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Design Considerations and Proposals for Compatible U. S. Subsidiary Coinage

By R. A. KIMBER and R. R. STOKES

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For 173 years since the Coinage Act of 1792, United States subsidiary coinage has contained 90 per cent silver. During the last decade, an increasing unbalance between supply and demand has jeopardized the availability of silver and made it necessary to consider alternative coinage materials. Because present coin-handling mechanisms are designed to discriminate between coinage and spurious materials, it is necessary to identify the functional properties of coins so that a compatible material system may be proposed. Compatibility is established by considering such items as weight, resistivity, diameter, thickness, wear, scrap, composition, coinability, corrosion and color. One alloy and four laminated metal systems, varying in silver content from 70 to 0 per cent, were found which satisfy the compatibility requirements. This paper discusses the design considerations for compatibility and proposes five metal systems which meet these considerations.

I. INTRODUCTION

Coin silver, as used in U. S. subsidiary coinage, is made of an alloy containing 90 per cent silver and 10 per cent copper. During the last decade, it has become increasingly evident that the availability of this alloy for the continued production of coinage is jeopardized by an increasing unbalance between the demand for, and the supply of silver.

The involvement of Bell Telephone Laboratories in this issue results from the use of 5, 10, and 25 cent coins in public telephones. In view

of the possible change in subsidiary coinage alloy, it became necessary to identify the functional properties of coins in their various applications and to specify accurately those properties which are essential to maintaining an operationally compatible coinage system. To this end, work was initiated on a study of coin properties and compatible metal systems for U. S. subsidiary coinage.

This paper records (i) the results of an investigation of the permissible deviations from the present coinage system for compatible performance in coin chutes, (ii) proposals for changes which represent improvements in the present coinage, and (iii) the analysis and description of various metal systems which could be employed. This information has been discussed and confirmed with various representatives of the vending industry and conveyed to the U. S. Mint and the Battelle Memorial Research Institute, consultant to the Mint, for their use.

II. COIN CONSIDERATIONS

2.1 *Functional Requirements*

The selection of alternative metal systems to provide compatible U. S. subsidiary coins involves a number of diverse considerations. There are practical questions relative to manufacture and procurement, psychological questions of public acceptance, and functional questions of serviceability in coin operated machines. It is the purpose of this paper to identify the functional requirements for subsidiary coins and to present specific criteria for insuring compatibility. This is facilitated by first discussing those factors which pertain to coin operated mechanisms.

2.1.1 *Weight*

The published weight limits on dimes and quarters as minted by the U. S. Department of the Treasury are 38.58 ± 1.5 grains (2.5 grams nominal) and 96.45 ± 3 grains (6.25 grams nominal), respectively. Coins in circulation naturally weigh less due to wear. To establish the reduction in size, a study was made of the physical properties of circulated coins which included data on changes in coin weight, diameter, and thickness. The sample included 1000 coins of each denomination, pro-rated to have representative numbers by date for the number of each remaining in circulation. Probability plots of the data showed that the population distribution was sufficiently normal to permit statistical estimation of the minimum and maximum values. The three-sigma limits, which theoretically include 99.73 per cent of the population, showed that the weight of dimes could range from a maximum minted

weight of 2.6 grams to a low of 2.31 grams, while the weight of quarters could similarly range from 6.44 to 5.74 grams.

In coin equipment, the lightest coin must have sufficient energy to operate one or more mechanisms. Since the quarter is two and a half times heavier than the dime, the half dollar is five times heavier, and each is required to accomplish similar functions, it is safe to assume these requirements will be satisfied for the quarter and half dollar if they are satisfied for the dime. To establish the lower acceptable weight for a dime, coins of various weights were deposited in new and existing coin mechanisms. The data show that dimes weighing less than 1.98 grams begin to malfunction and cause misregistration of coin deposit information.

The wear study showed that a minted dime of 2.50 grams could wear to a lower expected weight of 2.31 grams, experiencing in circulation a maximum weight loss of 0.19 grams. Assuming the same wear characteristics for an alternate metal, the minimum minted weight of a new dime should not be less than 2.17 grams to avoid encountering worn dimes as light as 1.98 grams. The 2.17 gram dime weighs approximately 87 per cent of the present nominal minted value of 2.5 grams.

As previously mentioned, the weights of new quarters and half dollars are not critical for coin mechanism operation. Since most metals from which coins might be made have densities which differ from coin silver by less than 20 per cent, it is not considered restrictive to limit, arbitrarily, the weights of new quarters and half dollars to 80 per cent of their present weights. The reason for not reducing the weight further is to exclude the use of aluminum and magnesium in coins because of the inability to separate them by eddy current means. This will be discussed next.

2.1.2 *Acceptance Number*

Acceptance number is the nomenclature for a measure of a coin's sensitivity to eddy current detection. The lower the number, the greater the sensitivity, although the relationship is not linear.

An eddy current detector consists of a ramp on which a coin, or coin substitute, can roll through a magnetic field as shown in Fig. 1. When a coin rolls through the magnetic field, each incremental mass of the material instantaneously rotates about that point on the coin periphery which touches the ramp, and each cuts the transverse magnetic flux lines with some instantaneous velocity. By the "generator" rule, eddy currents flow in the coin perpendicular to the instantaneous velocity vectors. The lower the resistivity of the material, the greater are the eddy currents for a given velocity and magnet strength.

where

- x = the linear progression of the center of the coin,
- t = the length of time the coin experiences the magnetic field,
- ϕ = total magnetic lines of flux,
- k = a numerical constant,
- σ = electrical resistivity,
- ρ = density, and
- a = the constant acceleration of a coin rolling down an incline.

A diagram showing the physical interrelationships of the members is given in Fig. 1. In eddy current terminology, the coefficient of the velocity term (dx/dt) is called the braking constant and is designated by K . Thus,

$$K = \frac{k\phi^2}{\sigma\rho}. \quad (2)$$

The coin material resistivity (σ) and density (ρ) appear in the denominator as a product ($\sigma\rho$) and is part of the rationale for so defining acceptance number. As this number decreases, the braking constant increases, consistent with the earlier observation that decreases in acceptance number result in increases in coin braking. Also, consistent with expectation, the braking constant increases with increasing magnet strength. Fortunately, the acceptance numbers of metals are sufficiently different to provide a basis for achieving discrimination between many of the common metals and U. S. coin silver. Table I lists a number of metals and their acceptance numbers. While the difference in acceptance numbers is not ideal, it is adequate, when used in conjunction with weighing, to permit a high degree of slug rejection.

To illustrate how physical separation actually occurs, reconsider the eddy current brake previously described as a ramp intersecting a transverse magnetic field (Fig. 1). If this ramp is terminated after braking occurs, so that coins can fall through free space, it is apparent that coins which have been slowed down more will assume vertical free fall closer to the end of the ramp. Coins which have been slowed down less will have more horizontal momentum and will travel further from the end of the ramp before assuming vertical free fall.

