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The Dynamics of High-Field Propagating Domains in Bulk Semiconductors

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This paper discusses the dynamics of high-field propagating domains in bulk semiconductors such as gallium arsenide. First, the origin of a high-field domain and its nucleation mechanism are discussed. Next, important properties of a steady-state high-field domain are briefly reviewed. Then, the "unequal" areas rule is derived to explain transient domain behavior. Domain buildup or decay speeds are discussed in detail, and conditions are presented under which two or more domains can exist simultaneously. Finally, the above discussions are applied to explain the high-field domain behavior in pulse circuits, variable frequency oscillators, waveform generators, and domain bypassing schemes. Numerical examples are also given to illustrate how fast these operations can be performed.

I. INTRODUCTION

In the past several years, it was found that several bulk semiconductors showed voltage-controlled differential negative resistance over a certain range of applied electric field. The cause of this negative resistance is vastly different from one material to another. For example, it is attributable to field dependent trapping effect in gold-doped germanium, to phonon-electron interaction in CdS and to inter-valley scattering mechanism in GaAs, InP, CdTe and ZnSe. Regardless of the origin, however, the voltage-controlled differential negative resistance effect nucleates a high field domain in the bulk.[1] Once it is nucleated, the domain travels toward the anode with almost constant velocity; e.g., 10^7 cm/sec for GaAs and 10^5 cm/sec for CdS.[2] As the domain is absorbed into the anode, another domain is nucleated in the bulk and the whole process repeats again.

Although the detailed mechanism of the negative resistance is still a subject of intense discussion, the high-field domain itself appears to have a great significance in future electronics. The objective of this paper is to clarify the dynamics of high-field propagating domains in bulk semiconductors. Since the high field domain in GaAs is presently best understood, we shall mostly concentrate on it. However, similar discussions must be possible for the high field domains in other materials as well.

II. DOMAIN NUCLEATION

The inter-valley scattering effect in GaAs is explained as follows.[3, 4] When electrons are accelerated to a certain drift velocity by an applied electric field inside the bulk, they acquire enough energy to jump into a different valley of the conduction band where the mobility is low compared to the original valley and their drift velocity is reduced. As the applied field increases, more and more electrons come into the low mobility state and on the average the electron drift velocity $v(E)$ decreases. Thus, the material exhibits the differential negative resistance effect as shown in Fig. 1 between E_p and E_v. If the field is increased further, the material shows positive resistance again since most of the electrons are now in the low mobility state and the transition effect fades away. Suppose that we attach two electrodes at the ends of the bulk and slowly increase the voltage between them until the inside field reaches some point in the negative resistance range. The field can stay there indefinitely if no disturbance is applied. How-

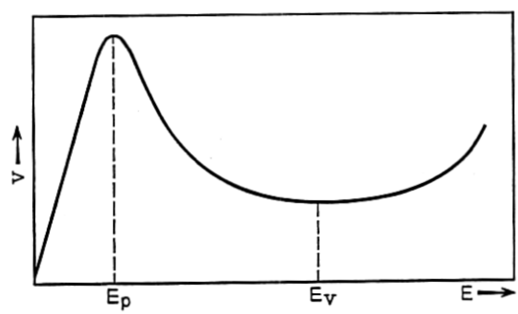

Fig. 1 — Electron velocity vs field inside GaAs.

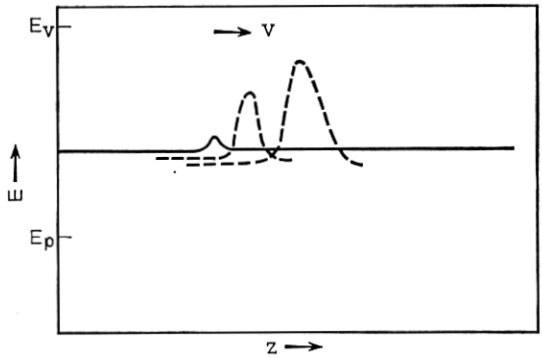

Fig. 2 — Nucleation of a high-field domain.

ever, suppose a small spatial disturbance is given to the field due to noise or due to other transient effect as shown by the solid line in Fig. 2. Then, the electrons in the disturbed region become slower than elsewhere. Hence, the electron density at the trailing edge of the disturbance increases while the leading edge is depleted. This increases the field in the region further and since the area under the field curve should be equal to the applied voltage, the field outside the region decreases slightly. Therefore, taking into account the motion of electrons as a whole toward the anode, the field distribution should look like one of the dotted lines in Fig. 2 sometime after the initial disturbance is applied. When the disturbance is fully grown it is called a high-field domain since the field there is higher than elsewhere. On the other hand, suppose that the opposite type of disturbance is initially given to the field as shown by the solid line in Fig. 3. Then, a similar reasoning to the above leads to a conclusion that the disturbance grows with time into a low field domain as illustrated by the dotted lines in Fig. 3. These two disturbances are equally likely to take place. However, if we raise the field from zero to the value just slightly above the threshold field, E_p in Fig. 1, the first type of disturbance grows but the second one does not since the disturbed portion in the latter is mostly in the positive resistance range. Therefore, under ordinary circumstances, the first one is expected to dominate. If the applied voltage is instantaneously increased to a large value and then gradually reduced, the second type will dominate.

Now since we saw that a small disturbance grew, let us consider how fast it grows initially. To do so, we have to investigate two equa-

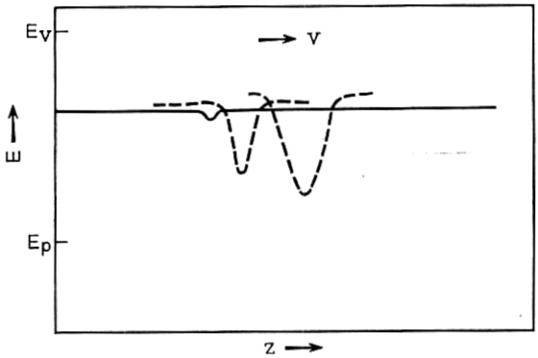

Fig. 3 — Nucleation of a low-field domain.

