Light Transmission in a Multiple Dielectric (Gaseous and Solid) Guide

By E. A. J. MARCATILI

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A low-loss transmission medium for optical frequencies consisting of a thin-wall dielectric tube that separates an internal high-density gas from an external low-density gas is proposed. Approximate transmission characteristics of the coaxial structure are derived via the analysis of a two-dimensional model.

The multiple dielectric guide behaves similarly to a dielectric waveguide, and the fundamental mode attenuation due to losses in the solid tube is calculated to be small even at the worst possible choice of the tube wall thickness.

Assuming (i) an internal diameter of 5×10^{-4} m, (ii) wall thickness of 10^{-5} m, (iii) loss in the solid dielectric of 1 neper/m, and (iv) internal and external gases, CO_2 and air, respectively, both at 20 atmospheres, the theoretical attenuation for the fundamental mode is of the order of 0.05 db/km and the attenuation of each mode is roughly four times smaller than that of the next higher one.

I. INTRODUCTION

Hollow metallic and dielectric waveguides for long distance optical transmission were suggested by the author and R. A. Schmeltzer.[1] An alternative, proposed by S. E. Miller, is the coaxial structure shown in Fig. 1. The internal medium $r \leqq a$, as well as the external one, $r \geqq b$, are gases of refractive indexes n_1 and n_3 such that $n_1 > n_3$. Both gases are kept apart by a dielectric tube of thickness $c = b - a$ and refractive index n_2.

Since n_1 and n_2 are both bigger than n_3, the structure will guide modes similarly to a dielectric waveguide, but it is of interest to find out if there are modes with most of their power traveling within the internal gas and not in the dielectric tube. If these modes exist, they

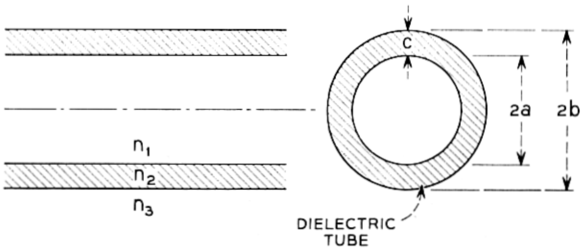

Fig. 1 — Multiple dielectric guide. $(n_3 - n_1)/(n_2 - n_1) \ll 1$.

will have relatively low losses originating from Rayleigh scattering, from scattering due to geometrical and dielectric imperfections, and from small absorption in the dielectric tube.

It is shown here that such modes indeed exist, and the losses due to absorption in the dielectric tube are calculated.

II. SOLUTION OF MAXWELL'S EQUATIONS

Instead of solving the equations for the three-dimensional cylindrical structure of Fig. 1, we solve them for the two-dimensional model of Fig. 2. The simplification is justified by the results obtained in Ref. 1; the propagation constants of modes in a hollow dielectric waveguide of circular cross section turn out to be comparable to the propagation constants of modes in the two-dimensional model, independently of the polarization.

We look for TE modes with electric field only along the y axis and independent of y. The field components are then, E_y, H_x, and H_z. The components in the three regions 1, 2, and 3, that will be needed to match the boundary conditions, are

$$\left.\begin{aligned} E_{y1} &= \cos k_1 x \\ H_{z1} &= \frac{k_1}{i\omega\mu} \sin k_1 x \\ E_{y2} &= A \cos (k_2 x + \varphi) \\ H_{z2} &= \frac{A k_2}{i\omega\mu} \sin (k_2 x + \varphi) \\ E_{y3} &= B\, e^{-ik_3 x} \\ H_{z3} &= \frac{B k_3}{\omega\mu} e^{-ik_3 x} \end{aligned}\right\} e^{-i\gamma z} \tag{1}$$

where A, B, and φ are constants to be determined; ω is the angular

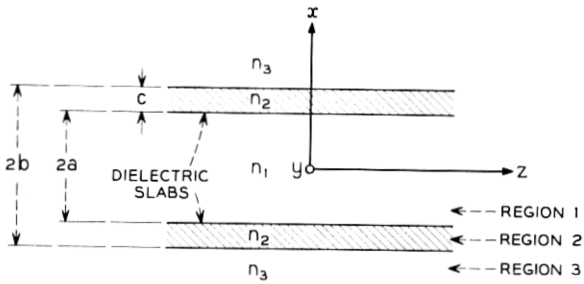

Fig. 2 — Two-dimensional version of the multiple dielectric waveguide.

frequency; μ is the magnetic permeability; k_1, k_2, and k_3 are the propagation constants along the x axis in the three regions and γ is the propagation constant along z. The electric field has been chosen symmetric with respect to the plane of symmetry $x = 0$, but the results will be extended later to antisymmetric modes.