In experimental studies it was noted that the free-fall path of coin silver alloy is closer to aluminum than it is to zinc. This does not provide optimum spacing for obtaining separation from both metals. Since new metal systems are being proposed to replace coin silver alloy, it seems judicious to select one with a higher acceptance number to obtain opti-

TABLE I—ACCEPTANCE NUMBER OF TYPICAL METAL SYSTEMS

Material System	Electrical Resistivity (microhm-cm)	Density (grams/cm ³)	Accept. No. (microhm gm/cm ²)
Aluminum	2.8	2.7	7.5
Magnesium	4.45	1.74	7.75
Aluminum base alloy	5.5	2.64	15.1
Silver, pure	1.60	10.5	16.8
Magnesium base alloy	10.0	1.78	17.8
U. S. coin silver alloy	2.1	10.3	21.6
40-58-2 silver-copper-zinc	2.3	9.5	21.8
70-27-3 silver-copper-zinc	2.25	9.8	22.1
97.5-2.5 copper-nickel	2.5	8.9	22.5
Zinc	5.92	7.13	42.2
70-30 brass	6.98	8.4	58.6
Nickel	6.84	8.9	60.8
			(magnetic)
Ingot iron	9.7	7.76	75.2
			(magnetic)
Grade A phosphor bronze	10.7	8.8	94.0
Columbium and alloys	13.1 min	8.6	122 min
Lead	20.6	11.2	231
18% nickel silver	29	8.7	252
Titanium	55	4.6	253
Zirconium	40 min	6.5	260 min
75-25 copper-nickel	32	8.9	285
95-5 nickel-silicon	38	8.55	325
18-8 stainless steel	79	8.0	632
90-10 nickel-chromium	80	8.7	696
Incoloy alloy 800	92	8.03	740
80-20 nickel-chromium	108	8.55	925
Nonconductors	∞	—	∞

num rejection capabilities. Experimental studies indicate that a coin with an acceptance number of approximately 25 gives optimum rejection capabilities between aluminum and zinc. It is interesting to note that the Mercury dime had an acceptance number of approximately 25. With improvements in the refining of silver, the acceptance number of the Roosevelt dime has dropped to its present value of 21.6. This point is illustrated in Fig. 2, where the acceptance curve for the Mercury dime is closer to that of zinc than the Roosevelt dime. From the discussion to ensue on compatible coinage metals systems, it will become evident that the option to adopt a different acceptance number can be realized in a number of different proposals.

Even with the acceptance number of coin silver being closer to aluminum than zinc, the adequacy of eddy current coin chutes for separating spurious slug materials is readily demonstrated. A nationwide sample of over $\frac{1}{2}$ million nonstandard deposits in public coin operated equipment which did not have eddy current coin chutes was collected,

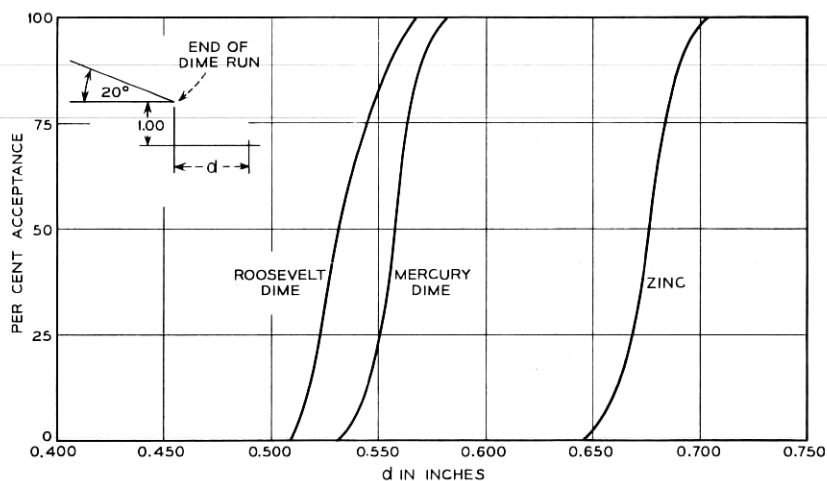


Fig. 2—Acceptance vs coin position. (Eddy current magnetic field characteristics: $\phi = 1860$ lines; gap = pass 0.058 inch, stop 0.060 inch.)

sorted and identified. Of the total, approximately 73 per cent were unredeemable slugs and the remainder were tokens and foreign coins. Excluding the sample of tokens, foreign and mutilated coins which are either partly or totally redeemable, the composition of the slug sample is shown in Table II. The information is presented in categories representing the principal separation means in coin chutes. It is seen from this analysis that approximately 99.8 per cent of the sample would be rejected in eddy current coin chutes.

2.1.3. Diameter

The diameters of newly minted dimes and quarters are 0.705 ± 0.003 and 0.955 ± 0.003 inches, respectively. The previously mentioned coin study showed that the diameter of dimes in circulation ranged between 0.698 and 0.706 inches, while those for the quarter ranged between 0.946 and 0.957 inches. In many coin mechanisms, diameter gauging is the principal means of separation.

Further, discrimination against foreign coins with compatible alloys is achieved primarily on a diameter gauging basis. In view of these factors and the tremendous amount of coin handling, sorting, storing, and dispensing equipment which would have to be changed, it is not advisable to change the present minted diameters.

TABLE II—MATERIAL COMPOSITION STUDY OF NONSTANDARD DEPOSITS COLLECTED IN PUBLIC COIN-OPERATED MECHANISMS NOT USING EDDY CURRENT COIN CHUTES

		No. Nonstandard Pieces	Per Cent Of Total
I. Would be rejected by eddy current coin chutes:			
Removed by magnetic trap:			
Iron	242,561		
Magnetic stainless steel	3,444		
	<u>246,005</u>	246,005	46.65
Removed by weighing:			
Aluminum	2,311		
Plastic	1,705		
	<u>4,016</u>	4,016	0.76
Removed by sizing:		92,041	17.46
Removed by eddy current rejection:			
Lead	12,326		
Brass	99,788		
Zinc	13,522		
Nonmagnetic stainless steel (10¢ & 25¢ sizes)	14,011		
Tokens	5,906		
Other	38,628		
	<u>184,181</u>	184,181	34.93
		<u>526,243</u>	99.80
			99.80
II. Would be accepted in eddy current coin chutes:			
Nonmagnetic stainless steel	1,036		0.20
		<u>1,036</u>	<u>0.20</u>
		<u>527,279</u>	<u>100.00</u>

2.1.4. Thickness

The thicknesses of newly minted dimes and quarters are 0.053 (+0.005, -0.003) and 0.067 (+0.005, -0.003) inches, respectively.¹ By the same criteria adopted for establishing minimum and maximum weights and diameters, the thicknesses of dimes in circulation range from 0.042 to 0.054 inches while those for quarters range from 0.053 to 0.072 inches.

In general, the thickness dimension is not a good measure of a coin. During use, mushrooming of the rim can cause increases in thicknesses;

in other cases, wear on the rim causes reduction in thicknesses. Further, many different foreign coins have substantially the same thicknesses. Thickness gauging is primarily useful in detecting mutilated coins, but is otherwise rather insensitive for regular gauging.

Although the coin gauge of the present coin mechanisms provides thickness gauging, the control is not so close that a small increase in thickness could not be tolerated. This may be necessary to help maintain the coin weight in a substitute metal system. Coin silver alloy has a density of 10.3 grams/cm³ while most of the metals which might feature in a new system have lower densities. Two principal contenders, copper and nickel, each have nearly equal densities of approximately 8.9 grams/cm³. Especially in the case of the dime, some change will have to be made to maintain the required weight.

