Techniques for Adaptive Equalization of Digital Communication Systems

By R. W. LUCKY

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A previous paper described a simple adjustment algorithm which could be employed to set the tap gains of a transversal filter for the equalization of data transmission systems. An automatic equalizer was shown which used this algorithm during a training period of test pulse transmission prior to actual data transmission. The present paper extends the utility of this automatic equalization system by permitting it to change settings during the data transmission period in response to changes in transmission channel characteristics. Three schemes for accomplishing this adaptive equalization without the use of test signals are described and evaluated analytically. The first such scheme uses periodic estimates of channel response based on the received data signal to adjust or update the transversal filter settings. The second system is entirely digital and employs a sequential testing procedure to make adjustments aperiodically as they are required by changing conditions. The third system uses information obtained from a forward-acting error correction system for the purposes of adaptive equalization. Of the three systems described, the second is not only theoretically superior, but is practically the simplest. Experimental results for this second system are described.

I. INTRODUCTION

A previous paper[1] has dealt with the problem of automatic equalization for data transmission systems. In that paper, it was assumed that a finite-length transversal filter was to be used to correct the pulse response of a baseband (VSB) system at the sampling instants. A simple control system was shown which could be used to adjust the tap gains of the transversal filter to optimum positions using a series of test pulses transmitted prior to actual data transmission. After this training period the control system is disconnected, the tap gains remain fixed, and normal data transmission ensues. This automatic equalization system has been used in conjunction with multilevel vestigial sideband modulation to achieve a rate of 9600 bits-per-second on private line voice facilities.[2,3,4]

Two limitations of this automatic equalization system are immediately apparent — it requires that test pulses be transmitted and it must be reset in another training period whenever the channel characteristics change. Other disadvantages of the present mode of operation include the long training period required to establish accurate final settings and the possibility of a nonlinear channel causing the transmission characteristics for data transmission to be slightly different from those for isolated pulse transmission.

For these reasons, it has been found advantageous to develop an equalizer capable of deriving its control signals directly from the transmitted data signal itself. Such an equalizer would be capable of tracking a time varying channel and would also circumvent the other difficulties associated with preset equalizer operation. To distinguish this equalizer from the previously described preset automatic equalizer, we shall call the tracking equalizer an *adaptive* equalizer. The purpose of the adaptive equalizer is to continually monitor channel conditions and to readjust itself when required so as to provide optimum equalization. To conserve signal power and bandwidth, the channel monitoring (or system identification) of the adaptive equalizer must be done using only the normal received data signal and without the benefit of added test information.

In this paper, we will present a few techniques which may be used to achieve adaptive equalization. Since these techniques are based on the use of a particular tap gain adjustment algorithm used in the preset equalizer, we will begin with a brief description of this equalizer and the algorithm.

II. PRINCIPLES OF TRANSVERSAL FILTER EQUALIZATION

In the preset automatic equalizer, a sequence of isolated test pulses is transmitted through the channel and demodulator. At the output of the demodulator, the pulse waveform is designated $x(t)$ as shown in Fig. 1. The pulse then passes through a $(2N + 1)$-tap transversal filter whose tap gain settings are $c_{-N}, \cdots, c_N$. The output pulse of the equalizer is

$$\mathfrak{h}(t) = \sum_{j=-N}^{N} c_j x(t - jT). \tag{1}$$

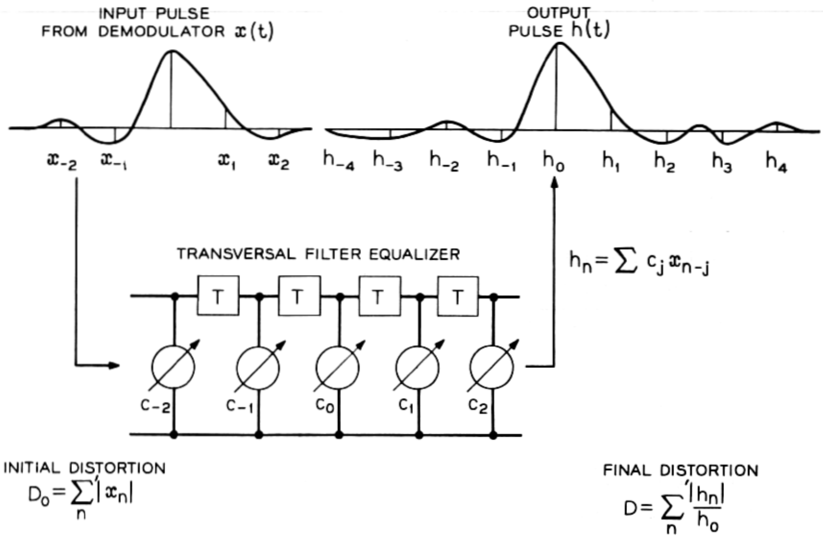

Fig. 1 — Transversal filter equalizer.

Since the output of the equalizer will be sampled at T second intervals during data transmission, we are only interested in the samples

$$\mathfrak{h}_n = \mathfrak{h}(nT)$$

of the output pulse. In terms of the input samples x_n we can write

$$\mathfrak{h}_n = \sum_{j=-N}^{N} c_j x_{n-j} . \tag{2}$$

The objective of the equalizer control circuitry is to set the tap gains such that the pulse distortion D is minimized, where we define*

$$D = \frac{1}{\mathfrak{h}_0} \sum_{n=-\infty}^{\infty}{}' \mid \mathfrak{h}_n \mid. \tag{3}$$

This criterion is equivalent to requiring that the equalizer maximize the eye opening.

We assume that the input is normalized so that $x_0 = 1$ and that the center tap c_0 is used to satisfy the practical constraint $\mathfrak{h}_0 = 1$. In Ref. 1 it was proved that if the initial distortion $D_0 < 1$, where

* Primes on summations indicate deletion of the zeroth term.

$$D_0 = \sum_{n=-\infty}^{\infty}{}' \mid x_n \mid, \tag{4}$$

then the output distortion D is at a minimum when the $2N$ tap gains c_j for $|j| \leqq N, j \neq 0$ are adjusted so that $h_n = 0$ for $|n| \leqq N, n \neq 0$. In other words, if a binary eye is open before equalization, then using the tap gains to force zeros in the output response is optimum.

Also, if the initial distortion is less than unity, it was shown that a simple iterative procedure could be used to obtain optimum tap gain settings. In this procedure each tap gain c_j is adjusted an amount $-\Delta$ sgn h_j after each test pulse. (The center tap gain c_0 is adjusted by $-\Delta$ sgn $(h_0 - 1)$.) Thus one simply inspects the polarities of the output pulse $h(t)$ at the sampling times and uses this polarity information to advance or retard counter-controlled attenuators along the tapped delay line.

In constructing an adaptive equalizer we shall use this same simple adjustment algorithm. The problem now becomes one of finding the polarities of the channel pulse response $h(t)$ without the benefit of test pulses. Once the decisions have been made as to the most likely polarities for the impulse response samples h_j, we make discrete adjustments in the corresponding tap gains c_j to correct the equalizer. (This is shown diagrammatically in Fig. 2.) The adjustments in tap gains can be made periodically or only as required by changes in transmission characteristics. Since the normal telephone channel's characteristics change very slowly, the decisions regarding the channel response polarities can be

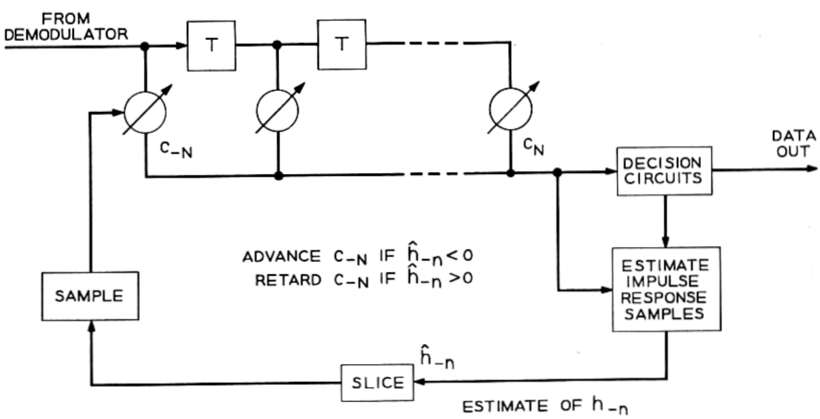

Fig. 2 — Adaptive equalizer.

made extremely accurate. In general the more accurate these decisions, the more precise the equalization which can be obtained, but with a corresponding increase in adaptive equalizer response, or settling time. We shall evaluate the accuracies and response times of the adaptive equalizers described in subsequent sections.

