Comparison Between a Gas Lens and Its Equivalent Thin Lens

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(Manuscript received June 23, 1966)

Gas lenses can be replaced by equivalent thin lenses. This paper shows a comparison between ray traiectories through 100 gas lenses and 100 equivalent thin lenses. The agreement is good enough to warrant the use of equivalent thin lenses for the study of the transmission properties of beam waveguides made of gas lenses.

I. INTRODUCTION

Gas lenses have been studied for their potential use as focusing elements in beam waveguides.[1,2,3,4] Two earlier papers[2,3] were concerned with the study of the optical properties of a particular gas lens (see Fig. 1) and came to the conclusion that certain types of gas lenses behave as optically thin lenses. The equivalent thin lens approximating the optical properties of the gas lens is not flat but deformed to fit the shape of the principal surface of the gas lens.

The definition of the equivalent thin lens is based on the optical properties of the gas lens for input rays parallel to the optical axis. For those rays the two lenses are optically equivalent by definition. This equivalence need not necessarily hold true for arbitrary input rays. To show that the equivalent lens can replace the gas lens for arbitrary input rays is the purpose of this paper. For the purpose of optical waveguides a gas lens can be replaced by an equivalent thin lens if the ray trajectories through many gas lenses coincide reasonably closely with the ray trajectories through the equivalent thin lenses. A computer simulated experiment was conducted to determine the ray trajectories through 100 gas lenses and through 100 equivalent lenses and to compare their results. It will be shown in this paper that the two ray trajectories are very nearly the same. This result allows us to use the equivalent thin lenses to study the light guidance properties of gas lenses. This replacement is particularly desirable to examine the wave optics properties

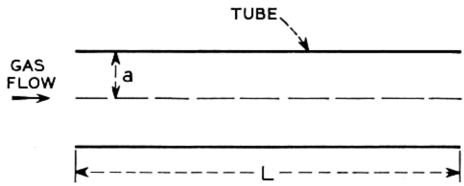

Fig. 1 — Tubular gas lens. A cool gas is blown into a warm tube.

of gas lenses since it would be prohibitively complicated to solve the problem of wave propagation through the actual gas lens.

II. RAY TRACING THROUGH GAS LENSES AND THIN LENSES

The details of determining the principal surface and focal length of a gas lens are discussed in Ref. 3. Typical results of the principal surface and the dependence of the focal length on ray position are shown in Figs. 2 and 3. Strictly speaking there are two principal surfaces. Since they coincide rather closely, however, only one will be considered.

The equivalent thin lens is assumed to have the shape of the principal surface of the gas lens, as shown in Fig. 2, and is assigned the focal length f of the gas lens with its dependence on radius as shown in Fig. 3.

Ray tracing through the gas lens is accomplished by numerical integration of the ray equation. Since rays are being traced through 100 gas lenses in succession, high accuracy is required. For that reason I used the exact ray equation instead of the approximation which was sufficient for the purpose of Ref. 3. The ray trajectory in the gas lens is obtained by numerical integration of the ray equation. This trajectory, however,

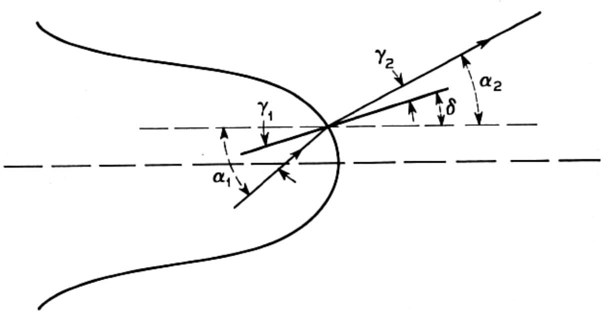

Fig. 2 — The principal surface of the tubular gas lens. The angles used for ray tracing are indicated.

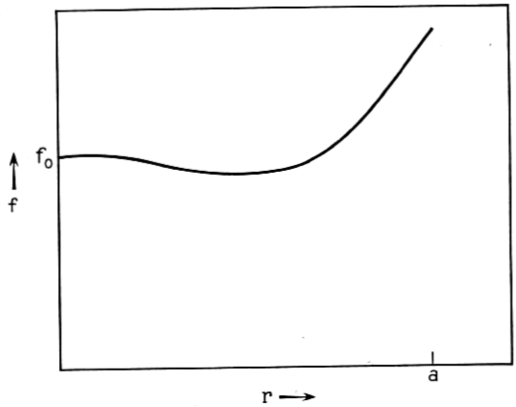

Fig. 3 — Focal length dependence on radius for the tubular gas lens under typical operating conditions.

cannot be used for comparison with the ray trajectory through the equivalent lenses. To compare the two trajectories, the ray entering each gas lens was extended in a straight line into the lens to find the point at which it intercepted the principal surface. This point was used for comparison with the ray trajectory through the thin lenses.

