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Information Rate of a Coaxial Cable with Various Modulation Systems

By J. R. PIERCE

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In contrast to inherently broadband media, such as radio, TE_{01} waveguide, or guided coherent light, the attenuation of a coaxial cable increases rapidly with frequency. Thus, while for broadband media broadband transmission schemes (FM or PCM, for example) decrease the power required for a given channel capacity, they would seem to be ill-suited to coaxial cable.

Idealized comparisons are made among digital systems which transmit pulses of various numbers of amplitudes or levels. These show multilevel digital pulse transmission or analog transmission to have greater channel capacity (in the sense of information theory) than digital pulse transmission. Practical difficulties or cost of instrumentation may, in particular instances, dictate the use of single-sideband frequency-division multiplex for efficient voice transmission or binary pulse transmission for efficient digital transmission. Multilevel pulse transmission is a possible alternative if problems of instrumentation can be overcome.

I. INTRODUCTION

In the very early days of information theory, it was proposed that broadband signals might be sent over a narrow-band medium by using more power. Most media (such as radio) are inherently broadband, and it has turned out that for broadband media the advantage lies in the other direction. Broadband modulation systems, such as FM or PCM transmitted by means of binary pulses, increase the signal-to-noise ratio for a given power and help to guard against distortion and interference. Indeed, there are strong arguments for the advantage of broadband modulation systems for any broadband medium, including radio, TE_{01} waveguide and guided optical transmission. With the rising importance of digital transmission, there are of course very strong arguments for digital forms of modulation, such as binary pulse transmission.

Coaxial cable (and other transmission lines) are unique in that the attenuation arises extremely rapidly with increasing frequency. Qualitatively, this suggests that broadband modulation systems may be unsuited to coaxial cable. What do the numbers show?

The purpose of this paper is to illustrate strong effects, not to make exhaustive comparisons or optimizations or to take the practical matters of details of circuit use and limitations of circuit art into account. To this end, a simple, particular case will be considered — a system using standard $\frac{3}{8}$-inch coaxial cable, with a repeater spacing of two miles. Over most of the useful frequency range the received power P_2 will be related to the transmitted power P_1 by*

$$P_2 = P_1 \exp\left[-(f/0.30 \times 10^6)^{\frac{1}{2}}\right].$$

Here f is the frequency in hertz. An average transmitter output power of a tenth of a watt will be assumed, and a repeater noise power density of 1.67×10^{-19} watts/hertz, corresponding to a receiver noise temperature of 12,100°K and a noise figure of 16.2 dB. The calculations would equally apply for a tenth the average power and a tenth the noise.

II. COMPARISONS FOR PERFECT INSTRUMENTATION

In this section we will compare channel capacities, in the sense of information theory, for various signal spectra and, in the case of digital transmission, for various encodings. It is assumed that there is no degradation due to imperfect amplification, imperfect regeneration, imperfect equalization or imperfect timing. The pulse rate of digital pulse systems is taken as $2B$, where B is a sharply limited bandwidth.

As a standard of comparison we will use the channel capacity for the best possible frequency distribution of transmitter power density. This results in a frequency distribution of signal-to-noise ratio which seems unsuited to any useful analog signal, multiplex voice or video. Further, we do not know a practical digital encoding which will realize or even closely approximate this ideal channel capacity.

* This assumes that the loss is due to skin resistance in the conductors. If this is so, there is an unavoidable nonlinear phase lag of $(\frac{1}{2})\ (f/0.30 \times 10^6)^{\frac{1}{2}}$ radians, which amounts to 13 radians or 740 degrees at 200 MHz.

We will also consider the rate or channel capacity for binary and multilevel digital pulse transmission with regeneration and for analog transmission with a flat signal-to-noise ratio. Details are given in Appendices A through D.

In Fig. 1, rate or channel capacity in megabits/second is plotted against bandwidth B in megahertz. The cross is the optimal channel capacity; the dashed line indicates this rate (1581 megabits/second).

The optimal power density is nearly constant over the band, so that the received power is concentrated at low frequencies for which the cable attenuation is low. If we make the transmitted power density increase with frequency so that the received power density and the signal-to-noise ratio at the receiver are constant, the transmitter power is mostly used at high frequencies where the attenuation of the cable is high. This causes a degradation of performance.

The upper solid curve of Fig. 1 applies to this case of constant signal-to-noise at the receiver. This curve shows the channel capacity, subject to this restriction of signal, as a function of bandwidth B. The channel capacity is given by the formula

$$R = B \log_2 (1 + (S/N)). \tag{1}$$

The maximum capacity is about 989 megabits/second at a bandwidth of 80 MHz. This is lower than the 1581 megabits/second for optimal transmitter power distribution, because the power has been concentrated at high frequencies where the attenuation of the cable is large.

