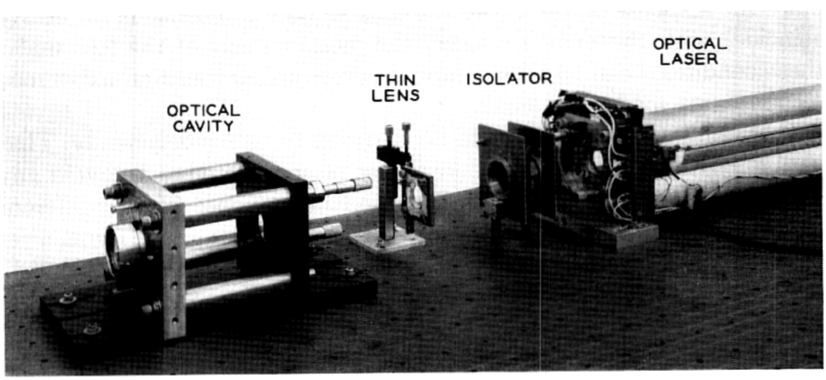

FIG. 3 — Photograph of the interferometer apparatus

second. To avoid air currents, both the laser and the optical cavity are enclosed separately in Plexiglas boxes.

The isolator between the laser and the cavity was found to be essential to prevent interaction between the two optical cavities. It is a circular polarizer consisting of a polaroid analyzer and a quarter-wavelength plate. This circular polarizer will absorb any light reflected back from the interferometer, as the sense of rotation of the circularly polarized light is inverted upon reflection and hence will not be transmitted back through the polaroid analyzer. For this type of isolator, all loss measurement must be made with circularly polarized light.

The photodetector is a standard electron multiplier phototube with an S-20 cathode. A bandpass optical filter, 200 Å wide, centered at 6300 Å, is placed between the source and the detector to eliminate most of the background light.

A 20-cycle triangular wave shape generator delivers 500 volts pp to the piezoelectric mirror drive. For this voltage, the motion of the mirror is about one micron, which is about three half-wavelengths of the 6328 Å laser source. The motion of the mirror is parallel and was checked by using an alignment telescope with a flat mirror placed in the movable mirror holder. The fringe pattern of the alignment telescope reflected back from the moving mirror showed no discernible change when 1000 volts dc or ac was applied to the driver. Therefore any mirror tilt variation must be less than 5 seconds of arc.

IV. EXPERIMENTAL PROCEDURE

When spherical mirrors are used in the interferometer, the ray of incident light must be on a line passing through the centers of curvature of both mirrors. With the aid of a thin optical lens to vary the incident angle, the cavity can readily be aligned to minimize the "off-axis" modes.[2,9]

The visible red gaseous laser, which was one meter long, was so adjusted that it oscillated only in the fundamental transverse mode and at several longitudinal modes.

In spite of all the precautions taken to eliminate building vibrations and air currents, the output pulse pattern on the output scope shows a considerable amount of time position jitter for any one pulse whenever the scope sweep speed is increased to be able to measure the half-power pulse widths. When the sweep speeds are made 10 μsec per cm to view a 30-μsec output pulse, the excursion of the time jitter is about ± 50 μsec, and the jitter frequency is about two cycles or less.

To overcome the effects of the time jitter, a photographic method was developed to determine the pulse width. For a single period of the 20-cycle drive on the piezoelectric driver, the time jitter is small. The half height of the output pulse was determined by using the following electronic circuits. In the amplifier following the photocell, a fast transistor switch, operating at a one-megacycle rate, reduces the voltage gain of the network periodically by a factor of two (6 db). With this arrangement, a single photograph shows simultaneously both the full-height and the half-height pulses, as shown in Fig. 4. The time position jitter in the pulse pattern is small enough that the fundamental pulse group spacing can be determined directly from the scope. The measurement of the finesse of the cavity does depend upon the accuracy of the various sweep rate calibrations. According to the manufacturer of the oscilloscope, these sweeps, once calibrated, should remain accurate to several per cent for several months.

