A Millimeter Wave, Two-Pole, Circular-Electric Mode, Channel-Dropping Filter Structure

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Interest in circular-electric mode channel-dropping filters has been stimulated by recent advances in the repeater art. This paper presents the theory and establishes design procedures for filters having two-pole maximally flat response functions. The basic structure uses mode-conversion resonators, i.e., the resonating mode is the TE_{02} circular electric mode, for three of the resonators. The rejection filter portion of the structure is conventional in that two resonators separated by an odd multiple of $\pi/2$ radians realize the desired characteristic. The branching filter is novel in that a rectangular waveguide is wrapped around a mode-conversion resonator and coupled to the TE_{02} resonating mode via a multiplicity of apertures. The rectangular guide is then resonated to permit realization of the two-pole branching filter. The theory developed is an extension of Marcatili's original work on mode-conversion resonators. The mode-conversion resonator parameters are related to the elements of a lumped constant prototype network thus extending the utility of mode-conversion resonators.

Experimental results are presented on several filter models. The agreement between theory and experiment is generally good. Four filters were developed for use in an all solid-state repeater experiment with successful results.

I. INTRODUCTION

The TE_{01} circular-electric mode in round waveguide has received considerable attention due to its low-loss characteristic. The problem of multiplexing in communication systems using this mode was first approached by Marcatili.[1] His scheme used low-loss, TE_{02} circular-electric mode resonators* to realize channel-dropping filters having single-pole, maximally flat responses. Interest in channel-dropping filters with multiple-pole responses was stimulated by recent advances in the repeater art. An all solid-state repeater having a 51.7 GHz carrier and operating at a bit rate of 306 megabits per second has been constructed.[2] The repeater performance is such that a 15-mile repeater spacing could be achieved using propagation in the TE_{01} circular-electric mode in two-inch i.d. round guide. Since high bit rates imply increased bandwidth, multiple-pole filters are required to maximize usable channel capacity.

A design procedure has been developed for diplexers having a two-pole, maximally flat amplitude response. Section II describes the structure. Section III presents some preliminary considerations involving arrays of such structures. Section IV outlines the design procedure. Experimental results are presented in Section V.

The major theoretical contribution of this work lies in relating the parameters of the structure to the elements of a low-pass prototype network. This extension of Marcatili's work makes it possible to utilize mode-conversion resonators in multiple-pole filter structures. The analysis is summarized in Appendix A.

The novelty in the physical structure lies primarily in the realization of the branching filter. (See Fig. 1.) A mode conversion resonator

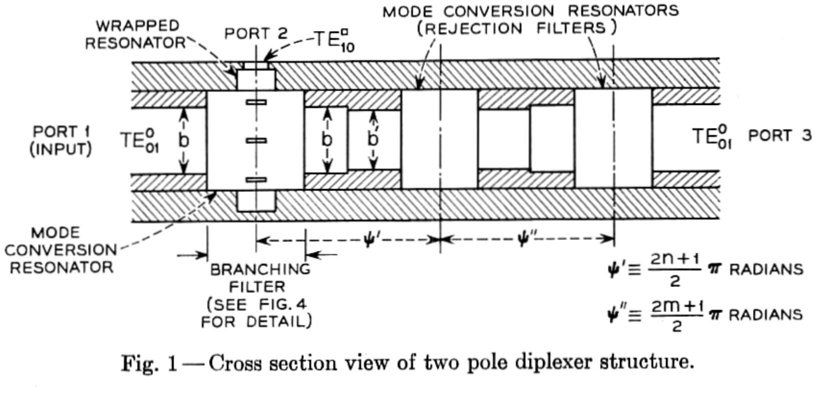

Fig. 1 — Cross section view of two pole diplexer structure.

is coupled to a wrapped rectangular waveguide to permit realization of a two-pole system. The rejection resonator portion of the structure is conventional in that two resonators separated by an odd multiple of $\pi/2$ radians is utilized to realize the two-pole characteristic.

* Throughout this paper, the TE_{02} mode resonators will be referred to as mode conversion resonators.

II. TWO-POLE FILTER STRUCTURE AND PROTOTYPE EQUIVALENT CIRCUIT

Fig. 1 shows the physical structure and identifies the resonant elements. The input and output guides are above cutoff for the TE_{01} mode and just below cutoff for the TE_{02} mode. The large guide sections are just above cutoff for the TE_{02} mode. The rectangular waveguide output is coupled to the mode-conversion resonator nearest the input port via a wrapped rectangular waveguide.