We cannot transmit a digitalized signal with the rate given by the upper curve of Fig. 1 because we don't know any practical means of encoding which will give a bit rate very close to the channel capacity. In

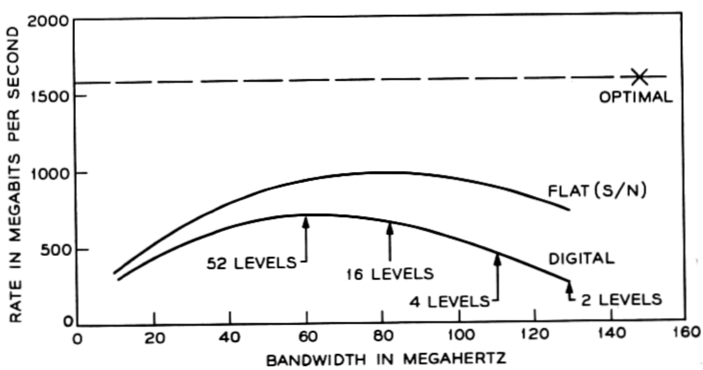

Fig. 1 — Information rate vs frequency for various forms of transmission.

practice, we can transmit digital pulses — either binary or multilevel. The lower curve of Fig. 1 is for pulse transmission with an error rate of one error per repeater in 10^{12} Nyquist intervals (one error in 10^8 for 10,000 repeaters). The optimum rate is about 701 megabits/second at a band width of 60 megahertz. This would call for 52 levels, an impractically large number. The performance for 2, 4, and 16 levels is indicated on the curve.

One can do a little better with pulse transmission by equalizing differently (see Appendix E).

One can compare these bit rates with the channel capacity for analog transmission in an unquantized transmission system. This is done in Appendix D. Equalization for flat signal-to-noise is assumed. A 4000-mile system with 2000 repeaters is assumed, so that the signal-to-noise ratio is only 1/2000 that for a single two-mile link. The bandwidth is taken as 20 MHz — about that of the L4 system, which transmits voice channels by single-sideband frequency-division multiplex.[1] The channel capacity is found to be 286 megabits/second.

It is of some interest to ask what the ideal bit rate would be for digital pulse transmission over such a 4000-mile analog system, with one error in 10^8. This is also computed in Appendix D; the bit rate is found to be 216 megabits/second and the optimum number of levels 42, which is of course impractically large.

These various results are displayed in Table I. We should remember that there is little received power in the upper part of the "148-megahertz" band of the optimal system.

We see that an analog system with a 20-megahertz bandwidth has a somewhat greater channel capacity than a binary digital system, but a smaller channel capacity than a digital system with four or more levels. As we might expect, the ideal bit rate for multilevel quantized transmission over the analog system is less than the ideal bit rate for binary digital transmission with regeneration. However, the difference is small.

Table I

System	Number of Levels	Bandwidth (MHz)	Rate or Channel Capacity (Mb/s)
Optimal	—	148	1581
Digital	2	129	258
Digital	4	111	444
Digital	16	82	656
Digital	52	60	701
Analog	—	20	286
Digital on Analog	42	20	216

III. SOME PRACTICAL CONSIDERATIONS

All the comparisons in Section II are made in terms of average power. This is chiefly because Shannon's simple formula for channel capacity[2]

$$R = B \log_2 (1 + (S/N))$$

holds only for average signal power (and additive Gaussian noise). In practice, the limitation on transmitter power is more likely to be a limitation on peak power than a limitation on average power. I do not believe that a comparison based on peak power would give results substantially different from those of Section II.

The comparisons of Section II are made assuming perfect instrumentation. An actual analog system will be inferior to an ideal system chiefly because of nonlinearity. An actual digital system can be inferior to an ideal system because of imperfect equalization in amplitude and phase, imperfect level control, imperfect timing, and imperfect regeneration.

In present practice, well-instrumented analog systems (such as L4) come closer to ideal performance than well-instrumented digital systems. There are good reasons for this. In digital transmission, equalization, level control, and recovery of timing are not easy. They are usually imperfect and sometimes substantially impair performance. Moreover, the complexity of a regenerative repeater increases as the number of levels is increased. Thus, in practice the comparison of digital and an analog system will be less favorable to digital than the comparison for ideal instrumentation, which is given in Table I.

Nonetheless, comparisons of various coaxial cable systems are not easy. For a given repeater spacing an analog system appears to be somewhat better than a binary digital system for speech transmission, but if we have to transmit digital signals over it the analog system will be considerably inferior to a binary digital system for this purpose. A multilevel digital system might be very considerably superior to either an analog system or a binary digital system for either speech or data transmission.

D. G. Holloway[3] and E. D. Sunde (in unpublished work[4]) have pointed out the advantages of multilevel digital transmission.

