THE BELL SYSTEM
TECHNICAL JOURNAL

VOLUME XLVI | MAY–JUNE 1967 | NUMBER 5

Unit-Cube Expression for Space-Charge Resistance

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(Manuscript received November 17, 1966)

A simple analysis shows that the unit-cube conductance is a figure of merit in semiconductor device design theory. The unit-cube conductance, G, is given by $2Kv_d$ where K is the permittivity of the semiconductor and v_d is the limiting drift velocity.

The space-charge resistance, R_{sc}, due to carrier generated under avalanche condition is derived for p-n junctions. It is found that for parallel-plane structure, $R_{sc} = 1/GN$, where N is the number of unit cubes in the depletion region with cube edge equal to the depletion width or $N = A/W^2$ where W is the depletion width and A the junction area. The disturbance in voltage caused by the space-charge effect is given by $I/GN = JW^2/G$ where I and J are the current and current density, respectively. Similar results are obtained for p-n junctions with coaxial-cylinder and concentric-sphere structures.

For silicon, the value of G is approximately 40 μmhos. The transconductance of a silicon surface-controlled avalanche transistor in terms of the unit-cube expression is about $12.5\ N$ μmhos.

A simple analysis of "avalanche resistance" can be given for the limiting case in which carriers are generated at one boundary surface of the depletion region of a p-n junction and travel across the depletion region with a limiting drift velocity v_d. Structures satisfying these conditions can be of the n^+pp^+ form. It will be shown that the quantity

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$2Kv_d$ (where K is the permittivity of the semiconductor) is a figure of merit in semiconductor device design theory which limits the performance of space-charge-limited devices. This quantity is a combination of "material constants" similar to $F_B v_d$ (where F_B is "breakdown field") which limits the frequency-power performance of transistors[1,2] and K/σ (σ is the conductivity) which limits the gain-bandwidth product[3] of solid-state devices.

For a structure in which the space-charge layer is bounded by parallel planes of area A and spacing W, it will be shown that the effective space-charge resistance can be interpreted as due to N unit-cube conductances in parallel, where the unit-cube conductance G is given by

$$G = 2Kv_d \tag{1}$$

and N is number of unit cubes in the depletion layer with cube edge equal to the depletion width, or

$$N = A/W^2. \tag{2}$$

The space-charge resistance is then given by

$$R_{sc} = 1/NG. \tag{3}$$

For coaxial-cylinder and concentric-sphere structures, similar results are obtained for the R_{sc}. The number of unit cubes (or curvilinear cubes), however, depends on the radius of the surface upon which avalanche occurs and the length of the cylinder (for the coaxial-cylinder structure). These functional dependences are derived below.

An interesting application of the space-charge resistance and the unit-cube expression is given for a surface-controlled avalanche transistor (SCAT).[4]

I. PARALLEL PLANE STRUCTURE

As represented in Fig. 1(a) the depletion layer of an n^+pp^+ structure extends through the p layer with a doping of N_a, and is bounded by the planes at $x = 0$ and $x = W$. When the applied voltage V is equal to the breakdown voltage V_B the electric field $E(x)$ has its maximum absolute value F_B at $x = 0$ and decreases to $F_B - (qN_aW/K)$ at $x = W$. This insures breakdown at $x = 0$. Furthermore, if $qN_aW/K < 0.9\, F_B$, then the field is everywhere $\geqq F_B/10$, so that holes have their limiting drift velocity v_d all across W.

The space-charge current, I, is given by

$$I = v_d \rho A, \tag{4}$$

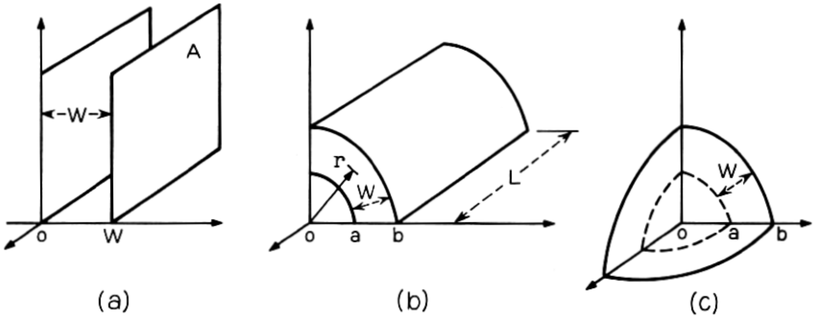

Fig. 1 — p-n junction geometry of (a) parallel plane, (b) coaxial cylinder, and (c) concentric sphere structures.