tions governing the phenomenon. The first one is a one-dimensional Poisson's equation

$$\frac{\partial E}{\partial z} = \frac{e}{\epsilon}(n - n_o), \tag{1}$$

where $-e$ is the electron charge, n the electron density, and n_o the donor density. The second equation is

$$J = env(E) - eD\frac{\partial n}{\partial z} + \epsilon\frac{\partial E}{\partial t}, \tag{2}$$

where J is the total current density, D the diffusion constant and ϵ the delectric constant of the material. The total current consists of the conduction, diffusion and displacement currents. The positive direction for J and E is taken to be from the anode to cathode and that for $v(E)$ from the cathode to anode. Suppose some disturbance is given inside the bulk and assume that the effect on the electron density has not yet reached z_1 and z_2, where z_1 is on the left and z_2 on the right-hand side of the disturbance. Then, from (1) we have

$$\frac{\partial E(z_1)}{\partial z} = \frac{\partial E(z_2)}{\partial z}.$$

Furthermore, $J = J(z_1) = J(z_2)$ gives

$$\epsilon\frac{\partial}{\partial t}\{E(z_1) - E(z_2)\} = en\{v(E(z_2)) - v(E(z_1))\}.$$

The initial condition is $E(z_1) - E(z_2) = 0$, hence $E(z_1)$ has to stay equal to $E(z_2)$. Let E_o be the field at z_1. Then,

$$E_o = E(z_1) = E(z_2).$$

In terms of E_o, the current through the device is given by

$$J = en_o v(E_o) + \epsilon \frac{\partial E_o}{\partial t}. \tag{3}$$

Substituting (1) and (3) into (2), we have

$$\frac{\partial}{\partial t}(E - E_o) = \frac{en_o}{\epsilon}\{v(E_o) - v(E)\} + D\frac{\partial^2 E}{\partial z^2} - v(E)\frac{\partial E}{\partial z}.$$

Integrating with respect to z from z_1 to z_2, we obtain

$$\frac{d}{dt}\int_{z_1}^{z_2}(E - E_o)\,dz = \int_{z_1}^{z_2}\frac{en_o}{\epsilon}\{v(E_o) - v(E)\}\,dz. \tag{4}$$

This equation holds even when the disturbance is large. However, when it is small, using the relation

$$v(E) \cong v(E_o) + \frac{dv}{dE}(E - E_o) \tag{5}$$

(4) becomes

$$\frac{d}{dt}\int_{z_1}^{z_2}(E - E_o)\,dz = -\left(\frac{en_o}{\epsilon}\frac{dv}{dE}\right)\int_{z_1}^{z_2}(E - E_o)\,dz. \tag{6}$$

Since e is positive and dv/dE is negative in the negative resistance range, the size of the disturbance

$$\left|\int_{z_1}^{z_2}(E - E_o)\,dz\right|$$

increases with time. The time constant is given by

$$\tau = \left(\frac{en_o}{\epsilon}\frac{dv}{dE}\right)^{-1}. \tag{7}$$

The negative value means the growth instead of decay.

III. STEADY STATE DOMAIN[5]†

Next, let us consider the domain in the steady state. When a domain travels with a constant speed without changing its shape, we call it

† This section closely follows Ref. 5. However, it is included here for completeness and continuity of discussion.

in the steady state. Let v_d be the velocity of the domain. Then, both n and E must be functions of a single variable $\xi = z - v_d t$ in the steady state. Therefore, we have

$$\frac{\partial E}{\partial z} = \frac{dE}{d\xi}, \qquad \frac{\partial E}{\partial t} = -v_d \frac{dE}{d\xi}, \qquad \frac{\partial n}{\partial z} = \frac{\partial n}{\partial \xi}.$$

Equation (1) becomes

$$\frac{dE}{d\xi} = \frac{e}{\epsilon}(n - n_o). \tag{8}$$

This shows that $n = n_o$ whenever $dE/d\xi = 0$ and vice versa. As a result, n and E should look like Fig. 4(a) and (b), respectively. Note that the maximum field is located at the neutral position. On the other hand, (2) becomes

$$D\frac{dn}{d\xi} = n(v - v_d) - n_o(v_o - v_d), \tag{9}$$

where v_o is the electron velocity outside the domain and use is made of $J = en_ov_o$. Eliminating ξ from (8) and (9), we have

$$D\frac{dn}{dE} = \frac{n(v - v_d) - n_o(v_o - v_d)}{\frac{e}{\epsilon}(n - n_o)}$$

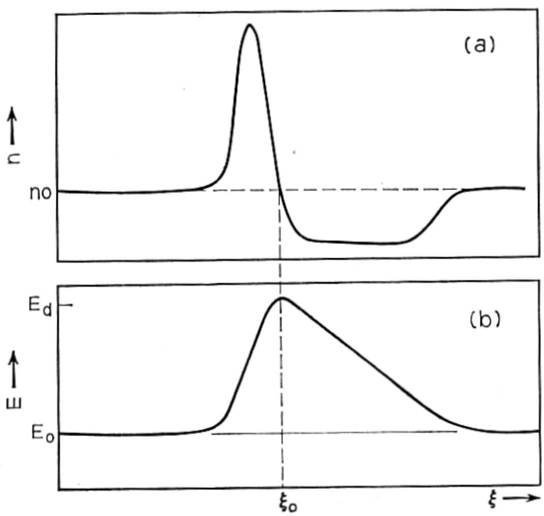

Fig. 4 — Steady-state high-field domain.

which is equivalent to

$$\frac{n - n_o}{nn_o}\frac{dn}{dE} = \frac{\epsilon}{en_oD}\left[v - v_d - \frac{n_o}{n}(v_o - v_d)\right]. \tag{10}$$

Integrating (10) with respect to E and remembering that $n = n_o$ when $E = E_o$, where E_o is the field outside the domain, we get

$$\frac{n}{n_o} - \ln\frac{n}{n_o} - 1 = \frac{\epsilon}{en_oD}\int_{E_o}^{E}\left[v - v_d - \frac{n_o}{n}(v_o - v_d)\right]dE.$$

The integration on the right-hand side can be carried out over the leading or trailing edge of the domain. However, when the upper limit is set equal to the peak field E_d of the domain, the integral should vanish in both cases since $n = n_o$ at the peak point. However, this is impossible unless $v_o = v_d$, since the integral of $(v - v_d)$ is the same while the integral of the remaining term is different in the two cases because $n < n_o$ over the leading edge and $n > n_o$ over the trailing edge. It follows from this that the velocity of the steady-state domain is equal to the electron velocity outside the domain. Furthermore, since

$$\int_{E_o}^{E_d}(v - v_d)\,dE = \int_{E_o}^{E_d}(v - v_o)\,dE = 0$$

for a given v_o, E_d can be determined by equating two shaded areas in Fig. 5. Thus, the relation v_o vs E_d should look like the broken line in Fig. 5. As the domain becomes larger, E_d increases from E_p and v_o decreases from the peak velocity v_p. However, E_d cannot exceed E_{dm}, the field corresponding to the intersection between the broken line and the solid line. This is because the equal areas rule cannot be satisfied beyond this point. When E_d reaches E_{dm}, the outside field becomes E_{om} and the velocity v_{om}.

