A High-Capacity Digital Light Deflector Using Wollaston Prisms

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A high-capacity digital light deflector (DLD) using Wollaston prisms as the passive elements is described. It is shown that, for a 1-cm aperture, approximately $4(10)^6$ resolvable positions with a crosstalk ratio of 17 dB are theoretically possible. A manually-operated model was constructed that gave $\frac{1}{4}(10)^6$ resolvable positions with a crosstalk ratio of 20 to 28 dB. The output positions of the model showed resolution approximately equal to that set by diffraction theory.

The problems associated with imperfect modulators are discussed and the characteristics of three different schemes of operation are calculated. Results from experiments with one such scheme, the reflection mode of operation, are given. They compare favorably with the calculations.

I. INTRODUCTION

A digital light deflector (DLD) is a device that can switch a light beam to a number of distinguishable positions and has been previously described by a number of authors.[1–4] Such a device can be made from a number of modulators and passive deflectors. The modulator, for this application, is one that is capable of switching the sense of polarization, and the deflector unit is a passive element which has different optical paths corresponding to the two senses of polarization. A basic unit of a DLD is shown in Fig. 1. It has been previously

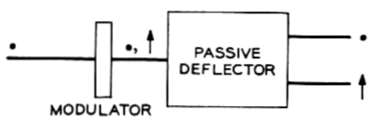


Fig. 1 — Basic unit in a digital light deflector.

shown[1–3] that n such units in series can generate 2^n distinguishable positions.

A modulator can be made from any material which can become birefringent with the application of an external signal. A minimum of π retardation is needed in order to switch the sense of polarization. Examples of modulators that have been considered for this application are Kerr cells,[5] stressed plate shutters,[6] and crystals such as KDP[3] and KTN[7,8] which exhibit an electro-optic effect. The Kerr cells, although very fast, cannot be used at high repetition rates because of heating difficulties. Stressed plate shutters, since they depend on mechanical strain, are limited to lower frequencies. The most attractive modulator materials are the electro-optic crystals and are the ones that are being seriously considered for the DLD.

The passive deflectors that have been considered for this application are uniformly thick sections of properly oriented uniaxial crystals such as calcite,[2,3] prisms of the same materials,[1] and Wollaston prisms.[4] The uniformly thick pieces of calcite are used with converging light for the maximum number of resolvable positions[2,3] but with such use suffer from aberrations that are caused by the variation of angle in the converging beam. A converging beam of light passing through a thick piece of calcite oriented for a displaced beam shows aberrations which for the most part appear like astigmatism. Prisms, when used with plane waves, can deviate the angle of the plane wave without distortion, and therefore, a DLD using prisms can give results that are limited only by diffraction theory. A DLD using prisms also uses much less birefringent material than one based on uniformly thick pieces of the same material. A disadvantage of simple prisms is that the difference in angle between the two oppositely polarized beams is only a small variation superimposed on the much larger normal type prism deflection. This difficulty can be minimized if the prisms are immersed in an oil whose index of refraction is near that of the prism. Wollaston prisms do not have the deflection associated with simple prisms—instead, the only deviation is that between the two oppositely polarized beams—and are therefore well suited for DLD use.

In this paper, the design of a high-capacity DLD using Wollaston prisms will be discussed along with experimental results which will show that this system does lead to densities limited primarily by diffraction effects. The problem of an imperfect modulator is also discussed, and calculations on several systems are given which relate the signal to background ratio to the modulation efficiency.

II. CAPACITY OF A DLD

Since a Wollaston prism is used to deflect a beam in angle, it is convenient to think of the operation of a DLD in terms of angular space. Later it will be convenient to place a lens after the DLD which will focus the beams of light, each with a different angle with respect to the axis of the lens, into corresponding points on the image plane.

The capacity of a DLD is determined by the values of two angles. One is the largest angle that is allowed in the system and the other is the minimum angular separation between adjacent positions. The capacity of the DLD is then just the square of this ratio. The minimum value is determined by either diffraction effects or imperfections in the optical system, and the largest value is determined by the maximum angular aperture of the system. First, let us consider the lower limit set only by diffraction theory. The light emerging from a circular aperture illuminated by a uniformly intense plane wave will have a spread of angles that is caused by diffraction. The intensity of the light as a function of angle is given by the well-known Airy function (Fig. 2).[9] The smallest deflection angle in a DLD must be sufficiently large so that the deflected beam must be resolved from the undeflected one. If we set the criterion that the two beams should be separated in angle such that in the far field the first dark ring of each beam overlap, then the minimum angle is given by 2.44 λ/D where λ is the wavelength of light and D is the diameter of the circular aperture (Fig. 2).

It is possible to estimate the crosstalk, e.g., the ratio of light within the first dark ring to the light within a circle of equal size as the first dark ring but displaced, by examining the Airy function. The light within the first dark ring contains 84 percent of the total energy,[9] and a ring displaced by one diameter (corresponding to a separation of the two directions of 2.44 λ/D) falls within an annulus of 1.22 λ/D to 3.66 λ/D which contains 10.6 percent of the total energy.[9] Since a circle can be surrounded by six circles of the same diameter, this displaced ring contains somewhat less than $\frac{1}{6}$ (10.6 percent) $=$ 1.77 percent. The crosstalk ratio is then $84/1.77 = 47$ or 16.7 dB. For a separation between the beams of twice the above, i.e., 4.88 λ/D, the crosstalk ratio can be estimated to be 210 or 23.2 dB.

If, in addition to diffraction effects, the wavefront is distorted further by some aberration, then the focused spot size will increase and thereby decrease the capacity of the DLD. The amount of wavefront distortion depends somewhat on the type of aberration, but for wavefront distortion of $\lambda/4$ or less the increase in the focal spot size is not very significant.[10] The aberrations in the DLD will result from inhomogeneities in the material and from poorly worked surfaces. It should be emphasized that the value of $\lambda/4$ is the maximum variation allowed after passing through the entire DLD, and therefore, the maximum variation for any individual unit is much less. For a DLD with 10^6 resolvable positions, the total number of units would be 20, i.e., 2^{20} is approximately 10^6; and therefore the maximum wavefront error in any unit should be less than $\lambda/4\sqrt{20} \cong \lambda/20$ where by using the square root we have

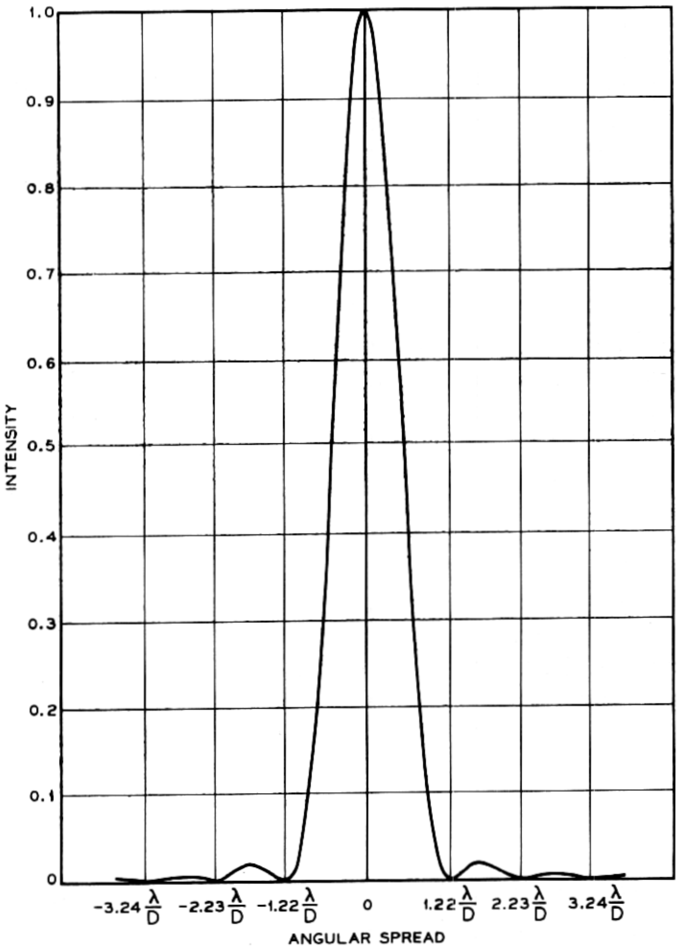


Fig. 2 — Far-field diffraction at a circular aperture (the Airy pattern).

assumed random irregularities. The requirement that a modulator have an optical distortion of less than $\lambda/20$, not allowing for any imperfections in the remainder of the optics, is extremely difficult and will probably represent a serious problem for some time to come. The high requirements placed on individual components is a direct result of the large number of such elements that must be placed in series for the complete DLD.

The maximum angular aperture of a DLD is limited by a number of effects: (*i*) the response of the Wollaston prisms, (*ii*) the walk-off of the beam as it is deflected to larger and larger angles, (*iii*) the angular aperture of the modulators, and (*iv*) the angular aperture of the output lens. These limitations will now be considered in more detail.

The deviation angle of a Wollaston prism is not constant but is a function of the incident angle (see Appendix A) and at some angle the deviation will vary sufficiently such that the array of angles is no longer uniformly spaced. Calculations based on the equations in Appendix A indicate that if the Wollaston prism with the smallest deviation is placed first in the DLD and the next largest second, etc., for a total of 20 stages and a maximum angle of deviation of 8°, the array is uniformly spaced to within 10 percent.