In order to obtain repeatable loss measurements, it is important to have the laser operate with a stable mode pattern output. At times, this was found to be difficult because adjacent longitudinal modes would

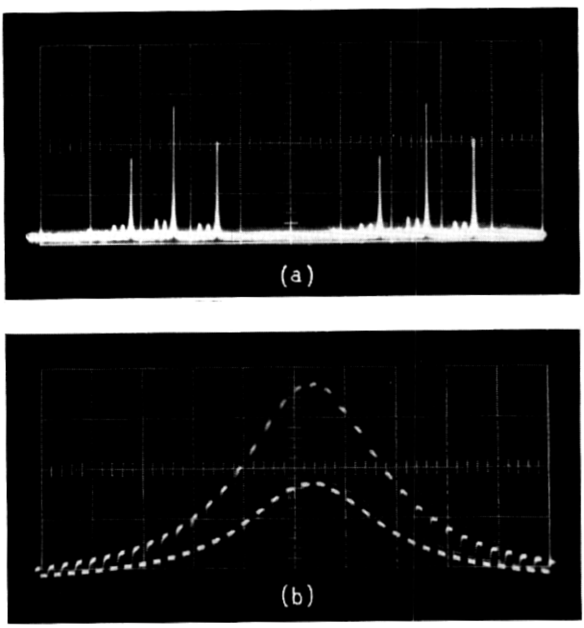

Fig. 4 — (a) Fundamental cavity spacing. Laser source has three output frequencies. Sweep rate is 2 msec per cm. (b) Half width of one of the cavity responses. Sweep rate is 10 μsec per cm. The indicated half width is 34 μsec.

compete with each other, thereby producing an erratic pulse pattern output from the interferometer. This condition was improved by reducing the length of the laser to 70 cm to separate the adjacent modes farther in frequency.

V. EXPERIMENTAL RESULTS

The reflectance was measured for a number of the multiple dielectric coated mirrors whose radius of curvature varied from 2 meters to infinity. The highest value of reflectance was 0.995 and the lowest was 0.940. The average mirror reflectance was about 0.990. The accuracy to which the reflectance of 0.990 can be repeated was about ± 0.001. Since the reflectance for an individual mirror was calculated from the three loss measurements for three mirrors taken two at a time, the error in the individual mirror reflectance was probably twice that of the single measurement, or ± 0.002 maximum. This leaves something to be desired. The largest source of error is in determining the half-power-height pulse width. The photographic method permits a determination of this width to about ± 5 per cent under ideal conditions.

The transmission loss through a fused quartz (Homosil) Brewster angle window with fairly high-quality surfaces was found to be about 0.25 per cent. These and all other transmission loss measurements were taken with two specimens at opposite Brewster angles in the optical cavity, so that the deflection of the light ray passing through the samples canceled out. The transmission loss through the quartz was measured by comparing the loss of a sandwich of three quartz blanks to that of just two windows, where an index of refraction matching liquid was used to overcome the surface irregularities at the interfaces in both cases. This loss was about 0.2 per cent $\pm$ 0.05 per cent for a $\frac{3}{4}$ cm optical path length in the Homosil. This would give a transmission loss of about 1.0 db per meter for high-quality optical Homosil. Using the same technique, the transmission loss through cast Plexiglas II, properly annealed, is about 2 db per meter.

VI. CONCLUSIONS

The interferometer method has proven to be capable of measuring very small optical losses. It requires a number of special precautions, such as a stable mode pattern output from the laser source, a careful and correct alignment of the interferometer, all possible reduction of the effects of building vibrations, and a large degree of optical isolation between the cavity and the laser. This method measures the optical loss at only one small spot in the cavity, and this is measured using circularly polarized light. It is believed that the system accuracy can be further increased by developing an improved method of measuring the finesse of the system.

VII. ACKNOWLEDGMENTS

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APPENDIX

The theory for the loss measuring optical cavity using plane mirrors is that of the Fabry-Perot interferometer, given many times before,[12] modified to include loss within the cavity, different reflectivities for the two mirrors, and the first-order effects of the velocity of the moving mirror. The exact theory for the effect of the moving mirror's velocity upon the interferometer response was developed. However, it was found that after neglecting some of the higher-order velocity terms, the same expression for the first-order velocity effects could be more readily obtained by assuming the mirror spacing to be fixed and by linearly varying the input frequency. Only the simpler theory will be given here. These first-order velocity effects for the interferometer, which has a Lorentzian frequency response, will be shown to be appreciably different from those previously calculated for the Gaussian filter.[13]

We assume that the incident light beam is collimated, monochromatic, and perpendicular to the mirrors. It is also assumed that the index of refraction of the space between the two mirrors is equal to that of free space, and that the mirrors have no loss. The list of all symbols and definition of all terms used in the following theory are given below:

$a = 2\pi(\Delta f/\Delta t)$ = angular sweep rate

$\alpha = (-4\pi vt/\lambda)$

B = half-power bandwidth

$\beta_0 \equiv 2m\pi v/c$

c = velocity of light in free space

d_0 = fixed mirror spacing

E_T = total combined electric field of all the output light rays assuming a unit input

g = power loss per single pass through any material within the cavity expressed as a ratio