III. PERIODIC EQUALIZER ADJUSTMENT

3.1 *Maximum Likelihood Estimation of Response Values*

Let us assume that at the end of every KT seconds we wish to make a decision as to the current most probable impulse response polarities and to effect an incremental adjustment of the equalizer. The demodulated and equalized voltage at time t_k is

$$y_k = \sum_{n=-\infty}^{\infty} a_n \mathfrak{h}_{k-n} + \eta_k , \tag{5}$$

where the a_n's are the input symbols chosen from an M-symbol alphabet, the h_n's are the samples of the overall (equalized) system impulse response, and the η_k's are noise samples.* We also make the following key assumptions.

(*i*) The noise samples η_k are independent, identically distributed Gaussian variables with variance σ^2.

(*ii*) The input data symbols are uncorrelated.

(*iii*) The probability of error is relatively small, so that for practical purposes the sequence $\{a_n\}$ is available at the output of the detector.

(*iv*) The channel response samples $\mathfrak{h}_n$ are essentially constant over the observation interval of KT seconds.

It is important to note that these assumptions are not made because they are true in a practical situation, but are made at this time to enable us to derive a reasonable system configuration and to assess its probable performance. Assumption (*ii*) is a particularly important one which we shall have to consider more carefully in predicting actual system performance.

Since we are using a decision-directed system — i.e., the detected symbols a_n are assumed correct — we can regard the signal samples y_k as being determined by noise and the set of parameters $\mathfrak{h}_n$. By an appro-

* It is true that the noise sequence $\{\eta_k\}$ must pass through the equalizer. However, since in practice, all the tap gains c_j of the equilizer except for c_0 are very small we assume that the statistics of the noise sequence are unaffected by the equalizer.

priate statistical technique we can make estimates of the response samples h_n from the signal samples y_k. Using the set of assumptions (*i*) through (*iv*), the probability of receiving the sequence $\{y_k\}$, $k = 1, \cdots, K$ for a particular choice of the parameter set $\mathfrak{h}_n$ is

$$p(\mathbf{y}|\mathbf{h}) = \prod_{k=1}^{K} \frac{\exp - \frac{1}{2\sigma^2}\left(y_k - \sum_{n=-\infty}^{\infty} a_n \mathfrak{h}_{k-n}\right)^2}{\sigma\sqrt{2\pi}}. \tag{6}$$

The likelihood function $L(\mathbf{y}|\mathbf{h})$ is the logarithm of $p(\mathbf{y}|\mathbf{h})$. Apart from a constant, this is

$$L(\mathbf{y}|\mathbf{h}) = \sum_{k=1}^{K} - \frac{1}{2\sigma^2}\left(y_k - \sum_{n=-\infty}^{\infty} a_n \mathfrak{h}_{k-n}\right)^2. \tag{7}$$

The maximum likelihood estimates of the $(2N + 1)$ response values of $\mathfrak{h}_j$, $j = -N, \cdots, +N$, needed to adjust the transversal filter tap gains are determined by the $(2N + 1)$ simultaneous equations $\partial L / \partial \mathfrak{h}_j = 0$. Thus

$$\sum_{k=1}^{K} a_{k-j}\left(y_k - \sum_{n=-\infty}^{\infty} a_n \hat{\mathfrak{h}}_{k-n}\right) = 0, \qquad \text{for } j = -N, \cdots, +N. \tag{8}$$

These equations are more conveniently rewritten in the form

$$\frac{1}{K}\sum_{k=1}^{K} a_{k-j} y_k - \sum_{n=-\infty}^{\infty} \hat{\mathfrak{h}}_n A_{nj} = 0, \tag{9}$$

where

$$A_{nj} = \frac{1}{K}\sum_{k=1}^{K} a_{k-n} a_{k-j}. \tag{10}$$

Because of assumption (*ii*) that the input symbols are uncorrelated, for a reasonably long averaging period K we can use $A_{nj} = S\delta_{nj}$ where S is the average signal power. This simplification yields the estimators

$$\hat{\mathfrak{h}}_j = \frac{1}{KS}\sum_{k=1}^{K} a_{k-j} y_k. \tag{11}$$

We could easily now find the variance of the estimates (11) under the assumption of random data, and we would discover that they are poorly converging estimates with typically about 50,000 samples y_k required before we could move the equalizer taps with any degree of confidence. This exorbitant settling time is caused by the presence of a large parameter, $\mathfrak{h}_0 \approx 1$, among a set of typically very small parameters.

Bear in mind that in normal operation the equalization will be very close to perfect and the samples $\mathfrak{h}_j$ for $j \neq 0$ will be generally much less than 0.01 in magnitude.

This difficulty is circumvented by estimating $(\mathfrak{h}_0 - 1)$ which is comparable in size to $\mathfrak{h}_j$ for $j \neq 0$ instead of directly estimating $\mathfrak{h}_0$. Therefore, we define

$$h_j = \begin{cases} \mathfrak{h}_j & \text{for} \quad j \neq 0, \\ \mathfrak{h}_0 - 1 & \text{for} \quad j = 0. \end{cases} \tag{12}$$

The samples h_j represent equalization error in the output pulse response. Following estimation of these values each of the taps c_j is advanced if h_j is negative and retarded if h_j is positive — the center tap being handled the same as any other tap.

Substitution of (12) in (7) gives the likelihood function

$$L(\mathbf{y} \mid \mathbf{h}) = \sum_{k=1}^{K} -\frac{1}{2\sigma^2}\left(y_k - a_k - \sum_{n=-\infty}^{\infty} a_n h_{k-n}\right)^2. \tag{13}$$

The quantities $(y_k - a_k)$ will be used frequently so we designate these e_k since they represent the error between the received sample y_k and the detected level a_k. The maximum likelihood estimates of the equalization error become

$$\hat{h}_j = \frac{1}{KS}\sum_{k=1}^{K} a_{k-j} e_k . \tag{14}$$

Fig. 3 shows a block diagram of an adaptive equalizer employing the estimates (14). In this system, the detected levels a_k are converted to analog form and subtracted from the received samples y_k to form the error samples e_k. The error samples e_k are then correlated simultaneously with each of the detected symbols a_{k-j} for $j = -N, \cdots, +N$. To accomplish this, the error samples e_k must be delayed NT seconds while the detected samples a_k are passed along a $(2N + 1)$-tap delay line. The delay line may consist of parallel shift registers since the samples a_k are in digital form. The outputs of the correlators (multipliers — lowpass filters) are sampled at K symbol intervals and appropriate actions are taken on the transversal filter attenuators.

3.2 *Performance of the Adaptive Equalizer Under Ideal Conditions*

In assessing the performance of an adaptive equalizer, we are primarily concerned with accuracy and settling time. The action of the equalizer

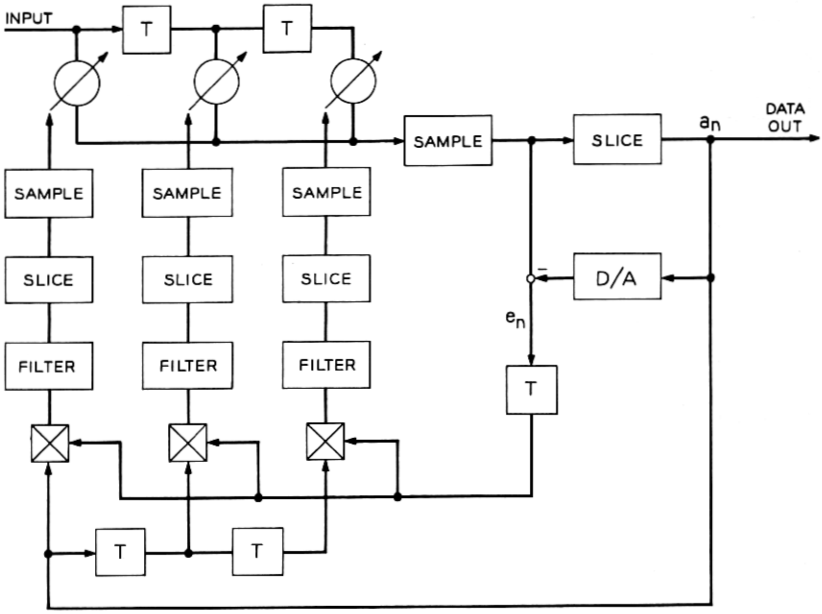

Fig. 3 — Periodically sampled adaptive equalizer.

is that of a multidimensional random walk with a bias toward correct equalization being supplied by the estimation circuitry. We shall make the assumption of small tap interaction and consider that each of the tap gains exhibits a one-dimensional walk independent of the action of other taps. The fundamental quantity involved in questions of accuracy and settling time is the probability $P(\text{sgn } \hat{h}_j = \text{sgn } h_j) = p_c$. For given noise and data statistics this probability is a function of the value of h_j and represents the probability of making a correct adjustment of tap c_j. When $| h_j |$ is large, i.e., equalization is poor, we expect that p_c will be close to unity, while for small $| h_j |$ the probability p_c approaches 0.5 and the tap gain tends to wander.

