Equation (6) is easily solved to give

$$p(k) = p(0)\left[1 + K \sum_{j=1}^{k} r^{2(j-1)}\right]. \tag{8}$$

Then

$$\lim_{k\to\infty} p(k) = p(0)\left[1 + \frac{K}{1 - r^2}\right]. \tag{9}$$

The bracketed term in (8) or (9) gives the enhancement of error probability due to errors made on all previous pulses.

To compare results obtained by application of (9) with those obtained experimentally by R. D. Howson,[2] we take $b = \frac{1}{4}$, $g_0 = \exp(-\frac{1}{8})$, and $g_1 = -g_0(1 - \exp(-\frac{1}{4}))$. Over a wide range of S/N ratios of interest, analysis based upon (9) predicts about a 0.6-dB S/N penalty of this system over the ideal case of no low frequency cutoff. Agreement with experiment is excellent. It should be noted that the enhancement term in (9) is very close to unity and the S/N penalty is due essentially to the reduction of the pulse peak by the low-frequency cut-off.

In a future paper we will show (*i*) how Zador's approach can be extended to a wider class of systems, and (*ii*) how the approximation given herein can be used and improved when necessary, to arrive at meaningful quantitative results.

REFERENCES

1. Zador, P. L., Error Probabilities in Data System Pulse Regenerator with DC Restoration, B.S.T.J., *45*, July, 1966, pp. 979–984.
2. Flavin, M. A., Gerrish, A. M., Howson, R. D., and Salz, J., unpublished work.

Application of Automatic Transversal Filters to the Problem of Echo Suppression

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(Manuscript received October 6, 1966)

Long-haul voice communication has long been subject to the problem of returned echo. The advent of synchronous satellite communication introduces increased delay as a degrading factor in the overall quality of two-way conversation. This compounds the problem in that the echo must be further attenuated if its annoyance level is not to be increased as the delay of the returned echo increases.

At the present, the problem of returned echo is alleviated by the insertion of attenuation into the path of the weaker signal, i.e., the echo path.[1] Given the larger delay inherent in synchronous satellite communication, a better technique is wanted. One attractive scheme is the use of a transversal filter to synthesize a replica of the echo, which is then subtracted from the actual echo so that the two signals cancel. An algorithm which allows this synthesis to be carried out automatically was discovered by B. F. Logan and the late J. L. Kelly, Jr.

A scheme for simulating echoes and the technique for suppressing them are shown in block form in Fig. 1. The person using the handset at the left of the figure would experience echo; no echo is simulated for the person using the handset on the right. The echo, which would normally be caused by an improperly balanced hybrid, was instead simulated using various linear networks. A tape recorder simulated the long delay re-

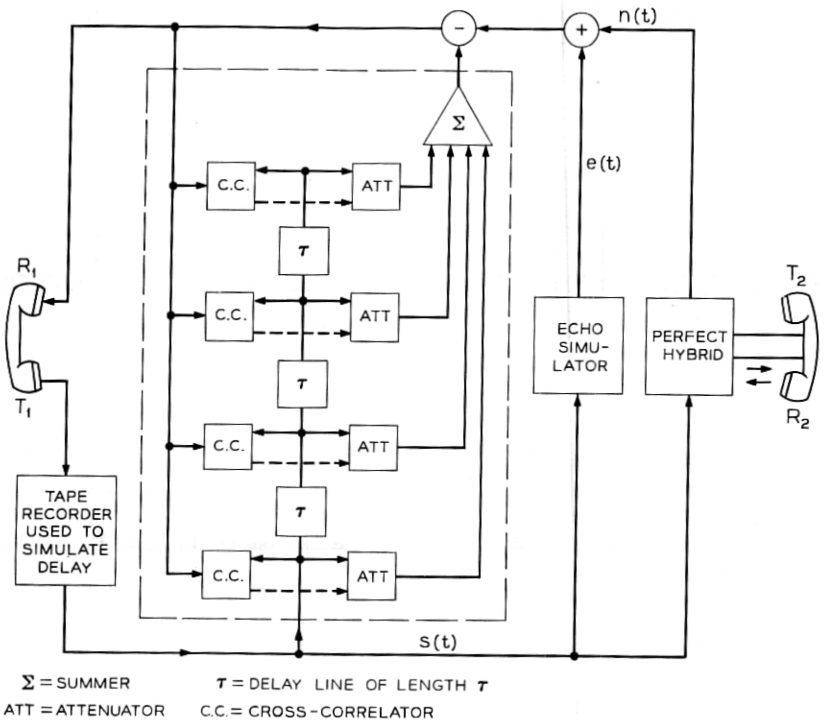

Fig. 1 — System for demonstration of automatic transversal echo suppressor.

sulting from satellite transmission. The automatic transversal filter is enclosed in dashed lines; its input is the delayed speech from T_1 and its function is the minimization of the echo delivered to R_1. The signal from T_2 merely appears as noise, $n(t)$, in this minimization procedure.