The relation between n and E for a domain can be determined from

$$\frac{n}{n_o} - \ln\frac{n}{n_o} - 1 = \frac{\epsilon}{en_oD}\int_{E_o}^{E}(v - v_o)\,dE. \tag{11}$$

With this known n vs E, the shape of the domain can be calculated using the integral of (8), i.e.,

$$\xi = \xi_o + \frac{\epsilon}{e}\int_{E_d}^{E}\frac{dE}{n - n_o},$$

where ξ_o indicates the position of E_d.

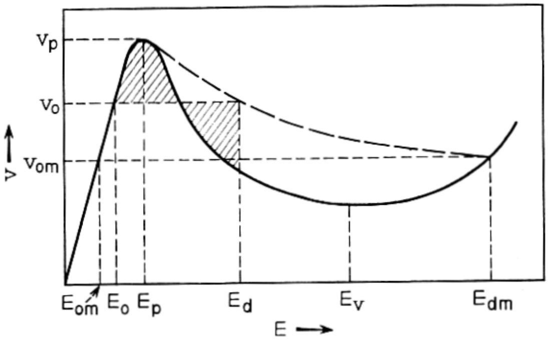

Fig. 5 — The broken line shows the domain velocity vs peak field.

If the diffusion constant is small, the right-hand side of (11) becomes large; hence, $n \gg n_o$ over most part of the trailing edge and $n \cong 0$ over most part of the leading edge. As a result, the trailing edge becomes considerably shorter than the leading edge.

The integral defined by

$$V_{ex} = \int_{\text{domain}} (E - E_o)\, d\xi$$

is called the domain excess voltage.[6] This amount of voltage is necessary to support the domain in addition to the voltage drop in the bulk given by $E_o l$, where l is the sample length. V_{ex} is a function of E_o. When E_o is close to E_p, E_d is also close to E_p, the domain is small and V_{ex} is small. As E_o decreases, E_d increases as we saw in Fig. 5 and V_{ex} increases. The highest field in the domain is limited by E_{dm}. However, V_{ex} can increase without limit by increasing the domain width. For E_o below E_{om}, there is no steady-state domain. As a result, V_{ex} as a function of E_o should look like the solid line in Fig. 6. The detailed shape changes with n_o. In general, for a given E_o, V_{ex} decreases with increasing doping.

Now, suppose we apply voltage V to a device with length l. Let us draw a straight line through (o, V) and $(V/l, o)$ in Fig. 6, as shown by the broken line and let P be the intersection with the V_{ex} curve. Since $V - V_{ex}$ is exactly equal to $E_o l$ for P, the terminal voltage becomes V if a domain determined by P exists in the device. Suppose somehow V_{ex} becomes higher than the value given by P. Then, for the steady state, $E_o l$ becomes smaller but not enough to compensate the increase in V_{ex}; hence, V_{ex} has to decrease. Similarly, when V_{ex} decreases, it has to return to P to satisfy the terminal voltage condition. Consequently, P represents the stable operating point. Thus, Fig. 6 can conveniently be utilized to determine V_{ex} and E_o of the steady-state domain. Note that if the applied voltage is high, V_{ex} is correspondingly high and E_o becomes almost equal to E_{om}. Then, the current density $en_o v(E_o)$ becomes almost constant regardless of a small variation in the applied voltage. Such a domain is called a saturated domain.

When V/l is slightly smaller than E_p but V is large, there exists two intersections between the solid and broken lines in Fig. 6. In this case, the field in the absence of a domain is smaller than E_p and no domain will be nucleated. However, if a domain exists, it can continue to travel without collapsing. Of these two intersections, only the one which corresponds to the smaller E_o represents the stable operating points. For the other intersection, if V_{ex} increases slightly, $E_o l$ decreases more than enough to compensate the increase in V_{ex}; hence, V_{ex} has to increase further until the stable operating point is reached. Similarly, if V_{ex} initially decreases, the domain will disappear. Suppose that the broken line is momentarily raised up to launch a domain by applying additional voltage to the terminal. As the additional voltage decreases, the intersection moves down and reaches the stable operating point without ambiguity. $E_p l$ is called the threshold voltage since it is necessary to launch a domain.

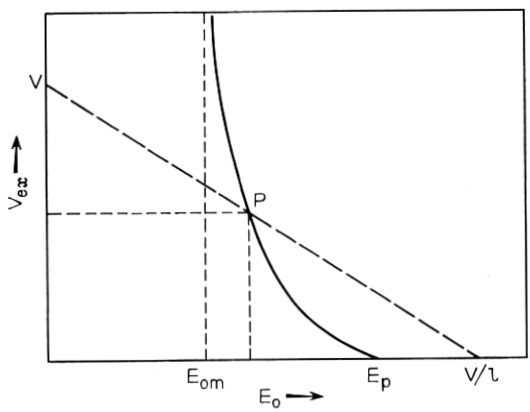

Fig. 6 — Domain excess voltage vs the outside field.

IV. TRANSIENT BEHAVIOR OF DOMAINS

Now, suppose that we apply a certain voltage higher than $E_p l$ to a device increasing from zero value. Since the field reaches the threshold value E_p first near the cathode because of high resistive layer or inhomogeneity generally existing there, a disturbance grows into a high-field domain near the cathode and travels toward the anode. As the disturbance grows, the cathode current decreases until it reaches $J \times S = en_o v(E_o)S$, where E_o is determined from Fig. 6 and S is the cross-sectional area of the device. This current is maintained until the high-field domain reaches the anode. As the domain disappears into the anode, the field E in the device and hence the current increases to keep the terminal voltage constant. When the field near the cathode reaches the threshold value again, another disturbance grows into a high field domain and the whole process repeats. As a result, the cathode current like that in Fig. 7 is obtained. The height of the peak current is determined by $en_o v(E_p)S$ plus the displacement current due to the rapid change in E. The displacement current initially increases as V_{ex} increases. However, the peak current tends to saturate with further increasing V_{ex}. Also note that for a given V_{ex} the longer sample gives less displacement current. Similar displacement current exists during the decrease of the current and it may show up as the overshoot of decreasing current.