The adaptive equalizer must be designed to keep the inevitable wander of the tap gain within bounds imposed by accuracy requirements. Generally, we would wish to take full advantage of the inherent accuracy of the attenuator setting apparatus, i.e., each attenuator is adjusted in steps of Δ. Thus, when the equalizer is in perfect adjustment each tap gain will have an error of about 0.5Δ, which means that each of the samples h_j will be about 0.5Δ in magnitude. At the end of K symbol durations, each of the gains will be increased or decreased by Δ. If $|h_j| = 0.5\Delta$ and a mistake is made in the polarity of $\hat{h}_j$, then the next value of h_j will be about 1.5Δ in magnitude and distortion will be considerably increased. We should generally design the system so that each tap gain spends a great majority of the time in the state where $|h_j| = 0.5\Delta$ and a small amount at 1.5Δ. Thus, for example, we may wish p_c to be 0.99 when $h_j = 0.5\Delta$.

We now evaluate $p_c(h_j)$ under the ideal conditions given in assumptions *(i)* through *(iv)*. When the averaging period K is large the estimates $\hat{h}_j$ become normally distributed. The mean value of $\hat{h}_j$ is

$$\bar{\hat{h}}_j = \frac{1}{KS} \sum_{k=1}^{K} \overline{a_{k-j} e_k}. \tag{15}$$

The error sample e_k may be written

$$e_k = y_k - a_k = \sum_{n=-\infty}^{\infty} a_n h_{k-n} + \eta_k \tag{16}$$

so that (15) becomes

$$\bar{\hat{h}}_j = \frac{1}{KS} \sum_{k=1}^{K} \sum_{n=-\infty}^{\infty} \overline{a_{k-j} a_n}\, h_{k-n} + \frac{1}{KS} \sum_{k=1}^{K} \overline{a_{k-j} \eta_k}. \tag{17}$$

Since $\overline{a_k a_j} = S\delta_{kj}$ we have the necessary result

$$\bar{\hat{h}}_j = h_j. \tag{18}$$

The variance, σ_j^2, of $\hat{h}_j$ may be evaluated in straightforward fashion.

$$\sigma_j^2 = \frac{1}{K^2 S^2} \sum_{k=1}^{K} \sum_{e=1}^{K} \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \overline{a_{k-j} a_n a_{e-j} a_m}\, h_{k-n} h_{e-m} + \frac{1}{K^2 S^2} \sum_{k=1}^{K} \sum_{e=1}^{K} \overline{a_{k-j} a_{e-j} \eta_k \eta_e} - h_j^2. \tag{19}$$

The four-fold sum in (19) may be partitioned into three sums involving pairwise equality in subscripts and an overlap term. A little manipulation yields

$$\sigma_j^2 = \frac{1}{K} \sum_{\substack{n=-\infty \\ n \neq j}}^{\infty} h_n^2 + \frac{2}{K^2} \sum_{n=1}^{K-1} (K - n) h_{j+n} h_{j-n} + \frac{\sigma^2}{KS}. \tag{20}$$

To get some sort of feel for the values involved, let's assume that $h_j = 0.5\Delta$ or -0.5Δ with equal probability when $|j| \leqq N$ and is zero otherwise. Then the expected value of σ_j^2 becomes

$$\overline{\sigma_j^2} = \frac{1}{K}\left(\frac{N\Delta^2}{2} + \frac{\sigma^2}{S}\right). \tag{21}$$

For p_c to be 0.99 when $h_j = 0.5\Delta$, we require that $0.5\Delta = 2.33\sigma_j$, so that for the zero noise condition

$$K \approx 10.8N. \tag{22}$$

For example, a 13-tap equalizer in the absence of noise requires about 140 symbols (0.058 sec. at a baud rate of 2400) to make a sufficiently accurate estimate. However, for a typical phone line application we require extremely accurate equalization so that Δ might typically be on the order of 0.0025 while the signal-to-noise ratio (S/σ^2) is about 30 db. In this case, the noise term in (21) completely swamps the "intersymbol clutter" term $N\Delta^2/2$ and the 0.99 accuracy at 0.5Δ condition means that

$$K \approx \frac{21.7\sigma^2}{\Delta^2 S}. \tag{23}$$

For this example, 3470 samples (1.45 sec) are required per step of equalization.

3.3 *Performance of the Adaptive Equalizer Under Adverse Conditions*

Now that the equalization system configuration has been established under the assumption of ideal conditions, it becomes necessary to judge deterioration when these conditions are not met in practice. Throughout this paper, the specific application is assumed to be high-speed, voice telephone channel, data transmission.

Assumption (*i*) regarding Gaussian noise is usually justified in practice (the impulsive noise is not a determining factor for high-speed transmission) and in any event is not a very crucial assumption. Assumption (*iii*) that the received symbols are detected correctly is generally amply satisfied during normal data transmission with error rates of 0.01 or less. Remember that the information from a thousand or more symbols may be averaged to make one decision regarding equalization. Thus, we only require that a large majority of decisions are correct. Obviously the system performance deteriorates and finally "breaks" as the error rate becomes higher and higher, but the analytical evaluation of the effect appears difficult. Experimental results will be mentioned in a later section of this paper, but it should be said here that the system will work well with error rates on the order of 0.1.

Once the equalizer begins to work, the error rate quickly drops back to a more normal value.

The time variation rate of the channel must be matched with the accuracy requirement to arrive at an averaging time KT seconds during which the channel does not vary greatly compared to the size step Δ being taken on the equalizer taps. In the phone line application, this appears simple even for very high accuracies since the transmission characteristics are not usually observed to continually change at any great rate.

The most troublesome of the four ideal assumptions is the one involving uncorrelated input symbols. During normal data transmission one would expect that the sequence $\{a_n\}$ would appear random over an interval of a thousand symbols, but unfortunately this is frequently not the case. Long steady sequences of ones or of zeros may be used to hold the line, or the dotting pattern of alternate ones and zeros may be employed for some such purpose. In any event where a short, repetitive pattern is transmitted the spectrum of the transmitted signal consists of a number of discrete lines. Obviously it is impossible at the receiver to extract any information about the channel's transmission characteristics except at a few discrete points. Any adaptive equalizer must be prepared to weather this period and await new random data upon which meaningful decisions can be made. Fortunately, it can be shown that the adaptive equalizer of Fig. 3 acts on whatever information is available in the received data and retains its settings through periods of bad sequences.

We return now to (17) and remove the assumption concerning uncorrelated data. Then

$$\bar{h}_j = \sum_{n=-\infty}^{\infty} h_n r_{j-n} \tag{24}$$

where r_j is the normalized autocorrelation function of the input data sequence (which is assumed to be stationary).

$$r_j = \lim_{K\to\infty} \frac{1}{2KS} \sum_{k=-K}^{K} a_k a_{k-j} \,. \tag{25}$$

For ideal equalization, we require that the action of the equalizer cause $h_n = 0$ for $|n| \leqq N$. The adaptive equalizer tries to accomplish this by forcing $\bar{h}_j = 0$ for $|j| \leqq N$. As can be seen from (24), a slight error may be introduced if $r_j \neq 0$ for $j \neq 0$. The only error involved, however, is to cause some influence of the samples h_n for $|n| > N$ on the tap settings. Generally, these samples which are outside the range of equalization are quite small (otherwise the number of equalizer taps needs to be increased) and their effect is only multiplied by the tails of the input autocorrelation function. Suppose, for example, that $h_n = 0$ for $|n| > N$. Then after equalization

$$\bar{\hat{h}}_j = \sum_{n=-N}^{N} h_n r_{j-n} = 0 \quad \text{for} \quad |j| \leqq N \tag{26}$$

and the only solution to this set of $(2N + 1)$ equations is the perfect state $h_n = 0$ for $|n| \leqq N$ provided the matrix

$$R = \begin{matrix} r_0 & r_1 \cdots r_{2N} \\ r_{-1} & r_0 & \\ \vdots & & \\ r_{-2N} & & r_0 \end{matrix} \tag{27}$$

is nonsingular.

If the data sequence consists of a repetitive pattern of period L symbols, then $r_j = r_{j+L}$ and the rank of R cannot be greater than L. Thus, if the period of pattern repetition L is less than the number of taps on the equalizer $(2N + 1)$, then the equalizer does not reach optimum settings. Consider then what settings the equalizer does reach. In view of $r_j = r_{j+L}$ we can rewrite (24) in the form

$$\bar{\hat{h}}_j = \sum_{n=0}^{L-1} r_n \left\{ \sum_{m=-\infty}^{\infty} h_{j-n-mL} \right\} \tag{28}$$

and

$$\bar{\hat{h}}_j = \bar{\hat{h}}_{j+L} . \tag{29}$$

Thus, only L taps of the equalizer represent independent feedback loops, while the other $(2N + 1 - L)$ tap gains are slaves to the L gains considered independent. In fact, they receive the identical error signals and are thus incremented identical amounts. The equalizer solves the L simultaneous equations to arrive at

$$\sum_{m=-\infty}^{\infty} h_{j-mL} = 0 \tag{30}$$

for $j = 0, \cdots, L - 1$.

