Observed 50 to 60 Gc/s Attenuation for the Circular Electric Wave in Dielectric-Coated Cylindrical Waveguide Bends

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A thin low-loss dielectric coating inside a 0.875-inch i.d. round waveguide was achieved by drawing a slightly oversize flexible tube of polyethylene into the waveguide. The plastic tube fits snugly to the guide wall. Several 14-foot lengths of waveguide were lined and loss measurements made. The lined pipes were 8 per cent lossier than the unlined. The increase in loss could be almost completely accounted for by the theory of H. Unger.

A 92° Fresnel integral bend (curvature a linear function of arc length) was constructed from a 14-foot section of the 0.875-inch i.d. lined waveguide. The insertion loss between 50 and 60 Gc/s had a value of about 0.1 db. This small value is close to the theoretical result for such bends.

I. INTRODUCTION

In a nonideal section of circular cylindrical waveguide, the circular electric wave TE_{01} couples to other modes of transmission which are permitted.[1] The strongest coupling for bends is to the TM_{11} mode since it is degenerate in a perfectly conducting straight waveguide. The degeneracy of the phase factors of the TE_{01} and TM_{11} modes in the uncoated cylindrical waveguide gives rise to the serious problem of an increase in loss upon bending. For example, in a 2-inch i.d. bare pipe at a 5.4-mm wavelength, the attenuation constant is doubled when the radius of curvature is a few miles.[1] The power transfer between the two modes can be reduced by removing the phase degeneracy. One technique for doing this is to introduce a thin layer of dielectric next to the wall of the waveguide. The electric field intensity of the TE_{01} mode vanishes at the wall but that of the TM_{11} mode has a large value there, hence, the effect on the propagation constant of the TM_{11} mode is larger than on the TE_{01} .

The solution of the characteristic equation for the normal modes of the circular cylindrical waveguide with dielectric coating obtained by Unger and Morgan[1,2] indicates there is an increase in attenuation of the TE_{01} mode attributable to dielectric loss and an added copper loss arising from the tendency of the electric field intensity to concentrate in the dielectric next to the wall. Attenuation measurements on such a straight length of cylindrical waveguide are reported in Section II and compared to theoretical predictions insofar as is possible.

A bend for which the curvature is a linear function of bend length follows the Fresnel integral curve. This is the elastic bend which is obtained by applying forces at the center and at the two ends of a rod. According to Unger, a bend in a dielectric-coated waveguide with such a curvature has a minimum of mode conversion, hence, minimum loss.[3] Measurements on a 92° bend with the linearly tapered curvature are discussed in Section III.

II. STRAIGHT DIELECTRIC-COATED WAVEGUIDE

Long straight sectious of dielectric-lined waveguide were prepared by using the technique of pulling a slightly oversize polyethylene liner into an oxygen-free copper waveguide. Only a small length of the liner was in contact with the copper wall during the pulling. The remainder was stretched enough elastically to have an outside diameter less than the inside diameter of the pipe. The liner expanded in place against the waveguide wall after the tension was removed.

The polyethylene sleeve was made by extrusion molding from a crease-resistant formulation of polyethylene and rubber.[4] The dielectric constant and loss tangent at 55.2 Gc/s of this material were measured[5] and are 2.25 and 1×10^{-4}. This means that the material is very good from the dielectric attenuation point of view. A finished extrusion tended to have small air pockets and trapped dust particles. Another difficulty existed with the extrusion. The inner die happened to be slightly eccentric with the outer during manufacture. A maximum wall thickness t_{max} of 0.0134 inch was measured on one side and a minimum t_{min} of 0.0095 inch on the other. Furthermore, the tubing had hash marks remaining from the water cooling used during extrusion.

The TE_{01} attenuation coefficient of the dielectric-coated waveguide was measured by observing the 3-db bandwidth of a cavity containing the test section. Any cavity technique for measuring the attenuation coefficient implies the use of a considerable amount of auxiliary equipment. The cavity quality test set which was used, operated between 50 and 60 Gc/s and used an M-1977 backward-wave oscillator.[5] The cavity quality was measured by sweeping the frequency of the oscillator through the cavity resonance and then observing the frequency difference between the half-power points of the transmission curve. The apparatus was made sensitive to dispersion rather than absorption so that the measurements would be relatively insensitive to drifts in amplifier gain. The cavity assembly consisted of a two-hole coupler constructed from helix waveguide, the long section of waveguide to be studied, a 12-inch section of helix waveguide, and a piston as a shorting termination. The piston had a port for removal of air from inside the cavity. A short section of helix in the cavity was necessary in order to remove unwanted modes. Mode interference is manifest by observing markedly increased bandwidth values for some positions of resonance for the piston; thus, the minimum measured TE_{01} bandwidth was selected as the most probably correct value.