To get a high-field domain, some initial disturbance in the field distribution is necessary. This may well be due to noise, if the field is increased very slowly. However, in most cases the following process is considered to dominate. Suppose there is a region where the donor density is slightly lower than elsewhere. For each value of the field strength well outside the region, there is a corresponding steady state

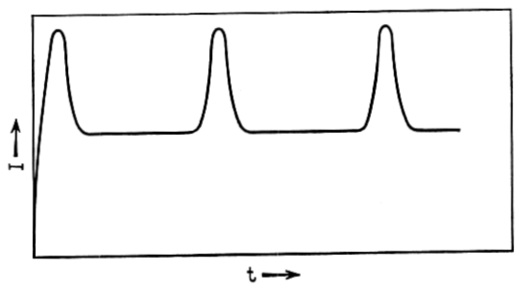

Fig. 7 — Cathode current of bulk semiconductor oscillator.

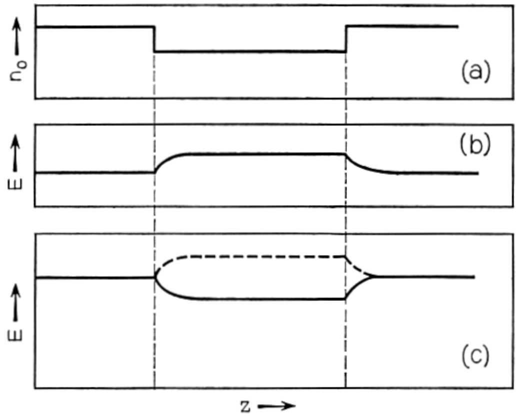

Fig. 8 — Origin of small disturbance in the field distribution.

field distribution. Except for some deviations near the ends of the region, the inside field E_i should be related to the outside field E_o through

$$n_{oi}v(E_i) = n_{oo}v(E_o),$$

where n_{oi} is the donor density in the region and n_{oo} outside the region.* Therefore, the field should look like the solid lines in Fig. 8(b) or (c) depending on whether it is in the positive or negative resistance range, respectively. However, if we change the outside field quickly, from the positive to negative range, since the displacement current has to be approximately equal everywhere and since the inside change is smaller than the outside, the inside field will overshoot the steady-state value as shown by the broken line in Fig. 8(c). As a result, a definite discrepancy from the steady-state value takes place which grows into a high-field domain. In practice, there may be a number

* In this case, corresponding to (4), we have

$$\frac{d}{dt}\int_{z_1}^{z_2}(E - E_o)\,dz = \int_{z_1}^{z_2}\frac{e}{\epsilon}\{n_{oo}v(E_o) - n_o v(E)\}\,dz,$$

which is equivalent to

$$\frac{d}{dt}\int_{z_1}^{z_2}(E - E_s)\,dz = \int_{z_1}^{z_2}\frac{en_o}{\epsilon}\{v(E_s) - v(E)\}\,dz,$$

where E_s indicates the steady-state field. Since this is identical to (4) except that E_0 is replaced by E_s, no major modification is required in our discussion of growth rate if V_{ex} is redefined by the integral on the left-hand side.

of such nucleating sites. However, only one domain will be fully grown since two steady-state domains cannot exist at a time as we shall discuss shortly.

So far, we have studied the case where the terminal voltage is kept constant. Next, let us consider the case with a constant current source. When we gradually increase the current from zero, except in the vicinity of the cathode, the electron velocity in the device follows the $v(E)$ vs E curve in Fig. 1 until it reaches $v(E_p)$. If we increase the current further and if the sample does not burn out, E probably jumps to a high value where $v(E)$ is equal to $v(E_p)$ and then follows the $v(E)$ vs E curve as before. If we decrease the current, the electron velocity follows the curve until it reaches $v(E_v)$ and then E jumps to a low value where $v(E) = v(E_v)$. There is hysteresis. However, during this process, the field is almost everywhere in the positive resistance range and the device is stable. Now, suppose somehow a high-field domain is already present. If the source happens to supply the same current as the steady-state domain requires, it will continue to travel. However, if the supply current is slightly different, say, larger than the domain current, the electron density increases at the trailing edge and decreases at the leading edge of the domain. As a result, V_{ex} increases. However, the larger the V_{ex} is the smaller the steady-state current becomes. Therefore, no steady state can be achieved. A similar argument holds for the case where the supply current is smaller than the domain current. In this case, V_{ex} decreases to zero. To consider the same problem from a different angle, let us draw a line representing the constant current condition in Fig. 6. It becomes a vertical line. The possible intersection with the V_{ex} curve gives an instable operating point which we discussed before. Thus, with a constant current source, a high-field domain is instable. The reason why two steady-state domains cannot exist simultaneously in a device can be seen in a similar manner. If one of them requires a larger current than the other, V_{ex} of the second one increases. As a result, the first one quickly disappears and all the excess voltage is absorbed by the second one requiring less current. We shall have more to say about this later.

Next, let us consider the situation where no high-field domain exists but the field is in the negative resistance range over a certain length of the device. Suppose a small disturbance exists inside the region. In this case E_o is fixed but (6) still shows that the initial disturbance grows with time with the time constant given by (7). As a result, the terminal voltage should fluctuate considerably.

In many cases, it is desirable to estimate how quickly domains can grow or decay. The starting point is (4). If the diffusion constant is small, the trailing edge is considerably shorter than the leading edge; hence, the primary contribution to the integral comes from the leading edge. Since $n \cong 0$ over the leading edge, we have

$$\int_{z_1}^{z_2} \frac{n_o e}{\epsilon} \{v(E_o) - v(E)\}\, dz \cong \int_{E_o}^{E_d} \{v(E_o) - v(E)\}\, dE,$$

where use is made of (1). Substituting the above results into (4) we obtain

$$\frac{dV_{ex}}{dt} \cong \int_{E_o}^{E_d} \{v(E_o) - v(E)\}\, dE. \qquad (12)$$

For the steady state, $dV_{ex}/dt = 0$ and (12) reduces to the equal areas rule discussed before. It is now obvious that if we draw a figure similar to Fig. 5 and if the lower-right shaded area is larger or smaller than the upper shaded area, the domain grows or decays, respectively. As the difference becomes larger, the rate of change of the domain size increases. It is also obvious why a domain is instable under a constant current condition. Since E_o is fixed if E_d increases slightly from the steady-state value, V_{ex} increases and hence E_d further increases and no steady state is reached.