Actually, this solution minimizes the data distortion for the particular sequence being transmitted. The received samples y_k may be written in terms of the equalization error samples h_n using (5) and (12).

$$y_k = \sum_{n=-\infty}^{\infty} a_n h_{k-n} + a_k + \eta_k . \tag{31}$$

But, since $a_k = a_{k+L}$, we have

$$y_k = \sum_{n-0}^{L-1} a_n \left\{ \sum_{m=-\infty}^{\infty} h_{k-n-mL} \right\} + a_k + \eta_k \tag{32}$$

and we see that an equivalent channel response could be defined as $\{g_n\}$ where

$$\begin{aligned} g_n &= \sum_{m=-\infty}^{\infty} h_{n-mL} \qquad 0 \leqq n \leqq L - 1 \\ g_n &= 0 \qquad\qquad\qquad \text{elsewhere.} \end{aligned} \tag{33}$$

The distortion for the equivalent channel is minimized by zeroing the samples g_n within the range of the equalizer, but this is precisely what (30) indicates is done.

If the equalizer is at a nominally perfect setting ($h_n = 0; | n | \leqq N$) when the repetitive sequence is begun, then the equalizer holds its settings over the time of periodic transmission. There is no possibility of a drift in settings since taps c_k and c_{k+L} are "locked" together and yet must maintain the solutions $g_n = 0$. There are L free taps and L independent equations for which the taps are initially at a solution. Thus, the equalizer can hold its settings over an indefinite time while periodic sequences are transmitted so long as the channel characteristics remain fixed.

If the channel response changes while periodic sequences are being transmitted then the equalizer will move to a new solution of (30) to minimize the distortion for the particular periodic sequence. However, this solution will not infer that $h_n = 0$ for $| n | \leqq N$ and the equalization will not be perfect when random data starts again.

The action of the equalizer for unfavorable data sequences can be summarized by saying that the system always attempts to do the best equalization possible for the data being transmitted. In no case is the equalization deteriorated because of the adaptive loop over the performance of a fixed equalizer.

IV. AN IMPROVED DIGITALIZED ADAPTIVE EQUALIZER

4.1 *Description*

There is a great premium attached to the use of digital circuits where possible for reasons of equipment cost and size. It is possible to considerably simplify the implementation of the adaptive equalizer of Fig. 3 by discarding linear concepts and using only polarity information throughout. Thus, instead of correlating the error signal e_k with the detected symbols a_{k-j} we add mod 2 the binary symbols corresponding to the polarities of e_k and a_{k-j}. The resulting simplification can be seen in Fig. 4. The symbol polarity sgn a_{k-j} is obtained by passing the most significant digit of the detected symbol (in binary format) through a shift register. In the Gray code commonly used for binary-to-multilevel conversion the first bit indicates polarity of the symbol.

The polarity of error sgn e_k can be produced by the simple expedient of adding an additional stage of slicing to the $\log_2 M$ slicers required for M-level transmission. Each stage of the $\log_2 M$ slicers "folds" the signal value about the last threshold, so that an extra stage simply produces automatically the polarity of the error sgn e_k. Fig. 4 illustrates a 16-level transmission system. Five stages of slicing are employed. The first four stages deliver the four detected bits indicating the received symbol a_k while the first and fifth bits are used for equalization purposes.

After passing the error polarity through an N-stage shift register

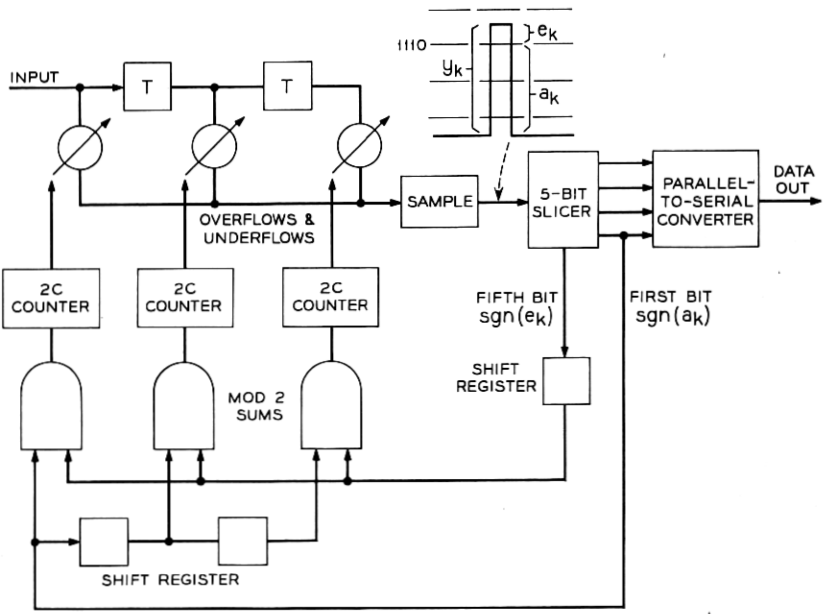

Fig. 4 — 3-tap digital adaptive equalizer.

(in order to be able to correlate with N future and N past symbol polarities), the error polarities and symbol polarities are summed using exclusive-OR circuits. At this point we are able to easily introduce an improvement over the low-pass filter and periodic slicer used in the equalizer of Fig. 3. The problem with periodic slicing is that a compromise time interval KT must be chosen for averaging which is based on the most critical situation of near perfect equalization. The equalizer moves just as slowly when equalization is poor and the correlator output samples need much less averaging for a given accuracy.

A sequential testing procedure is clearly called for in this application. In a sequential test, the interval between decisions is determined by the input data itself. Instead of averaging data over a KT second interval and then sampling to determine polarity, running sums of the exclusive-OR outputs are kept. Positive and negative thresholds are set and the tests are terminated whenever these thresholds are crossed. This procedure is most easily implemented in digital form using up-down binary counters whose capacities of $2C$ counts determine the decision threshold value. Whenever a one is emitted from the exclusive-OR the counter advances one count, while a zero retards the counter one count. When the counter overflows we decide the polarity of h_k is positive and reduce the gain of on tap c_k. The counter is then reset to the center position of C counts. Similarly, an underflow adjusts c_k one step higher and resets the counters. The $2C$ storage counters are of course tied directly to the up-down counters which control the tap gain to accomplish this task in a most simple manner.

Thus, the equalizer of Fig. 4 is surprisingly simple. It requires only an N-stage shift register, a slicer, and $(2N + 1)$ binary counters of capacity $2C$ in order to convert a preset equalizer to the adaptive mode. Since the storage counters are used for averaging during stepup for the preset equalizer we finally arrive at an adaptive equalizer which costs almost nothing more than a preset equalizer.

The question arises as to why the preset mode (test pulses before transmission) is needed at all. In many cases it may not be needed provided a period of initial equalization is alloted during which data is transmitted, but not used due to its unreliability. As we shall find, the adaptive equalizer can accomplish a given degree of accuracy in equalization in less time than a preset equalizer providing the error rate is not too high. However, during initial setup the error rate is generally so high that the adaptive equalizer operates very slowly or not at all. Thus, a short period of test pulses can be profitably used to bring the error rate down to manageable values before adaptive equalization is begun.

4.2 *Analytical Evaluation*

In this section we will evaluate the probability of correct adjustment of tap c_j and the average time required for an adjustment. The probability of correct adjustment p_c depends on the size $(2C)$ of the storage counters and on the probability of an up-count p, and of a down-count q, on the jth storage counter.

The kth count of the jth counter is obtained by multiplying the polarities of e_k and a_{k-j}. Let us assume for convenience that h_j is positive (for negative h_j the situation is, of course, entirely similar). The probability of a correct adjustment is then the probability of an overflow occurring before an underflow. The probability of an up-count is

$$p = P(e_k > 0, a_{k-j} > 0) + P(e_k < 0, a_{k-j} < 0). \tag{34}$$

These probabilities are identical so we use

$$p = 2P(e_k > 0, a_{k-j} > 0). \tag{35}$$

Equation (35) can be rewritten in terms of the conditional probability

$$p = 2P(e_k > 0 \mid a_{k-j} > 0)P(a_{k-j} > 0). \tag{36}$$

The symbols a_j will be taken as independent and equally likely to assume any of the M values. Take $2d$ as the distance between adjacent levels, so that d is the distance from any level to the nearest slicing (decision) threshold. The amplitudes a_j can assume are then $d(2i - 1)$ for $i = -M/2 + 1, \cdots, M/2$. Since a_{k-j} can be positive or negative with equal likelihood, (36) becomes

$$p = P(e_k > 0 \mid a_{k-j} > 0) \tag{37}$$

$$p = \frac{2}{M} \sum_{i=1}^{M/2} P[e_k > 0 \mid a_{k-j} = d(2i - 1)]. \tag{38}$$

Now we need to evaluate the conditional probabilities in (38). With equalization error samples h_n at times nT and noise samples η_n, the received voltage at time kT is

$$y_k = \sum_n a_n h_{k-n} + a_k + \eta_k . \tag{39}$$

The error voltage e_k is

$$e_k = \sum_{n=-\infty}^{\infty} a_n h_{k-n} + \eta_k . \tag{40}$$

We remove the term involving a_{k-j} in (40) to obtain

$$e_k = a_{k-j} h_j + [\sum_{n \neq k-j} a_n h_{k-n} + \eta_k]. \tag{41}$$

The assumption is made that the sum of the intersymbol interference and noise (the terms in brackets) is Gaussian distributed, with mean zero and variance σ^2. The error e_k is then Gaussian with mean $a_{k-j}h_j$, and variance σ^2.