The long section of waveguide under test has a quality factor Q_2 given by the expression[6]

$$1/Q_2 = 2\alpha_2\beta_2/(\beta_{c2}^2 + \beta_2^2). \tag{1}$$

The parameters α_2 and β_2 are the real and imaginary portions of the propagation constant of the TE_{01} wave in the waveguide section under consideration. The factor β_{c2} is the free-space propagation constant corresponding to the TE_{01} cutoff frequency. The reciprocal cavity quality factor in terms of the measured 3-db frequency bandwidth Δf_m is $1/Q_m = \Delta f_m/f$ and is the sum of two portions. The first is that associated with the intrinsic attenuation of the waveguide, $\Delta f_2/f = 1/Q_2$. The second is associated with the attenuation at the ends of the cavity, b/L, where b is a constant characteristic of the ends and L the length of the cavity. The resulting expression for the measured cavity quality factor is

$$\frac{1}{Q_m} = \frac{\Delta f_m}{f} = \frac{\Delta f_2}{f} + \frac{b}{L}. \tag{2}$$

The expression for the attenuation factor of the test section becomes

$$\alpha_2 = \frac{\beta_{c2}^2 + \beta_2^2}{2\beta_2 f}\left(\Delta f_m - \frac{fb}{L}\right). \tag{3}$$

Unlined evacuated cylindrical copper waveguides were measured first in order to establish the end loss corrections and the loss characteristics of the bare copper pipe to be used later for lining and in making bends. These parameters were found by measuring the cavity bandwidth as a function of length. The intrinsic bandwidth of the test section was given directly from a plot of Δf_m versus $1/L$ by the intercept and the end loss factor calculated from the slope. The attenuation coefficients in Fig. 3 of the bare copper pipes agree well with the results of King.[7]

The losses due to air were determined using a long section of unlined copper waveguide open to the atmosphere. These measurements were needed in order to correct the measurements taken on the dielectric-coated waveguide. The dielectric-coated waveguide could not be evacuated since an occasional small pocket of air between the liner and the copper wall caused the liner to collapse upon evacuation. The plot of the measured atmospheric losses versus frequency in Fig. 1 is in satisfactory agreement with the results of others.[8]

An initial check of the placement of the dielectric coating inside the guide was made by fabricating many sections of various lengths and then measuring the bandwidth of the cavity formed from each. The bandwidth measurements were made at a fixed frequency near 55 Gc/s (Fig. 2). Of interest are the points associated with the bare pipe. These are not scattered appreciably, which means that all of the bare pipes are quite uniform. The circled points associated with the liner show considerable scattering. Apparently the liner was not in close enough contact with the copper wall. Any blister in the liner causes the lined pipe to be lossy. The liner contact with the wall was improved by alternately cooling and warming for several cycles. A new set of measurements were made, and the decrease in attenuation indicated by the triangular points was observed. Next, a 14-foot section of the shrunken dielectric-coated guide was measured across the frequency band from 50 to 60 Gc/s. The increase in the attenuation coefficient of the waveguide due to the liner

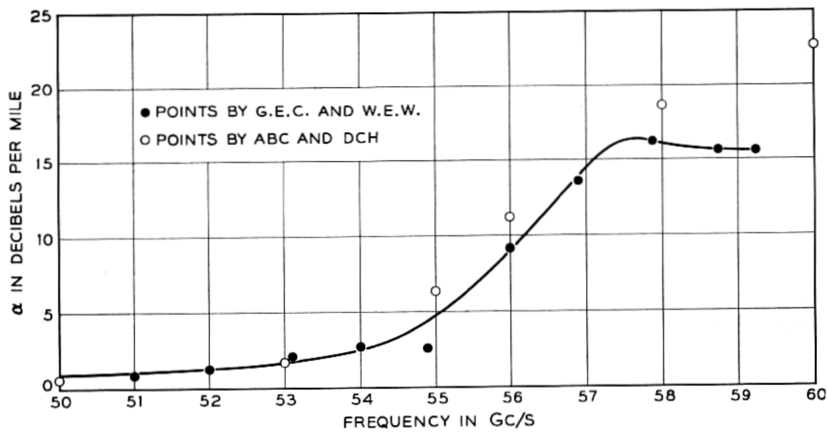

Fig. 1 — Attenuation of air in 0.875-inch i.d. waveguide.