For simplicity, we refer the refractive indexes to that of the internal gas. Therefore,

$$n_1 = 1. \tag{2}$$

The propagation constants in the three media are related to the propagation constant in free space $k = \omega\sqrt{\mu\epsilon}$ by

$$\begin{aligned} k_1^2 &= k^2 - \gamma^2 \\ k_2^2 &= k^2 n_2^2 - \gamma^2 \\ k_3^2 &= k^2 n_3^2 - \gamma^2. \end{aligned} \tag{3}$$

From (1) the continuity of tangential components at the discontinuities ($H_{z1}/E_{y1} = H_{z2}/E_{y2}$ at $x = \pm a$ and $H_{z2}/E_{y2} = H_{z3}/E_{y3}$ at $x = \pm b$) yield two equations

$$\begin{aligned} k_1 \tan k_1 a &= k_2 \tan (k_2 a + \varphi) \\ k_2 \tan (k_2 b + \varphi) &= i k_3 \end{aligned} \tag{4}$$

which, together with those in (3), are enough to calculate the five unknowns k_1, k_2, k_3, γ, and φ. In particular, the characteristic equation which determines γ is derived from (4) eliminating φ between the two equations:

$$k_1 \tan k_1 a = \frac{-k_2 \tan k_2 c + i\, k_3}{1 + i \dfrac{k_3}{k_2} \tan k_2 c}. \tag{5}$$

Substituting k_1, k_2, and k_3 by their equivalents (3), we have a transcendental equation in γ. To solve it in closed form we make several assumptions:

(*i*) The propagation constant is close to that of a plane wave in medium (1) Fig. 2. Therefore,

$$\gamma = k(1 - \delta) \tag{6}$$

and

$$\delta \ll 1. \tag{7}$$

Powers of δ bigger than one will be neglected.

(*ii*) Most of the power flows within the internal gas and consequently the electric field is close to zero in the vicinity of $x = \pm a$. Then from (1), (3), and (6)

$$k_1 a = ka\sqrt{2\delta} \cong (2p + 1)\frac{\pi}{2} \tag{8}$$

where p is an integer.

Since $\delta \ll 1$, it follows from this equation that the free space wavelength must be much shorter than the internal gas gap. That is,

$$\frac{\lambda}{a} \ll 1. \tag{9}$$

With these assumptions and some algebra, the value of δ derived from (5) and substituted in (6) yields the propagation constant

$$\gamma = k\left\{1 - \frac{1}{2}\left(\frac{\pi q}{2ka}\right)^2 \cdot \left[1 + \frac{2}{ka}\,\frac{1 + i\sqrt{\dfrac{n_3^2 - 1 + \left(\dfrac{\pi q}{2ka}\right)^2}{n_2^2 - 1}}\tan kc\sqrt{n_2^2 - 1}}{\sqrt{n_2^2 - 1}\tan kc\sqrt{n_2^2 - 1} - i\sqrt{n_3^2 - 1 + \left(\dfrac{\pi q}{2ka}\right)^2}}\right]\right\}. \tag{10}$$

The integer $2p + 1$ has been replaced by another integer, q. If q is odd, the propagation constant (10) is that of a symmetric mode. If q is even, the propagation constant is that of an antisymmetric mode (electric field zero in the plane of symmetry $x = 0$, Fig. 2).

III. DISCUSSION OF PARTICULAR CASES AND CONCLUSIONS

As expected, if n_2 is real and $n_3^2 < 1 - (\pi q/ka)^2$ * the propagation constant γ given in (10) is real, that is, the mode of order q propagates without loss. Nevertheless, the dielectric sheets separating the gases have some loss and their refractive index can be rewritten

$$n_2 = n\left(1 - i\,\frac{\alpha}{kn}\right) \tag{11}$$

where n is the real part of the refractive index and α is the plane wave attenuation constant in the dielectric. Since the light attenuation in a low-loss solid dielectric is of the order of 1 db/m,

$$\frac{\alpha}{kn} \ll 1, \tag{12}$$

and powers of α/kn bigger than one will be neglected.

Now, let us substitute the value n_2 given in (11) into equation (10) and furthermore, let us consider two extreme cases of slab thickness.

3.1 *First Case*

The dielectric slabs are resonant: each slab contains an integer number s of half wavelengths. This requires

$$kc\sqrt{n^2 - 1} = s\pi. \tag{13}$$

Then,

$$\gamma_1 = k\left[1 - \frac{1}{2}\left(\frac{\lambda q}{4a}\right)^2\left(1 - \frac{2}{ka}\,\frac{\sqrt{1 - n_3^2} - i\alpha nc}{1 - n_3^2 + \alpha^2 n^2 c^2}\right)\right]. \tag{14}$$

3.2 *Second Case*

The slabs are antiresonant, that is, each contains an odd number $2s + 1$, of quarter wavelengths, which requires that

$$kc\sqrt{n^2 - 1} = (2s + 1)\,\frac{\pi}{2}, \tag{15}$$

and consequently,

$$\gamma_2 = k\left[1 - \frac{1}{2}\left(\frac{\lambda q}{4a}\right)^2\left(1 - \frac{2}{ka}\,\frac{\sqrt{1 - n_3^2} - i\alpha nc}{n^2 - 1}\right)\right]. \tag{16}$$

* The reader is reminded that n_2 and n_3 are refractive indexes normalized to n_1.