The probability density of e_k is sketched in Fig. 5. The conditional probabilities in (38) should be interpreted as the probability that, given given a_{k-j} was at level i, we *decide* that e_k is positive. If e_k crosses the decision threshold on the right-hand side of Fig. 5, it will appear to the receiver that e_k was negative since a_k will be incorrectly received as the next higher symbol. If e_k crosses the decision threshold on the left, e_k is interpreted as being positive. Thus, the conditional probability may be written (refer to Fig. 5)

$$P(e_k > 0 \mid a_{k-j}) = \tfrac{1}{2} + p_1 - (p_2 - p_3) \tag{42}$$

where

$$p_1 = \frac{1}{\sqrt{2\pi}\sigma} \int_{-a_{k-j}h_j}^{0} \exp\left(-\frac{x^2}{2\sigma^2}\right) dx \tag{43}$$

$$(p_2 - p_3) = \frac{1}{\sqrt{2\pi}\sigma} \int_{d-a_{k-j}h_j}^{d+a_{k-j}h_j} \exp\left(-\frac{x^2}{2\sigma^2}\right) dx. \tag{44}$$

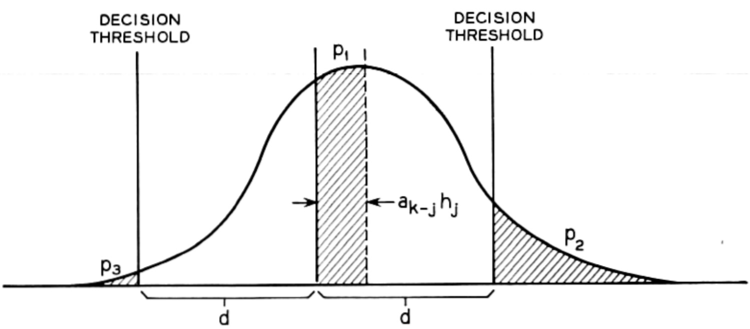

Fig. 5 — The probability density $p(e_k \mid a_{k-j})$.

With the equalization even fairly good (as it must be if meaningful data are being transmitted), $a_{k-j}h_j$ must be a small number so that the ranges of integration in (43) and (44) are small. Thus, we make the approximations

$$p_1 \cong \frac{a_{k-j}h_j}{\sqrt{2\pi}\sigma} \tag{45}$$

$$(p_2 - p_3) \cong \frac{2a_{k-j}h_j \exp\left(-\frac{d^2}{2\sigma^2}\right)}{\sqrt{2\pi}\sigma}. \tag{46}$$

Using these approximations we can write

$$P[e_k > 0 \mid a_{k-j} = d(2i-1)] = \tfrac{1}{2} + \frac{d(2i-1)h_j}{\sqrt{2\pi}\sigma}\left[1 - 2\exp\left(-\frac{d^2}{2\sigma^2}\right)\right]. \tag{47}$$

This expression is substituted into (38) and the summation is easily performed over i to give

$$p = \tfrac{1}{2} + \left[1 - 2\exp\left(-\frac{d^2}{2\sigma^2}\right)\right]\frac{h_j dM}{2\sigma\sqrt{2\pi}}. \tag{48}$$

For normal operation the exponential term in the brackets is small in comparison with unity and may be neglected. This term gives the contribution due to slightly more errors being made in the right-hand region (p_2) than in the left-hand region (p_3). An examination of the relative probabilities p and $(p_2 - p_3)$ without the approximations (45) and (46) shows that $(p_2 - p_3)$ is generally small in comparison with p, even for error rates if 0.01 and higher. Thus, the system is able to estimate the polarity of h_j even when the eye is completely closed.

The final approximation for the probability p of a correct step in the jth counter becomes

$$p = \tfrac{1}{2} + \frac{h_j dM}{2\sigma\sqrt{2\pi}}. \tag{49}$$

Generally, this probability is only a little larger than 0.5, hence many counts must be averaged before a decision can be made as to whether $p > \frac{1}{2}$, in which case $h_j > 0$, or whether $p < \frac{1}{2}$ and consequently $h_j < 0$. This averaging is best done using the $2C$-count storage devices, since these devices effect a sequential test of the two hypotheses $p > \frac{1}{2}$ and $p < \frac{1}{2}$. This sequential test will require less time on the average for a given accuracy than straightforward averaging with a fixed sample size as in the adaptive equalizer of Fig. 3. In addition, it is more easily implemented than the former technique.

The probability p_c of an overflow before an underflow when the probability of an up-count is p and the probability of a down-count is $q = 1 - p$ is taken from Feller's analysis of the problem of the gambler's ruin.[5] For a $2C$ counter initially set to its midpoint of C, Feller gives

$$p_c = 1 - \frac{(q/p)^{2C} - (q/p)^C}{(q/p)^{2C} - 1}. \tag{50}$$

If p_c is to be close to unity, $(q/p)^{2C}$ must be small compared with $(q/p)^C$, so we approximate (50) as

$$p_c \approx 1 - (q/p)^C. \tag{51}$$

We can further simplify this expression since q and p are both very close to 0.5. Writing

$$\left.\begin{aligned} p &= 0.5 + \varepsilon \\ q &= 0.5 - \varepsilon \end{aligned}\right\} \tag{52}$$

we obtain

$$(q/p) \approx 1 - 4\varepsilon \tag{53}$$

and we use

$$(1 - 4\varepsilon)^C = \exp\,[C \log\,(1 - 4\varepsilon)] \tag{54}$$

to obtain the approximation

$$p_c = 1 - \exp\,(-4C\varepsilon). \tag{55}$$

Finally, we substitute the value of ε from (49).

$$p_c = 1 - \exp\left(-\frac{2Ch_j dM}{\sigma\sqrt{2\pi}}\right). \tag{56}$$

The average number of counts required for an overflow (or underflow) is also given by Feller.

$$\bar{n} = \frac{C}{(q-p)} - \frac{2C}{(q-p)}\left[\frac{1 - (q/p)^C}{1 - (q/p)^{2C}}\right]. \tag{57}$$

Using the same approximations in (57) as we used in (51) results in the approximation

$$\bar{n} \cong \frac{C\sigma\sqrt{2\pi}}{dMh_j}. \tag{58}$$

In order to be able to compare this system with the previous adaptive equalization system which uses linear techniques and averages over a fixed interval of K symbols, we need to find p_c in terms of h_j, $\bar{n}$ and S/σ^2. The signal power S for an M-level system with separation $2d$ is

$$S = \frac{2}{M}\sum_{i=1}^{M/2} d^2(2i - 1)^2 = \frac{d^2}{3}(M^2 - 1). \tag{59}$$

Now combining (59), (58), and (56) we arrive at (for $M \gg 1$)

$$p_c = 1 - \exp\left(-\frac{3}{\pi}\,\bar{n}h_j^2\,\frac{S}{\sigma^2}\right) \quad \text{(digitalized)} \tag{60}$$

whereas an equivalent approximation for the previous equalizer is

$$p_c = 1 - \frac{\sigma}{h_j\sqrt{2\pi KS}}\exp\left(-\tfrac{1}{2}Kh_j^2\,\frac{S}{\sigma^2}\right). \tag{61}$$

As expected, this comparison shows the previous system requires about twice as much time to achieve a given degree of accuracy as does the digitalized system. For the specific example used earlier 3090 symbols (1.29 seconds) are required to achieve an accuracy $p_c = 0.99$ when $h_j = 0.5\Delta$, $\Delta = 0.0025$, and $S/\sigma^2 = 1000$.

This does not mean that the digitalized equalizer operates twice as fast as the previous equalizer. Actually, it operates much faster than that. The average time of 3090 symbols is required only when $h_j = 0.5\Delta$ and since this represents perfect equalization we don't care how long the equalizer takes to move to the state -0.5Δ. When the gain c_j is out of equalization by a single step h_j is approximately 1.5Δ and according to (58) the time $\bar{n}$ required is only a third as much — or 1030 symbols. Similarly, when c_j is 2 steps out $\bar{n}$ becomes 618 symbols. If the equalizer is turned on when equalization is relatively poor the steps are taken in nearly the minimum time of C symbols. The counter capacity may be calculated from the accuracy requirement using (56). For our example we obtain $C = 85$. (Of course, either a 7-stage ($2C = 128$) or an 8-stage ($2C = 256$) would have to be used in practice.)