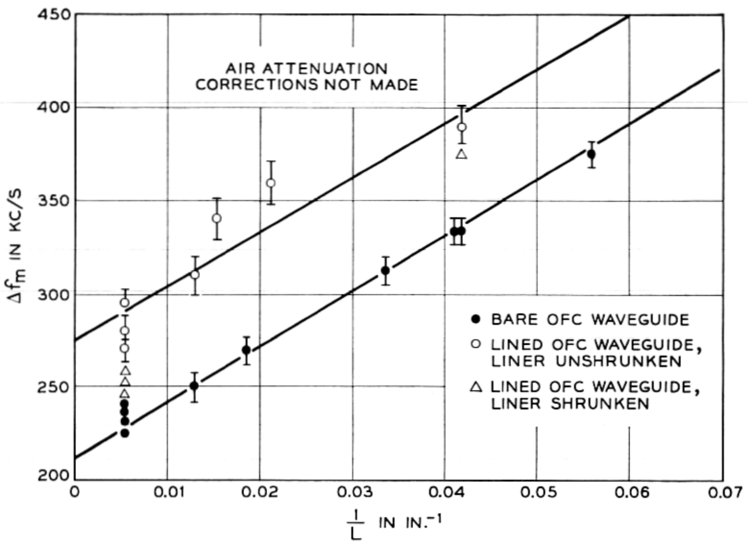

Fig. 2 — Cavity bandwidth as a function of reciprocal length.

$\Delta\alpha_l$ was calculated by subtracting the losses due to air α_a and the measured unlined waveguide α_{01} from the measured attenuation coefficient α_m at each frequency. The results for this section of coated guide are given in Fig. 3.

The increase in the TE_{01} attenuation coefficient of the dielectric-lined waveguide due to the dielectric loss in the liner is given by Unger as

$$\Delta\alpha_D = 1.835 \times 10^5 \frac{k_0^2 p_{01}'^2 t^3}{\beta_{01} a^3} \epsilon'' \text{ db/mile}, \tag{4}$$

and the increase in copper loss due to the liner being in contact with the waveguide wall as

$$\Delta\alpha_\omega = \alpha_{01}(\epsilon' - 1)k_0^2 t^2. \tag{5}$$

In these equations, $\epsilon^* = \epsilon' - j\epsilon''$ is the dielectric constant and t the thickness of the coating, a the radius of the waveguide, k_0 the phase propagation constant in unbounded vacuo, p_{01}' is the first root of the Bessel equation $J_0'(p_{01}'a) = 0$, and β_{01} is the phase propagation coefficient of the TE_{01} wave in the waveguide. Equations (4) and (5) are perturbation expressions for which β_{01} of the lined waveguide is assumed to be the same as that of the bare pipe.

An analysis of the increase in attenuation coefficient of the dielectric-

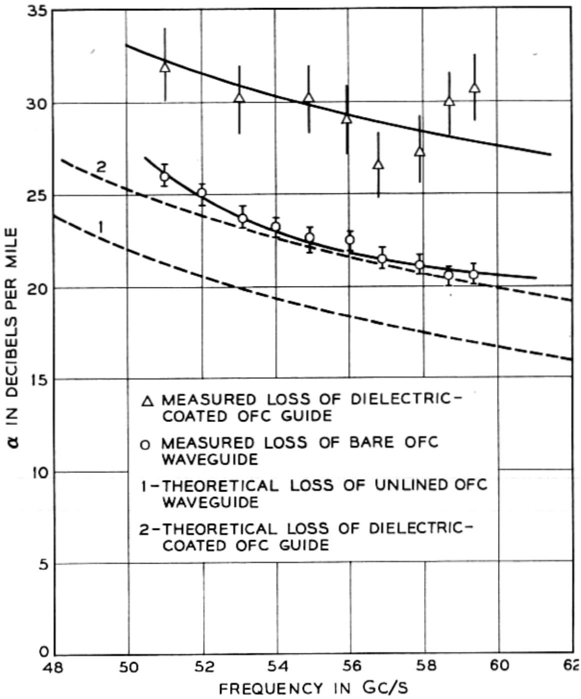

Fig. 3 — Attenuation of coated and uncoated waveguide.