2.1.5. *Wear*

Due to the nature of coin handling, coins wear primarily in thickness. Pearson product-moment correlations obtained from a sample coin study on coin wear indicates that there is a +0.46 correlation between coin weight and thickness, and practically no correlation between coin weight and diameter for coins in circulation. Because the wear occurs principally on the coined surfaces, it is apparent that a percentage change in coin thickness does not produce the same percentage change in coin weight as would be experienced with a solid disc. Over the range of expected coined surface wear, the loss of weight is estimated to be approximately one third of what would be experienced with a solid disc having the same thickness reduction. This was determined by comparing the coin weight losses with changes in coin thicknesses.

2.2 *Criteria for Compatibility*

From the previous section on factors pertaining to coin operated mechanisms, it is possible to identify a set of numerical constraints on coinage which forms the basis for an objective definition of compatibility. These constraints call for maintaining the present diameter, establishing a tolerance range for the acceptance number, and specifying values for the minimum weight and maximum thickness. The controlling physical dimensions for material and compatibility with U. S. dimes, quarters, and half-dollars are given in Table III.

2.3 *Laminated Coinage*

From the previous discussion on the relative uniqueness of the acceptance numbers of various metals, it is not surprising that the number

TABLE III—PHYSICAL CHARACTERISTICS FOR COMPATIBLE
U. S. SUBSIDIARY COINAGE

	10¢	25¢	50¢
Diameter (inch)	0.705 ± 0.003	0.955 ± 0.003	1.205 ± 0.003
Acceptance number (microhm-gm/cm ²)	25 ⁺¹ _{-3.5}	25 ⁺¹ _{-3.5}	25 ⁺¹ _{-3.5}
Weight (grams)	2.17 min	5.00 min	10.0 min
Thickness after coining (inch)	0.061 max	0.072 max	0.091 max

of existing alloys which satisfy the eddy current considerations is limited. When additional requirements like appearance, corrosion, and manufacturability, to be discussed later, are added, the number of possible choices is even more restricted. In view of this, attention has been directed to combining various metals in a laminate structure and utilizing the different material properties to achieve the equivalent acceptance number of coin silver alloy. One can get an intuitive understanding of the approach by considering a low resistivity metal, which permits high eddy currents and provides braking action to oppose the coin motion, laminated to a high density metal which provides inertia to assist the coin motion. By establishing the correct laminate thickness, it is possible to control the resulting acceptance number. This approach has the additional attraction of using common metals in an uncommon way to offer protection against counterfeiting.

The equation of motion for a laminated coin rolling without slipping through a magnetic field, given below, was derived and experimentally verified as presented in Appendix A.

$$\frac{d^2x}{dt^2} + \frac{k\phi^2 \sum_{i=1}^n \left(\frac{\tau_i}{\sigma_i} \right)}{\sum_{i=1}^n (\rho_i \tau_i)} \cdot \frac{dx}{dt} - a = 0, \quad (3)$$

where

- τ_i = thickness of the i th lamella,
- σ_i = resistivity of the i th lamella,
- ρ_i = density of the i th lamella,
- n = number of lamellae,

and all other terms are the same as defined in (2).

A comparison of this equation with that for the homogeneous coin

shows that it differs only in the braking constant. Since the two equations have the same form, they have the same general solution. If the two braking constants were constrained to be equal, the equations would have the same specific solution and the velocities of the homogeneous and laminated coins leaving the magnetic field would be equal. This is the condition for compatible eddy current materials. When the two braking constants are equated and all common terms are cleared, the following equation results and provides the constraint relation on the section thicknesses (τ_i) of the laminated coin to achieve the equivalent acceptance number ($\sigma\rho$) of a homogeneous coin.

$$(\sigma\rho)_{\text{eff}} = \frac{\sum_{i=1}^n \rho_i \tau_i}{\sum_{i=1}^n \frac{\tau_i}{\sigma_i}} \quad (4)$$

This equation has been verified experimentally and used in the design of the laminated coins proposed in a later section for compatible coin substitutes.

2.4 Additional Coinage Considerations

2.4.1. Scrap

One important consideration in the production of coins is the efficient utilization of scrap. Present punch press operations produce a material stock skeleton containing 30 to 35 per cent scrap. This is reclaimed by melting, casting, and rerolling. The same procedure can be employed for any single element or alloy substitute.

In the use of laminate strip stock, the scrap reclamation problem places two constraints on the choice of the metals for laminate sections. First, when laminated scrap is melted, all of the constituent metals in the various laminate sections appear in the resultant alloy. Because refining is too costly, it is necessary to be able to enrich the reclaimed stock by adding one or more of the constituent metals to produce an additional quantity of one of the required laminate alloys. Second, in the enriching process it is necessary that the reconstructed alloy be produced in quantities that can be completely used to prevent stockpiling.

These two constraints lead to two conclusions. First, the constituent elements used in one laminate section must appear in the second laminate metal which is planned for reconstruction. Second, the percentage

of the reconstructed laminate used in the coin must exceed the percentage of scrap produced. In the simplest case, should one laminate be a pure metal, like zinc, the other laminate material must be a zinc bearing alloy and must constitute at least 35 per cent of the coin volume. The specific details of these constraints will be discussed for the proposed laminate metal systems in Section III.

2.4.2. *Composition*

From an eddy current view, the metallic elements must work into some metal system which achieves the required acceptance number and remains essentially non-magnetic. Should the proposed metal system require alloying, some of the most promising pure metals like silver and nickel cannot be used together except in very small quantities because of the tendency to crack during rolling. Lead is excluded in laminated structures as a high density replacement for silver because it fails to bond. The phase relationships of copper-nickel and copper-aluminum alloys cause the resistivities of both to rise radically above the resistivity of either pure metal and limit their utility.

In addition to the scrap problem previously discussed and the considerations of corrosion, coinability, and color, to follow, the materials should be in good supply at reasonable cost. Zinc, copper, and nickel are all attractive in this regard.

2.4.3. *Coinability*

Two of the principal factors which contribute to the appearance of a coin are the amount of relief and sharpness of the coined surfaces. By these criteria, American coins rank among the finest in the world, and there is strong interest in maintaining the present high standard. To achieve this objective, it is necessary to employ a metal system which has an adequately low initial hardness and which will not work-harden during coining by an amount which would prevent complete filling of the characters on the coined surfaces.

Presently, coin blanks have to be capable of having an initial hardness of 28 to 29 Rockwell B in the annealed condition and not rising above a hardness of 76 to 77 Rockwell B in the fully-coined condition. This places a double constraint on the selection of coin substitute metals.

Alloys which might be considered for laminated coins, such as stainless steel, silver-copper-nickel alloy, copper-zinc, and silver-copper-zinc alloys having more than 30 per cent zinc, are too hard in the annealed condition for coining. Other choices for coin substitutes which are coin-

able like the silver-copper and copper-nickel alloys, change rapidly in hardness with the addition of certain trace elements used to control the alloy resistivity. Specifically, the addition of 0.1 per cent silver to copper-nickel alloys raises the hardness from 26 Rockwell B to about 60 Rockwell B and 1 per cent silver raises the hardness to 70 Rockwell B. Such factors are important with respect to both manufacturability and acceptability.

2.4.4. *Corrosion*

A further constraint is placed on the choice of materials by corrosion. Any metal system which is acceptable for subsidiary coinage must neither tarnish in a manner to change appreciably the appearance of the coin after minting, nor produce corrosive residues which could soil the property of a possessor, or become dislodged in coin mechanisms and eventually cause malfunctions.