$g_0 \equiv [g(1 - R_1)(1 - R_2)]^{\frac{1}{2}}$
I_T = total output light intensity assuming unit light input
I_m = maximum output from the interferometer assuming a unit light input
λ = free-space wavelength of light source
$m \approx (2d_0/\lambda)$ = large integer
R_1, R_2 = power reflectance of the two mirrors expressed as a ratio
t = real time
T_p = pulse width in time of the output pulse at half peak power level
T_c = fundamental pulse group spacing in time
$v = (\Delta d/\Delta t)$ = velocity of moving mirror
$x = g(R_1R_2)^{\frac{1}{2}}$.

Let the instantaneous angular frequency input be

$$\omega = \omega_0 - at. \tag{1}$$

Then the instantaneous phase is $\varphi = \omega_0 t - (at^2/2)$.

If the fixed mirror spacing is d_0 and the input light ray is normal to the mirrors as indicated in Fig. 5, then the total time delay for the nth output ray is

$$\tau_n + \tau_0 = 2n \frac{d_0}{c} + \frac{d_0}{c}$$

where the delay for the initial ray is

$$\tau_0 = \frac{d_0}{c}.$$

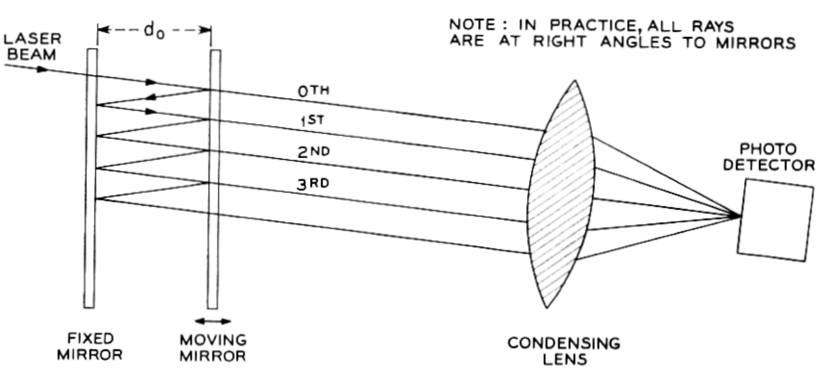

Fig. 5 — Light path through interferometer mirrors to output.

Thus the phase of the nth output ray is

$$\varphi_n = \omega_0 \left[t - \frac{2nd_0}{c} - \frac{d_0}{c} \right] - \frac{a}{2} \left[t - \frac{2nd_0}{c} - \frac{d_0}{c} \right]^2 .$$

The voltage for the nth output ray is

$$E_n = g_0 x^n \exp(-j\varphi_n).$$

The term independent of n can be neglected since its magnitude is unity. Thus the total output voltage is given by

$$E_T = \sum_{n=0}^{\infty} g_0 x^n \exp \left\{ -j \frac{2nd_0}{c} \left[\omega_0 - a\left(t - \frac{d_0}{c} \right) \right] - j2an^2 \frac{d_0^2}{c^2} \right\}. \quad (2)$$

The relation between the rate of change of the angular frequency, a, and the velocity of the mirror may be found as follows

$$a = 2\pi \frac{\Delta f}{\Delta t} = \frac{2\pi}{\Delta t} \frac{2\Delta d}{\lambda} \frac{c}{2d_0} = \frac{\omega_0 v}{d_0} . \quad (3)$$

Now, the velocity of the mirror is so small that in any reasonable length of time the variation in the mirror spacing is very small compared to the initial spacing. Hence, the exponent of (2) can be written as

$$\frac{2nd_0}{c} \left[\omega_0 - a\left(t - \frac{d_0}{c} \right) \right] + 2an^2 \frac{d_0^2}{c^2} = n[2m\pi + \alpha] + n^2\beta_0 \quad (4)$$

where α is small,

$$m \approx 2d_0/\lambda = \text{large integer},$$

and

$$\beta_0 = 2\pi mv/c.$$

Thus from (2) and (4), the total output voltage is given by

$$E_T = \sum_{n=0}^{\infty} g_0[x \exp(-j\alpha)]n \exp(-j\beta_0 n^2).$$

Now

$$I_T = | E_T |^2.$$

Thus

$$I_T \approx \left[\sum_{n=0}^{\infty} g_0 x^n \cos(\alpha n + \beta_0 n^2) \right]^2 + \left[\sum_{n=0}^{\infty} g_0 x^n \sin(\alpha n + \beta_0 n^2) \right]^2 . \quad (5)$$