coated waveguide due to the coating is dependent upon knowing the magnitude of the coating thickness. However, the inside and outside radii associated with the sleeve were slightly eccentric. A first approach to the problem of eccentricity can be made by assuming that the functions for calculating the attenuation coefficient in cylindrical geometry are changed only negligibly by the introduction of the eccentricity. In such case the average values for the quantities t, t^2, and t^3 can be used,

$$\bar{t} = \tfrac{1}{2}(t_{\max} + t_{\min}), \tag{6}$$

$$\overline{t^2} = \bar{t}^2 + \frac{c^2}{2}, \tag{7}$$

$$\overline{t^3} = \bar{t}^3 + \frac{3c^2}{2}\,\bar{t}, \tag{8}$$

where c is the distance of separation of centers

$$c = \tfrac{1}{2}(t_{\max} - t_{\min}). \tag{9}$$

Substitution of the measured values of t_{max} and t_{min} into (6), (7), (8), and (9) indicates that the arithmetic average is suitable for use in the calculations. The measured average wall thickness for the tubing used in this work was 0.0115 inch and the separation of centers was 0.0020 inch.

The values for $\Delta\alpha_D$ and $\Delta\alpha_\omega$ were calculated for several frequencies and their sum is compared in Fig. 4 to the values measured for a long section of the dielectric-coated waveguide. There is apparently some conversion of TE_{01} to other modes of propagation.

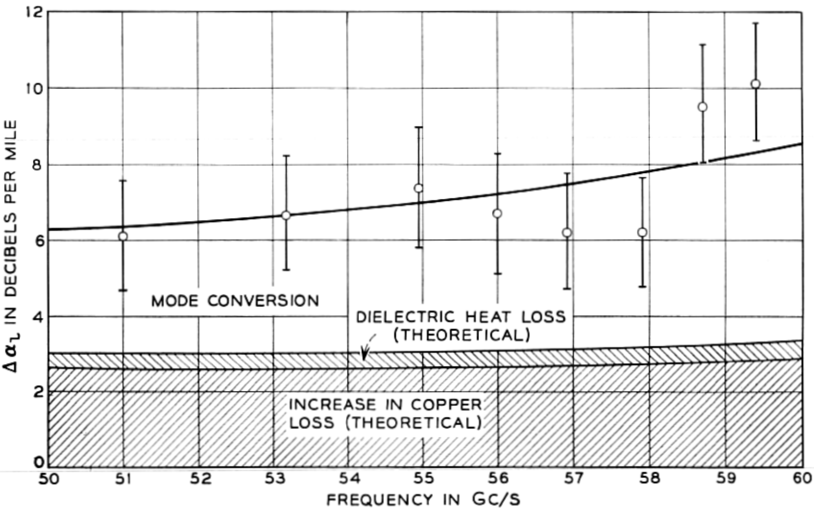

Fig. 4—Increase in attenuation $\Delta\alpha_l$ of 0.875-inch i.d. copper waveguide due to linear.

III. DIELECTRIC-COATED WAVEGUIDE BEND

A typical bend was constructed to fit into an existing waveguide transmission system. This required that the angle of bend θ_0 be 92° and the length l_b be 14.135 feet. In order to design a linearly tapered bend with a given bend angle and length, consider a bend with the curvature being an arbitrary function of the length of arc z taken along the axis of the pipe $K = f(z)$. The curvature of the bend described in the rectangular coordinate system (Γ, Σ) of Fig. 5 is given by the expression

$$K = \left[1 + \left(\frac{d\Gamma}{d\Sigma}\right)^2\right]^{-\frac{3}{2}} \frac{d^2\Gamma}{d\Sigma^2}. \qquad (10)$$

Of supplementary use is the equation for the differential length of arc,

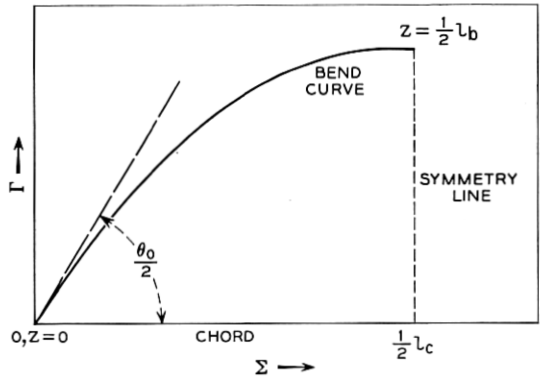

Fig. 5 — Coordinate system for half of the bend.

$$dz = \left[1 + \left(\frac{d\Gamma}{d\Sigma}\right)^2\right]^{\frac{1}{2}} dS. \tag{11}$$

