The Nonlinearity of the Reverse Current-Voltage Characteristics of a p-n Junction Near Avalanche Breakdown

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For nonlinear applications such as high-speed switching, a device figure of merit is γ, the ratio of the second derivative to the first derivative of the current-voltage (I-V) characteristic, or $\gamma \equiv (d^2I/dV^2)/(dI/dV)$. At room temperature, the value of γ for an ideal forward-bias Schottky diode is about 40 V^{-1}. It is shown that although the ideal reverse breakdown characteristic could give a value of γ greater than 40 V^{-1}, because of the statistical distribution of impurities, the effect of space-charge resistance, and other complications, much lower values of γ are expected. Furthermore, the nonlinear characteristic is noisy, relatively slow, and causes some power consumption. It appears, therefore, that this nonlinearity is not likely to supersede Schottky barrier diodes in high-speed switching applications. It does not, however, rule out the possibility of microwave generation application.

For nonlinear applications such as high-speed switching, a device figure of merit is γ, the ratio of the second derivative to the first derivative of the current-voltage characteristic. The value of γ is a measure of the degree of nonlinearity, normalized to the operating admittance level. It is used here to compare the nonlinearity of a reverse-biased diode near breakdown to that of a forward-biased p-n junction or a Schottky barrier.

The current-voltage characteristic and γ for a forward-biased p-n junction or a Schottky barrier are given by

$$I = I_0[e^{qV/nkT} - 1] \tag{1}$$

$$\gamma \equiv (d^2I/dV^2)/(dI/dV) = \frac{q}{nkT}. \tag{2}$$

At room temperature, the value of γ for an ideal Schottky barriers ($n = 1$) is about 40 V^{-1} independent of bias.

The present note is undertaken to answer the following question: Is it possible at room temperature to have a value of γ greater than 40 V^{-1} for a p-n junction near its avalanche breakdown voltage? The answer is "yes" for an ideal isothermal breakdown characteristic and "no" for practical considerations.

Near avalanche breakdown the isothermal current-voltage characteristic without space-charge effect is given by

$$I = I_0 M = \frac{I_0}{1 - \int_0^W \alpha \exp\left[-\int_0^x (\alpha - \beta)\, dx'\right] dx}, \tag{3}$$

where M is the multiplication factor, W the depletion width depending on the applied voltage, and α and β are the ionization rates of electrons and holes, respectively.[1]

The value of γ computed from (3) is plotted in Fig. 1 [curve (a)] as a function of M for a silicon p^+n junction with n-type background doping of 5×10^{16} cm^{-3} and a breakdown voltage of 19.48 volts.[2] It is clear that the value of γ exceeds 40 V^{-1} at $M \cong 65$ corresponding to a voltage of 19.43 volts approximately 50 mV ($\equiv \Delta V$) smaller than the breakdown voltage. For a background doping of 1.5×10^{17} cm^{-3}, the breakdown voltage is 9.546 volts.[2] The value of γ exceeds 40 V^{-1} at $M \cong 50$ [curve (b)], corresponding to a voltage some 50 mV smaller than the breakdown voltage. Similar results are obtained for other dopings and different semiconductors.

Now, let us consider the space-charge effect. The space charge of holes and electrons produced by the avalanche generates a counter emf which reduces the field across the multiplying region.[3] Since the voltage reduction is proportional to the current, it is represented as a resistance; furthermore, it increases as the square of the length of the region in which the space charges accumulate.[4] For the above devices [curve (a)] with an n region approximately equal to the depletion width at breakdown, and a circular area of one mil in diameter, the equivalent space-charge resistance R_{sc} is about 50 ohms. The incorporation of the space-charge resistance will increase the applied voltage for any current level, thus, reducing the value of γ at any given M. The computed result is also shown in Fig. 1 [curve (c)]. One notices that γ reaches a maximum value of about 500 V^{-1} at an $M \cong 10^3$. For the above device area with a saturation current density of