In the case of laminates, additional care must be exercised to insure that the adjacent laminates are adequately close in the electromotive series to minimize rim corrosion. In the use of dissimilar materials, it is to be expected that some corrosion will occur. However, the effects of this at a low level are adequately offset by the effective polishing which coins experience in use.

2.4.5. *Color*

Since the coin silver alloy used in higher value subsidiary coinage has been "white," or "silvery," since the first coinage act in 1792, the American public is accustomed to white coins. How far a new metal system can depart from this standard is a subjective question.

Silver-copper alloys with decreasing silver content take on a progressively increasing pink tone which finally becomes a rich bronze color. A number of independent subjective evaluations set a lower silver content limit, concluding that at or above a 70 per cent silver content, the alloys look subjectively similar to the present coinage. The American public has also grown accustomed to the material appearance of the U. S. five cent coin which is 75-25 Cupro-Nickel. While this alloy has a different "whiteness," it is apparently acceptable for a non-silver standard. It is felt that any new metal system should retain white coined surfaces.

III. METAL SYSTEMS FOR COMPATIBLE COINAGE

This section includes five proposals for metal systems which provide eddy current compatibility with existing U. S. silver alloy coinage. One basic alloy and four laminate solutions are presented.

3.1 Alloys

In surveying the extensive number of existing alloys for possible compatible systems, the most severe requirement — correct acceptance number — was initially used. This criterion was applied to a listing of alloys presented in The Bureau of Standards Publication, *The Mechanical Properties of Metals and Alloys*. Alloys found which satisfy this requirement are:

- (i) aluminum alloys
- (ii) magnesium alloys
- (iii) copper-zinc alloys with less than 5 per cent zinc
- (iv) copper-nickel alloys with less than 3 per cent nickel
- (v) silver-copper alloys
- (vi) silver-copper-zinc alloys with less than 5 per cent zinc.

The copper-aluminum alloys are excluded here because resistivities of the alloys are high, yielding unacceptably high acceptance numbers.

There are other criteria which further reduce the possibilities for acceptable alloys. The aluminum and magnesium alloys do not have sufficient weight to dependably rotate the coin separating devices in existing coin operated mechanisms. The copper-zinc and copper-nickel alloys are coppery in color and tend to tarnish. Therefore, these alloys are eliminated from further consideration.

The silver-copper alloy meets the weight requirement for all percentage compositions. Fig. 3 shows the variations of resistivity-density, and acceptance number for different amounts of silver. The acceptance number is everywhere between 21.5 and 23 microhm-gm/cm² over the range of 26-74 to 94-6 silver-copper. As previously discussed, the color of the silver-copper alloys changes from a bright, shiny white as the silver content is reduced, requiring that all alloys having less than 70 per cent silver be eliminated because of the color requirement.

Although zinc is added as a whitener, the silver-copper-zinc alloys have much the same properties as silver-copper alloys. Since its corrosion properties are the same, the silver-copper-zinc series must contain at least 70 per cent silver to prevent corrosion. Therefore, the only compatible alloy system found is comprised of silver-copper alloys which contain more than 70 per cent silver.

3.2 Laminates

A laminated coin has a more complex set of constraints than an alloy coin as noted in the coin considerations previously reviewed. A large

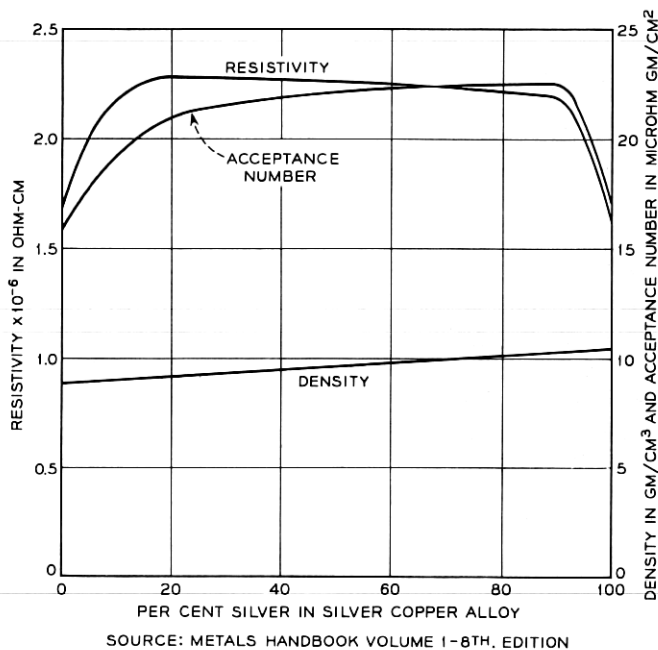


Fig. 3 — Resistivity and density of silver-copper alloys.

number of metal systems were investigated, and this section presents those few combinations which satisfy all of the restrictive conditions.

In the interest of achieving the simplest coin to fabricate, each laminated coin to be presented consists of three lamella sections. In each case, the two outside lamellae are white alloys to satisfy the color requirement, and are the same thickness to insure symmetrical eddy current performance. The core in each case is a copper alloy.

3.2.1 70-30 Silver-copper sections laminated on a 30-70 silver-copper core. (Representative)

In the discussion on alloys it was mentioned that all silver-copper alloys with silver contents between 26 and 94 per cent have acceptance numbers exceeding 21.5. The lower limit on silver content was set at 70 per cent for reasons of color and corrosion. This produced a coin material with an acceptance number of 22.5 which contained 70 per cent silver.

A laminated coin could be made with 70-30 silver-copper alloy on the outside layers for reasons previously cited, and reduced silver content

alloy used in the core to achieve an over-all lower silver content in the coin.

The lower limit for the over-all silver content is determined by the minimum acceptance number. Assuming a dime coin blank thickness of 0.039 inch and minimum practical thicknesses of 0.007 inch for each outside laminate, the lowest silver content which can be used in the core and maintain an acceptance number of 21.5 is 20 per cent. This gives the coin an over-all silver content of 37.9 per cent.

Using this approach, it is possible to obtain a family of coins by letting the core contain higher silver than the 20 per cent minimum. For purposes of illustration, if a coin bearing 50 per cent silver were required, Fig. 4 would apply. To illustrate its use, if it is given that the outer laminates should be 70-30 silver-copper, the designer is free to select either the section thicknesses or the per cent silver in the core. For example, if the center core thickness is desired to be half of the total thickness, the core must contain 30 per cent silver. In this series of possible coins, there is no scrap problem since the melted scrap can be divided into two parts and each enriched to the desired value.

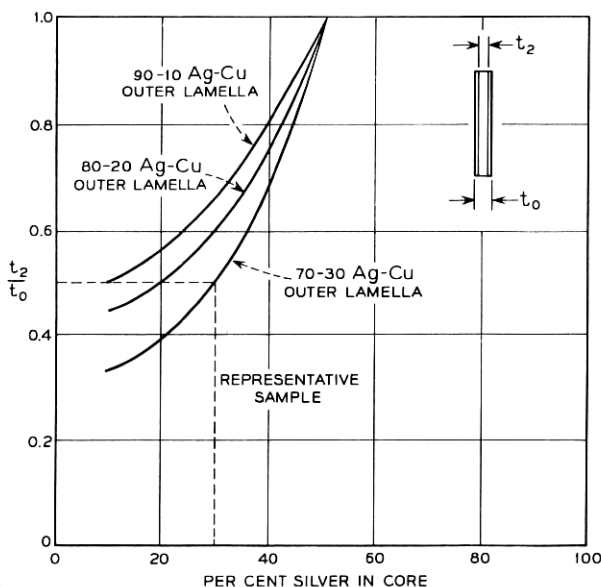


Fig. 4—Laminated coin proportions. (Ag-Cu, Ag-Cu, Ag-Cu laminate. 50 per cent silver.)