As the beam traverses through the DLD, the deflection angle can become larger and larger and unless the apertures of the prisms and modulators are very large, the beam will eventually strike the sides of the apparatus. It is clear that the Wollaston prism with the largest deviation angle should be placed last in the DLD in order to minimize the spreading of the beams. With this arrangement approximately ½ of the beam is intercepted by the apparatus for a 20-stage deflector with a maximum deviation angle of 8°. This latter figure is not a result applicable in all cases because it depends on the specific lengths of the elements in a DLD.

The relative retardation in a modulator is also a function of the incident angle. How rapidly this function varies depends on the type of modulator. In KDP and similar crystals the angular aperture is very small (less than 1 minute of arc for 30 dB extinction between crossed polarizers for a crystal thickness of 0.089 inches[11]) since these crystals are uniaxial, with a large birefringence, and will, therefore, give large relative retardation for even small angles away from the crystal axis. Techniques are reported that compensate the birefringence[11,12] in KDP but the degree of success is not made clear. Because of the limited angular aperture, crystals in the class with KDP are not considered attractive for the DLD. In CuCl and other crystals in this class, the angular aperture can be very large because they are cubic crystals in the field-free case and are therefore optically isotropic. Angular apertures of ±25° have been reported[13] for this material. KTN is also a cubic crystal, but the electro-optic effect in this crystal is quadratic in contrast to the linear effect in most other materials useful as modulators. Therefore, KTN is usually biased by a dc voltage in order to reduce the value of the modulation voltage, and this bias reduces the angular aperture of the modulator; however, as shown in Appendix B, the angular aperture can still be ±10° for reasonable bias fields.

The lens at the output of the DLD will focus the beams to points on the image plane. If this lens is not perfect, the spots will be larger than that calculated by diffraction theory, and the capacity of the DLD will be reduced. Since the choice of lens will depend on the application of the DLD, it is not possible to state very precisely what the angular aperture could be; however, ±10° seems reasonable for most applications.

The calculations on the capacity of the DLD have been based on a plane wave of uniform intensity resulting in an Airy pattern in the far field. By placing a filter in such a beam which attenuates the light as a function of the radial distance, it is possible to greatly reduce the energy in the rings at an expense of slightly increasing the size of the central disk.[14] If such a filter could be effectively incorporated in the DLD, the crosstalk between resolvable positions could be greatly reduced without significantly reducing the overall capacity.

As an illustrative example, we will calculate the number of resolvable positions assuming that the minimum angle is set by diffraction theory which corresponds to 2.44 λ/D for 16.7-dB crosstalk and to 4.88 λ/D for 23.2-dB crosstalk, and that the maximum angle is ±10°. For a wavelength of 6000Å and an aperture of 1 cm, this corresponds to a two-dimensional array of approximately $4(10)^6$ resolvable positions with a crosstalk ratio of 16.7 dB and $(10)^6$ positions with a crosstalk of 23.2 dB. These two cases imply 22 basic units in the first case and 20 units in the latter. If one can learn to use much larger angles, then the number of positions will increase; however, on the other hand, if components are optically imperfect so that diffraction-limited performance is not possible, then these numbers will be reduced.

Thus far the DLD has only been considered in conjunction with a diffraction-limited beam; however, images may also be transmitted through the deflector. Since it takes a number of diffraction-limited points to make up an image, the capacity of a DLD in terms of images will obviously be less.

III. PERFORMANCE OF A MANUALLY-OPERATED DLD

At the present time it is not possible to construct a large capacity DLD using electro-optic modulators since these materials are not available in the quantity and quality required. In order to check the performance of this system, it is necessary to replace the electro-optic modulators with half-wave plates and thereby replace electronic activation with mechanical rotation.

A system as shown in Fig. 3 was constructed consisting of 18 mica half-wave plates, 7 pair of quartz Wollaston prisms, and 2 pair made from calcite. The aperture of the system was 18 mm and the wavelength was 6328 Å. The smallest deviation angle in the system was 1 minute and the largest was 4°; the smallest deflection angle corresponds to 8.3 λ/D, which is a separation somewhat larger than that considered earlier in this paper. The aperture of the pinhole was (0.001 inches which is larger, by a factor of approximately 4, than that required to give a diffraction-limited divergence to the wave emerging from lens 1. This system when used with an aperture of this size should be considered to be deflecting an image rather than operating with a diffraction-limited beam. The purpose of using a spot of this size is that the ratio of light in the central disk to that in the diffraction rings is much higher than when a diffraction-limited beam is used, hence the crosstalk between adjacent positions should decrease when compared to the diffraction-limited case.

Fig. 4 is a picture of a focal plane taken with this apparatus. It shows the $2^{18} \approx \frac{1}{4}(10)^6$ resolvable positions. This picture was taken by setting each half-wave plate in the halfway position so that light

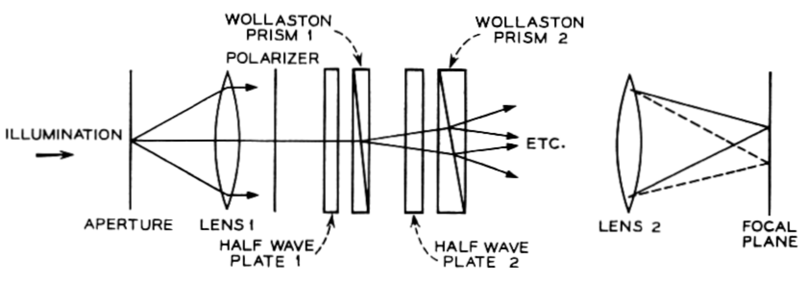


Fig. 3 — Arrangement of elements for a high capacity DLD.

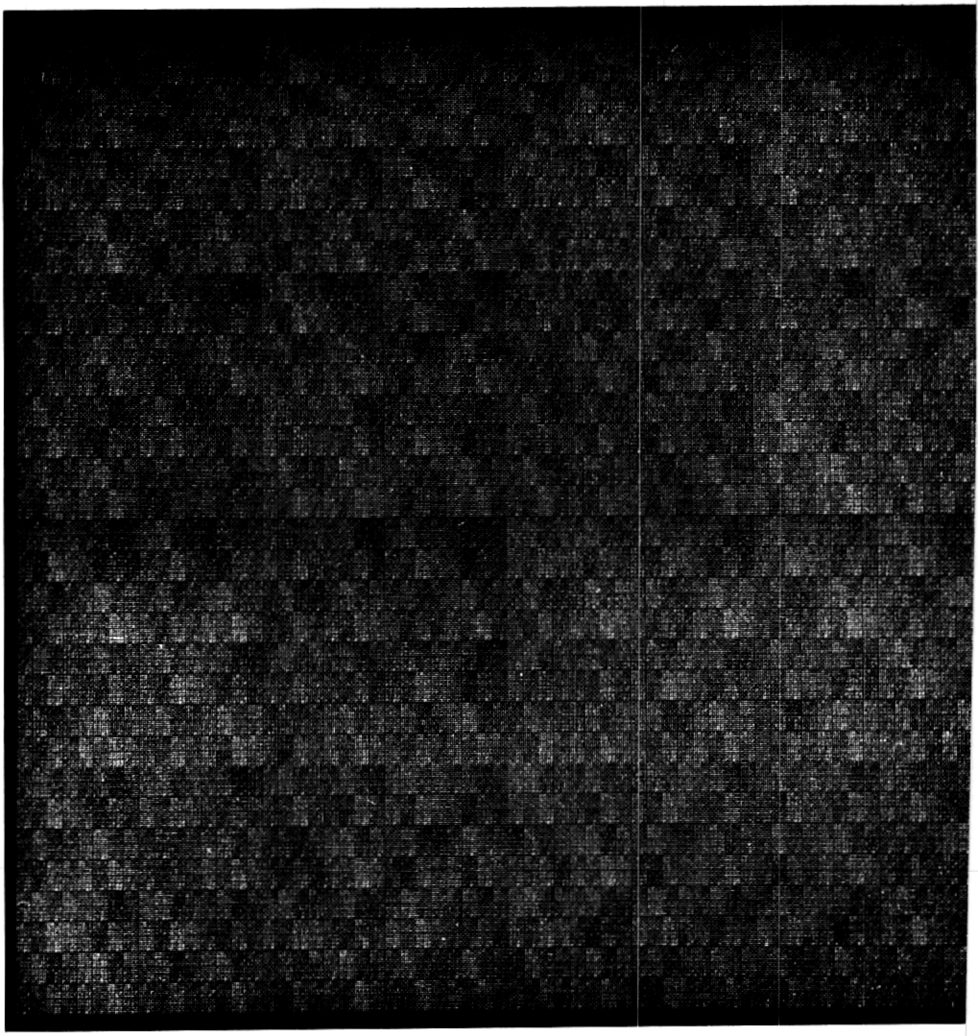


Fig. 4 — $2^{18} \approx \frac{1}{4}(10)^6$ resolvable positions of the experimental apparatus.

was divided equally into both polarizations. In this way all 2^{18} positions are simultaneously illuminated. Fig. 5 is an enlargement of an arbitrarily-selected subsection of Fig. 4 and shows the resolution much more clearly.