A qualitative description of the behavior of the structure is obtained as follows. First, consider an individual rejection resonator. A signal incident in the TE_{01} mode is coupled to the TE_{02} mode via a symmetrical diameter discontinuity. Since the input and output guides are below cutoff for the TE_{02} mode, the power in that mode is trapped in the large diameter region. Marcatili's analysis of the structure shows that at resonance the transverse mid-plane of the resonator is effectively a short circuit.[1] The center frequency and bandwidth are dependent on the length of the resonator and the ratio of the input guide to resonator guide diameters. The details of the relationship are given by Marcatili.[1] Now, in the structure of Fig. 1, the mid-planes of adjacent mode-conversion resonators are electrically separated by odd multiples of $\pi/2$ radians. Hence, at resonance, the rejection resonator pair presents an open circuit at the mid-plane of the input mode-conversion resonator. All of the incident TE_{01} mode power appears at the rectangular waveguide output when the various coupling coefficients are properly chosen.

Further insight into the electrical behavior of the structure is obtained by considering the prototype network shown in Fig. 2. The prototype network consists of complementary admittances connected in shunt. The elements of the network have been chosen to yield a two-pole, maximally flat insertion loss response between ports 1 and 2 while maintaining a constant input admittance as a function of fre-

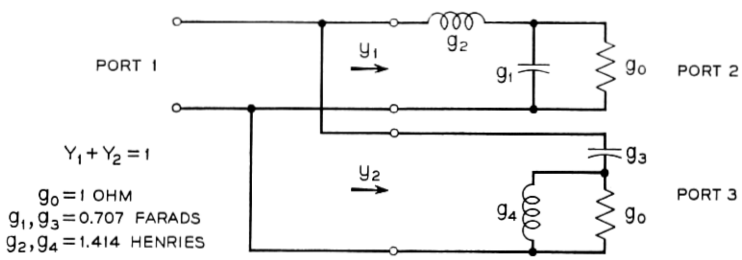

Fig. 2 — Prototype network for a two-pole diplexer.

quency.[4, 5] Total power transfer occurs at zero frequency, and half-power transfer occurs at an input angular frequency of one radian per second. The prototype network is converted to a network having total power transfer at some frequency ω_0 through use of the angular frequency mapping function

$$\omega' = Q_L\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right), \tag{1}$$

where

ω' = angular frequency of the prototype network
ω = angular frequency of the desired network
$Q_L = \omega_0/(\omega_1 - \omega_2)$
ω_1, ω_2 = half power angular frequencies of the desired network.

The relationship between the prototype network parameters and the parameters of the structure shown in Fig. 1 are discussed in detail in Appendix A. For the purpose of obtaining a qualitative understanding of electrical behavior it is sufficient to state that the performance of the microwave structure will be identical to that of the frequency mapped prototype network subject only to the approximations involved in relating their respective parameters.

III. PRELIMINARY CONSIDERATIONS

The general problem is to develop an array of channel-dropping filters. The use of mode-conversion resonators results in several fundamental performance limitations as described below.

First, consider the case where the input guides to all filters in the array have the same diameter. Since the input guide must be below cutoff for the TE_{02} mode at the highest significant frequency, the diameter, b, of the input guide is restricted to

$$b < 7.016\lambda_u/\pi, \tag{2}$$

where λ_u is the free-space wavelength at the highest significant frequency, f_u. At the same time, the TE_{01} mode at the lowest significant frequency in the overall system must be passed requiring that

$$b > 3.832\lambda_L/\pi, \tag{3}$$

where λ_L is the free-space wavelength at the lowest significant frequency, f_L. Combining (2) and (3) gives

$$\Delta f < 0.8309 f_L , \tag{4}$$

where Δf is the bandwidth of the filter array.

The next case to be considered is that in which successive filters have varying input guide diameters. This arrangement would permit maximization of the intrinsic Q's of each of the resonators. (A short taper is required to interconnect the filters.)

Now assume that the lowest frequency channel is to be dropped first. For the first channel-dropping filter in the array to operate properly, it must be terminated in a matched impedance throughout its passband. Hence, the smallest diameter guide in the system must be above cutoff to the TE_{01}-mode at the lowest significant frequency to be dropped. This argument leads to the same restrictions on the overall bandwidth of the array as given by (4).