where ρ is the carrier-charge density and A the area. Since E at $x = 0$ is assumed to be equal to F_B, the disturbance $\Delta E(x)$ in the electric field due to ρ is

$$\Delta E(x) = \frac{Ix}{AKv_d} \tag{5}$$

so that the disturbance in voltage caused by the carriers (i.e., the average field times W) is obtained by integrating $\Delta E(x)$

$$\Delta V_B = I\left(\frac{W^2}{A}\right)\left(\frac{1}{2Kv_d}\right) = \frac{I}{NG}. \tag{6}$$

The total voltage is thus

$$V = V_B + \Delta V_B = \left(F_B W - \frac{qN_a W^2}{2K}\right) + \frac{I}{NG} = V_B + IR_{sc}, \tag{7}$$

which verifies the interpretation of G and N.

II. COAXIAL CYLINDER STRUCTURE

Consider first that the maximum field occurs at the inner surface. As shown in Fig. 1(b) the depletion layer extends through the intrinsic region of an n^+ip^+ coaxial-cylinder structure and is bounded by the cylinders of radii $r = a$ and $r = b$. When $V = V_B$, the electric field $E(r)$ has its maximum absolute value F_B at $r = a$, and decreases to $F_B a/b$ at $r = b$.

The space-charge current per unit length, I/L is given by

$$\frac{I}{L} = 2\pi r \rho v_d \tag{8}$$

so that ρ varies as $1/r$. Integrating Poisson's equation leads to a disturbance $\Delta E(r)$ in the electric field and ΔV_B in the voltage due to ρ given by

$$\Delta E(r) = \frac{\rho(r - a)}{K} = \frac{I}{2\pi K v_d L}\left(1 - \frac{a}{r}\right) \tag{9}$$

and

$$\Delta V_B = \frac{I}{(2Kv_d)\pi L}\left[b - a - a \ln\left(\frac{b}{a}\right)\right] \equiv \frac{I}{NG} = IR_{sc}\,, \tag{10}$$

where

$$\frac{1}{N} = \frac{b}{\pi L}\left(1 - \frac{a}{b} - \frac{a}{b}\ln\frac{b}{a}\right) = \frac{\left[2b^2\left(1 - \frac{a}{b} - \frac{a}{b}\ln\frac{b}{a}\right)\right]}{2\pi b L} \equiv \frac{A_c}{2\pi b L}. \tag{11}$$

A_c is the area on the outer cylinder surface that corresponds to one unit-cube conductance G.

$$\begin{aligned} A_c &\to (b - a)^2, \quad &&\text{for} \quad a \to b \\ A_c &\to 2b^2, &&\text{for} \quad a \to 0. \end{aligned} \tag{12}$$

Equation (10) may be interpreted as the resistance of N unit curvilinear cubes in parallel. These cubes are formed by intersection of equipotential surfaces with the orthogonal family of electric field lines. Each cube has a conductance $(2Kv_d)$, and the number of cubes N is given by (11). The area A_c approaches $(b - a)^2$ when $a \to b$, and approaches $2(b - a)^2$ when $a \to 0$, and consequently remains finite even as the inner cylinder approaches a line.

The maximum field may be caused to occur on the outer surface $r = b$ by adjusting the chemical charges in the depletion layer appropriately, such as a p^+pn^+ structure with the pn^+ junction at $r = b$, the p^+p boundary at $r = a$. In this case, the area A_c approaches $(b - a)^2$ when $a \to b$, and approaches $2b^2 \ln (b/a)$ when $a \to 0$. Hence, the space-charge resistance has the same value as given by (10) and (11) when $a \to b$, but approaches infinity as $a \to 0$.