Fig. 6 is an enlargement of a single position taken under two cases: (*i*) the pattern on the top illustrates the focused beam with the DLD removed from the system, and (*ii*) the pattern on the bottom is the same focused beam with the DLD in the system. The degradation of the pattern on the bottom is a result of the optical imperfections in the many elements that make up the DLD. A comparison of the two patterns shows that the DLD did not increase the size of the central disk by an appreciable factor but did make it much more irregular. Fig. 6 was overexposed in order to show the weaker diffraction rings much more clearly.

The crosstalk ratio between adjacent positions was measured by first setting the DLD so that only one position was present at the focal plane. A 0.001-inch aperture was placed at the focal point, adjusted in position for maximum light transmission, and the amount of light was measured by a detector. The aperture was then moved to an adjacent position, and the amount of light passing through the opening was again measured. The ratio of these two numbers is the crosstalk. The measurements were made for various settings of the DLD and the values ranged from 20 to 28 dB. The range in the measurements is presumably due to imperfections in the optical system which cause the focused spot to be irregular and unsymmetric in shape. The irregular shape is also evident from Fig. 6.

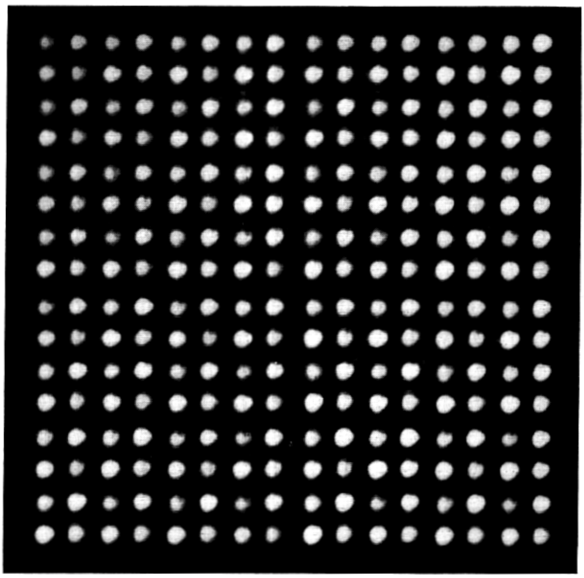


Fig. 5 — An enlargement of a section of Fig. 4.

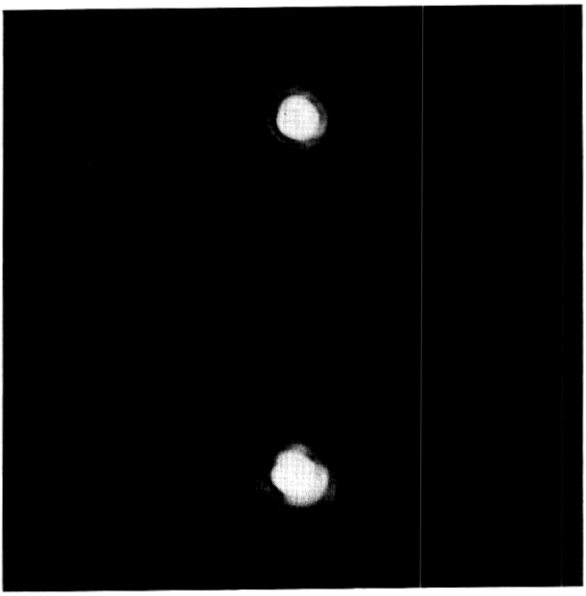


Fig. 6 — The degradation of the focused beam pattern by the DLD. (The pattern on the top was taken without the DLD in the system and that on the bottom with the DLD in the system.)

The performance of this system is probably worse than the $\lambda/4$ tolerance discussed previously in the paper but is probably not much worse than a wave or so; this latter figure was not measured directly but was estimated from diagrams which show spot patterns as a function of various aberrations.[10,15]

IV. DLD PERFORMANCE RESULTING FROM IMPERFECT MODULATORS

It is anticipated that an electronically activated modulator will be the weak link in DLD performance for some time to come; it therefore is important to know how an inefficient modulator will affect the performance of a DLD. In this study, we assume that the Wollaston prisms in the DLD are perfect and that the modulator can be characterized by a single term, E, which is defined as

$$E = \frac{a}{b} = \frac{\text{light intensity in the desired polarization}}{\text{light intensity in the undesired polarization}} \tag{1}$$

with

$$a + b = 1.$$

A perfect modulator by this definition has an infinite efficiency.

With perfect modulators the image plane would have one bright spot at the desired position and the remaining 2^n–1 positions would be completely dark. With imperfect modulators, and for simplicity we assume that they are all imperfect to the same degree, some light will fall on every position. The resulting intensity distribution on the focal plane has been studied by others[16,17] and is also given in Appendix C. It is shown there that the intensities in the focal plane of an n unit DLD can be generated by the expansion of $a^n[1 + (1/E)]^n$ where the first term a^n gives the intensity of the desired position, the second term $n(a^n/E)$ implies n positions with intensity a^n/E, the third term $[n(n-1)/2](a^n/E^2)$ implies $n(n-1)/2$ positions of intensity a^n/E^2, etc. The sum of all the coefficients in the expansion of $(1 - 1/E)^n$ is equal to 2^n so that each position in the focal plane can be assigned to one of these terms.

It can also be established that the polarization of the even powers of E, i.e., a^n, a^n/E^2, a^n/E^4, etc. have the opposite sense of polarization from that of the odd powers of E, i.e., a^n/E, a^n/E^3, etc. This result can be determined from the basic definition of the efficiency (1).

The requirement on the modulator efficiency is determined by the particular application of the DLD. If the deflector will be used to accomplish localized heating or welding, to supply energy for a switch, or to be used as a printout or display, then the ratio of the intensity at the desired location to that at the next highest position is important. This ratio must be sufficiently large so that the intensity at the desired location must be great enough to cause the reaction, and yet the intensity at the next brightest position must be less than that to cause the reaction. For example, it is possible that for processes that depend on heating 10 dB is a sufficient ratio, whereas for a visual display 30 dB or greater may be necessary so that the eye will not be confused by multiple images. The intensity in the desired position, as previously stated, is a^n and that in the next highest case is a^n/E for the opposite polarization and a^n/E^2 for the same polarization. Therefore, this ratio is $1/E$ when there is no polarization selection and $1/E^2$ when polarization selection is used. In general, it should always be possible to eliminate the opposite polarization so that the first troublesome term will be a^n/E^2.

The DLD can also be used as a memory device.[2] In this application a memory is placed in the focal plane which is constructed such that at each of the focused points an opaque or transparent spot is present. This code is suitable for a binary organized memory where, for example, the opaque position can represent 0 and the transparent position can represent 1. The nature of the position can be read by placing a detector behind the memory and then directing a light beam to the desired location. The presence of an opaque position is determined by no response at the detector, and similarly a transparent position will result in a positive response.

When the modulators are imperfect, the undesired beams of light will also strike the memory plane and have some chance of reaching the detector with the possibility of causing erroneous results. For error-free operation the detector must receive more light when the DLD is addressed to a transparent position than when it is addressed to an opaque position, and for future reference let us call the ratio of these two intensities the signal-to-background ratio, R. The least light that can reach the detector when the DLD is addressed to a transparent position is a^n, i.e., the main beam alone, while the most light that can reach the detector when the DLD is addressed to an opaque position is $I_{tot} - a^n$, i.e., all the light except for the main beam. The minimum signal-to-background ratio is then

$$R_{min} = \frac{a^n}{I_{tot} - a^n}. \tag{2}$$

The R_{min} ratio defined by (2) is not an unreasonable minimum in that a memory plane can be designed to give ratios very close to the values calculated by this equation.

Three different ways of interrogating the memory will be discussed, and for each case calculations will be made for the signal-to-background ratios. The first case is where all of the light is allowed to strike the memory plane; the second uses a polarization selection before the memory so that only light polarized in the same sense as the main beam will reach the memory plane; and the third is the reflection mode of operation. To prevent confusion the subscripts 1, 2, 3 will be used on the R_{min} ratios for the three cases mentioned.

4.1 *Case 1—All Light From The DLD Allowed to Reach The Memory*

In this case $I_{tot} = 1$ since $a + b = 1$. Therefore,

$$(R_{min})_1 = \frac{a^n}{1 - a^n} = \frac{1}{\left(1 + \frac{1}{E}\right)^n - 1}$$

$$= \frac{1}{\frac{n}{E} + \frac{n(n-1)}{2!}\left(\frac{1}{E^2}\right) + \frac{n(n-1)(n-2)}{3!}\left(\frac{1}{E^3}\right) + \cdots}. \tag{3}$$

The signal-to-background ratio for this case is plotted as a function of modulator efficiency, E, for several values of n in Fig. 7. It shows that for an $n = 20$ DLD with an $(R_{\min})_1$ ratio of 5 dB, a modulator efficiency of 18.6 dB is required.