3.2.2 70-27-3 Silver-copper-zinc sections laminated on a 90-10 copper-zinc core

The second laminated coin to be considered consists of two 70-27-3 silver-copper-zinc outer sections laminated on a 90-10 copper-zinc core and contains 44.5 per cent silver. The addition of zinc to copper in a ratio of one to nine increases the acceptance number to 34.5. The presence of zinc in the core complicates the reclamation problem by requiring that zinc also appear in the outer laminate material as discussed earlier in the section on scrap. Further, it needs to appear in the ratio of one to nine with the copper to simplify the melt-enriching process without causing stock piling. Alloys of 90-9-1 and 80-18-2 silver-copper-zinc were tried in addition to the proposed 70-27-3 composition. All work equally well from an eddy current view, but the 90 per cent silver alloy definitely leads to a scrap build up and the 80 per cent silver alloy situation was marginal. With the 70-27-3 alloy in the outer laminates and the 90-10 copper-zinc core, a coin is achieved which is completely compatible when the section thicknesses are properly controlled.

To visualize the effect of section thicknesses on the acceptance number, one might imagine the center thickness becoming increasingly thin, in which case the acceptance number would equal that of the outside material. Conversely, with increasing thickness of the inside section, the acceptance number approaches that of the center laminate. Depending on the thicknesses of the sections, the acceptance number of the coin could assume any value between the two limiting values of the constituent laminate alloys. To achieve the desired acceptance number of 25, it is obvious that one laminate material must have an acceptance number which is lower and the other material one that is higher. Obviously, as the outside laminates wear, the influence of the inner laminate increases and the composite acceptance number shifts towards that of the core material.

To determine what the section thicknesses should be to obtain a desired acceptance number, use is made of (4) which expresses the constraint relation. Since each coin consists of only two laminate materials, and since the cut blank thickness for a coin is constant (t_0), it is convenient to express the thickness of one laminate in terms of the other. In this case where the core material has the higher acceptance number, the inner laminate thickness is called t_2 , and (4) takes the form

$$(\sigma\rho)_{\text{eff}} = \frac{[\rho_L(t_0 - t_2) + \rho_H t_2]}{\left[\frac{(t_0 - t_2)}{\sigma_L} + \frac{t_2}{\sigma_H} \right]}, \quad (5)$$

where

- ρ_L = density of the lower acceptance number material,
- ρ_H = density of the higher acceptance number material,
- σ_L = resistivity of the lower acceptance number material,
- σ_H = resistivity of the higher acceptance number material.

Since (5) is a function of only one independent variable, t_2 , it is possible to plot a curve for the acceptance numbers which result from varying t_2 . The resulting design chart is shown for this metal system in Fig. 5.

Since the cut blank thicknesses (t_0) for dimes, quarters, and half dollars are different, three separate design curves are shown for the three coin denominations and labeled $t_0 = 0.039$, $t_0 = 0.052$, and $t_0 = 0.066$, respectively. To obtain a coin acceptance number of 25, one has only to read the core material thicknesses, t_2 , from the chart. The laminate thicknesses for the three denominations are given in Table IV.

Before leaving the design chart, it is appropriate to consider the change in acceptance number with wear. As a laminated coin wears, the outside sections are reduced in thickness while the core thickness is unchanged. This alters the ratio of the original thicknesses and causes a shift in the acceptance number. As the lower resistivity material on the out-

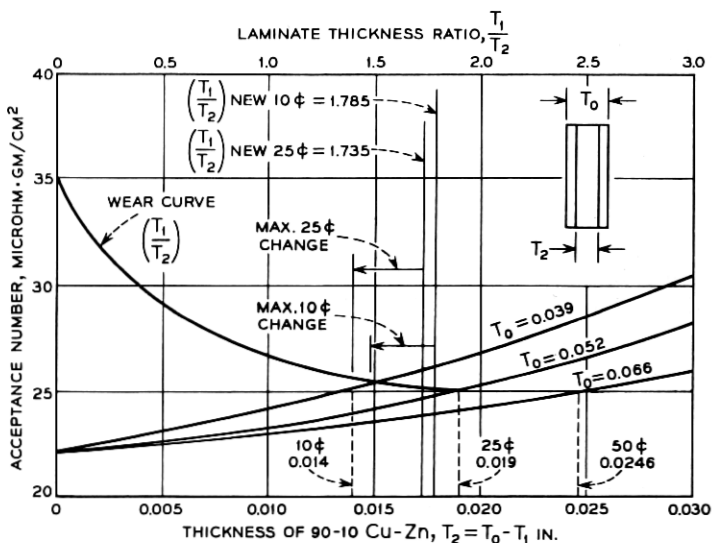


Fig. 5—Laminated coin proposal no. 2. (70-27-3 Ag-Cu-Zn [$R = 2.25 \times 10^{-6}$ ohm-cm, $D = 9.8$ gm/cm³] and 90-10 Cu-Zn [$R = 3.92$ ohm cm, $D = 8.8$ gm/cm³].)

TABLE IV

Coin	70-27-3 Silver-Copper-Zinc	90-10 Copper-Zinc	70-27-3 Silver-Copper-Zinc
	$\frac{t_1}{2}$ (in.)	t_2 (center) (in.)	$\frac{t_1}{2}$ (in.)
10¢	0.0125	0.014	0.0125
25¢	0.0165	0.019	0.0165
50¢	0.0207	0.0246	0.0207

side wears away, the acceptance number increases. Conversely, the acceptance number decreases if the denser material is used on the outside and wears away. To show this effect, a wear curve was constructed by holding the core thickness constant and calculating the acceptance number for reduced clad thicknesses.

Referring to the wear curve of Fig. 5, the ratio of combined outer laminate thicknesses to inner laminate thickness, t_1/t_2 , is 1.785 for a newly minted 10-cent coin and this value on the wear curve coincides with an acceptance number of 25. From the study on coin wear, the maximum reduction in dime thickness was estimated to be 0.012 inches. Recalling that the weight decreased at one-third the rate of the thickness reduction, the maximum dime wear would result in the equivalent solid disc loss of 0.004 inch total, or 0.002 inch per side. Since this wear occurs on the two outer laminate sections, it reduces the combined outer section thicknesses t_1 to 0.021 inch, and the t_1/t_2 ratio to 1.50. Referring to the wear curve, this change would cause the acceptance number to rise to 25.2, which is an insignificant change. The wear limits for the dime and quarter acceptance number change differ as a function of the relative amount of wear and the initial thickness ratio. No wear curve is given for the 50-cent piece since wear data are not available.

As mentioned previously, the coins under discussion here contain approximately 44.5 per cent silver. The specific compositions of the three subsidiary coins are given in Table V.