In both cases, the phase constant is very close to

$$\text{Re}\,\gamma_1 \cong \text{Re}\,\gamma_2 \cong k\,[1 - \tfrac{1}{2}\,(\lambda q/4a)^2], \tag{17}$$

which coincides with that of TE_{qo} modes far from cut-off in oversize rectangular waveguides.

In general, since $n_3 \cong 1$,

$$n^2 - 1 \gg 1 - n_3^2 + \alpha^2 n^2 c^2. \tag{18}$$

Therefore, the attenuation constant

$$\text{Im}\,\gamma_1 = -\left(\frac{\lambda q}{4a}\right)^2 \frac{\alpha n c}{a} \frac{1}{1 - n_3^2 + \alpha^2 n^2 c^2} \tag{19}$$

in the first case (resonant slabs) is larger. Small attenuation requires n_3 as large as possible,

$$\frac{a}{\lambda} \gg 1, \tag{20}$$

and

$$\frac{\alpha n c}{1 - n_3^2} \ll 1. \tag{21}$$

These inequalities indicate that for small losses the internal gap must be large compared to the wavelength and that the attenuation through the solid dielectric thickness c must be small compared to the difference between refractive indices of the two gases.

Since the attenuation constant is proportional to q^2, the attenuation discrimination between the q_1th and the q_2th mode is $(q_1/q_2)^2$. Better discrimination is predicted with modes in hollow metallic optical waveguides.[1]

Let us put in some numbers. For

$$a = 0.25\ 10^{-3}\ \text{m},$$
$$\lambda = 10^{-6}\ \text{m},$$
$$\alpha = 1\ \text{neper/m},$$
$$n = 1.5,$$
$$c = 10^{-5}\ \text{m},$$

and using CO_2 and air for the internal and external gases, respectively, both at 20 atmospheres, which yield

$$1 - n_3^2 \cong 0.01,$$

the attenuation in db for the fundamental mode would be

$$8.686\ \text{Im}\,\gamma_1 = 0.052\ \text{db/km}.$$

In order to get an idea about how critical it is to maintain the wall thickness uniformity, we calculate the propagation constant of modes assuming an extreme case in which the external surface of the dielectric is very rough and scatters. This transmission medium is equivalent to that of Fig. 2 with dielectric slabs infinitely thick. Then the propagation constant is derived from (10) making $n_3 = n_2 = n$. Thus,

$$\gamma_3 = k\left[1 - \frac{1}{2}\left(\frac{\lambda q}{4a}\right)^2\left(1 + \frac{i2}{ka\sqrt{n^2-1}}\right)\right]. \qquad (22)$$

The ratio between the attenuation constants of the medium with infinite and finite dielectric slab is

$$\frac{\operatorname{Im}\gamma_3}{\operatorname{Im}\gamma_1} = \frac{1 - n_3^2}{\alpha nc\sqrt{n^2-1}} = 595.$$

The fundamental mode in the guide made with CO_2 backed with hard dielectric is 595 times more lossy than the mode in the guide made with CO_2 and air separated by the hard dielectric.

For comparison, we reproduce some of the results obtained for hollow metallic and dielectric waveguides in Ref. 1. An aluminum straight waveguide of 0.25 mm radius operating at $\lambda = 1\ \mu$ was calculated to have a TE_{01} mode loss of 1.8 db/km, and this loss doubled for the case where the axis of the guide had a radius of curvature of 48 m. Using glass instead of aluminum, the lowest loss mode EH_{11} was calculated to be attenuated 118 db/km.

Therefore, from the example in this paper, we conclude that if the wall thickness can be perfectly controlled, the multiple dielectric waveguide should be approximately one order of magnitude less lossy than the hollow metallic waveguide; at the other extreme, if the wall thickness cannot be controlled at all, the multiple dielectric waveguide should be roughly one order of magnitude lossier than the metallic waveguide.

We have only scratched the surface of the problem and the results are attractive. Many questions though remain unanswered. What thickness uniformity, and consequently, loss can be achieved in practice, what is the loss increase due to (*i*) the presence of supports for the dielectric tube, (*ii*) lack of straightness, and (*iii*) unwanted thermal gradients, etc.

REFERENCES

1. Marcatili, E. A. J. and Schmeltzer, R. A., Hollow Metallic and Dielectric Waveguides for Long Distance Optical Transmission and Lasers, B.S.T.J., *43*, July, 1964, pp. 1783–1810.