4.2 *Case 2—Polarization Selection Before The Memory*

If polarization selection is used after the DLD, which would require an additional modulator and polarizer, the odd powers of E can be cancelled from (2) and the $(R_{\min})_2$ ratio becomes

$$(R_{\min})_2 = \frac{1}{\frac{n(n-1)}{2!}\left(\frac{1}{E^2}\right) + \frac{n(n-1)(n-2)(n-3)}{4!}\left(\frac{1}{E^4}\right) + \cdots}. \tag{4}$$

This ratio, $(R_{\min})_2$, is plotted as a function of efficiency in Fig. 8. It shows that for $n = 20$ and $(R_{\min})_2 = 5$ dB, a modulator with an efficiency of 14.0 dB is required. This case represents an improvement of 4.6 dB over the first case.

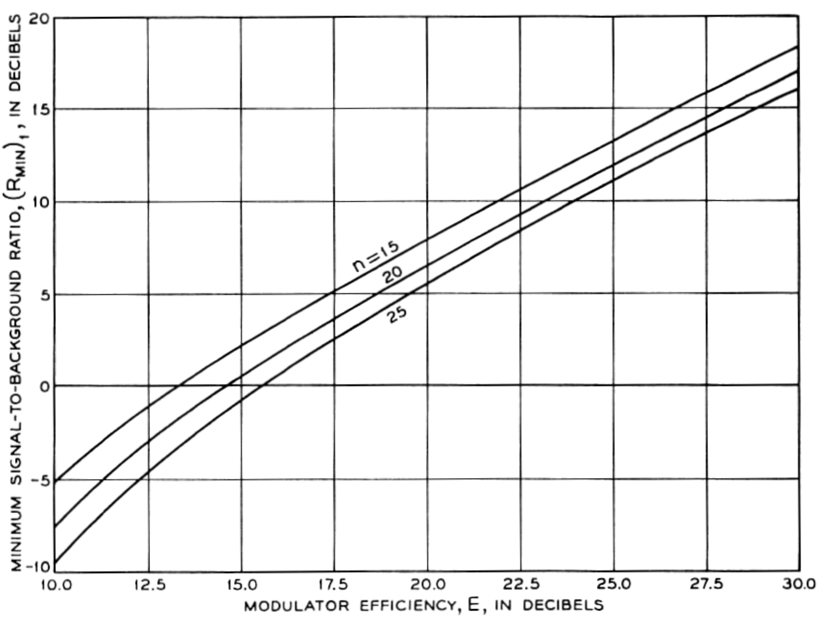


Fig. 7 — Minimum detector ratio versus modulator efficiency for a DLD where the detector is placed behind the focal plane.

4.3 *Case 3—Reflection Mode of Operation*

An alternate way[18] of reading the memory plane is shown in Fig. 9. In this case, the light is reflected from a mirror located just behind the memory and is redirected through the DLD to be detected after passing through a second aperture. The second aperture eliminates a large part of the background and therefore the ratio, $R_{\min}$, for the same modulator efficiency is considerably improved. The derivation of the $R_{\min}$ ratio for this reflecting mode of operation, $(R_{\min})_3$, is given in Appendix D and only the result is shown here:

$$(R_{\min})_3 = \frac{1}{n\left(\frac{1}{E^2}\right) + \frac{n(n-1)}{2}\left(\frac{1}{E^4}\right) + \cdots}. \tag{5}$$

$(R_{\min})_3$ is plotted as a function of E in Fig. 10. With this mode of operation a modulator with an efficiency of only 9.1 dB is required to give an $(R_{\min})_3$ ratio of 5 dB or better. The reflection mode of operation therefore represents an improvement, when measured by reduced requirements on the modulator, of 9.5 dB over the first case and 4.9 dB

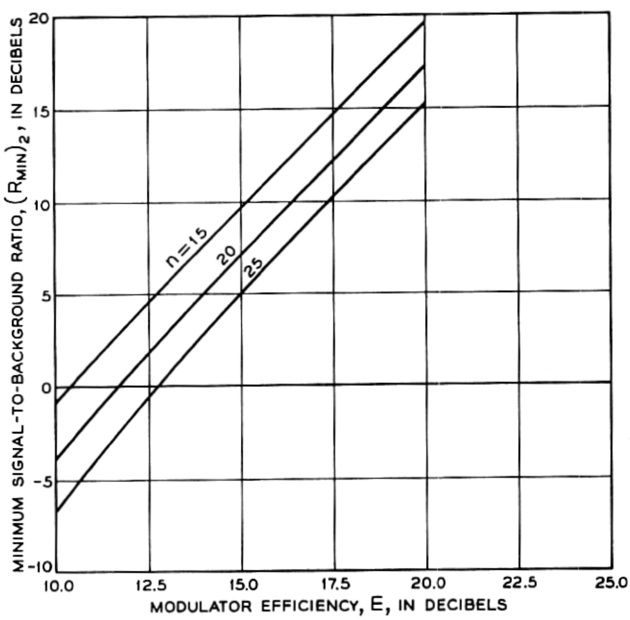


Fig. 8 — Minimum detector ratio vs modulator efficiency for a DLD with the detector placed behind the focal plane and with polarization selection.

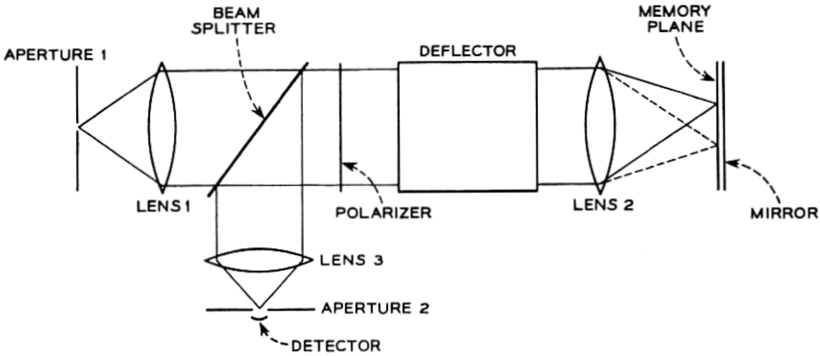


Fig. 9 — Arrangement of elements for the DLD using the reflection mode of operation.

over the second case. It should be emphasized that the improvement ratios given here are strongly dependent on the choice of the $R_{\min}$ ratio, which was taken to be 5 dB in this paper. For larger $R_{\min}$ ratios the improvement in the E ratio would be even greater and vice versa.

A disadvantage of the reflection mode is that a low f-number lens must be used at the output of the DLD. The reason for this is that the light reflected from the plane mirror must enter the output end of the DLD, and this requires that the focal plane be approximately ½ the linear dimension of the aperture of the DLD. One can show that this requires a lens of f:1.5 or so if the angular spread of the DLD is $\pm 10°$. If a lens is designed to have a spherical focal surface and a spherical mirror is used as the reflector, then the lens can have any f number.

Any light reflected from the surfaces in the DLD when used in the reflection mode can be prevented from entering the aperture near the detector by giving a slight tilt to the elements that make up the DLD. It is possible to choose an angle such that no reflection is centered on the aperture.

V. EXPERIMENTS WITH THE MANUALLY-OPERATED DLD EQUIPPED WITH POOR MODULATORS

The experiments that will be described in this section make use of the $n = 18$ manually-operated DLD where the half-wave plates have been substituted for the electro-optic modulators. This is the same apparatus as used in Section III.

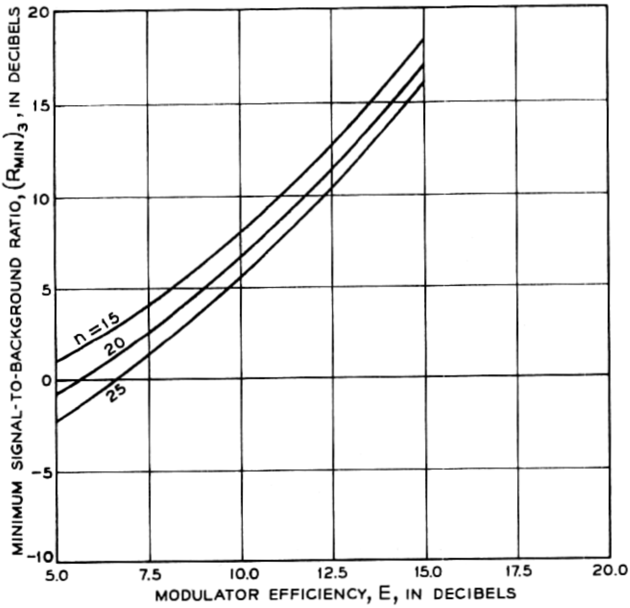


Fig. 10 — Minimum detector ratio versus modulator efficiency using the reflection mode of operation.

Ideal half-wave plates have the property that if the angle between the polarization direction and the axis of the half-wave plate is θ then the plane of polarization emerging from the plate will be rotated by 2θ from its original direction. For maximum efficiency, the half-wave plates are oriented at $\theta = 0$ if no switching action is desired and at $\theta = 45°$ if the other polarization is desired. The practical maximum efficiency for the split mica plates used in this experiment ranged from 30 to 40 dB, which is high enough to give almost perfect DLD behavior. In order to simulate poor modulators, the wave plates are set at $\theta = \epsilon$ for the predominantly unswitched case and at $\theta = 45° - \epsilon$ for the predominantly switched case. The angle ϵ can then be adjusted to achieve any degree of modulator efficiency.