The longest average time before an equalizer change is required when $h_j = 0$ and, consequently, $p_c = 0.5$. Here a long average time is desirable since disturbing the equalizer is detrimental. The time in such case is C^2 symbols — in our example about 7200 symbols. Thus, the average time between equalizer adjustments varies between C (85) and C^2 (7200) symbols with short times used when urgency of movement is greatest and longer times used when leisurely adjustment is possible and, in fact, necessary because of stringent accuracy requirements.

V. ADAPTIVE EQUALIZATION USING ERROR CONTROL INFORMATION

5.1 *Description*

In many applications for adaptive equalization a forward-acting error correction system will be associated with the data transmission system. When the objective of system design is high-speed transmission, the modem is generally operated at an unacceptably high error rate. A detection-retransmission system cannot be solely relied upon, since the high error rate would necessitate constant requests for retransmission.

In the exploratory VSB system described in Refs. 2, 3, and 4, a (200, 175) Bose-Chaudhuri code with a minimum distance of 8 was used for triple error correction. In the event of a detectable error pattern containing more than three errors, a retransmission request was made. Using triple error correction with a modem error rate of roughly 2×10^{-3}, the frequency of requests for retransmission was low enough to not have appreciably affected the throughput of the system.

When the equalization is imperfect the error rate is naturally increased, but moreover the data system becomes pattern sensitive. Some patterns of input data are more likely to result in errors than other patterns because of the memory of the system. Given that an error has occurred, it is quite likely that such a bad pattern was transmitted. Since the bad patterns are simply related to the system impulse response we have the interesting possibility of using the information available in the error correction system for the purpose of adjusting the equalizer.

Briefly, a scheme based on this principle works as follows. Whenever an error is corrected by the error control unit, the direction of the error and the polarities of the surrounding symbols are observed. By "direction" of the error is meant whether a symbol has been changed to a higher amplitude level (+) or to a lower level (−). If the direction of the error is positive the polarities of the surrounding symbols are taken directly to the equalizer. If the direction of the error is negative all symbol polarities are inverted. These polarities are used to either advance or retard counters attached to the variable attenuators of the equalizer. The attenuators are incremented positively or negatively whenever the corresponding counters underflow or overflow. A block diagram of this system is shown in Fig. 6.

For a specific example, suppose that 16-level transmission is used. The incoming binary data train is converted 4 bits at a time into the 16 symbols using a Gray code. Suppose that at the receiver the following sequence is received

$$0011 \quad 1011 \quad 1\text{X}10 \quad 0110 \quad 0001.$$

The error control locates and corrects an error in the bit marked with an X. As part of the error correction procedure the entire 200-bit word has been stored, so the polarities of the symbols surrounding the error are readily available. In fact, with a Gray code the polarity of the amplitude level corresponding to a given 4-bit symbol is determined by the first bit. Therefore, the polarities of the two symbols either side of the error are, in order,

$$- + \text{(error)} - -.$$

The symbol which was in error was 1010 changed to 1110. From a table of the Gray code we can determine that this error carried amplitude level 13 into amplitude level 12. Thus, the direction of the error is nega-

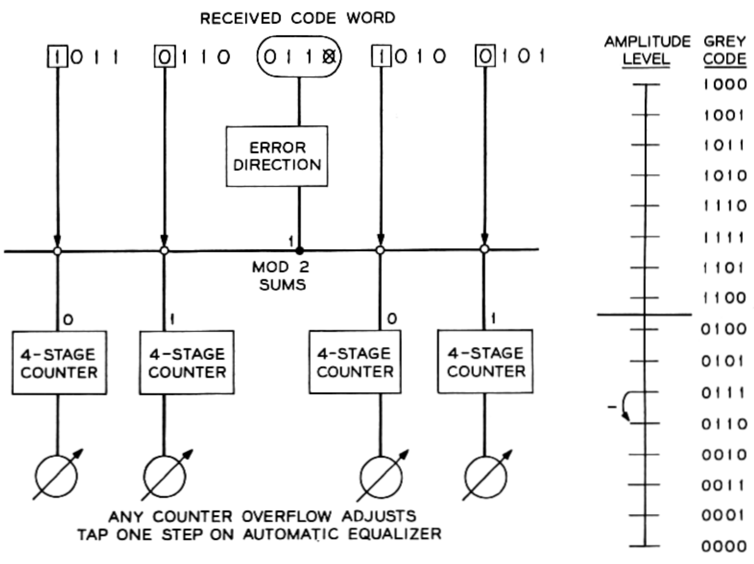

Fig. 6 — Example error control — adaptive equalizer coordination ($2C = 16$).

tive and the symbol polarities are reversed and used to increment counters.

Notice that in this system the storage counters are only incremented when errors occur. If no errors are being made, then the equalizer settings are not changed. Therefore, the counters are changed much less frequently in this system than in the previous adaptive equalizer. However, when the counters are changed, we shall find that the changes are more reliable and that smaller counters may be used to effect a comparably reliable statistical test.

5.2 *Analytical Evaluation*

Again we are going to evaluate the probability p of counting in the correct direction on the storage counter attached to attenuator c_j. We suppose that h_j is positive and with a spacing $2d$ between levels we have

$$p = P(a_{k-j} > 0 \mid e_k > d) + P(a_{k-j} < 0 \mid e_k < -d) \tag{62}$$

$$p = 2P(a_{k-j} > 0 \mid e_k > d). \tag{63}$$

With the M possible symbols equally likely we can write (63) in the form

$$p = \frac{\sum_{i=1}^{m/2} P[e_k > d \mid a_{k-j} = d(2i-1)]}{MP(e_k > d)}. \tag{64}$$

As in Section 4.2, we write the error voltage e_k

$$e_k = a_{k-j}h_j + [\sum_{n \neq k-j} a_n h_{k-n} + \eta_k] \tag{65}$$

and assume the bracketed term is Gaussian, mean zero, variance σ^2.

Thus,

$$P[e_k > d \mid a_{k-j} = d(2i-1)] = \frac{1}{\sqrt{2\pi}\sigma} \int_{d-d(2i-1)h_j}^{\infty} \exp\left(-\frac{x^2}{2\sigma^2}\right) dx \tag{66}$$

and for reasonably small error rates we make the approximation

$$P[e_k > d \mid a_{k-j} = d(2i-1)] \cong \frac{\sigma}{\sqrt{2\pi}\, d[1-(2i-1)h_j]} \exp\left(-\frac{d^2[1-(2i-1)h_j]^2}{2\sigma^2}\right). \tag{67}$$

If we also assume that when a_{k-j} is a random variable, e_k is Gaussian distributed and make the same approximation involved in (67), we arrive at

$$P(e_k > d) \cong \frac{\sqrt{\sigma^2 + Sh_j}}{\sqrt{2\pi}\, d} \exp\left(-\frac{d^2}{2(\sigma^2 + Sh_j)}\right). \tag{68}$$

Presumably $\sigma^2 \gg Sh_j$ when the system is near perfect equalization, so we drop the Sh_j terms. Equations (68) and (67) are then inserted into (64) to get

$$p = \frac{1}{M} \sum_{i=1}^{M/2} \frac{1}{[1 - (2i - 1)h_j]} \exp\left(-\frac{d^2[1 - (2i - 1)h_j]^2 + d^2}{2\sigma^2}\right). \tag{69}$$

Our aim in evaluating (69) is an approximation which is accurate to terms linear in h_j, a small number. The denominator in (69) does not contribute terms on this order, so we are able to sum the geometric series giving

$$p = \frac{1}{M} \exp\left(\frac{d^2 h_j}{\sigma^2}\right) \left[\frac{1 - \exp\left(\frac{Md^2 h_j}{\sigma^2}\right)}{1 - \exp\left(\frac{2d^2 h_j}{\sigma^2}\right)}\right]. \tag{70}$$

Finally, we retain only terms linear in h_j to obtain the result

$$p = \tfrac{1}{2} + \frac{Md^2 h_j}{4\sigma^2}. \tag{71}$$

Equation (71) is similar in form to (49) for the probability of a correct count in the digitalized equalizer. The principal difference is that (71) involves the threshold-to-noise ratio squared (d^2/σ^2) whereas (49) uses the ratio (d/σ). Thus, p in (71) is considerably more reliable than the probability for a correct count in the digitalized adaptive equalizer. However, the counts in the error control system occur at a much slower rate, namely $2P(e_k > d)$.