The following calculation shows that there is no scrap build up with these coins. Assuming there is 100 pounds of laminate material before the coins are blanked, there will be 35 pounds of scrap to be melted. For the dime, this 35 pounds of scrap will consist of 15.75 pounds of silver, 17.32 pounds of copper, and 1.93 pounds of zinc. To achieve a new alloy in the same proportions as the outer lamella (70-27-3) only silver needs to be added to the melt, since the copper to zinc ratio re-

TABLE V—COIN COMPOSITION

	10¢	25¢	50¢
% Silver	45	44.4	44.0
% Copper	49.5	50.0	50.4
% Zinc	5.5	5.6	5.6
Total	100.0	100.0	100.0

mains nine to one. The weight of outer material obtained from the addition of silver is found from the following ratio

$$W_s = \frac{(\% \text{ Copper in melt}) (35 \text{ lbs scrap})}{\% \text{ Copper in outer layer}} = \frac{17.32}{0.27} = 64.2 \text{ lbs.}$$

The weight of the outer lamella in 100 pounds of laminate is

$$W_0 = \frac{t_1}{t_2} \times 100 = \frac{0.025}{0.039} (100) = 64.2 \text{ lbs.}$$

Since the two weights are equal, there will be no scrap build up.

3.2.3 40-50-5-5 Silver-copper-nickel-zinc sections laminated on a copper core

The third proposal for a compatible metal system consists of two outer laminates of 40-50-5-5 silver-copper-nickel-zinc alloy bonded to a core of copper. The 40-50-5-5 silver-copper-nickel-zinc alloy is commonly known as Swedish Coin Silver. It is used in five Swedish coins, has an attractive white appearance and has fair coinability. As an alloy, the material has an acceptance number of 57.2 which is too high. As an outside lamella of a laminated coin with a copper core, it is an attractive material proposal for achieving an acceptance number of 25. The coin contains 19.9 per cent silver.

The design and wear curves for this coin were constructed as in the case of the previous coin, and are plotted in Fig. 6. In this case, since the material with the lower acceptance number is used in the core, the curve is plotted with the thickness, t_1 , as the independent variable.

The coin meets all of the cited requirements and is attractive as a coin of low silver content. The section thickness for each denomination is given in Table VI with a listing of the various coin compositions.

3.2.4 75-25 Copper-nickel sections laminated on a copper core

This fourth proposal consists of two 75-25 copper-nickel outer sections laminated on a copper core and contains no silver. The 75-25 cupro-

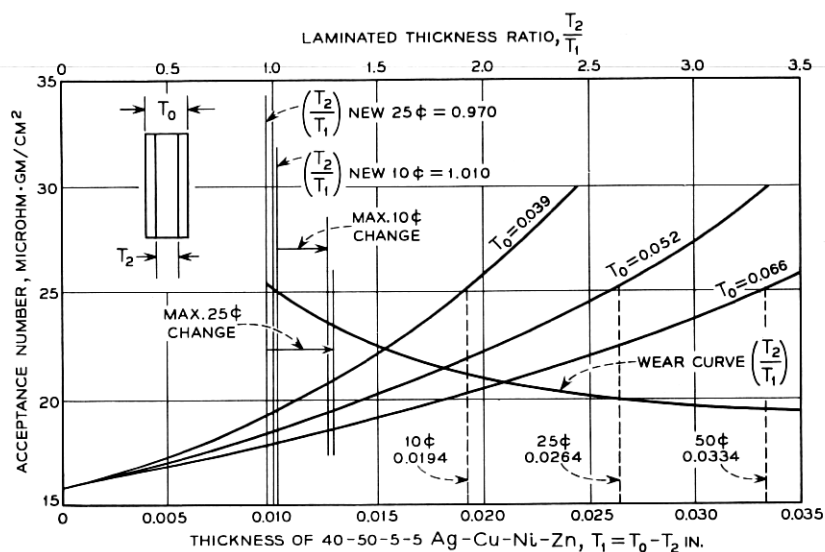


Fig. 6—Laminated coin proposal no. 3. (40-50-5-5 Ag-Cu-Ni-Zn [$R = 6.05 \times 10^{-6}$ ohm-cm, $D = 9.45$ gm/cm³] and Cu [$R = 1.74 \times 10^{-6}$ ohm-cm, $D = 8.94$ gm/cm³].)

TABLE VI

Coin	40-50-5-5 Silver-Copper-Nickel-Zinc $\frac{t_1}{2}$ (in.)	Copper t_2 (center) (in.)	40-50-5-5 Silver-Copper-Nickel-Zinc $\frac{t_1}{2}$ (in.)
10¢	0.0097	0.0196	0.0097
25¢	0.0132	0.0256	0.0132
50¢	0.0167	0.0326	0.0167

COIN COMPOSITION			
	10¢	25¢	50¢
% Silver	19.9	20.3	20.2
% Copper	75.1	74.6	74.8
% Nickel	2.5	2.55	2.5
% Zinc	2.5	2.55	2.5
Total	100.0	100.0	100.0

nickel is presently used in the coining of U. S. five cent pieces. It is an easy material to fabricate, is adequately white and has an acceptance number of 286. Because of this high value, it must be fabricated with thicker sections of a core material with a low acceptance number to achieve the desired acceptance number of 25. The acceptable composition percentages are given in Table VII.

The loss of outer laminate thickness due to wear has a more pronounced effect on the change in acceptance number than for the other laminates. Because of the faster relative change of the thickness ratio, the acceptance number is reduced after maximum wear to 22.5. This is not considered objectionable because the acceptance number remains within the proposed limits for compatible coinage. Experimental results with this coin composition confirmed the expectation that it yields better rejection than does present coin-silver.

IV. SUMMARY

After examining the properties of U. S. silver alloy coins and the operating requirements of typical coin handling mechanisms, it was concluded that the coin diameters should not be changed. The dime, quarter, and half-dollar minimum newly minted weights should be 2.17, 5.00, and 10.0 grams, respectively. The coined thicknesses of the quarter and half-dollar should be retained, and the dime thickness could be increased to 0.061 inch maximum to more easily achieve the weight objective.

Further, an acceptance number of 25 with a maximum and minimum limit of 26 and 21.5, respectively, would permit a distinct improvement in the ability to reject slugs.

It is possible to design a number of substitute metal systems which have eddy current compatibility with U. S. silver alloy coinage. After identifying the considerations which enter into the design of compatible coinage, five proposals have been made which satisfy these conditions. They differ in silver content from a high of 70 per cent to a low of zero, offering freedom in the final selection.

TABLE VII—COIN COMPOSITION

	10¢	25¢	50¢
% Nickel	9.9	10.1	10.2
% Copper	90.1	89.9	89.8
Total	100.0	100.0	100.0

TABLE VIII—COMPATIBLE COIN PROPOSALS IN ORDER OF MERIT

Order of Merit Metal System	Accept. No. Before Wear	Accept. No. After Max. Dime Wear	% Nominal Dime Weight Advantage Over Minimum
1. 70-27-3 Silver-Copper-Zinc Laminates on 90-10 Copper-Zinc	25.0	25.2	7.7%
2. 40-50-5-5 Silver-Copper-Nickel-Zinc Sections Laminated on a Copper Core	25.0	23.0	3.8%
3. 75-25 Copper-Nickel Sections Laminated on a Copper Core	25.0	22.5	0.7%
4. Silver-Copper Alloys With at Least 70 Per Cent Silver	22.5	22.5	12.9%
5. 70-30 Silver-Copper Sections Laminated on a 30-70 Silver-Copper Core	22.1	21.6	9.7%

From a coin-operated mechanism point of view, there are three factors which provide a basis for comparing the five material systems: weight, acceptance number, and resistance to changes in acceptance number with wear. Because the minimum weight requirement is achieved with all five proposals, the ranking is based principally on the ability to obtain an acceptance number of 25 and to resist changes therein with wear, and secondarily, on the ability to obtain as much dime weight above the minimum as possible. Judging by these standards and using the dime data as a basis of comparison, it is possible to rank the five compatible coin proposals in an order of merit as listed in Table VIII.