To illustrate the behavior of a DLD under the influence of poor modulators, the half-wave plates were set for $E = 10$ dB, and a picture, which is shown in Fig. 11, was taken at the focal plane. This picture shows some characteristics which will now be enumerated:

(*i*) The desired spot is shown as the brightest point in the upper left-hand quadrant and is vertically polarized.

(*ii*) Some of the 18 points whose intensity is $1/E$ of the main beam are shown in the lower right-hand quadrant and are horizontally polarized.

(*iii*) Some of the 153 points whose intensity is $1/E^2$ of the main beam are shown in the upper left-hand quadrant and are vertically polarized.

(*iv*) Some of the 816 points whose intensity is $1/E^3$ of the main beam are shown in the lower right-hand quadrant and are horizontally polarized.

Fig. 11 — Focal plane intensity distribution with modulators set at $E = 10$ dB.

(*v*) The points in the upper left-hand quadrant are vertically polarized and those in the lower right-hand quadrant are horizontally polarized. A polarization selector in this case would eliminate the entire lower side. It will always eliminate the side that is opposite to the one that contains the main beam.

(*vi*) There are no points with significant intensity ($> 1/E^{n/2}$) in either the upper-right or lower-left quadrant.

As all of the focal plane is not shown in this figure, some of the background positions are missing in order that an enlargement could be presented. In this exposure there is a total of approximately 4.6 times more light energy in the background than in the main beam.

The performance of the reflection mode of operation was compared to the theoretical calculation by making use of the properly misoriented half wave plates. For two reasons (5) cannot be directly used: (*i*) In this experiment a memory was used that contained only one opaque position and for such a case the signal-to-background ratio using this equation is not accurate, and (*ii*) equation (5) assumes that the opaque positions are also perfectly absorbing which is not valid for this experiment in that the opaque position reflected 4 percent of the incident power. A signal-to-background ratio for this particular experiment can be calculated as follows. When the DLD is addressed to a transparent position the light reaching the mirror is I_{tot} and the light striking the mirror when the DLD is addressed to the opaque position is $I_{tot} - (1 - \Gamma)a^n$ where Γ is the power reflection coefficient. The ratio of these two values is

$$(R_{exp})_3 = \frac{I_{tot}}{I_{tot} - (1 - \Gamma)a^n} = \frac{I_{tot} - (1 - \Gamma)a^n}{I_{tot} - (1 - \Gamma)a^n} + \frac{(1 - \Gamma)a^n}{I_{tot} - (1 - \Gamma)a^n}$$

$$= 1 + \frac{(1 - \Gamma)a^n}{\dfrac{a^n}{(R_{min})_3} + \Gamma a^n} = 1 + \frac{(1 - \Gamma)}{\dfrac{1}{(R_{min})_3} + \Gamma}. \tag{6}$$

The calculated and experimental values of $(R_{exp})_3$ are summarized in Table I. The first column lists the modulator efficiency, the second column contains the calculated $(R_{exp})_3$ and the third column lists the measured values.

The agreement between the calculated and measured quantities agree very well and indicate that the behavior of the reflection mode of operation is adequately understood.

TABLE I

Modulator efficiency	Signal-to-background ratio for the reflection mode experiment, $(R_{exp})_3$	
dB	Calculated in dB	Measured in dB
30	14.0	14
20	13.8	14
10	7.0	8

VI. PARALLELLING THE OUTPUT

In the DLD thus far discussed, only one memory in the output focal plane is used (see Fig. 3), and therefore the memory is read one bit at a time. For some applications it may be advantageous to parallel the output as shown in Fig. 12 in order to increase the bit capacity of the DLD. With this scheme the number of bits read with each setting of the DLD is equal to the number of memory planes; the memory now corresponds to one with word organization. Fig. 12 shows four such memory planes but by going to a three-dimensional array it is possible to parallel 30 to 40 such planes and still use only one output lens providing the maximum angular aperture of the DLD is limited to a total angle of 12° or so. If additional lenses are employed, then any number of memory planes can be incorporated.

This scheme of paralleling is directly applicable to cases 1 and 2, which were discussed in Section IV, but will not work for the reflection mode since there is no way to distinguish between different memory planes when the light is redirected through the DLD. In order to parallel the output of the reflection mode, three different schemes have been devised: (*i*) to use different wavelengths for each memory plane and separate the colors before and after the DLD,[19] (*ii*) to modulate a monochromatic beam at each memory plane with a different frequency and then to separate the different frequencies after reflection through the DLD,[20] and (*iii*) to arrange the memory planes to have different distances between the output of the DLD and the memory plane and to use a short pulse of light; the different planes can now be read since each plane will return the pulse to the detector at a different time.[21]

One difficulty that can arise in the paralleling schemes is that very low *f*-number lenses must be used to refocus the output plane into repeated images (Fig. 13). The first lense placed after the DLD can

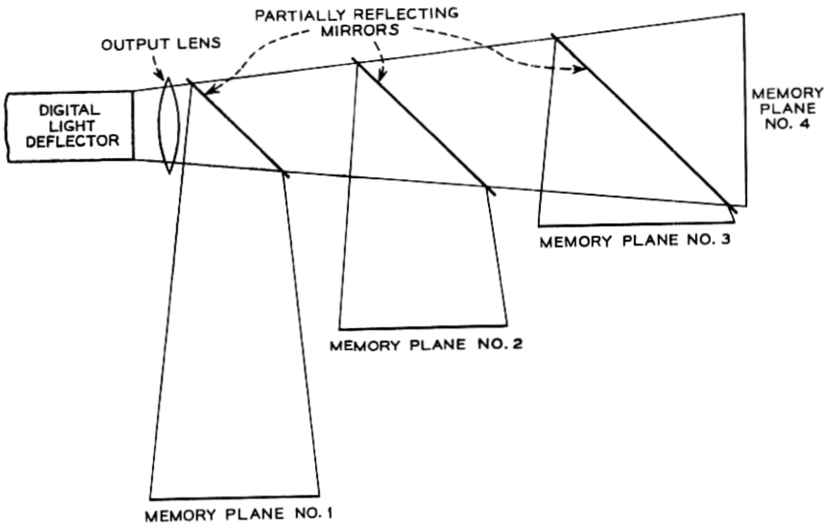


Fig. 12 — One method of paralleling the output of the DLD.

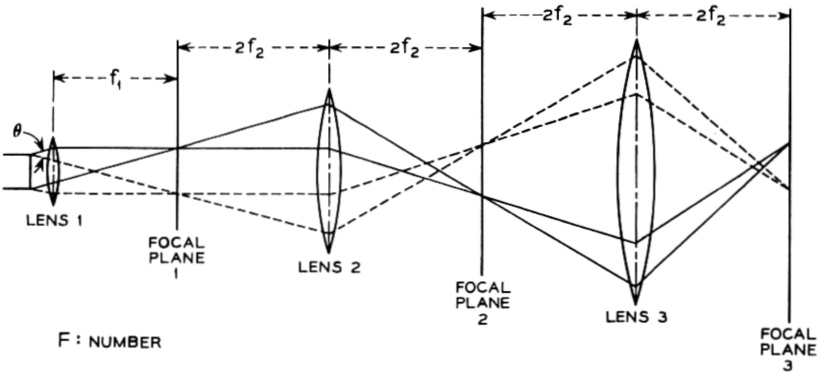
have a reasonable f number since the light beams are still confined to the aperture of the DLD. The second lens must have f:1 or so if the output of the DLD has a total angle of approximately 12°. Additional lenses must have even lower f numbers (Fig. 13). The practical solution to this problem is to perform all of the paralleling within the first focal length. This scheme will not work with the reflection mode where the time of flight varies for each memory plane since one lens implies only one distance between it and the various memory planes.

VII. MEMORY MEDIA

The problem of reading a memory has been discussed in earlier sections of this paper; we will now consider the problem, which is again primarily the result of imperfect modulators, of using the DLD to write into a memory. Two general types of materials will be considered for use as a memory medium. One is a medium where the process is linear in terms of total exposure, i.e., the effect on the medium of n pulses of light of intensity I/n, each lasting for a time ΔT, is the same for any value of n; and the second is one which has a threshold in terms of the light intensity. An example of the first type is a photographic film and of the second is a memory based on a transparent ferrimagnetic garnet at its compensation temperature.[22]

A linear medium has the disadvantage, for this application, in that it integrates the light striking its surface. Therefore, when the photographic film is being exposed by a DLD with poor modulators, the problem of the background light must be considered. This problem is different from that considered in Section IV because in that case the DLD was set for one address and the question was asked what is the light intensity distribution over the whole focal plane. In this case, we ask what is the total amount of light energy striking one position on the memory plane when the DLD is addressed to all of the positions. When the DLD is set for one address, the total exposure over the whole plane is $a^n[1 + (1/E)]^n \Delta T$ where ΔT is the duration of the exposure. This result is evident from the discussion in Section IV. When one sits at a position and the DLD is addressed to all positions and dwells at each position for the same time ΔT, the total exposure at that position is also $a^n[1 + (1/E)]^n \Delta T$. This latter result has been previously published[17] and is also proven in Appendix C.