When $E_d - E_o$ is small, using (5), (12) becomes

$$\frac{dV_{ex}}{dt} = \int_{E_o}^{E_d} \left(-\frac{dv}{dE}\right)(E - E_o)\, dE = \left(-\frac{dv}{dE}\right)\tfrac{1}{2}(E_d - E_o)^2. \qquad (13)$$

However, if we assume that the leading edge w is depleted, by integrating (1) twice, we have

$$\tfrac{1}{2}(E_d - E_o)^2 = \frac{1}{2}\left(\frac{n_o w}{\epsilon} e\right)^2 = \frac{n_o e}{\epsilon}\left(\frac{1}{2}\frac{n_o e}{\epsilon} w^2\right) \cong \frac{n_o e}{\epsilon} V_{ex}\,. \qquad (14)$$

Substituting (14) into (13), we obtain

$$\frac{dV_{ex}}{dt} \cong \left(-\frac{dv}{dE}\right)\frac{n_o e}{\epsilon} V_{ex}$$

which gives the same answer as (6). Therefore, provided that E_d is calculated from V_{ex} through (14), the simple formula (12) is also applicable for small disturbances whose electron density distribution is quite different from the one assumed in the derivation.

Now suppose that there are two regions in a device where the donor density is slightly lower than elsewhere. If the terminal voltage is increased to a certain value to bring the field into the negative resistance range, the field disturbances at these two regions start to grow exponentially and at the same time the field E_o outside the disturbances decreases to compensate the increase of the field in the disturbances. After a while the disturbances become large enough so that we can call them domains. The domain from the larger nucleation site of the two is larger than the other but both continue to grow until E_0 reaches a point where the right-hand side of (12) becomes zero for the smaller domain. Then, the smaller domain stops growing and starts to decay because the larger one continues its growth. As a result, the smaller domain will finally disappear and the one from the larger nucleation site remains. However, suppose that the difference in size between the nucleation sites is small. Then, when the smaller domain stops growing, the right-hand side of (12) for the larger one is also small; hence, some time elapses before the larger one absorbs the excess voltage from the smaller one. If the larger domain is on the anode side of the smaller one, it may reach the anode before the absorption is completed. If this happens, the smaller domain starts to grow again as the larger one disappears into the anode. As a result, if there are several nucleation sites of comparable size in a device, one period is not completed until the largest remaining domain becomes smaller than the initial disturbance the largest nucleation site can give.

Summarizing the above discussion, we list some of the important properties of a domain as follows:

(*i*) A high-field domain is nucleated when the field reaches the negative resistance range from the lower side of the v vs E characteristic.

(*ii*) With a constant voltage source, the domain reaches a steady state. However, with a constant current source, no steady-state domain is realized.

(*iii*) The steady-state domain velocity is equal to the electron drift velocity outside the domain.

(*iv*) For a given terminal voltage, the domain excess voltage and the outside field can be determined from Fig. 6.

(*v*) The transient behavior of a domain can be approximately determined by (12).

In the above discussion, the diffusion constant was assumed to be independent of the field. However, many theoretical studies indicate that it is a function of E. In that case, the equal areas rule no longer holds and the domain velocity may not be equal to the electron velocity.* Furthermore, an inhomogeneity in the material may create

* If $D\partial n/\partial z$ is replaced by $\partial/\partial z\ (Dn)$ in (2), (4) follows even when D is a function of E. Therefore, (12) is considered to be a valid approximation as long as the magnitude of D is small.

an extremely high but localized field within the bulk which ionizes deep donors or even generates hole-electron pairs. Then, the effective n_o becomes larger and V_{ex} lower. However, the following discussions will not be affected.

V. PULSE CIRCUITS

Let us consider the *I-V* characteristic of a device with the possible presence of a domain. Since for each *I*, the terminal voltage is equal to V_{ex} plus the voltage drop in the device, it should look like Fig. 9. There is hysteresis, but otherwise the curve is very similar to the *I-V* characteristic of a tunnel diode. Therefore, many pulse circuits developed in connection with tunnel diodes are also applicable with the present devices; e.g., relaxation oscillator, monostable circuit and pulse inverter. The main difference is that once a domain is launched, it determines the device current rather than the external circuits and after a while the domain disappears at the anode.

VI. VARIABLE FREQUENCY OSCILLATOR[7]

Suppose the device has the form of a trapezoid as shown in Fig. 10. If the taper is gradual, high-field domains are expected to have similar properties to one in a uniform device. Therefore, assuming that the relation between the domain excess voltage and the field immediately outside the domain is given by Fig. 6, let us consider how to obtain the outside field as a function of the domain position. First assume

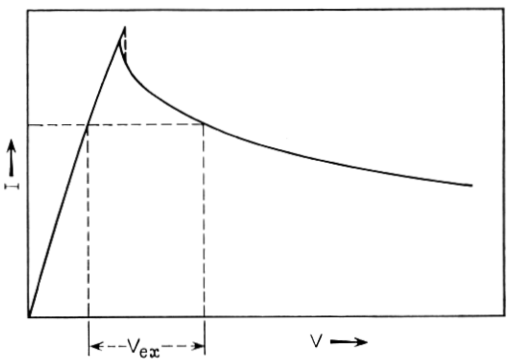

Fig. 9 — *I-V* characteristic of a uniform device.

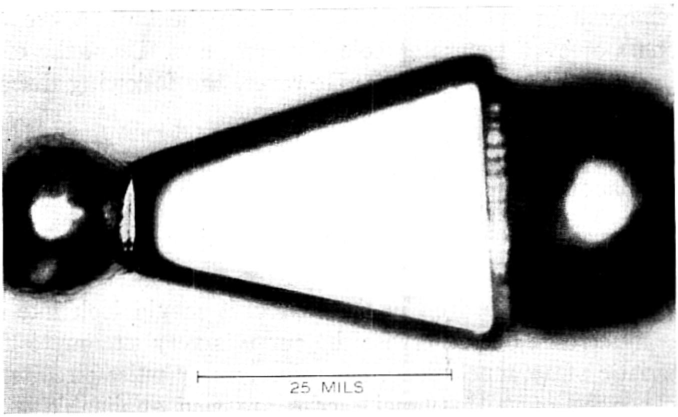

Fig. 10 — An example of trapezoidal oscillators.