If the highest frequency channel is the first to be dropped, then the argument leading to (2) and (3) again applies resulting in the bandwidth restriction (4).

In the case of varying input guide diameters, there are other limitations on the array performance which *are* dependent on the order in which the channels are dropped. If the highest frequency channel is the first to be dropped, then the diameter discontinuities produce a small residual return loss at out-of-band frequencies over and above that produced by a conventional resonator. This is true of each successive filter in the array. The effect can become cummulative for certain filter-to-filter spacings. If the lowest frequency channel is the first to be dropped, then the discontinuities cause conversion of power from the incident TE_{01}-mode to the TE_{02}-mode which can now propagate. In addition, the bandwidth of the array is further restricted by the possibility of TE_{03q} resonances in the mode-conversion resonators. This occurs when the input and output guides to a given resonator are just below cutoff for the TE_{03}-mode and the large diameter region is just above the TE_{03}-mode cutoff. The diameter discontinuity again produces mode conversion with the TE_{03}-mode power trapped in the resonator. The ratio of the TE_{03} to TE_{02}-mode cutoff frequencies is

$$\frac{f_{c03}}{f_{c02}} = 1.45.$$

Hence, this problem occurs in the vicinity of 1.45 times the resonant frequency of the first filter in the array.

A more detailed analysis to select the optimum dropping order is beyond the scope of this paper. In the following, it is assumed that the above restrictions have been observed prior to establishing the design for a given filter in the array.

IV. DESIGN PROCEDURE

The necessary design equations are derived in Appendix A. It is possible to begin with those results and design the three mode-conversion resonators as described by Marcatili. However, a sufficiently accurate and more rapid approach is to use the data obtained by C. N. Tanga[6] shown here as Fig. 3.* Equations (6), (19), and (20)

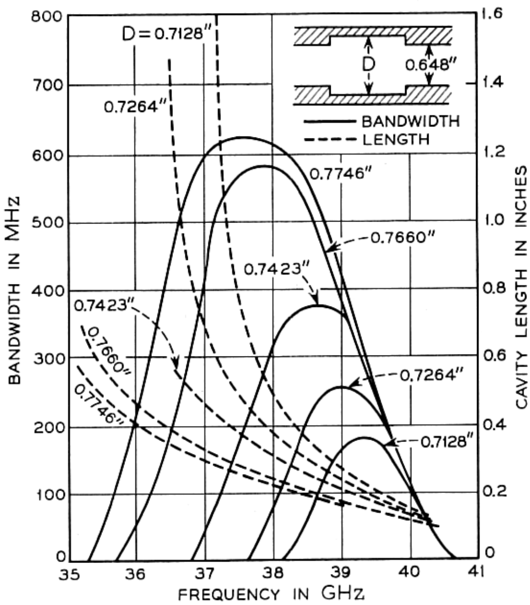

Fig. 3—Design chart for $TE_{01}{}^0$ – $TE_{02}{}^0$ mode conversion resonators (from ref. 6).

are used to compute the required external Q's of the resonators. The data shown in Fig. 3 are used to obtain a design scaled to a lower frequency. Frequency scaling techniques are applied to convert the design to the frequency of interest.

That portion of the structure of Fig. 1 consisting of the first mode-

* While the range of the parameter D is somewhat restricted, the data is believed sufficient for most practical applications.

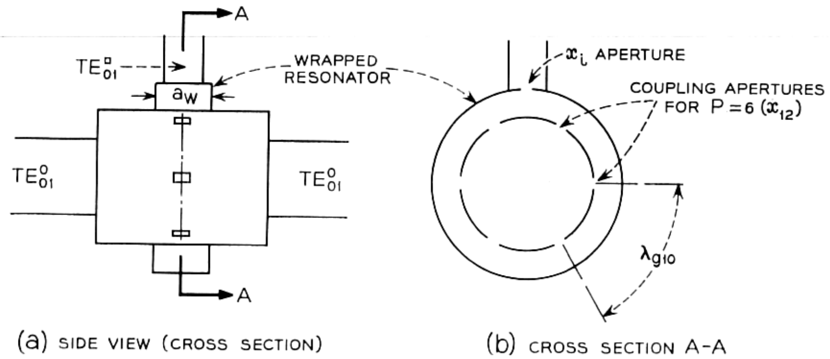

Fig. 4 — Physical structure of branching filter.