The following calculations, which represent worst cases for writing, can be performed. The first case that will be considered is the situation where all of the light is allowed to strike the memory plane and the second case makes use of polarization selection. In both cases it will be assumed that all points will be addressed, except for one.



$$\text{LENS 1} = \frac{D}{f_1}$$

$$\text{LENS 2} = \frac{1}{2\left[\left(\frac{f_1}{2f_2} + 1\right)\theta + \frac{1}{F_1}\right]}$$

$$\text{LENS 3} = \frac{1}{2\left[\left(\frac{3f_1}{2f_2} + 1\right)\theta + \frac{1}{F_1}\right]}$$

$$\text{LENS N} = \frac{1}{2\left[\left(\frac{2N-3}{2}\,\frac{f_1}{f_2} + 1\right)\theta + \frac{1}{F_1}\right]}$$

Fig. 13 — Lens requirements for re-imaging the output focal plane of the DLD.

7.1 *Case 1—All Light from the DLD Allowed to Reach the Memory*

In this case the light striking any of the addressed positions will have an exposure of nearly $a^n(1 + 1/E)^n \Delta T$ and the exposure at the one position that was not addressed will be $[a^n(1 + 1/E)^n - a^n]\Delta T$, and the ratio of these two values is

$$M_1 = \frac{a^n\left(1 + \frac{1}{E}\right)^n}{a^n\left(1 + \frac{1}{E}\right)^n - a^n} = \frac{a^n\left(1 + \frac{1}{E}\right)^n - a^n}{a^n\left(1 + \frac{1}{E}\right)^n - a^n} + \frac{a^n}{a^n\left(1 + \frac{1}{E}\right)^n - a^n} \tag{7}$$

$$= 1 + (R_{\min})_1 .$$

7.2 *Case 2—Polarization Selection Before the Memory*

In the same way as in the first case one can derive

$$M_2 = 1 + (R_{\min})_2 . \tag{8}$$

The exposure ratios M_1 and M_2 are plotted as a function of modulator efficiency in Figs. 14 and 15. These plots can be used to determine the minimum modulator efficiency required for a certain exposure ratio.

From the exposure ratio and the properties of the medium, e.g., photographic film, the density ratios of the positions can be calculated. These two ratios do not have to be the same, as a material such as photographic film can be very linear in terms of exposure but the exposure vs film density can be very nonlinear.

For a theshold medium the problem is much simpler. The only requirement is that the most intense beam must be greater than threshold and that the next highest position be less than threshold. As mentioned in Section IV, this ratio is $1/E$ when no polarization selection is used and $1/E^2$ when polarization selection is used.

VIII. ANALOG CONTRASTED WITH DIGITAL DEFLECTION

The same large number of resolvable positions as described in this paper could, in principle, be achieved by means of an analog deflection; an example of this is a prism of an electro-optic material with electrodes placed on the parallel surfaces. It is necessary to induce an increase of 2π retardation along the base of such a prism in order to deflect the beam by one resolvable position. Therefore, for a 10^6 position deflector it is necessary to have $2(10)^3\pi$ along the X bank and

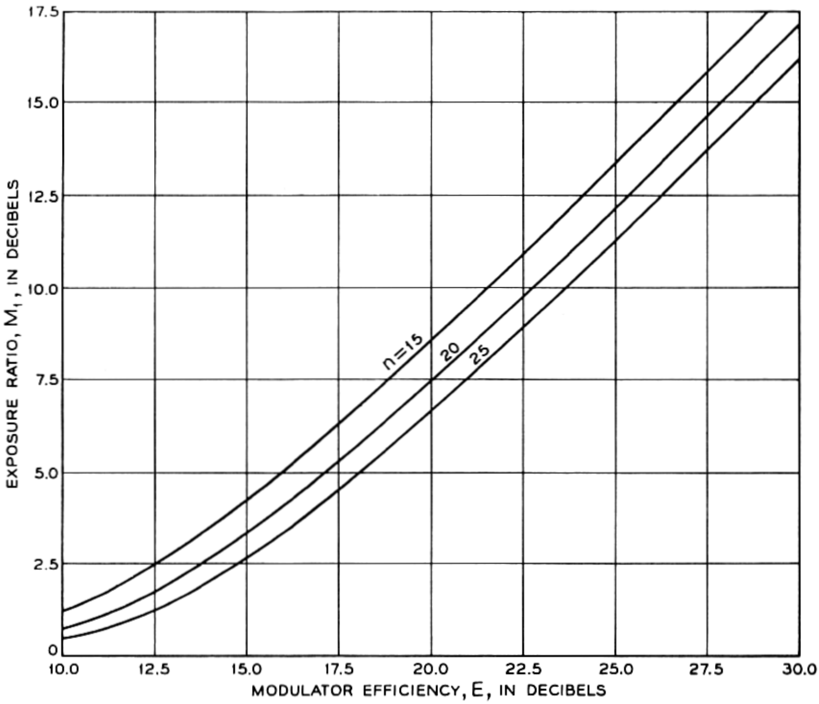


Fig. 14 — Exposure ratio vs modulator efficiency for a DLD where all of the light is allowed to reach the memory plane.

$2(10)^3\pi$ along the Y bank for a total of $4(10)^3\pi$ total retardation. With a DLD, a total retardation of 20π can accomplish the same number of resolvable positions. Therefore, it is evident that the DLD makes very efficient use of the variable retardation. The reason for this efficiency is that the DLD makes use of the fixed retardation in the passive elements whereas the analog deflector must generate all of the retardation. In addition, the DLD can be designed for any separation between the beams and still not require any more than the 20π variable retardation. The analog reflector, on the other hand, cannot separate the beams any further without supplying additional retardation.

IX. CONCLUSION

The construction and characteristics of a high-capacity DLD have been described, and it has been demonstrated that the number of

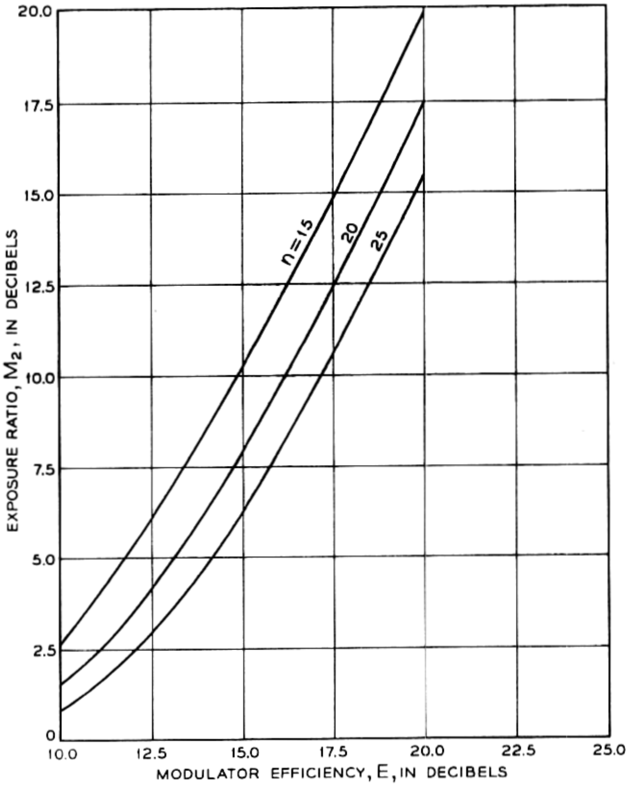


Fig. 15 — Exposure ratio vs modulator efficiency for a DLD where polarization selection is used.

resolvable positions that can be attained is reasonably close to that allowed by diffraction theory. The effect of imperfect modulators on the performance of the DLD has also been discussed.

The discussion presented here does not mention the problems associated with the high-speed switching of an electro-optic modulator, which is a problem that must be solved if the DLD is to have broad application. This problem has been studied by S. K. Kurtz.[23]

X. ACKNOWLEDGMENTS

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APPENDIX A

Optical Properties of a Wollaston Prism

The formulas necessary to trace the two wave normals through a Wollaston prism in a direction as shown in Fig. 16 are given below:

$$\begin{aligned} \sin\beta &= \frac{1}{n_o}\sin\gamma \\ \tan\alpha &= \frac{n_o}{n_e}\tan\beta \\ \sin(\theta+\epsilon) &= \frac{\sin\beta}{\sin\alpha}\sin(\theta+\alpha) \\ \sin\alpha &= n_o\sin\epsilon \end{aligned} \tag{9}$$

. . . .

$$\begin{aligned} \sin\beta &= \frac{1}{n_o}\sin\gamma \\ \sin(\theta+\delta) &= \frac{n_o}{n_e}\sin(\theta+\beta) \\ \sin b &= n_e\sin\delta, \end{aligned} \tag{10}$$

where the symbols are defined in Fig. 16.