that the outside field is E_o when the domain is located at $z = \xi$. The conduction current is given by $env(E_o)S(\xi)$. Neglecting the diffusion and displacement currents outside the domain, the field at arbitrary z except the domain location can be determined from Fig. 1, utilizing

$$I = env(E_o)S(\xi) = env(E)S(z).$$

Once the field is determined as a function of z, the area under it plus $V_{ex}(E_o)$ gives the terminal voltage. Keeping this in mind, draw a family of field curves with the terminal current as a parameter as shown in Fig. 11. The highest field curve needed corresponds to $I = env(E_p)S(0)$ and is called the threshold line and the area under it the threshold voltage V_{th}. Calculate the area under each curve and add V_{ex} for each point on the $E - z$ plane to obtain the terminal voltage V. From this, we obtain a family of constant terminal voltage contours as shown in Fig. 12. Note that a straight line $E = E_{om}$ corresponds to $V = \infty$. Also note that there is no contour crossing the vertical axis above a certain value E_c which is determined as follows. On the V_{ex} vs E_o plane draw a curve representing the area between the threshold line and various field curves as a function of the field at the cathode. This curve intersects the E axis as well as the V_{ex} curve at E_p. The other intersection with V_{ex}, if any, gives E_c as shown in Fig. 13. Suppose we apply a certain terminal voltage slightly larger than V_{th} and a domain starts to form at the cathode. Then, the field inside the device drops. But if it is higher than E_c at the cathode, the decrease of the area exceeds the corresponding V_{ex}; hence, the domain

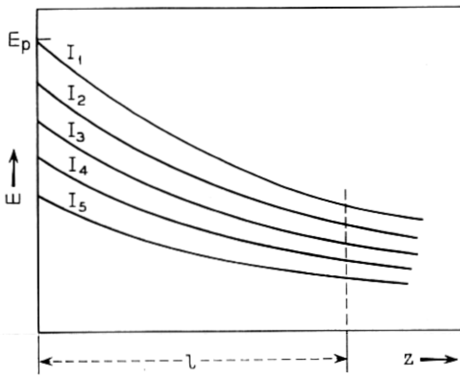

Fig. 11 — Field curves in a tapered device.

further grows until it becomes E_c. If the device length is sufficiently short, E_c may not exist depending on the shape of V_{ex} vs E_o.

As the domain travels toward the anode, the field immediately outside the domain follows one of the constant voltage contours in Fig. 12. When the contour intersects the threshold line, the field at the cathode reaches the threshold value E_p and a new domain starts to nucleate. Since the new domain in the narrow region grows according to the unequal areas rule given by (12), the old one in the wider region quickly decays and a new period begins. However, the cathode current does not decrease as quickly as in a uniform device since the old domain tends to keep the cathode field up as it decays. Some contours passing through the vicinity of E_c may turn back before they cross the threshold line. In such a case, at the turning point, the terminal

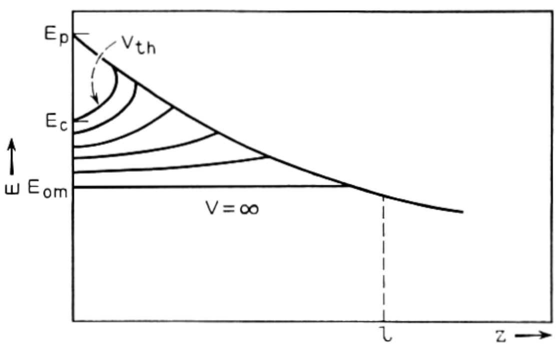

Fig. 12 — Constant terminal voltage contours.

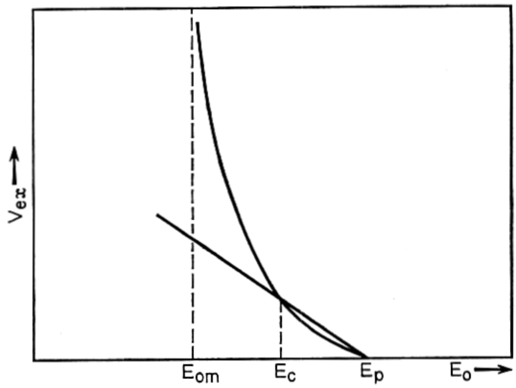

Fig. 13 — Explanation for the method to obtain E_c.

voltage becomes insufficient to support the domain due to the voltage drop in the bulk. The domain quickly decays and the field rises to the threshold line to start a new period as before. Note that the maximum current at the end of a period is approximately $env(E_p)S(0)$ regardless of the terminal voltage. In general, as the terminal voltage increases, the turning point or the intersection is further away from the cathode and the domain velocity becomes slower. As a result, the period gets longer. This effect gives a voltage-controlled variable frequency oscillation as shown in Fig. 14. If the straight line $E = E_{om}$ intersects the threshold line within the device, no matter how large terminal voltage is (of course within a certain limit), the domain cannot reach the anode. On the other hand, if the intersection is outside the device, the domain can reach the anode generating a pulse similar to the one shown in Fig. 7 at the end of a period.

VII. CURRENT WAVEFORM GENERATORS[8]

For a saturated domain, the velocity is constant and the current density at the domain is constant. If the cross-sectional area of the device varies with the distance from the cathode, the cathode current also varies as the domain moves along the sample since it is proportional to the current density times the area. Provided that the variation of the cross section is gradual, the current waveform should become an exact replica of the shape of the device excluding the pulse corresponding to the domain disappearance at the anode. In order to start the domain at or near the cathode, the field there should

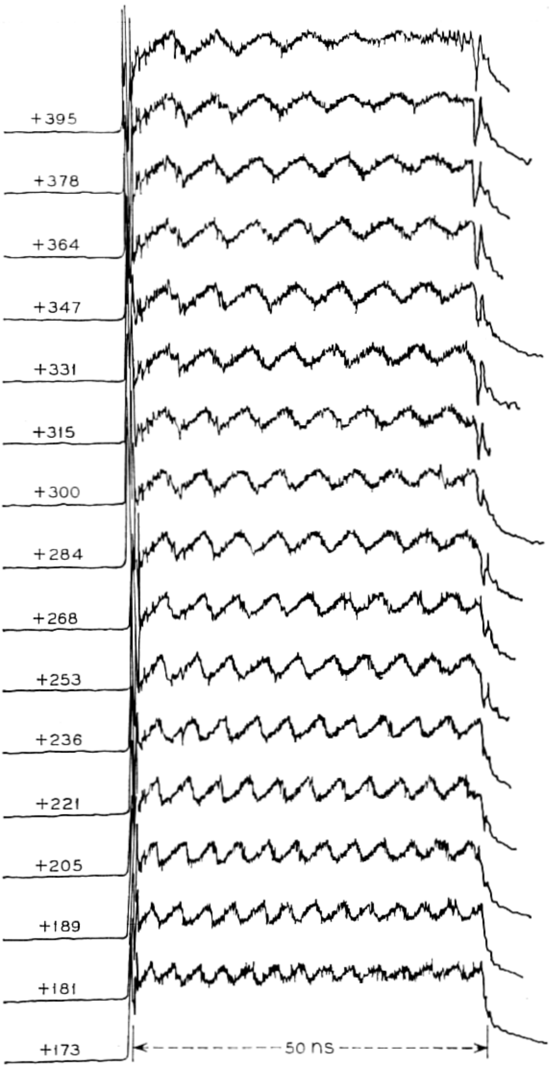

Fig. 14 — Variable frequency oscillation of a trapezoidal oscillator. The supply voltages are indicated on the left-hand side.