The above expression is a good approximation to that obtained by the exact theory for $v/c < 10^{-8}$. At the present time, the series in (5) can only be summed numerically, since any other approximation results in a slowly convergent infinite series. The number of terms required by the series may be calculated as follows:

$$\frac{I_T}{g_0^2} \leqq \left[\sum_{n=0}^{\infty} x^n\right]^2 = \left[\sum_{n=0}^{N-1} x^n + \sum_{n=N}^{\infty} x^n\right]^2$$
$$\leqq \frac{(1 - x^N)^2 + 2x^N(1 - x^N) + x^{2N}}{(1 + x)^2} .$$

Thus the ratio of the total error in stopping the series after $N - 1$ terms to the actual value is

$$\epsilon = 2x^N - x^{2N}.$$

The number of terms required is then given by

$$N = \frac{\log (2/\epsilon)}{\log (1/x)} \approx \frac{\log (2/\epsilon)}{1 - x}. \tag{6}$$

Thus if

$$\epsilon = 1/1000 \quad \text{and} \quad x = 0.9975,$$
$$N = 3040.$$

Hence for $x \approx 1$ a very large number of terms must be used in the series.

The series in (5) was computed for a number of values of x, and β_0, some of which are given in Fig. 1. From these values, it was found that the reduction in the maximum of the output response, I_p/I_m, and the increase in the half-height pulse width, T_m/T_p, produced by the mirror velocity can be expressed as

$$(I_m/I_p)^2 = T_m/T_p \approx \{1 + [13.8\ \beta_0^2/(1 - x)^4]\}^{\frac{1}{2}}. \tag{7}$$

The above expression matches the computed values only when the first-order velocity effects are the order of a few per cent. For higher values, the computed results are less than those given by (7).

The relation given in (7) may be compared to that developed for a Gaussian filter which might be used in a spectrum analyzer. It has been shown[13] that the loss in sensitivity, S/S_0, and the increase in apparent bandwidth, B_m/B, produced by sweeping the frequency in the spectrum analyzer is given by

$$\left(\frac{S_0}{S}\right) = \frac{B_m}{B} = \left[1 + 0.195\left(\frac{1}{B^2}\frac{\Delta f}{\Delta t}\right)^2\right]^{\frac{1}{2}} \tag{8}$$

where the above relation was calculated on a power basis.

In the interferometer, the mirror spacing is varied linearly with time. Hence the ratio of any two time intervals may be replaced by the ratio of their appropriate frequency differences. Thus from (3) and (11), it can be readily shown that

$$\frac{1}{B^2}\frac{\Delta f}{\Delta t} = \frac{\pi \beta_0}{(1-x)^2}. \tag{8a}$$

From (8a), (7) becomes

$$\left(\frac{I_m}{I_p}\right)^2 = \frac{T_m}{T_p} = \left[1 + 1.34\left(\frac{1}{B^2}\frac{\Delta f}{\Delta t}\right)^2\right]^{\frac{1}{2}}$$

which is an order of magnitude different from the relation given in (8).

When the mirror velocity is small enough to be neglected, the output from the interferometer is given by the first terms of (5). Thus for $\beta_0 = 0$,

$$I_T = \frac{g_0^2}{1 - 2x\cos\alpha + x^2} = \frac{g_0^2}{(1-x)^2 + 4x\sin^2(\alpha/2)}. \tag{9}$$

Now the maximum value of I_T occurs at $\alpha = 2\pi k$, $k = 0, 1, 2, 3, \cdots$, hence

$$I_m = (I_T)_{\max} = \frac{g_0^2}{(1-x)^2} \equiv \frac{g(1-R_1)(1-R_2)}{[1-g(R_1R_2)^{\frac{1}{2}}]^2}. \tag{10}$$

From these relations, it can readily be shown that for x nearly unity the finesse of the system is given by[12]

$$F \equiv \frac{T_c}{T_p} = \frac{\pi\sqrt{x}}{1-x}.$$

The solution for x is then

$$x \equiv g(R_1R_2) \approx 1 - \frac{\pi T_p}{T_c} + \frac{1}{2}\left(\frac{\pi T_p}{T_c}\right)^2 + \cdots. \tag{11}$$

This equation relates the optical loss to measurable quantities for the pulse response of the interferometer. The time function for the output pulse can be obtained from (9),

$$I_T \approx \frac{I_m}{1 + (4t^2/T_p^2)} \tag{12}$$

for x close to unity.

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