IV. ACKNOWLEDGMENT

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APPENDIX A

Optimum Regenerative System

Raisbeck[5] found the optimal power distribution and channel capacity (in the sense of information theory[2]) for a channel for which the average output power density $p'(f)$ is related to the average input power density $p(f)$ by

$$p'(f) = p(f) \exp [-(f/f_0)^{\frac{1}{2}}] \tag{2}$$

and in which the white noise power density at the output is N. He found that

$$\begin{aligned} p(f) &= k - N \exp [(f/f_0)^{\frac{1}{2}}], \qquad & f \leqq B \\ p(f) &= 0, & f \geqq B. \end{aligned} \tag{3}$$

He defines the parameter u as

$$u = k/N. \tag{4}$$

The total power P_o, the channel capacity C, which is the maximum possible value of the bit rate R for the power and the medium, and the bandwidth utilized in transmission, B, are given by

$$P_o = Nf_0(u \ln^2 u - 2u \ln u + 2u - 2) \tag{5}$$

$$C = f_0(\tfrac{1}{3} \log_2 e) \ln^3 u \tag{6}$$

$$B = f_0 \ln^2 u. \tag{7}$$

The transmitter power density $p(f)$ in watts per cycle is

$$p(f) = N\{u - \exp [(f/f_0)^{\frac{1}{2}}]\}. \tag{8}$$

This is nearly constant over most of the band, and falls rapidly to zero at the top of the band.

We will assume

$$\begin{aligned} P_o &= 0.1 \text{ watt} \\ f_0 &= 0.30 \times 10^6 \\ N &= 1.67 \times 10^{-19} \\ P_o/Nf_0 &= 2 \times 10^{12}. \end{aligned}$$

The value of N chosen corresponds to a noise temperature of 12,100 degrees Kelvin, or to a noise figure of about 16.2 dB.

For these figures,

$$u = 4.435 \times 10^9$$
$$C = 1581 \times 10^6 \text{ bits/second}$$
$$B = 148.0 \times 10^6.$$

APPENDIX B

Constant Signal-to-Noise Ratio at the Receiver

We assume that for a transmitted power P_o of frequency f the received power P_1 is

$$P_1 = P_o \exp\,[-(f/f_0)^{\frac{1}{2}}]. \tag{9}$$

Suppose that we deliberately make the received power density constant with frequency. To do this we must make the transmitted power density $p_o(f)$

$$p_o(f) = \frac{P_o \exp\,[(f/f_0)^{\frac{1}{2}}]}{2f_0\{\exp\,[(B/f_0)^{\frac{1}{2}}][(B/f_0)^{\frac{1}{2}} - 1] + 1\}}. \tag{10}$$

Here P_o is the total transmitted power and B is the highest frequency at which power is transmitted — the bandwidth.

If the receiver noise power density has a constant value N, the signal-to-noise ratio (S/N) at the receiver will be

$$(S/N) = \frac{P_o}{2Nf_0\{\exp\,[(B/f_0)^{\frac{1}{2}}][B/f_0)^{\frac{1}{2}} - 1] + 1\}}. \tag{11}$$

APPENDIX C

The Penalty for Digital Pulse Transmission

The bit rates computed in Appendices A and B are the limiting rates for the specified average power and noise densities. To approach them closely in digital transmission would require elaborate, error-correcting encoding. Suppose that instead of this we simply transmit digital pulses, with a signal-to-noise ratio great enough to insure a very low error rate, and without resorting to error correction.

Let us first consider binary transmission in which the pulse voltage is $\pm V/2$, where V is the voltage difference between levels. If $V/2$ is the peak pulse voltage of a $\sin x/x$ pulse, the average signal power is $V^2/4$. If the ratio of this average signal power to the average power of Gaussian noise is 50, there will be an error rate of one in 10^{12} for one repeater or 1 in 10^8 for 10,000 repeaters; this seems a reasonable rate.

For multilevel pulse transmission we say that the error rate will be nearly constant if we keep the ratio of the square of the level spacing, V^2, to the mean square noise voltage constant. This is nearly true for the number of errors in level when the error rate is low. The corresponding number of errors in the binary stream depends on how multilevel-to-binary encoding is done. For a Gray code an error of one level in the multilevel code will cause an error of only one bit in the corresponding binary code.

In computing the average power in the multilevel case we will assume that all levels are equally likely. Then for the same level spacing V the ratio of P_n, the power for n levels to P_2, the power for two levels, is

$$(P_n/P_2) = (n^2 - 1)/3. \tag{12}$$

The ratio of average signal power to average noise power, (S/N), will be

$$(S/N) = 50(P_n/P_2). \tag{13}$$

The rate r in bits per Nyquist interval will be

$$r = \log_2 n \text{ bits/Nyquist interval.} \tag{14}$$

The theoretical limiting rate for a flat signal-to-noise ratio is, in bits per Nyquist interval,[2]

$$c = (\tfrac{1}{2}) \log_2 (1 + (S/N)) \text{ bits/Nyquist interval.} \tag{15}$$

In Table II, (S/N), r, c and $(c - r)$ are given for several values of n.