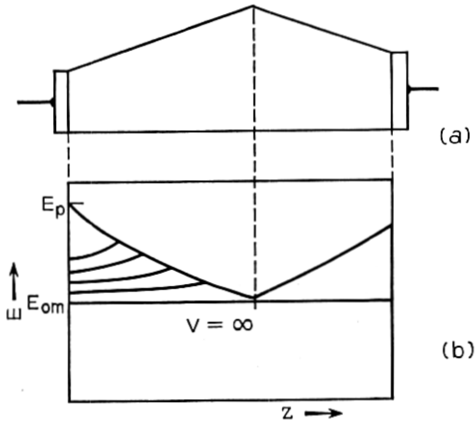

Fig. 15 — Explanation for the sudden change in frequency observed in certain nonuniform devices.

reach E_p before it does elsewhere. This limits the smallest cross-sectional area we can make. In some cases because of the high resistivity layer at the cathode, this minimum can be about 80 percent of the cathode cross section. In order to let the domain arrive at the anode, the threshold field line should never be lower than E_{om} anywhere in the device. This limits the largest cross-sectional area we can make. However, letting the threshold line cross the E_{om} limit, the pulse due to the domain disappearance at the anode can be eliminated. In some waveform generators, the frequency of oscillation suddenly increases as we decrease the terminal voltage. For instance, the constant terminal voltage contours for the device shown in Fig. 15(a) should look like Fig. 15(b). Therefore, if the terminal voltage is reduced slightly, the domain travels only halfway through and the frequency rises approximately one octave as shown in Fig. 16.

VIII. DOMAIN BYPASSING[9]

So far, we have considered the shaping of an active bulk region. However, a similar effect can be obtained by changing the doping density. In addition to this, there is an interesting method of getting similar or more versatile functions by means of bypass circuits. To explain the principle, let us consider the simplest case illustrated in Fig. 17. There are two additional ohmic contacts attached to the active bulk region. Let D_1 be the part between the cathode and the first

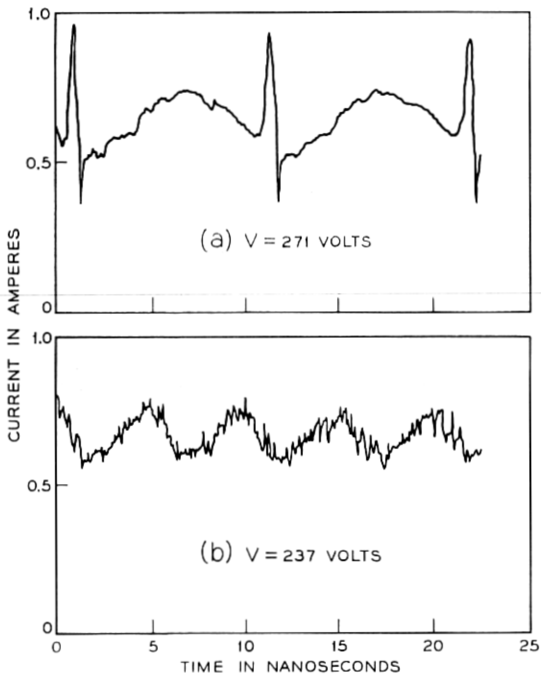

Fig. 16 — Current waveforms observed with a device illustrated in Fig. 15.

contact, D_2 between two contacts, and D_3 the remainder. Let d indicate the length of D_2 and h the device thickness. If a saturated domain is in D_1, the cathode current is given by $en_o v(E_{om})S$. When the domain moves into D_2, the current becomes approximately $en_o v(E_{om})S + (V_{ex}+E_{om}d)/R$, where R is the resistance of the interconnection between the additional contacts. Finally, when the domain gets into D_3, the current returns to $en_o v(E_{om})S$ again. Therefore, the cathode current should look like Fig. 18(a). In addition to regular pulses,

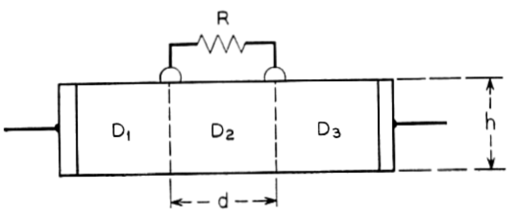

Fig. 17 — Domain bypassing scheme.

there are broad pulses corresponding to $(V_{ex}+E_{om}d)/R$. Now, suppose the interconnection is open circuited, then the additional pulses disappear as shown in Fig. 18(b). Thus, we get a means of controlling the waveform from the outside. The height of the additional pulses, however, cannot be made higher than the regular pulses. If we try to get higher pulses by decreasing R, the field at the cathode reaches the threshold value making a new domain nucleate there and the old one in D_2 disappear. The sharpness of the pulse edge is limited probably due to the spreading resistance of the contacts. As the thickness h of the bulk gets thinner, the definition is expected to improve.

There are many variations of the above scheme. By providing several contacts and interconnecting them through appropriate networks, various waveforms can be realized. Alternatively, contacting a relatively large electrode through a high resistive layer on one side of the active region and shaping the electrode, again produces various waveforms. Another interesting possible application is a photosenser. Suppose R is photosensitive. Then, the height of the additional pulses varies with illumination. If many such photoelements are lined up along a relatively long device, the domain provides automatic scanning action. A solid-state videcon might be a possibility, if a two-dimensional array of photoelements is made by placing many such devices side by side and the domain in each device is triggered by the pulse in the adjacent device.