V. ACKNOWLEDGMENTS

These proposals for compatible substitute coinage and the supporting technical information were submitted to Miss Eva Adams, Director of the U. S. Mint.

Appreciation is expressed to members of the U. S. Mint, the Battelle Memorial Institute, the Metals and Controls Division of Texas Instrument Company, and the Bell Telephone Laboratories' Metallurgical Engineering Department for their cooperative assistance.

APPENDIX A

Derivation of Eddy Current Equations

A.1 Solid Disc

The equation of motion for a homogeneous disc, rolling without slipping through a magnetic field, was derived by Messrs. L. Veith and C. F. Wiebusch using classical mechanics considerations and issued in

an internal Bell Telephone Laboratories memorandum dated September, 1940. The essentials of these equations have, subsequently, been confirmed by derivations based on energy considerations and dimensional analysis. Because the latter technique affords a simple and direct approach to the equation of motion and emphasizes the important parameters from the onset, it is used in the following derivation. The authors are indebted to Mr. J. P. Runyon for suggesting its use in this paper.

Consider a homogeneous conductive disc rolling down a ramp without slipping and cutting a transverse magnetic field. The parameters, governing the motion of the disc, are defined below.

θ = angle of inclination of ramp.

x = linear progression of the center of the disc.

ϕ = magnetic lines of flux of the transverse magnetic field.

ρ = density of the disc.

σ = electrical resistivity of the disc.

a = initial acceleration of the disc.

The basic equation of motion for the dynamic system represented here is of the form

$$\frac{d^2x}{dt^2} + K_1 \frac{dx}{dt} + K_2x + K_3 = 0. \quad (6)$$

An inspection of the equation shows that the constant, K_3 , must have the dimensions of an acceleration, hence it is the initial acceleration, a , of the disc which is opposite to that due to the eddy current action. Further, since the disc is moving through a constant magnetic field, it is assumed that the motion is not a function of the displacement, x , and that the coefficient K_2 is zero. These considerations reduce the form of the equation to

$$\frac{d^2x}{dt^2} + K_1 \frac{dx}{dt} - a = 0. \quad (7)$$

Since the first and third terms have the dimensions of acceleration, $[LT^{-2}]$, where L is length and T is time, the second term must also have the same dimensions. Further, since the dx/dt part of the second term is a velocity with the dimensions of $[LT^{-1}]$, the coefficient K_1 must have the dimensions of $[T^{-1}]$. In eddy current terminology, the coefficient of the velocity term, K_1 , is known as the braking constant.

From physical considerations, the braking constant is a function of the magnet and disc properties, specifically, the magnetic lines of flux, disc resistivity and density. This can be expressed in the form

$$K_1 \propto \phi^\delta \rho^\gamma \sigma^\lambda. \quad (8)$$

Since ϕ has the dimensions of $[ML^2/TQ]$ where M represents mass units and Q represents charge units, ρ has the dimensions of $[M/L^3]$, σ has the dimensions of $[ML^3/TQ^2]$ and knowing that their product must have the dimension of $[T^{-1}]$, (8) can be written in the form

$$\left[\frac{1}{T}\right] = \left[\frac{ML^2}{TQ}\right]^{\delta} \left[\frac{M}{L^3}\right]^{\gamma} \left[\frac{ML^3}{TQ^2}\right]^{\lambda}. \quad (9)$$

Recombining

$$\left[\frac{1}{T}\right] = M^{(\delta+\gamma+\lambda)} T^{(-\delta-\lambda)} Q^{(-\delta-2\lambda)} L^{(2\delta-3\gamma+3\lambda)}. \quad (10)$$

Equating exponents on both sides of the equation leads to the following four equations:

$$-\delta - \lambda = -1 \quad (11)$$

$$\delta + \gamma + \lambda = 0 \quad (12)$$

$$-\delta - 2\lambda = 0 \quad (13)$$

$$2\delta - 3\gamma + 3\lambda = 0. \quad (14)$$

From the simultaneous solution of (11), (12), and (13),

$$\lambda = -1$$

$$\delta = 2$$

$$\gamma = -1.$$

Substitution in (8) yields the braking constant

$$K_1 \propto \frac{\phi^2}{\sigma\rho}.$$

Therefore,

$$K_1 = k \frac{\phi^2}{\sigma\rho}, \quad (15)$$

and from (7) and (15), the equation of motion for a homogeneous disc rolling without slipping through a magnet field is

$$\frac{d^2x}{dt^2} + k \frac{\phi^2}{\sigma\rho} \frac{dx}{dt} - a = 0. \quad (16)$$

This equation assumes that the entire face area of the conductive disc is uniformly influenced by the magnetic field. In practice, this is rarely the case. Consequently, the braking constant is typically modified to account

for the area and position of the disc relative to the area and position of the magnetic field. In addition, the edge effect of the disc entering and leaving the field must be considered. These factors are generally determined from geometric considerations and empirically obtained results in specific applications. In no event do these modifying coefficients reduce the generality of the basic equation of motion, given in (16).

A.2 Laminated Disc

The equation of motion for a laminated disc, rolling without slipping through a magnetic field, has the same basic form as that for a solid disc. An examination of (16) shows that all of the disc parameters appear in the braking constant. The specific form of the equation of motion for the laminated disc can be obtained by substituting appropriate expressions for the laminated disc resistivity and density in the braking constant. To facilitate this step, it is convenient to express the braking constant in terms of the disc mass and resistance.

The resistance of the conductive disc to eddy currents can be obtained from the general resistance equation

$$R = \frac{\sigma l}{A}. \quad (17)$$

With the coin center velocity parallel to the ramp and the magnetic field transverse to it, the direction of current flow is normal to the ramp along conductive length b_1 , the height of the disc equal to the magnet dimension normal to the ramp. The cross-section area normal to the current is the product of the disc thickness, τ , and the width of disc equal to the length of the magnet, parallel to the ramp, b_2 . Hence,

$$R = \frac{\sigma b_1}{\tau b_2}. \quad (18)$$

The density of the solid disc is simply $\rho = \text{mass/volume} = m/A_c\tau$. Substituting these expressions into the braking constant of (15) yields

$$K_1 = \frac{k\phi^2}{\left(\frac{R\tau b_2}{b_1}\right)\left(\frac{m}{A_c\tau}\right)} = \frac{k\phi^2 A_c b_1}{R m b_2}. \quad (19)$$

To obtain the appropriate expressions for the mass and resistance of a laminated coin, one can immediately write the mass equation

$$m_L = A_c(\rho_1\tau_1 + \rho_2\tau_2 + \cdots \rho_n\tau_n) = A_c \sum_{i=1}^n (\rho_i\tau_i) \quad (20)$$

where $A_c = \text{face area of the coin}$.