Ray tracing through warped thin lenses has to be done with care since it is easy to violate laws of nature. One might be tempted to use the usual procedure for straight thin lenses and simply break each ray entering the lens at a distance r from the optical axis by an angle β, which is independent of the input angle, according to

$$\tan \beta = -\frac{r}{f}. \tag{1}$$

It was pointed out in Ref. 5 that (1) violates Liouville's theorem of statistical mechanics and that one has to use the equation

$$\sin \gamma_1 = \sin \gamma_2 + F(r). \tag{2}$$

The angle γ_1 is formed between the input ray and the direction normal to the lens surface and γ_2 is the angle between the normal direction and the output ray, Fig. 2. To compute the ray trajectory through the thin lens we have to determine the angle γ_1 from the input angle α_1 of the ray with respect to the optical axis and the angle δ of the lens normal with respect to the optical axis,

$$\gamma_1 = \alpha_1 - \delta. \tag{3}$$

Then we determine γ_2 from (2) and obtain α_2 from the equation

$$\alpha_2 = \gamma_2 + \delta.$$

The function $F(r)$ in (2) is determined from the known focal length of the lens. If $\alpha_1 = 0$, we obtain from (3) $\gamma_1' = -\delta$. The angle α_2 for an input ray parallel to the optical axis is known from the focal length of the lens

$$\tan \alpha_2' = -\frac{r}{f},$$

so that

$$\gamma_2' = \alpha_2' - \delta.$$

The function $F(r)$ is therefore, determined from

$$F(r) = \sin \gamma_1' - \sin \gamma_2'. \tag{5}$$

This complicated procedure does not lend itself easily to the formulation of a difference equation to determine the ray trajectories. An analytical solution for the ray trajectories through warped thin lenses cannot be obtained as easily as for thin straight lenses.[6] However, numerical ray tracing with the help of an electronic computer is only slightly more involved and time consuming as for thin straight lenses.

The results of ray tracings through gas lenses and equivalent thin lenses are shown in Figs. 4 and 5. The solid curve is the gas lens ray trajectory, the broken curve is the corresponding ray trajectory through the equivalent thin lenses. The points entered in these curves are the points of intersection of the (extended) rays with the principal surface of the gas lens or with the equivalent thin lens. These points were connected by straight lines. This procedure represents the ray trajectory through the thin lenses exactly. For the gas lenses it gives the exact ray trajectory only outside of the lenses. The two figures show the ray trajectories only from lens 62 to 100, the agreement is better at the beginning of the trajectory.

The two trajectories agree very well in Fig. 4. If the radius of the gas lens tubes is assumed as $a = 3$ mm the ratio of lens spacing D to lens radius a ($D/a = 1200$ for Fig. 4) corresponds to lenses spaced 3.6 m apart. Fig. 5 was computed with a ration $D/a = 330$ so that with $a = 3$ mm the lens spacing would be $D = 0.99$ m. Even for lenses spaced that close the concept of equivalent thin lenses works quite well.

These results show that the gas lenses can be replaced by equivalent

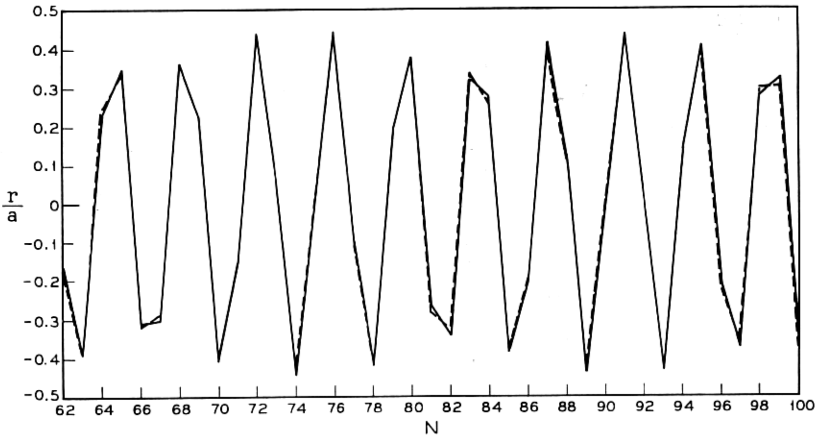

Fig. 4 — Comparison of ray trajectories through gas lenses and equivalent thin lenses. n = lens number, a = radius of gas lens, D = lens spacing, f_o = focal length of rays close to the optical axis, L = length of gas lens. $D/a = 1200$, $D/f_o = 2.16$, $L/a = 50$.

thin lenses. This replacement does not simplify the problem of ray tracing or of tracing wave field through the gas lenses sufficiently to make it accessible to an analytic treatment but it simplifies the numerical treatment greatly and reduces the time of numerical calculations to an economically acceptable level.

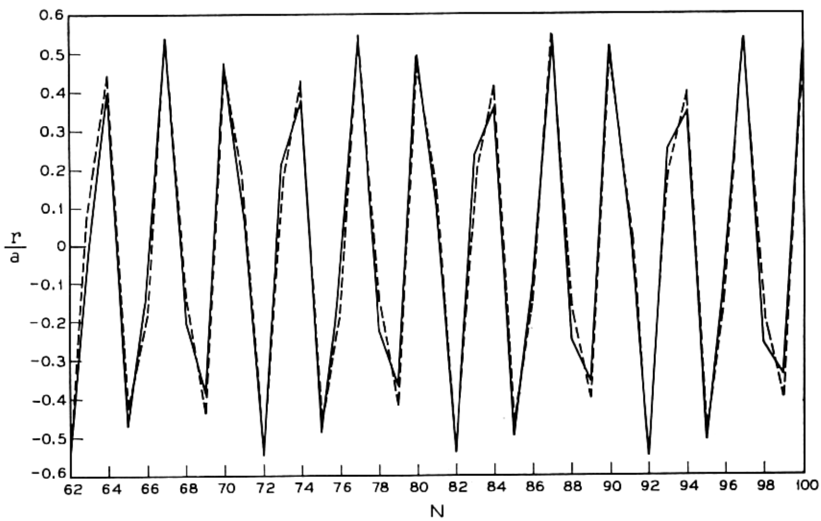

Fig. 5 — Same as Fig. 4 with $D/a = 330$, $D/f_o = 2.74$, $L/a = 50$.

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