Table II

n	(S/N)	r	c	$c - r$
2	50	1	2.83	1.83
3	133.3	1.59	3.54	1.95
4	250	2	3.98	1.98
8	1050	3	5.01	2.01
16	4250	4	6.03	2.03

For larger values of n, $(c - r)$ is 2.03.

Thus, for a flat signal-to-noise ratio, the penalty for using digital pulse instead of optimum encoding is about two bits per Nyquist interval. If the bandwidth is B, this means a reduction of rate below optimum of about

$$4B \text{ bits/second.}$$

The penalty for using digital pulse transmission is a shade less than this for binary, and a shade more for large numbers of levels.

The lowest curve in Fig. 1 shows the rate for digital pulse transmission as a function of frequency. The curve is, of course, meaningful only for integer values of n — it has been drawn as a continuous curve merely for sake of appearance. This digital pulse transmission curve falls below the middle curve (ideal rate for flat signal to noise) because of the digital pulse transmission penalty.

The maximum rate for multilevel digital pulse transmission is about 700 bits/second for a bandwidth of 60 MHz. This requires a number of levels (52) which seems impractically large. Rates and bandwidths for various numbers of levels are given in Table III.

APPENDIX D

Binary Digital Transmission Compared with a 20-Megahertz Channel

It is of some interest to try to compare binary digital pulse transmission with a 20-MHz analog channel (the approximate bandwidth of L4).[1]

According to (11) of Appendix B, for $(P/Nf_0) = 2 \times 10^{12}$, the signal-to-noise ratio of a 20-megahertz channel is 3.96×10^7. This is, however, for one two-mile link. If we do not use a regenerative system, noise will accumulate. For a 4000-mile system the noise will be 2000 times as great and the signal-to-noise ratio will be 1.98×10^4. The corresponding channel capacity will be 286 megabits/second. This is slightly larger than the 258-megabit rate for binary digital transmission for the same value of (P/Nf_0).

The conclusion must be that for the repeater spacing, attenuation, power and noise assumed, for transmission of analog signals, the cost of going to the large bandwidth needed for binary transmission, together

TABLE III

n, number of levels	B, bandwidth (MHz)	Mb/s
2	129	258
4	111	444
16	82	656
52	60	701

with the digital pulse transmission penalty, a little more than outweighs the accumulation of noise in a nonregenerative system. As an example, for voice transmission, single-sideband frequency-division transmission will give more voice channels than binary digital transmission for the same repeater spacing.

It is of interest to compute the ideal rate at which we could transmit multilevel digital pulses over this analog system with an error of one in 10^8. For binary transmission and an error rate of one in 10^8 the required signal-to-noise ratio is 32. The signal-to-noise ratio for the 4000-mile, 20-MHz analog system is 1.98×10^4, or 620 times as great. According to (12) of Appendix C, this should allow the number of levels n to be $n = 42$, and $\log_2 42 = 5.4$. Hence, for a 20-MHz bandwidth the ideal transmission rate is $(2)(20)(5.4) = 216$ megabits/second.

Transmission of 42 levels is impractical; transmission of 16 levels might be practical, and for 40 million pulses a second this would mean 160 megabits/second.

APPENDIX E

Optimum Power Density for Digital Pulse Transmission

A channel so equalized as to give a flat signal-to-noise ratio in the received pulse train is not quite optimum for digital pulse transmission. The optimum power distribution is that which will give the greatest signal-to-noise ratio when the received signal is finally equalized to give a flat transmission band of some width B. It can be shown that if the ratio of received power P_1 to transmitted power P_o is

$$P_1 = P_o \exp [-(f/f_0)^{\frac{1}{2}}], \tag{16}$$

the optimum transmitter power density $p(f)$ is

$$p(f) = \frac{P_o \exp [(f/4f_0)^{\frac{1}{2}}]}{8f_o\{\exp [(B/4f_0)^{\frac{1}{2}}][(B/4f_0)^{\frac{1}{2}} - 1] + 1\}}. \tag{17}$$

At the receiver, equalization of the signal for flat overall frequency response will result in a noise density which rises with frequency. The overall signal-to-noise ratio S/N will be

$$(S/N) = \frac{P_o(B/4f_0)}{16Nf_o\{\exp [(B/4f_0)^{\frac{1}{2}}][(B/4f_0)^{\frac{1}{2}} - 1] + 1\}^2}. \tag{18}$$

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