III. CONCENTRIC SPHERE STRUCTURE

As shown in Fig. 1(c), the depletion layer extends through the intrinsic region of an n^+ip^+ concentric sphere structure and is bounded by the spheres of radii $r = a$ and $r = b$. When $V = V_B$, the electric field $E(r)$ has its maximum value F_B at the inner surface $r = a$, and decreases to $F_B a^2/b^2$ at $r = b$.

The space-charge current is given by

$$I = 4\pi r^2 \rho v_d \ . \tag{13}$$

The quantities $\Delta E(r)$ and ΔV_B are given as follows:

$$\Delta E(r) = \frac{\rho(r-a)}{Kr^2} = \frac{I}{4\pi K v_d r}\left(1 - \frac{a}{r}\right) \tag{14}$$

$$\Delta V_B = \frac{I}{(2Kv_d)2\pi}\left(\ln\frac{b}{a} - 1 + \frac{a}{b}\right) \equiv \frac{I}{NG} = IR_{sc} \ , \tag{15}$$

where

$$\frac{1}{N} = \frac{\left(\ln\dfrac{b}{a} - 1 + \dfrac{a}{b}\right)}{2\pi} \equiv \frac{A_s}{4\pi b^2} \tag{16}$$

$$A_s = 2b^2\left(\ln\frac{b}{a} - 1 + \frac{a}{b}\right)$$

$$\begin{aligned} A_s &\to (b-a)^2, \quad &\text{for} \quad a \to b \\ A_s &\to \infty, \quad &\text{for} \quad a \to 0 \end{aligned} \tag{17}$$

and A_s is quantity of area on the outer sphere surface that corresponds to one curvilinear unit-cube conductance G. When a is finite or $a \to b$, (15) may be interpreted as the resistance of N unit curvilinear cubes in parallel, where each cube has a conductance $(2Kv_d)$ and the number of cubes N is given by (16). Unlike the n^+ip^+ coaxial-cylinder case, R_{sc} of the concentric-sphere structure approaches infinite resistance as the inner sphere approaches a point.

F_B can occur on the outer sphere for a p^+pn^+ structure, for example. The results of A_s are the same for both limiting cases as given in (17).

An interesting application of the unit-cube expression is that for a surface-controlled avalanche transistor (SCAT)[4] with a total junction perimeter of P and a space-charge layer W. Because the space-charge resistance is finite for an n^+ip^+ coaxial cylinder structure as the inner cylinder approaches a line [see (12)], it is feasible to make calculations for SCAT on the basis of an avalanche line source. There are $N = P/W$ such unit cubes around the edge, and the total transconductance, g_m, is[4]

$$g_m = \left(\frac{2}{\pi}\right)Kv_d\left(\frac{P}{W}\right) , \tag{18}$$

TABLE I—UNIT-CUBE EXPRESSIONS ($W \equiv b - a$)

Structures	Parallel plane	Coaxial cylinders		Concentric spheres
N (No. of unit cubes)	$\frac{A}{W^2}$	$a \to b$: $\frac{2\pi Lb}{W^2}$	$a \to 0$: $\frac{\pi L}{b}$	$a \to b$: $\frac{4\pi b^2}{W^2}$
Cube edge (cm)	W	W	$\sqrt{2b}$	W
R_{sc} (ohms)	$\frac{1}{GN} = \frac{1}{(2Kv_d)N}$			

or

$$g_m = \left(\frac{1}{\pi}\right)(2Kv_dN) \equiv \frac{1}{\pi R_{sc}}. \tag{19}$$

For a silicon SCAT with a device geometry of $P = 1000\ \mu$ and $W = 0.5\ \mu$, there are 2000 unit cubes with cube edge 0.5 μ. The space-charge resistance is 12.5 ohms, and the transconductance is 25,400 μmhos.

Another application is to calculate the voltage disturbance ΔV_B in a Read diode.[5] For a silicon Read diode with drift region of 10 μm and an operating current density of 1000 amp/cm^2, the value of ΔV_B, as obtained from (6), is approximately 25 volts.

A summary of the number of unit cubes and other pertinent quantities is presented in Table I. It has been shown that the unit-cube conductance $(2Kv_d)$ is a figure of merit in semiconductor device design theory. The unit-cube expressions are shown to be useful for calculation of the space-charge resistance.

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