A useful approximate formula for calculating the total deviation angle of a Wollaston prism, Δ, ($\Delta = a + b$) for perpendicular incidence, $\gamma = 0$ is

$$\Delta = (\,|a| + |b|\,)_{\gamma=0} = 2\,|n_o - n_e|\tan\theta + \cdots . \tag{11}$$

The variation of Δ with respect to a variation in γ at perpendicular incidence, $(\partial\Delta/\partial\gamma)_{\gamma=0}$, can be calculated from (9) and (10) to be

$$\left(\frac{\partial\Delta}{\partial\gamma}\right)_{\gamma=0} = \cos\theta\left\{\frac{\cos\delta}{\cos b\,\cos(\theta+\delta)} - \frac{\cos\epsilon}{\cos a\,\cos(\theta+\epsilon)}\right\}, \tag{12}$$

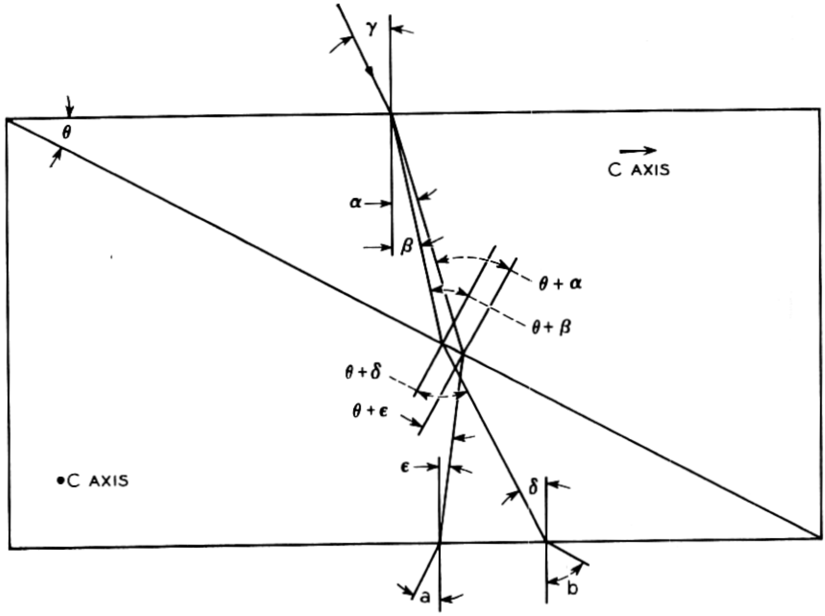


Fig. 16 — Diagram for wave normal paths in a Wollaston prism.

which to a good approximation can be reduced to

$$\left(\frac{\partial \Delta}{\partial \gamma}\right)_{\gamma=0} = (n_o - n_e)\left(\frac{1}{n_e} + \frac{1}{n_o}\right) \tan^2 \theta. \tag{13}$$

APPENDIX B

Angular Aperture of a Modulator

This calculation is valid for materials that are cubic, and therefore optically isotropic, in the absence of an electric field and become uniaxial, with the optic axis parallel to the electric field, in the presence of an electric field.

Fig. 17 describes the placement of the crystal with respect to the incident radiation. The xy plane is the first surface of the modulator, and the second surface is parallel to the first and passes through the point Z equals $-T$. The induced C axis of the crystal is parallel to the y axis. The light ray makes an angle γ with the z axis, and the intersection of the plane of incidence with the xy plane makes an angle α with the x axis. The relative retardation between the extraordinary and ordinary ray can be calculated to be

$$R(e - o) = \left\{ n_e \left[1 - \frac{\cos^2 \alpha \sin^2 \gamma}{n_e^2} - \frac{\sin^2 \alpha \sin^2 \gamma}{n_o^2} \right]^{\frac{1}{2}} - n_o \left[1 - \frac{\sin^2 \gamma}{n_o^2} \right]^{\frac{1}{2}} \right\} \frac{2\pi T}{\lambda_o}, \tag{14}$$

where n_o and n_e are the ordinary and extraordinary indices of refraction and λ_o is the free-space wavelength of the light. Equation (14) reduces the familiar $(n_e - n_o)(2\pi T/\lambda_o)$ for perpendicular incidence.

Since n_o and n_e are nearly the same in this case, we will expand (14) in terms of powers of $(n_e - n_o)$ and drop terms containing $(n_e - n_o)^2$ and higher. We will also expand $\sin \gamma$ using a power series in γ. The result of these substitutions is

$$\begin{aligned} R(e - o) = \Bigg\{ 1 &- \frac{1}{n_o^2} (\cos^2 \alpha - \tfrac{1}{2}) \gamma^2 \\ &- \left[\frac{1}{3n_o^2} (\cos^2 \alpha - \tfrac{1}{2}) - \frac{1}{n_o^4} \left(\frac{\cos^2 \alpha}{2} - \frac{1}{8} \right) \right] \gamma^4 \\ &+ \left[\frac{2}{45n_o^2} (\cos^2 \alpha - \tfrac{1}{2}) - \frac{2}{3n_o^4} \left(\frac{\cos^2 \alpha}{2} - \frac{1}{8} \right) \right. \\ &\left. + \frac{1}{n_o^6} \left(\frac{3 \cos^2 \alpha}{8} - \frac{1}{16} \right) \right] \gamma^6 \Bigg\} \frac{2\pi T(n_e - n_o)}{\lambda_o}. \end{aligned} \tag{15}$$

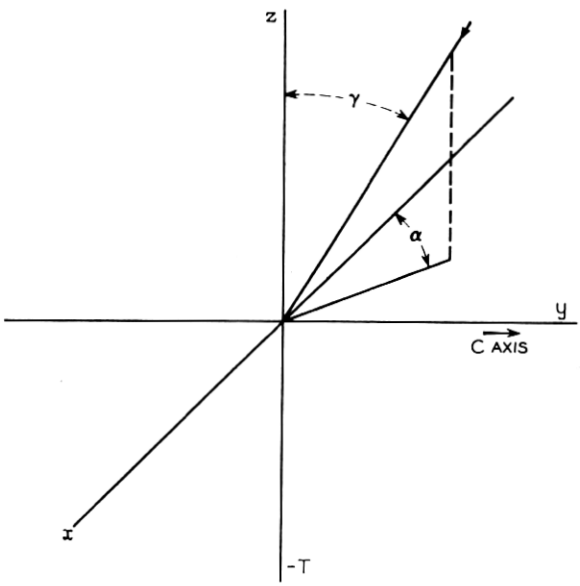


Fig. 17 — Coordinate axes showing the placement of the uniaxial crystal with respect to the incident radiation.

Equations (14) and (15) can be used to describe the interference pattern obtained with a uniaxial crystal whose C axis is parallel to the crystal surface is placed between crossed polarizers. This pattern can be observed in any standard text on optics.[24]

In order that a modulator switch the sense of polarization, it is necessary that the retardation be changed by π. In general, the retardation will be changed from $N\pi$ to $(N + 1)\pi$ with the application of an electric field. The retardation as a function of incident angle for the largest retardation, $(N + 1)\pi$, using terms only up to γ^2, becomes

$$R = (N + 1)\pi\left\{1 + \frac{1}{n_o^2}(\cos^2 \alpha - \tfrac{1}{2})\gamma^2\right\}. \tag{16}$$

The angular aperture of a modulator is determined by the value of γ where the change in retardation from π becomes serious enough to cause unwanted behavior. If we call this change in retardation ΔR, then the value of γ that corresponds to this ΔR can be calculated from (16), i.e.,

$$\gamma = \left[\frac{n_o^2 \Delta R}{(N + 1)\pi(\cos^2 \alpha - \frac{1}{2})}\right]^{\frac{1}{2}}. \tag{17}$$

From (17) one can see that the angular aperture for a given material is inversely proportional to $\sqrt{N + 1}$ so that we can write

$$\gamma_{\text{biased}} = \frac{\gamma_{\text{unbiased}}}{\sqrt{N + 1}}, \tag{18}$$

i.e., to the same degree of performance the angular aperture of a modulator biased to $N\pi$ is decreased by the factor $1/\sqrt{N + 1}$ of the unbiased case.

For a material with a linear electro-optic effect, it is most sensible to use $N = 0$ in order to use the lowest voltages; this will result in the maximum angular aperture. For a quadratic material like KTN, it is sometimes more efficient to use a biasing dc voltage in order to reduce the modulation voltage. In that case, an N of 10 or 20 might be used. If we decide that the maximum retardation error is $\Delta R = 0.0206\pi$, corresponding to a minimum extinction of 30 dB between polarizers, then the angular aperture of KTN is $\pm 26°$ in the unbiased case, $\pm 7.8°$ for $N = 10$, and $\pm 5.7°$ for $N = 20$.

The angular aperture of biased KTN can be increased by placing a properly oriented positive uniaxial crystal such as quartz in series with the biased KTN modulator. This technique can be used to eliminate the terms in γ^2 from the total retardation and thereby increase the angular aperture to approximately that of the unbiased case.