conversion resonator and the wrapped resonator is defined as the branching filter. See Fig. 4. The wrapped resonator of the branching filter is designed using equations (5) and (11). The additional requirement that the coupling apertures are to be separated by one guide wavelength in the wrapped structure at resonance must also be observed. This is controlled primarily by the width, a_w, of the wrapped resonator and the diameter of branching-filter mode-conversion resonator established earlier. The thickness of the wall in which the coupling apertures are placed is also significant.

The coupling aperture dimensions are determined by (18) and (16). Finally, the magnitude of the normalized coupling reactance required at the output of the wrapped resonator, (X_i/R_1), is determined from (10).

The lengths of the guide sections separating the mode-conversion resonators is determined so as to provide an odd multiple of 90 electrical degrees separation between resonators. The calculations required are similar to those given by Marcatili for single-pole filters.[1] Experimental work has shown that a minimum separation of 7 $\pi/2$ radians is required to avoid fringing field interaction.

Note that the two rejection resonators are not identical in that their respective input-to-output guide diameter ratio must be different. (See Appendix A.2). Three methods of realizing this result are possible:

(*i*) The resonator diameters can be made the same in which case either a small step or taper is required in the diameters of the connecting lines. The small step case is indicated in Fig. 1 in exaggerated form. In practice, the difference between the diameters b and b' is only a few tenths of a percent.

(*ii*) The input guide diameters can be made the same in which case the resonator diameters are different.

(*iii*) Both input guide diameters and resonator diameters can be different for the two.

V. EXPERIMENTAL RESULTS

The procedures outlined above were used to develop several channel-dropping filters. All of the filters had the physical form of Fig. 1 with six coupling apertures in the branching filter. The data of Fig. 3 was used to compute the required dimensions of all mode conversion resonators. In developing the filters, the first step was to construct the branching and rejection filters separately. The measured characteristics of the individual parts were then compared with theory through use of the transfer coefficients of Appendix B. The first model developed showed good correlation for the separate parts. However, the complete filter characteristic was found to have less than the theoretical bandwidth. Subsequent measurements indicated that the separation between branching and rejection filters (ψ' of Fig. 1) was too small. This produced interaction between resonators which we believe produced the bandwidth discrepancy. It was found experimentally that a minimum separation of $7\ \pi/2$ radians was required to inhibit this interaction.

Figs. 5 and 6 show the characteristics of two other filter models. The theoretical and measured 3 dB bandwidths were, respectively, 1388 and 1210 mHz for the filter of Fig. 5 and 1130 and 1144 mHz for the filter of Fig. 6.

The filter in Fig. 5 had resonator separations of

$$\psi' = 7\pi/2$$

$$\psi'' = 5\pi/2.$$

Some interaction effects were noted between the rejection resonators. The separation ψ'' was increased to $7\ \pi/2$ for the filter of Fig. 6. Note that the out-of-band return loss was much improved for the latter arrangement.

The insertion loss of all filters tested was of the order of 0.5 dB at mid-band.

The finite return loss observed at the circular waveguide input port is believed to be due primarily to dimensional tolerance problems. For example, the data of Fig. 3 indicates that a mode conversion resonator having an input guide diameter of 0.493 inches and a cavity diameter of 0.579 inches should have a 400 mHz bandwidth when resonant at