A counter of capacity $2C$ is used to store the counts from the error correction circuitry. The equations of Feller may again be used with suitable approximations to find the probability p_c of a correct equalizer adjustment and the average number of counts $\bar{n}$ required before a correction.

$$p_c = 1 - \exp\left(-\frac{Ch_j d^2 M}{\sigma^2}\right) \tag{72}$$

$$\bar{n} = \frac{2C\sigma^2}{Md^2 h_j}. \tag{73}$$

In order to get the average number of symbols required before an equalizer adjustment, we must multiply $\bar{n}$ by the average number of symbols per error, which is approximately [from (68)]

$$n_0 = \frac{\sqrt{2\pi}\, d}{2\sigma e^{-d^2/2\sigma^2}}. \tag{74}$$

The comparison of this system with the previous system becomes quite complicated because of the dependence of the error control system upon the number of levels M and on the error rate of the system (for which our approximation is only valid when equalization is exact). In general, it seems that for a given accuracy of equalization, the previous digitalized equalizer will require less time per adjustment. To follow through with our example we assume $h_j = 0.5\Delta$, $\Delta = 0.0025$, $S/\sigma^2 = 10^3$, and now $M = 16$. Since

$$S/\sigma^2 = \frac{d^2}{3\sigma^2}(M^2 - 1) \tag{75}$$

the threshold-to-noise ratio is

$$d^2/\sigma^2 = 11.76. \tag{76}$$

The probability of error for the system is about 6.5×10^{-4} and $n_0 = 1540$ symbols per error. From (72), we find that a counter capacity $2C = 40$ is required to ensure $p_c = 0.99$. This may be compared with $2C = 170$ for the previous system. The number of counts per adjustment $\bar{n} = 170$, but $n_0\bar{n} = 2.62 \times 10^5$ symbols per adjustment.

This comparison is somewhat unfair to the error control system since it must be pointed out that the speed of movement at $h_j = 0.5\Delta$ is immaterial. This condition merely determines the counter sizes necessary to meet accuracy requirements. We are much more concerned with the equalizer response when the equalization is imperfect. In this case, not only is $\bar{n}$ inversely proportional to h_j, but the error rate also increases so that counts are made more frequently. Even if one is willing to stretch the point quite a bit, it does seem that the error control equalizer coordination is unattractive in comparison with the previous adaptive system. Nevertheless the system implementation is quite simple and the concept sufficiently intriguing that perhaps a use can be found for such a coordination.

VI. EXPERIMENTAL RESULTS

Three systems for adaptive equalization of digital data systems have been described and analyzed. One of these three systems, the digitalized adaptive equalizer described in Section IV, appears to be much more attractive than the other two, both from the standpoint of instrumentation ease and of performance. Therefore, although the error control coordinated system has also been constructed, only the digitalized adaptive system has been subjected to extensive testing.

The system constructed used a 13-tap delay line and a tap increment Δ of 0.0025, although this increment is tapered to considerably smaller values near the outside taps of the delay line. In line with the example values computed in Section IV, eight stages were used in the storage counters, resulting in a capacity of $2C = 256$ counts for each tap.

The system was tested in conjunction with the 9600 bit-per-second, 16-level VSB system described by F. K. Becker in Ref. 2. A good pictorial demonstration of the adaptive equalizer is shown in the sequence of photos in Fig. 7 where the VSB system is transmitting at 4800 bits-per-second, using only four levels. The reduced speed here is to enable us to easily discern the distinct levels in the eye pictures of Fig. 7. The first photo shows the normal, equalized eye pattern. The intensified dots on this picture indicate the position of the sampling time which in this first photo has been artificially moved to the right, completely out of the eye opening. This is equivalent to the sudden introduction of a constant delay into the transmission channel. It can be observed in this first photo that the error rate would be relatively high due to the mistiming.

In the subsequent pictures the timing position has been left fixed while the adaptive equalizer changes its settings to move the entire eye pattern to the right effecting a reequalization of the system. At first the pattern moves quite rapidly since decisions are made quickly by the testing counters. As the eye approaches the timing position again decisions are made at a slower rate and the movement slows. The entire process takes only about a few seconds in spite of the quite abnormally large disturbance of the transmission characteristic.

Fig. 8 shows a sequence of photographs of a 16-level folded eye picture following a sudden change in transmission characteristics. If the 16-level eye were shown in the same format as the 4-level eyes of Fig. 4, the 15 "holes" in the eye diagram would be too small to distinguish. Therefore, all 15 "holes" have been superimposed by folding the 16-level eye diagram over and over until the picture resembles a binary eye diagram. In the first second of operation, from 0 to 1 second, the eye is recaptured by the adaptive equalizer.

At the positions shown in the first photos of Figs. 7 and 8 the equalizer

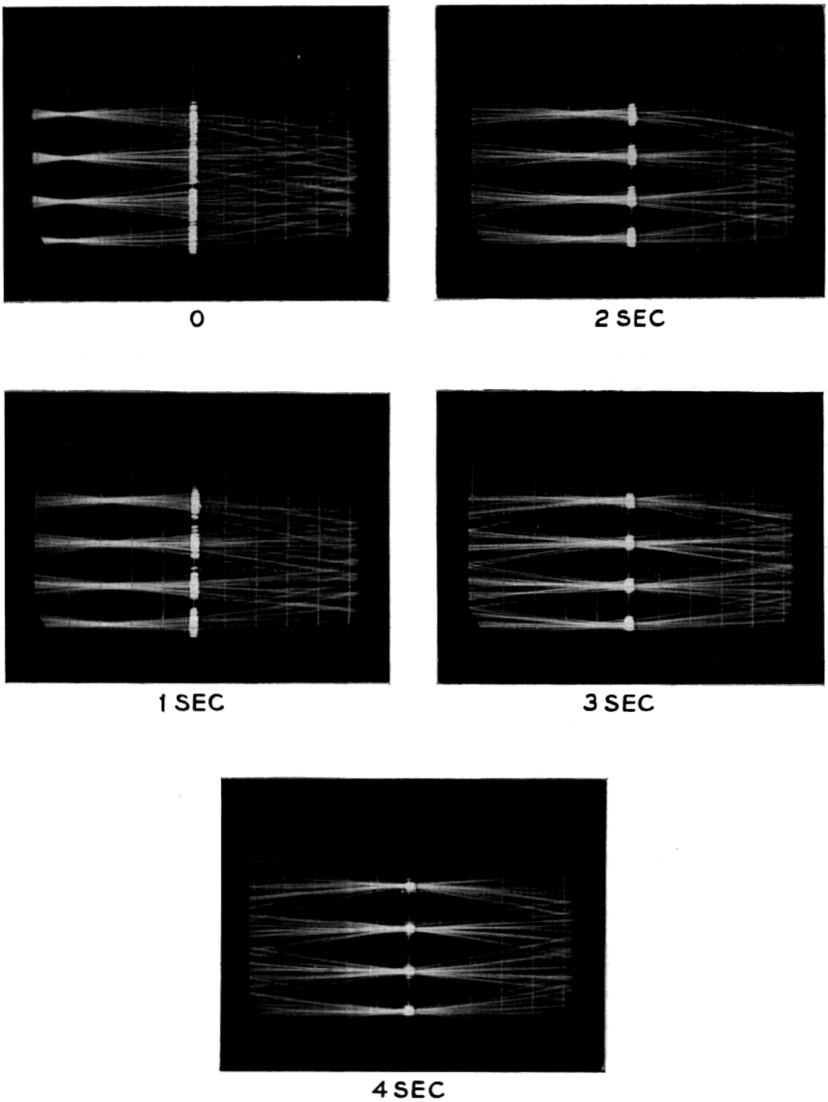

Fig. 7 — Four-level eye picture after timing is abruptly displaced.