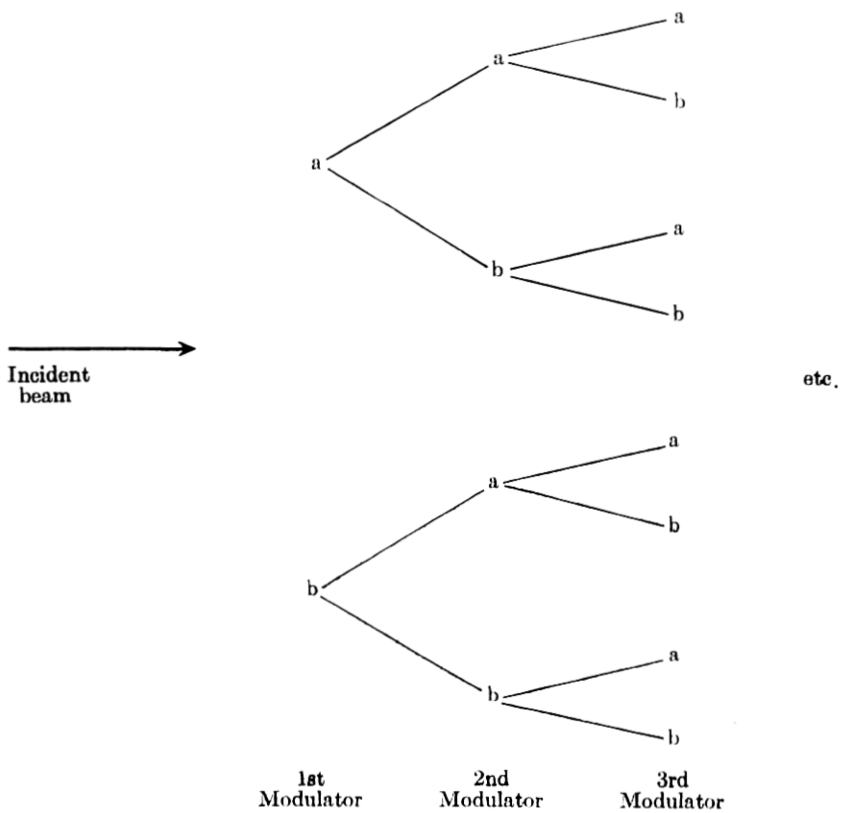
APPENDIX C

Intensity Distribution in a DLD

Assume that we have a DLD consisting of n stages made up of modulators with an efficiency, $E = a/b$, $a + b = 1$. We again assume that only the modulators are imperfect, and that every modulator can be characterized by the same efficiency.

Any light beam incident to a modulator is broken up into two beams, an "a" beam for the desired polarization and a "b" beam for the oppositely polarized position. A table can be made up which lists the total number of paths through the DLD (Table II). Since we have a choice at each modulator as to whether an a or b path is taken, the different paths are characterized by all possible combinations of the

TABLE II



a and b terms. Let us consider the paths containing an r number of b terms and an $(n - r)$ number of a terms. The total number of possible ways of grouping these terms is given by

$$\frac{n!}{(n - r)!\,r!} \tag{19}$$

and the associated intensity of these beams is $a^{(n-r)}b^r$. The total number of all paths is then given by summing r through its range

$$\sum_{r=0}^{n} \frac{n!}{(n - r)!\,r!}\, a^{(n-r)} b^r \tag{20}$$

where the intensity terms have been included. These terms can also be generated by

$$(a + b)^n \tag{21}$$

since the terms $n!/(n - r)!r!$ are also the coefficients of the binomial expansion.

From the definition of E, (21) can also be written

$$a^n\left(1 + \frac{1}{E}\right)^n. \tag{22}$$

To derive (22), the DLD was set for one address and the intensity at each point was determined. We now ask what are the different intensities that arrive at a particular position when the DLD is addressed to all possible positions.

In order to address the DLD to every position, the state of the modulators can be arranged according to Table III. In this table, 0 means no change in the state of polarization and 1 refers to a change

Table III

Modulator number			
etc.	$(n - 2)$	$(n - 1)$	n
	0	0	0
	0	0	1
	0	1	0
	0	1	1
	1	0	0
	1	0	1
	1	1	0
	1	1	1
	etc.	etc.	etc.

TABLE IV

Address of main beam and setting of DLD, A_0	00101
Address of the position at which we wish to compare intensity, A_n	00010
Intensity division at each modulator between A_0 and A_n	1 1 (b/a) (b/a) (b/a)
Intensity at A_0	a^5
Intensity at A_n	a^2b^3

to the opposite sense of polarization. Table III is a partial listing. The complete table is made by first writing the nth column which consists of alternating 0's and 1's for a total number of 2^n entries; the $(n - 1)$th column is written by entering pairs (2^1) of the 0's and 1's for a total of 2^n; the $(n - 2)$th column by entering 2^2 of 0's and 1's, etc. The sequence of addresses in Table III would place the main beam once at each location on the focal plane. The 0's and 1's that appear in any horizontal row is the address of that beam.

We must now be able to compare intensities between that of the main beam, which we shall call A_o, and some arbitrary position, which we shall call A_n. Table IV illustrates the technique. Table IV was constructed by using a rule that sets the intensity ratio at 1 for modulators that have the same setting and b/a at modulators that have different settings.

We now wish to determine all of the intensities at some arbitrary position, say $A_n = \ldots 010$ while the DLD is addressed to all positions. Using Table III which lists all of the addresses and Table IV which illustrates the comparison rule, we can construct a table (Table V) which lists the intensity ratios at $A_n = \ldots 010$.

Table V is similar to Table III in appearance in that for any vertical column a 0, 1 in Table III is changed into 1, b/a or b/a, 1 to make Table V.

Table V is one that lists all possible combinations of the entries 1, b/a and therefore is calculable from the same general formula as that deduced for Table II, i.e.,

$$\left(1 + \frac{b}{a}\right)^n. \tag{23}$$

It is evident that Table V would not change, except for a different ordering of the horizontal rows, no matter what the particular address of A_n. Therefore, (23) is the same for all points A_n.

Equation (23) will list the intensity ratios between A_o and some point A_n. If we multiply though by the intensity of $A_o = a^n$, then (22) becomes

$$(a + b)^n \tag{24}$$

which is the desired result.

TABLE V—LIST OF INTENSITY RATIOS AT POINT ···010 WHEN DLD IS ADDRESSED TO ALL POSITIONS

Modulator number			
etc.	$(n - 2)$	$(n - 1)$	n
	1	b/a	1
	1	b/a	b/a
	1	1	1
	1	1	b/a
	b/a	b/a	1
	b/a	b/a	b/a
	b/a	1	1
	b/a	1	b/a
	etc.	etc.	etc.

APPENDIX D

*R Ratio For The Reflection Mode Of Operation**

A beam of light traveling through the DLD breaks up into 2^n exit beams due to the imperfect modulators. These intensities are given by the terms in the expansion of $(a + b)^n$ as shown in Appendix C. We now need to ask how much of the light comes back through the second aperture after being reflected from the focal plane (see Fig. 9).

Let us consider one term of the expansion of $(a + b)^n$, say $a^{n-r}b^r$. We state that this system is reciprocal and that if $a^{n-r}b^r$ of the incident beam exits the DLD, then if unity power were directed through the DLD in exactly the opposite direction the same fraction of power, i.e., $a^{n-r}b^r$, will pass through the aperture.

Thus, for an n unit DLD there will be 2^n exit terms and each of these terms, for example $a^{n-r}b^r$, will generate one term that contributes

* This appendix represents the results of calculations performed jointly by J. T. Sibilia and the author.

to the intensity at the second aperture; and for the example above that corresponds to $(a^{n-r}b^r)^2$. The total number of terms that exit through the second aperture is then the sum of the squares of the exit terms and can be generated by $(a^2 + b^2)^n$.

Before we can add up the 2^n terms in the second aperture, we must know something about the relative phases of the terms. Each of the 2^n exit terms in the expansion of $(a + b)^n$ traverses through a different optical path length in the DLD. The reason for this is because light traverses through some of the prisms as an ordinary ray and others as an extraordinary ray, and the combinations of such paths are different for each of the 2^n exit beams. Thus, unless the DLD has been specifically designed to the contrary, each path has a different phase delay in passing through the DLD.

A term in the expansion $(a^2 + b^2)^n$ such as $a^{2(n-r)}b^{2r}$ represents an E field of $[a^{2(n-r)}b^{2r}]^{\frac{1}{2}}$ and a phase factor φ. Consider the sum of all the $n!/(n-r)!r!$ terms of the type $a^{2(n-r)}b^{2r}$

$$[a^{2(n-r)}b^{2r}]^{\frac{1}{2}}(\varphi_1 + \varphi_2 + \cdots + \varphi_{n!/(n-r)!r!}). \tag{25}$$

The intensity of the sum of all $a^{2(n-r)}b^{2n}$ terms is given by the square of (25)

$$a^{2(n-r)}b^{2r}(\varphi_1 + \varphi_2 + \cdots \varphi_{n!/(n-r)!r!})^2. \tag{26}$$

A series of phase terms such as in (26) can add as follows:

$$(\varphi_1 + \varphi_2 + \cdots + \varphi_{n!/(n-r)!r!})^2 = \frac{n!}{(n-r)!r!} \quad \text{for random phases}$$

$$\left[\frac{n!}{(n-r)!r!}\right]^2 \quad \text{for the same phase.}$$

As explained earlier, all of the phases are, in general, different, and so we will use the random phase addition. The intensity at the second aperture is then the sum of all of the terms in $(a^2 + b^2)^n$.

The R ratio, the ratio of the light from the main beam, a^{2n}, to the light from the remaining positions, $(a^2 + b^2)^n - a^{2n}$, is then

$$R_3 = \frac{a^{2n}}{(a^2 + b^2)^n - a^{2n}}$$

$$= \frac{1}{\left(1 + \frac{b^2}{a^2}\right)^n - 1} = \frac{1}{n\frac{1}{E^2} + \frac{n(n-1)}{2}\frac{1}{E^4} + \cdots} \tag{27}$$

which is the same as that used in Section IV.

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