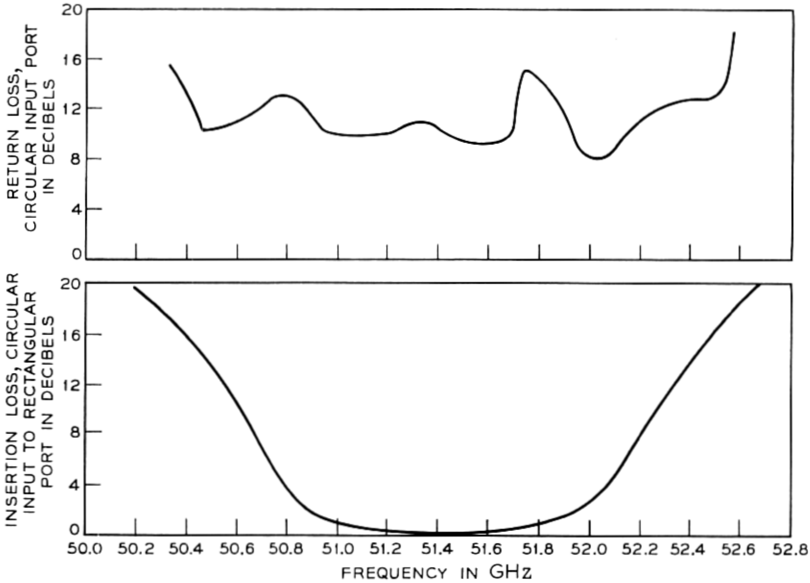

Fig. 5—Frequency response of experimental dropping filter ($\psi' = 7\pi/2$, $\psi'' = 5\pi/2$).

51.7 GHz. A one mil decrease in the input guide diameter would result in a 419 mHz bandwidth at 51.7 GHz. This rapid variation of electrical characteristics with dimensions makes precision machining absolutely necessary.

Another critical parameter was the width of the wrapped resonator. The height of the wrapped resonator for the filters described was 0.074 inch. This resulted in a variation of resonant frequency of the wrapped resonator of about 300 mHz per mil change in the resonator width. Hence, extreme care was required in machining the branching filter to obtain the desired resonant frequency.

Four filters with characteristics very nearly identical to those shown in Fig. 6 were successfully incorporated into the circuitry of the solid-state repeater described in the Introduction.[2]

VI. CONCLUSIONS

Marcatili's original work has been extended to permit realization of two-pole channel-dropping filters using low-loss mode-conversion resonators. The theory developed relates the mode-conversion resonator parameters to those of a lumped constant prototype network. This

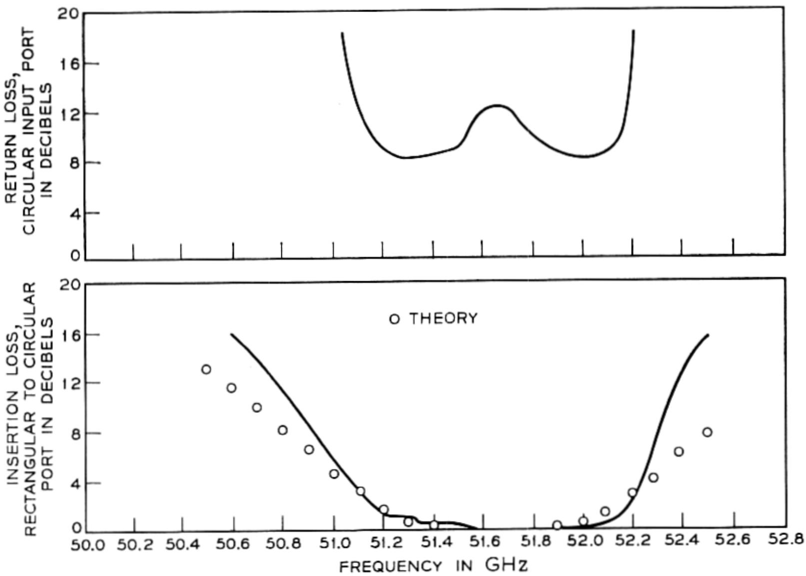

Fig. 6—Frequency response of experimental dropping filter ($\psi' = \psi'' = 7\pi/2$).

step leads to a straightforward design procedure which permits rapid determination of structural dimensions. The method can be readily applied to multiple-pole structures once the exact form of the additional resonators required in the branching filter has been determined. The latter might take the form of additional wrapped resonators or additional resonators in the rectangular waveguide output.

The design procedure evolved has been shown to yield filters whose characteristics compare well with theory. Dimensional tolerances have been shown to be extremely important.