Equation (10) is solvable in terms of the length of bend under the conditions that at the origin $z = 0$, $K = 0$, and $d\Gamma/d\Sigma \rfloor_{z=0} = \tan \theta_0/2$. The solution is then:

$$\Sigma = \int_0^z \cos\left(\frac{\theta_0}{2} + \int_0^z K dz_1\right) dz \tag{12}$$

$$\Gamma = \int_0^z \sin\left(\frac{\theta_0}{2} + \int_0^z K dz\right) dz. \tag{13}$$

The chord length l_c of a symmetrical bend of length l_b is

$$l_c = 2 \int_0^{l_b/2} \cos\left(\frac{\theta_0}{2} + \int_0^z K dz\right) dz. \tag{14}$$

When the curvature is made a linear function of length of arc, $K = -k'z$ with k' being a constant, the result is a Fresnel integral curve. The curvature parameter k' can be evaluated in terms of design parameters from the definition of the differential bend angle and the radius of curvature,

$$d\theta = \frac{1}{R} dz = K dz = -k' z dz. \tag{15}$$

The half bend angle occurs over the half-length of the pipe; thus,

$$k' = 4\theta_0/l_b^2. \tag{16}$$

Equations (12), (13), and (16) can be used to calculate the coordinate points for a Fresnel integral bend of arbitrary length and bend angle.

A centerline curve was calculated and a bend constructed to fit the desired design parameters. A convenient technique for making a bend with linearly tapered curvature is to form the waveguide around a wooden jig. The waveguide is then clamped in place. The attenuation of the bend was measured using the resonant cavity technique described previously and the results are plotted in Fig. 6. The attenuation was found to decrease slightly with increasing frequency. Theoretical values for the attenuation of the bend can be calculated from the equations of Unger and Morgan[2,3] and are shown in Fig. 6.

IV. CONCLUSIONS

The straight waveguide with liner had a slightly greater TE_{01} transmission loss than the bare pipe. Part of the loss, as suggested by Unger and Morgan, arises from dielectric heat loss and an increase in copper loss. The remaining portion of the added loss could probably be decreased because the liner was not as smooth and did not fit as snugly to the wall as desired. Extrusion of the dielectric directly inside the waveguide should yield improved performance.

The Fresnel integral bend as constructed had an insertion loss of about 0.1 db between 50 and 60 Gc/s, which is close to the theoretical

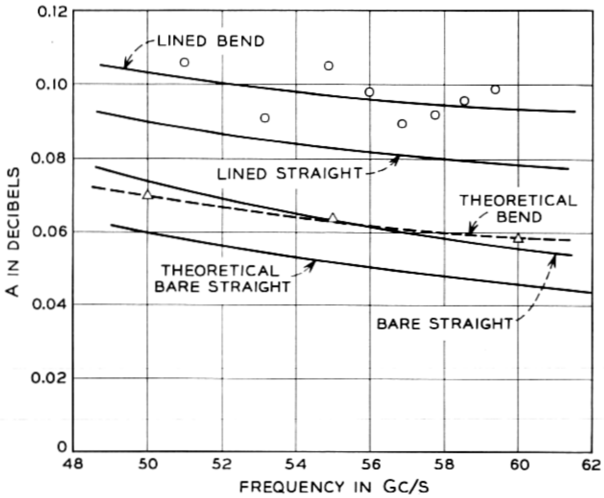

Fig. 6 — Total attenuation of the lined bend compared to straight section of equal length.

value. Lining the round waveguide removed the TE_{01} and TM_{11} mode degeneracy and most of the large loss associated with bending. This enabled the construction of a new and useful device.

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REFERENCES

1. Unger, H. G., Circular Electric Wave Transmission in Dielectric-Coated Waveguide, B.S.T.J., *36*, Sept., 1957, p. 1253.
2. Morgan, S. P., Theory of Curved Circular Waveguide Containing an Inhomogeneous Dielectric, B.S.T.J., *36*, Sept., 1957, p. 1209.
3. Unger, H. G., Normal Mode Bends for Circular Electric Waves, B.S.T.J., *36*, Sept., 1957, p. 1292.
4. Aeroflex-P, Anchor Plastics Company.
5. Conklin, G. E., Reduction of Dielectric Loss in Polyethylene, J. Applied Phys., *35*, 1964, p. 3228.
6. von Hipple, A., *Dielectrics and Waves*, John Wiley & Sons, New York, 1954.
7. King, A. P., Observed 5–6 mm Attenuation for Circular Electric Wave in Small and Medium-Sized Pipe, B.S.T.J., *35*, Sept., 1956, p. 1115.
8. Crawford, A. B. and Hogg, D. C., Measurement of Atmospheric Attenuation at Millimeter Wavelengths, B.S.T.J., *35*, July, 1956, p. 907.