Mathematically, the transversal filter's function is the minimization of the mean-squared value of the echo delivered to R_1. This is achieved through the approximation of $e(t)$ by the sum of the weighted, delayed versions of $s(t)$. Thus, the canceler strives to minimize

$$I = \int_{-\infty}^{\infty} \left[e(t) - \sum_{j=0}^{N} c_j s(t - j\tau) \right]^2 dt, \tag{1}$$

where c_j is the weighting (attentuator setting) associated with the jth tap and τ is the tap spacing, usually the reciprocal of twice the highest frequency of interest.

The attenuator settings (which may be positive or negative) can be calculated by partial differentiation of I by the various c_j's. Specifically,

$$\frac{\partial I}{\partial c_k} = -2 \int_{-\infty}^{\infty} \left[e(t) - \sum_{j=0}^{N} c_j s(t - j\tau) \right] [s(t - k\tau)] \, dt. \tag{2}$$

Note that the second equation states that the partial derivative of I with respect to the tap gain c_k is given by the cross-correlation of the signal at the kth tap on the delay line with the signal delivered to R_1. The optimum settings for all the tap weighting coefficients occur when all the partial derivatives are zero.

Assuming a reasonable spectrum for the signal $s(t)$, it can be shown that the integral I is a convex function of the tap gains. Given this fact, the information contained in the various partial derivatives is sufficient to point the way toward the unique minimum of I.

An experimental implementation of the echo canceler is built around a general-purpose automatic equalizer intended for the reduction of linear distortion in communication channels.[2] The attenuators are digitally-controlled, resistive ladder-networks. In the implementation, the information obtained by cross-correlation is used to increase or decrease the attenuator setting by a constant increment. The cross-correlation coefficients are then recalculated and the attenuators again changed. The attenuators are permitted to change their setting only when the $s(t)$ signal exceeds a threshold. The attenuators have infinite memory and so retain their setting during long lapses in the speech originating at T_1. A simple RC low-pass filter provides a sufficient approximation to the integration indicated in the second equation.

The signal originating at T_2, despite the fact that it may well be several times the size of the echo, $e(t)$, produces only small perturbations in the attenuator settings. This is a result of the powerful cross-correlation detection used to set the attenuators. In the experimental implementation described, a second speaker at T_2 did not perceptibly degrade echo suppression.

Another feature of the system is that it is inherently adaptive. If the characteristics of the transmission channel should change, the scheme automatically makes the necessary modifications as long as the speaker level is above the threshold. This is true provided that the change in the channel characteristics occurs at a very slow rate.

Early results from this implementation indicate that echo suppression of some 20 dB is attainable. Further evaluation is necessary to accurately predict behavior on real channels. The settling time for this experimental echo canceller was in the order of two seconds. The settling time is dependent on the echo-to-interfering-noise ratio.

An accompanying brief by A. J. Presti and M. M. Sondhi describes a different implementation of an echo canceller based upon similar principles.

There are a number of engineering problems which must be solved before adaptive echo cancellation becomes a practical reality. One of these is the long, distortion-free delay which the echo canceler must supply. The magnitude of this delay depends on the echo delay. Another problem arises when there are several reflection points (hybrids) in the echo path. A third problem is that the apparatus tracks a change in echo path only if the change is slow and the signal threshold is exceeded. Given the solution to these problems, however, the future of this technique of echo cancellation is a promising one.

REFERENCES

1. Holman, E. W. and Suhocki, V. P., A New Echo Suppressor, Bell Laboratories Record, April, 1966, pp. 139–142.
2. Lucky, R. W. and Rudin, H. R., Generalized Automatic Equalization for Communication Channels, Proc. IEEE, March, 1966, pp. 439–440.