The total resistance of the laminated disc consists of the resistances of each lamination added as parallel resistors.

$$R_L = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}} = \frac{1}{\sum_{i=1}^n \frac{1}{R_i}} \quad (21)$$

where

$$R_i = \frac{\sigma_i b_1}{\tau_i b_2}$$

$$\therefore R_L = \frac{b_1}{b_2} \frac{1}{\sum_{i=1}^n \frac{\tau_i}{\sigma_i}} \quad (22)$$

Substituting in (19), the braking constant for a laminated disc takes the form

$$K_{1(\text{eff})} = \frac{k\phi^2 \sum_{i=1}^n \left(\frac{\tau_i}{\sigma_i} \right)}{\sum_{i=1}^n (\rho_i \tau_i)} \quad (23)$$

Hence, the equation of motion for the laminated disc is

$$\frac{d^2 x}{dt^2} + \frac{k\phi^2 \sum_{i=1}^n \left(\frac{\tau_i}{\sigma_i} \right)}{\sum_{i=1}^n (\rho_i \tau_i)} \frac{dx}{dt} - a = 0. \quad (24)$$

A.3 *Equivalence of Solid and Laminated Discs*

Since the equations of motion for the homogeneous and laminated discs have the same form, they have the same solution and will differ in specific trajectories as a function of the braking constants only. The functional equivalence of the homogeneous and laminated discs can be obtained by equating the braking constants of (15) and (23), yielding

$$\frac{k\phi^2}{\sigma\rho} = \frac{k\phi^2 \left(\sum_{i=1}^n \frac{\tau_i}{\sigma_i} \right)}{\sum_{i=1}^n \rho_i \tau_i} \quad (25)$$

Equation (25) reduces directly to the desired constraint relation.

$$\frac{1}{\sigma\rho} = \frac{\sum_{i=1}^n \frac{\tau_i}{\sigma_i}}{\sum_{i=1}^n \rho_i \tau_i} \quad (26)$$

Equation (26) is in terms of the physical and material properties of the coins only, and establishes the equivalence between the two coins. While the resistivity-density product of a homogeneous disc is independent of thickness, it is readily seen that the value for the laminated coin is not. The expression correctly reduces to an identity for the case of a single laminate.

Putting (26) in a slightly different form,

$$(\sigma\rho)_{\text{eff}} = \frac{\sum_{i=1}^n \rho_i \tau_i}{\sum_{i=1}^n \frac{\tau_i}{\sigma_i}} \quad (27)$$

it is apparent that the effective resistivity-density product for any laminated coin can be calculated with this equation.

A.4 Experimental Verification of Equations

To verify the equivalence of the laminar resistivity-density product, it was necessary to fabricate laminated coins using (27) and to test their dynamic performance against homogeneous coins of known parameters in an eddy current coin-sampling device. For this purpose, it was decided to build a laminar coin to simulate the resistivity-density product of zinc. Zinc was selected because it represents the next highest acceptance number above coin silver that the chute normally rejects.

The resistivity of zinc, determined with a Magnaflux Conductivity Meter (Model FM-100), is recorded in Table IX, along with the measured density. The acceptance number of this material is given as 42.3.

TABLE IX

Material	Resistivity $\times 10^{-8}$ ohm-cm	Density gm/cm ³	Acceptance Number ($\sigma\rho$) $\times 10^{-8}$ gm-ohm/cm ²
Copper	1.74	8.86	15.4
Zinc	6.0	7.06	42.3
Phosphor Bronze (Grade A)	9.0	8.86	79.7

TABLE X

Material		Resistivity $\times 10^{-6}$ ohm cm		Density gm/cm ³		Thickness inch		Acceptance Number ($\sigma\rho$) $\times 10^{-6}$ gm-ohm/cm ²
1	2	σ_1	σ_2	ρ_1	ρ_2	t_1	t_2	
Phosphor Bronze	Copper	9.0	1.74	8.86	8.86	0.039	0.011	41.5

An examination of (27) indicates that the two materials selected to simulate a given coin must have acceptance numbers on each side of the desired value. For this reason, phosphor bronze (Grade A) with an acceptance number of 79.7 was laminated with copper to produce a zinc disc equivalent. The particular laminar thicknesses used in the simulated test discs are given in Table X, where t_2 is the thickness of the center laminate and t_1 is the combined thickness of the two, equal outside laminates. The tolerance on the three laminar section thicknesses was held to ± 0.0005 inch each and the resulting calculated acceptance number of the clad disc was within 1.9 per cent of the homogeneous coin value. This deviation is considerably smaller than the discrimination capability of the eddy current test instrument. Therefore, the laminated discs were judged to be adequate test samples for comparative dynamic testing.

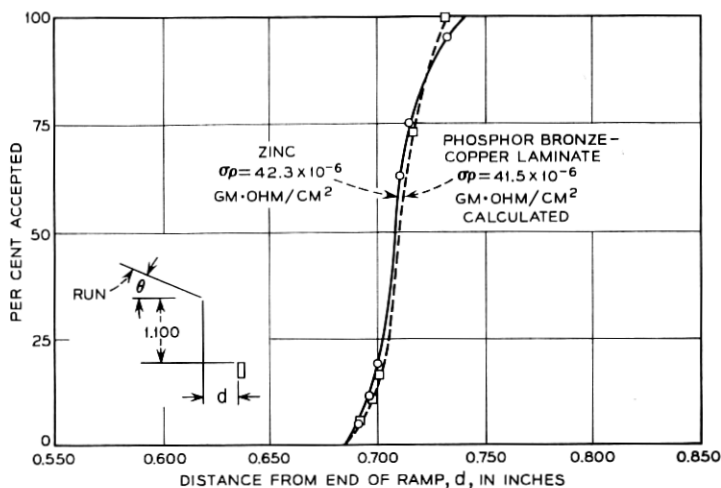


Fig. 7—Acceptance vs coin position. (Magnet strength: $\phi = 1800$ lines; $\theta = 20^\circ$; gap = pass 0.058 inch, stop 0.060 inch; coin diameter = 0.705 inch.)

It should be emphasized that the acceptance number of the laminated coin was calculated with the equation this experiment was designed to verify. The 1.9 per cent agreement, arrived at above, cannot be interpreted as a demonstration of equivalence. Satisfactory verification requires that the laminated coin exhibit the same dynamic behavior in the eddy current chute as the coin it was designed to simulate.

To evaluate the eddy current behavior of the laminated coins, a standard coin chute with the eddy current magnet adjusted to the nominal recommended flux strength of 1800 lines was mounted vertically. The accept mechanism was initially adjusted toward the front of the chute to insure that all deposited coins would be rejected. By progressively adjusting the mechanism toward the rear of the chute and dropping each coin and laminate equivalent 100 times at each setting, the results were obtained, as shown in Fig. 7, with the per cent acceptance plotted as a function of the distance, d , from the end of the run.

While there is some difference at the end points in the transition from low to high per cent acceptance, the mid-value acceptance characteristics of the solid coin and laminate disc equivalent approximate each other within the experimental accuracy of the test. This supports the conclusion that (27) is correct and provides an equivalence between single laminar and multiple laminate coins.

The end point transitional effect, referred to in the last paragraph, manifests itself as a sharper acceptance characteristic for the laminated disc. This means that the discrimination properties of this coin in eddy current detectors are superior to the homogeneous coin it simulates. Stated differently, the variation in trajectories which the laminated coin exhibits after leaving the run is less than for the companion homogeneous coin. This suggests that the laminated coin could be used not only to obtain a compatible eddy current coin alloy, but also, to achieve improved rejection characteristics.

REFERENCE

1. Annual Report of the Director of the Mint, Fiscal Year Ended June 30, 1962, U. S. Government Printing Office, Washington, 1963, p. 50.