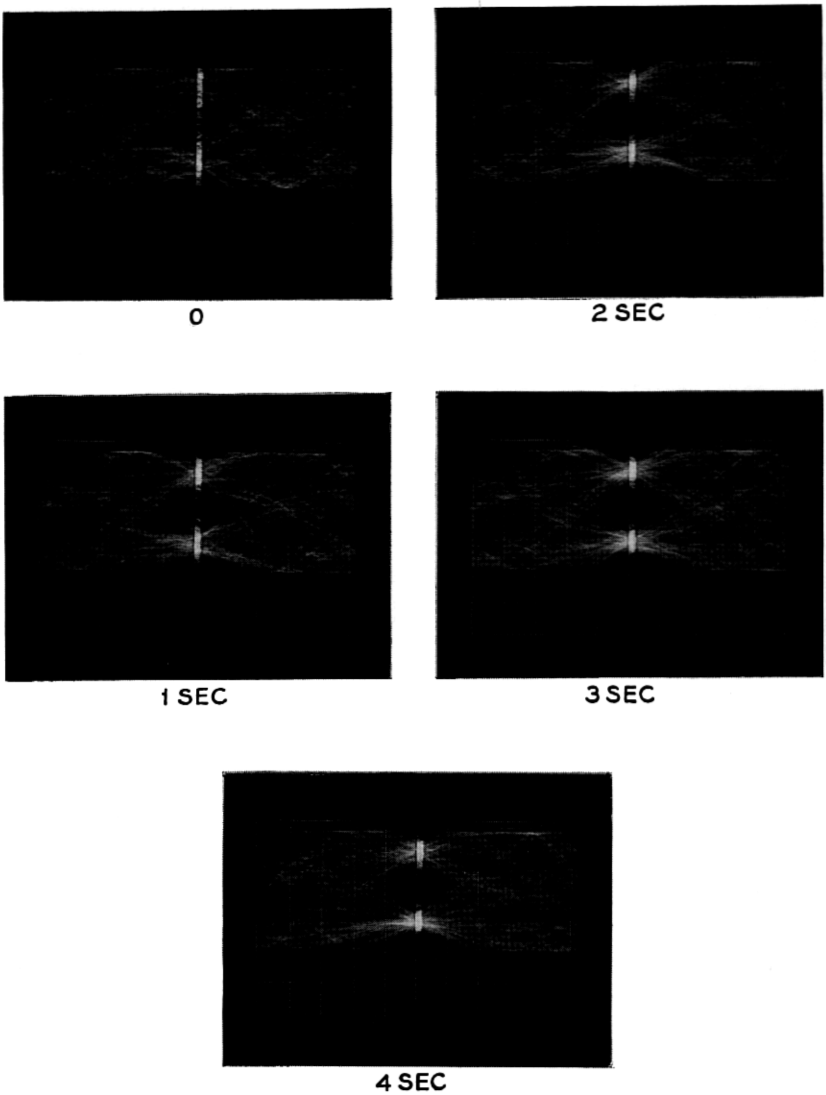

Fig. 8 — 16-level, folded eye picture after abrupt change in transmission characteristics.

acts quickly and decisively. It is possible to disturb the channel so badly that no semblance of an eye opening is left and the error probability is nearly 0.5. The mathematics of the equalizer operation here are quite complex and give little insight toward performance evaluation. It has been observed experimentally that for binary (2-level) operation it has been impossible to find a setting or disturbance from which the equalizer will not eventually converge. Sometimes the eventuality takes as long as a minute as the equalizer makes slow (on the order of C^2) decisions — a sure indication of inaccuracy. After a period of what seems to be random hunting the equalizer reaches a position which it suddenly recognizes. The decisions come very quickly (on the order of C) and the eye appears seemingly from nowhere. Finally, perfect equalization is approached and the decisions once more come very slowly.

For higher level systems, 8- and 16-level, positions can be found from which the equalizer will not in all probability converge in a time that one is willing to wait around and watch. There seem to exist certain stable states where, for example, a 16-level eye pattern exists where 8-level transmission is being used — each level being split into two balanced levels. It should be emphasized that these conditions never occur during normal data transmission when the equalizer has an open eye to begin with and only has to track this eye through changing transmission characteristics. They are relevant, however, for the acquisition period if the equalizer is turned on without any setup period. Such procedure is not recommended for the higher level systems. (It will be obvious to the reader that it is possible to send a pattern of "outside" levels — i.e., a 2-level signal to start a higher level transmission. Also, it is possible to use quasi-random pattern generators, say maximal-length shift registers, at both transmitter and receiver to start transmission without worrying about transmission errors.)

A series of error performance runs was made on two test facilities — one looped via K-carrier to Boston from Holmdel, New Jersey and the other looped to Chicago via LMX-1 carrier from Holmdel. A number of 2 minute runs were made at 9600 bits-per-second at various times of day on each facility. In every case a control run of 2 minutes (1.152×10^6 bits) was made using preset equalization as described in Ref. 3. The results of these runs are plotted in Fig. 9 with preset error rate as the abscissa and adaptive error rate as the ordinate. In all cases, the error rate was diminished by the use of adaptive equalization. Even in the worst case the improvement factor was over three, while the best case represented an improvement factor of 50 and the average factor was approximately 10.

The importance of this improvement factor in the performance of this system cannot be overemphasized. While the order-of-magnitude improvement may not seem significant, it must be pointed out that this is raw error rate previous to error correction. If the curves relating customer error rate to raw error rate for the error control system used in

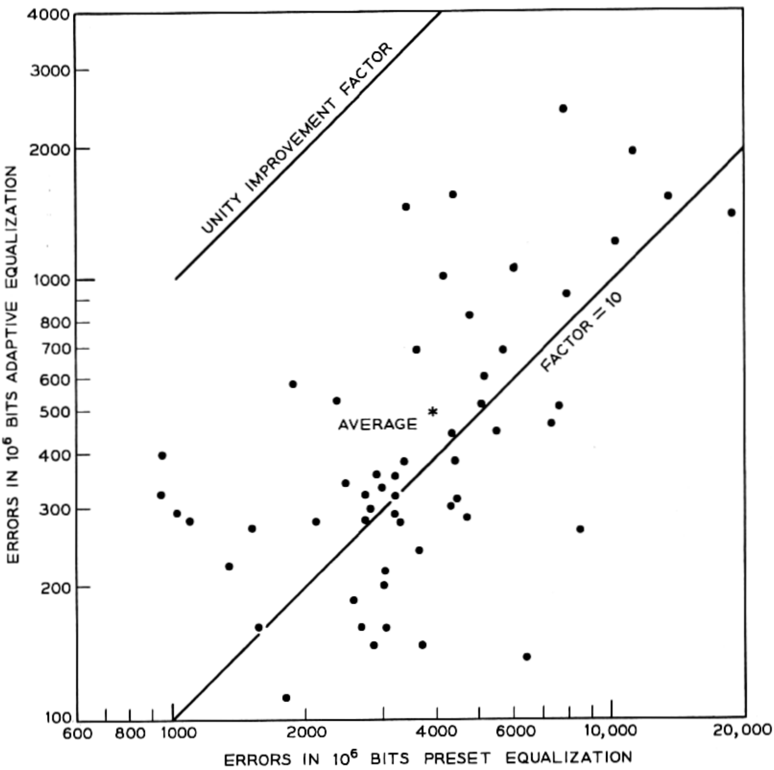

Fig. 9 — Error rates for adaptive and preset equalization on test runs.

conjunction with the VSB system which are shown in Ref. 4 are examined, it will be seen that the customer error rate is an extremely sensitive function of the raw error rate. An improvement of an order of magnitude in raw error rate results in an improvement of 2 or 3 orders of magnitude in the customer error rate. The customer error rates are not shown in Fig. 9 since in many cases the error rate for preset equalization was so high as to preclude synchronization, while frequently the adaptive error rate was too low to get any customer errors at all in the two-minute runs.

The question arises as to where the improvement comes from. First the improvement factor does *not* come from tracking the changing channel characteristics over the two-minute period. The error rate is virtually unchanged if the adaptive equalizer is turned off after a few second's operation. The improvement comes from a number of other factors, chief of which is the improved setting accuracy. The preset equalizer operates for a period of 7 seconds during which 100 test pulses per second are transmitted. After this period an equilibrium distribution of equalizer positions has been reached in which the average tap error is about Δ, whereas with the adaptive equalizer 2400 symbols per second are used to extract information and the average tap error has been designed to be close to the minimum value of 0.5Δ.

The other factors, whose importance has not been quantitatively assessed, are the possibility of a nonlinearity in the transmission channel so that different characteristics are presented to the data signals than to test pulses, the possibility that the test pulses have some overlap, and the possibility of some bias in the examination of test pulse sample polarities. There seems to be no question that adaptive equalization using received data is superior to test pulse equalization.

Finally, an attempt was made to measure the time variation of some of the test lines. A critical measure of this variation can be achieved by averaging in low-pass filters the outputs of the exclusive-OR circuits in Fig. 4. The filter outputs are extremely sensitive indicators of the parameters h_j, although they are difficult to calibrate since the proportionality constant is a function of the noise variance [see (49)]. The recorder outputs from these filters are purposely not reproduced here lest the reader grant too much significance to the time variation records obtained. Generally, about a peak variation of one percent was found on the center 3 taps (h_{-1}, h_0, and h_1) and a negligible amount on other taps. The period of variation varied from about 10 seconds to a few minutes and then grew short again periodically. It was easily demonstrated that the variation was due to phase wander in the carrier system. Such slow phase wander was subsequently tracked using more sophisticated phase control apparatus.

A record of the exact setting reached by each tap gain control using adaptive equalizer setup was made periodically over a period of several weeks. Within statistical error the values reached were constant with the exception of the three central, phase-sensitive taps. (The reason these taps are phase sensitive is that the quadrature pulse ideally has nonzero values only at these points.)

The transmission facilities have been designed through the years to be relatively insensitive to temperature and humidity changes. It would seem that even for the most critical current data transmission usage, the telephone channel's phase and amplitude characteristics can be assumed to be time invariant. Thus, it seemed that adaptive equalization was more useful as a refining device after test pulse equalization and as an insurance system rather than as a tracking equalizer for a significantly time varying channel. Nevertheless, the potentialities of the system considerably exceeded the demands of the environment and even so its benefits were quite strikingly apparent.

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