VII. ACKNOWLEDGMENTS

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APPENDIX A

Derivation of the Design Equations

A.1 *Branching Filter*

The branching filter consists of a $TE_{01} - TE_{02}$ mode-conversion resonator coupled to a wrapped rectangular waveguide resonator. The physical parameters of the latter are to be related to that portion of the network of Fig. 2 consisting of the elements g_0, g_1, and g_2. The parameters of interest are the external Q's, Q_e, of the two resonators and the coefficient of coupling between them, k_{12}. Using the notation of Chapter 8, Ref. 4, there is obtained

$$Q_{ew} = g_0 g_1 \omega_1' Q_L = \frac{\omega_1' Q_L}{\sqrt{2}} \tag{5}$$

$$Q_{em} = g_2 g_0 \omega_1' Q_L = \sqrt{2}\,\omega_1' Q_L \tag{6}$$

$$k_{12} = \frac{1}{\omega_1' Q_L \sqrt{g_1 g_2}} = \frac{1}{\omega_1' Q_L}, \tag{7}$$

where ω_1' is the bandpass edge angular frequency of the prototype network, and Q_L is the desired loaded Q of the diplexer. The subscripts w and m refer to the wrapped resonator and the mode-conversion resonator, respectively.

The next step is to derive the expressions for the external Q's and coupling coefficient in terms of the physical parameters of the structure. An analysis of Marcatili's results shows that

$$Q_{em} = \overline{Q_L}\ , \tag{8}$$

where $\bar{Q}_L$ is given by Marcatili's Equation (70).[1]

The external Q of the wrapped resonator, Q_{ew} is obtained from the defining equation

$$Q_{ew} = \frac{\left.\dfrac{\omega_0}{2}\dfrac{\partial X_1}{\partial \omega}\right|_{\omega_0}}{R_c}, \tag{9}$$

where

ω_0 = angular resonant frequency,
X_1 = resonator reactance function, and
R_c = coupled resistance.

The structure of Fig. 4(b) is used to evaluate X_1 and R_c. The resulting expressions when substituted in (9) yield

$$Q_{ew} = \frac{Z_{10}\theta_0(\lambda g_{10}/\lambda)^2}{4(X_i^2/R_1)}, \tag{10}$$

where

Z_{10} = characteristic impedance of the wrapped guide,
λg_{10} = guide wavelength of the wrapped guide at resonance,

X_i = reactance of the input iris,
R_1 = characteristic impedance of the input guide, and
θ_0 = length of the wrapped structure in radians at resonance.

In general, $\theta_0 = \pi p$ where p is the number of coupling apertures. For loose coupling

$$Q_{ew} = \frac{2\pi p(\lambda g_{10}/\lambda)^2}{c^2}, \tag{11}$$

where

$$c^2 = 8(X_i^2/R_1 Z_{10}) \tag{12}$$

is the power coupling coefficient between guides.

The coupling coefficient k_{12} is given in terms of the resonator parameters by

$$k_{12} = \frac{X_{12}}{\sqrt{x_1 x_2}}, \tag{13}$$

where

X_{12} = coupling reactance,

$x_1 = \left.\frac{\omega_0}{2}\frac{dX_1}{d\omega}\right|_{\omega_0}$ = slope parameter of the first resonator, and

$x_2 = \left.\frac{\omega_0}{2}\frac{dX_2}{d\omega}\right|_{\omega_0}$ = slope parameter of the second resonator.

To evaluate k_{12}, requires calculation of X_{12} as it is related to the physical structure. The slope parameters are known from the external Q calculations. For loose coupling, X_{12} is related to the power coupling coefficient between the TE_{02} waveguide and the TE_{10} wrapped waveguide, $|\Gamma_{12}|^2$, by,

$$|\Gamma_{12}|^2 \cong \frac{X_{12}^2}{2Z_{02}Z_{10}}, \tag{14}$$

where Z_{02} is the characteristic impedance of the TE_{02} waveguide.

If the coupling apertures are assumed sufficiently small so that the electromagnetic field is essentially constant over the aperture, then Bethe's small hole coupling theory can be applied to yield

$$|\Gamma_{12}|^2 = \frac{18p^2k^2G}{abR^4}\frac{\lambda_{g02}}{\lambda_{g10}}M^2, \tag{15}$$

where

a = wrapped guide width,
b = wrapped guide height,
p = number of apertures,
k = 7.016,
G = 0.07075,
R = radius of TE_{02} guide,
λ_{g02} = TE_{02} guide wavelength,
λ_{g10} = wrapped guide wavelength, and
M = magnetic polarizability of the aperture.