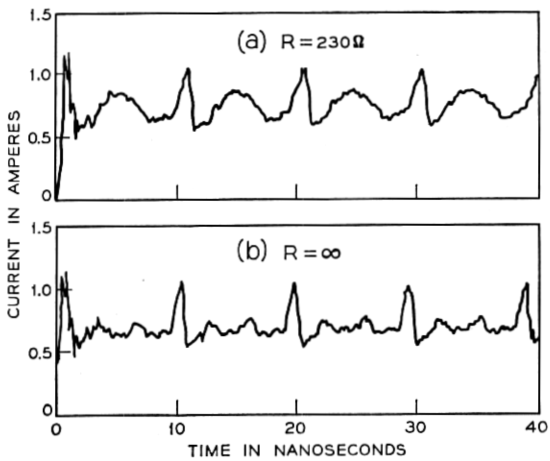

Fig. 18 — Waveforms observed with a device illustrated in Fig. 17.

IX. NUMERICAL EXAMPLES

The domain velocity for GaAs is approximately 10^7 cm/sec. If the device length is 1 mm, 100 μ, or 10 μ, the oscillation frequency is about 100 MHz, 1 GHz, or 10 GHz, respectively. The terminal voltage necessary to launch a domain in a uniform device is $E_p l$. Since E_p is approximately 3000 V/cm, 300 volts are required for 100 MHz samples, 30 volts for 1 GHz samples, and 3 volts for 10 GHz samples. The current is given by $env(E)S$. If $S = 0.3 \times 0.3$ mm² and $n_o = 5 \times 10^{14}/$ cm³

$$I = env(E)S = 1.6 \times 10^{-19} \times 5 \times 10^{20} \times 10^{5} \times 0.9 \times 10^{-7} = 0.7\text{A}.$$

The peak current is about twice this value.

The domain thickness is a function of doping as well as V_{ex}. For simplicity, let us assume that the diffusion constant is small and hence the thickness is approximately given by w. Then, as we calculated in (14),

$$w = \sqrt{\frac{2V_{ex}\epsilon}{en_o}}.$$

For $V_{ex} = 50$ volts and $n_o = 5 \times 10^{14}/\text{cm}^3$, since $\epsilon \cong 12.5 \times 8.85 \times 10^{-12}$ F/m for GaAs

$$w = \sqrt{\frac{2 \times 50 \times 12.5 \times 8.85 \times 10^{-12}}{1.6 \times 10^{-19} \times 5 \times 10^{20}}} = 12 \times 10^{-6}\text{m}$$

or $w = 12$ μ. For larger n_o, w becomes smaller.

The time constant of the initial growth of disturbances depends on the value of $|dv/dE|$. There are many theories and experiments suggesting various values ranging from 300 to 10,000 cm²/secV. This fact alone may well indicate how little we know about GaAs. For the moment, let us assume 3000 cm²/secV. Then,

$$|\tau| = \left| \frac{\epsilon}{en_o \dfrac{dv}{dE}} \right| = \frac{12.5 \times 8.85 \times 10^{-12}}{1.6 \times 10^{-19} \times 5 \times 10^{20} \times 0.3} \cong 4.5\text{ps}.$$

This is the time for a small disturbance to become about 2.7 times the original size. If this rate continues, the size becomes 1000 times within 32 ps and more than 100,000 times within 50 ps. In comparison, the time for a domain to disappear after its leading edge reaches the anode is of the order of w/v. For a domain 12 μ long, it is of the order of 120 ps.

If there is a region extending out a length b where the doping is reduced α percent, the initial size of the disturbance can be shown from Fig. 8 to be about $2E_p b\alpha \times 0.01$. For 10 percent inhomogeneity and $b = 5\ \mu$, we have

$$2 \times 300{,}000 \times 5 \times 10^{-6} \times 0.1 = 30 \times 10^{-2} \text{ volts.}$$

The corresponding $(E_d - E_o)$ calculated through (14) is given by

$$(E_d - E_o) = \sqrt{\frac{2n_o e}{\epsilon} V_{ex}} = 6.5 \times 10^3 \text{ V/cm}$$

when $n_o = 5 \times 10^{14}/\text{cm}^3$. This is large and we have to use (12) for the growth rate calculation. For simplicity, let us consider the fastest possible growth. From the area consideration, Fig. 5 shows that the fastest growth takes place when E_o is fixed at E_p. Then, the integral in (12) can be roughly approximated by $(E_d - E_p)\ \{v(E_p) - v(E_v)\}$ provided that there is a broad and flat valley region. Since $E_o = E_p$,

$$E_d - E_p \cong \sqrt{\frac{2n_o e}{\epsilon} V_{ex}}\ .$$

Thus, (12) becomes

$$\frac{dV_{ex}}{dt} \cong \sqrt{\frac{2n_o e}{\epsilon} V_{ex}}\{v(E_p) - v(E_v)\}.$$

The solution is given by

$$V_{ex} = \frac{n_o e}{2\epsilon}\{v(E_p) - v(E_v)\}^2(t + c)^2,$$

where c is a constant determined by the initial value of V_{ex}. If V_{ex} is small at $t = 0$, c can be neglected. For GaAs, $v(E_p) - v(E_v) \cong 10^7$ cm/sec and if $n_o = 5 \times 10^{14}/cm^3$ the above expression gives

$$V_{ex} = 3.6 \times 10^{21}(t + c)^2.$$

Since $V_{ex} \cong 0.3$ volts at $t = 0$, c is approximately 9 ps. V_{ex} becomes approximately 36 volts when $t + c = 100$ ps or $t = 91$ ps. If the terminal voltage increases slower than the above rate, E_o decreases and the growth of V_{ex} closely follows the terminal voltage. On the other hand if the terminal voltage increases faster, then E_o has to increase to accommodate the balance of the voltage since the growth rate of V_{ex} is no longer able to follow the applied voltage.

Finally, suppose that a steady-state domain exists and the terminal voltage is suddenly decreased so that E_o is decreased by β percent.

In this case, the right-hand side of (12) is approximated by

$$-0.01\beta E_o \frac{dv(E_o)}{dE} \times (E_d - E_o) \cong -0.01\beta E_o \frac{dv(E_o)}{dE} \sqrt{\frac{2n_o e}{\epsilon} V_{ex}} \, .$$

For $V_{ex} = 50$ V, $n_o = 5 \times 10^{14}/\text{cm}^3$, $dv(E_o)/dE \cong 6000$ cm^2/V, $\beta =$ 10 percent and $E_o = 1500$ V/cm, we have

$$\frac{dV_{ex}}{dt} = -0.1 \times 150{,}000 \times 0.6$$
$$\times \sqrt{\frac{2 \times 5 \times 10^{20} \times 1.6 \times 10^{-19}}{12.5 \times 8.85 \times 10^{-12}} 50} \cong -7.6 \times 10^{10} \text{ V/sec.}$$

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