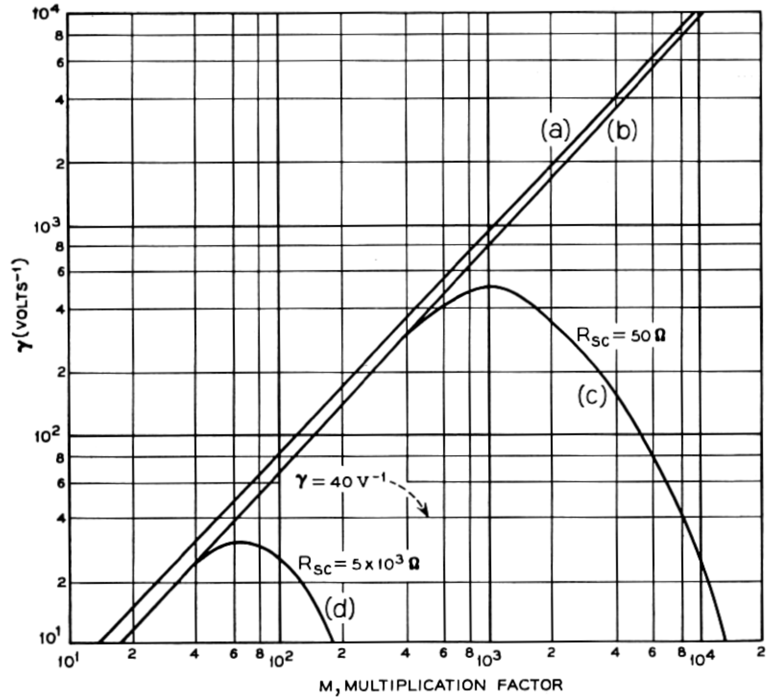

Fig. 1 — γ versus multiplication factor. (a) p^+n junction in silicon with background doping 5×10^{16} cm^{-3} and without space-charge effect. (b) p^+n junction in silicon with background doping 1.5×10^{17} cm^{-3} and without space-charge effect. (c) Same as (a) with 50-ohms space-charge resistance. (d) Same as (a) with 5000 ohms space-charge resistance.

about 10^{-9} amp/cm^2, the current at $M = 10^3$ is still very small, of the order of nanoamperes. For larger M, γ decreases rapidly to zero within 10 mV. If one increases the width of the n region, R_{sc} also increases; eventually for large enough R_{sc}, such as shown in Fig. 1 curve (d) the value of γ will be smaller than 40 V^{-1} for any M.

The effects of statistical spatial fluctuations of donor and acceptor ions on the breakdown voltages have been considered by Shockley.[5] It is found that this randomness leads to a characteristic fluctuation voltage of about 300 mV in silicon. The effect of these fluctuations in a p-n junction is to produce local regions with breakdown about a few hundred millivolts lower than the average in uncompensated material, and larger by $[(N_d + N_a)/(N_d - N_a)]^{1/2}$ in compensated material.

It is clear that this voltage fluctuation is an order of magnitude larger than the value of ΔV which is the difference between the breakdown voltage ($M \rightarrow \infty$) and the voltage at which $M \cong 50$. Consequently, in a practical junction with this effect present, the curvature will be very much less than the ideal calculation of Fig. 1.

We conclude that the normalized curvature γ could be large for a junction near breakdown, but only if the junctions were ideal. It would be possible for γ to exceed 40/V provided space charge, heating, microplasma and statistical effects are small. This ideal high curvature would be limited to regions within a few millivolts of breakdown, where the multiplication ratio M is high, $\approx$50 or more.

Practical considerations severely limit the use of this nonlinearity:

(*i*) Because of statistical fluctuations in impurity distributions, there is a characteristic spatial fluctuation in breakdown voltage of a few hundred millivolts in silicon, as pointed out by Shockley.[5] This alone drastically reduces γ below the ideal value.

(*ii*) Series resistance, either ohmic or space charge, limits the region of high curvature to small currents, typically a few microamperes for a 1-mil diameter junction. Therefore, with usual shunt capacitances, the response time tends to be slower than for a good Schottky Barrier diode.

(*iii*) Microplasmas, if present, would cause erratic curvature and high noise.

(*iv*) The noise is high compared to a forward-biased or zero-biased Schottky Barrier. So is the power dissipation.

(*v*) Negative resistance complications are possible.[6]

In summary, although the ideal reverse breakdown characteristic could give a value of γ greater than 40 V^{-1}, because of the statistical distribution of impurities, the effect of space-charge resistance, and other complications above, much lower values of γ are expected. Furthermore, the nonlinear characteristic is noisy, relatively slow, and associates with some power consumption. It appears therefore, that this nonlinearity is not likely to supersede Schottky Barrier diodes in high-speed switching applications. It does not, however, rule out the possibility of microwave generation application. This latter application is not discussed here; it would utilize not only the large nonlinear resistance of the I-V characteristic near breakdown but also the nonlinear reactance and negative conductance due to the interaction between transit time and the avalanche process in the device.[6]

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