The above assumes the apertures to be λ_{g10} apart in the wrapped structure. The magnetic polarizability of the aperture is determined from

$$M = \frac{M_0}{1 - (\lambda_{ca}/\lambda)^2} 10^{-[(2.73tA/\lambda_{ca})\sqrt{1-(\lambda_{ca}/\lambda)^2}]}, \tag{16}$$

where

M_0 = static polarizability of the aperture,
λ_{ca} = cutoff wavelength of the dominant aperture mode,
t = aperture thickness, and
A = empirical constant.

Equation (16) permits use of Bethe's small hole theory for large apertures.[4] Ref. 4 contains an excellent collection of data on M_0 for various types of apertures.

The above analysis when rearranged yields

$$k_{12} = \sqrt{\frac{2Z_{02}Z_{10}}{x_1x_2}} \mid \Gamma_{12} \mid = \frac{1}{\omega_1' Q_L \sqrt{g_1g_2}} \tag{17}$$

from which

$$M = \frac{R^2}{6pkG^{\frac{1}{2}}\omega_1' Q_L} \sqrt{\left(\frac{ab}{g_1g_2}\right)\left(\frac{\lambda_{g10}}{\lambda_{g02}}\right)\left(\frac{x_1}{Z_{10}}\right)\left(\frac{x_2}{Z_{02}}\right)}. \tag{18}$$

The latter then represents the relationship between the required aperture dimensions and the characteristics of the filter.

A.2 Rejection Filters

From Ref. 4, it is found that the loaded Q's of the rejection filters must be

$$Q_{L1} = \sqrt{2}\, Q_L \tag{19}$$

$$Q_{L2} = 2\sqrt{2}\, Q_L \,, \tag{20}$$

where Q_{L1} applies to the rejection filter nearest the branching filter. The loaded Q's of the rejection filters in terms of the physical parameters are given by Marcatili's Equation (139).[1]

APPENDIX B

Transfer Functions and Reflection Coefficients

The transfer coefficients and reflection coefficients, of the individual diplexer networks are of interest since the experimental development of a diplexer usually requires adjustments on the separate networks. Figs. 7 and 8 show the individual networks with the ports of interest identified. The analysis is straightforward yielding the following results:

$$S_{ab} = \frac{2\sqrt{2}}{D} \tag{21}$$

$$S_{bb} = \frac{(1 - 2g_1g_2\omega'^2) + j\omega'(g_1 - 2g_2)}{D} \tag{22}$$

$$S_{aa} = S_{cc} = \frac{1 + jg_1\omega'}{D} \tag{23}$$

$$S_{ac} = \frac{2(1 - g_1g_2\omega'^2) + jg_2\omega'}{D} \tag{24}$$

$$S_{bc} = \frac{2}{D} \,,$$

where

$$D = (3 - 2g_1g_2\omega'^2) + j(g_1 + 2g_2)\omega' \tag{25}$$

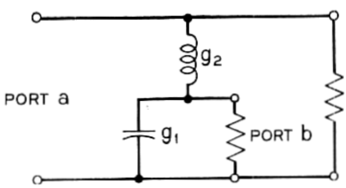

Fig. 7 — Branching filter prototype.

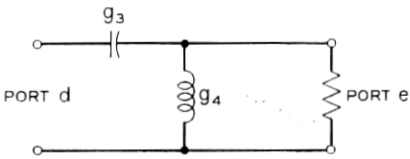

Fig. 8—Rejection filter prototype.

and

$$S_{de} = -\frac{2g_3g_4\omega'^2}{(1 - 2g_3g_4\omega'^2) + j(g_3 + g_4)\omega'}. \tag{26}$$

The above expressions were used in Section IV to compare theory and experiment with an assumed frequency mapping function

$$\omega' = \omega_1' Q_L\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right). \tag{27}$$

Note that (27) represents a narrow bandwidth approximation to the actual waveguide resonator frequency variation. The accuracy of the result will probably be sufficient for filter bandwidths up to a few percent. It should also be noted that Marcatili obtained good correlation between theory and experiment using the approximate mapping function

$$\omega' = 2Q_L\left(\frac{f - f_0}{f_0}\right)$$

for filters having bandwidths of the order of one percent.[1] The latter mapping function has been shown to yield good results in the frequency range where

$$\frac{|\,2(f - f_0)\,|}{f_0} \ll \left(\frac{\lambda_0}{\lambda_{g02}}\right)^2,$$

where λ_0 is the free space wavelength at frequency f_0.

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