Stability Considerations in Lossless Varactor Frequency Multipliers

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A general analysis of stability conditions of pumped nonlinear systems is presented in this paper. The type of instability investigated for these systems is that which causes spurious tones to appear at any point in the system in the vicinity of an appropriate harmonic carrier. A set of stability criteria that assure stability for the system has been given in terms of scattering parameters of the system. These criteria have then been applied to investigate the stability of lossless varactor harmonic generators that have been shown in this paper to be potentially unstable systems. It is then investigated for these multipliers how instability arises, and how it can be avoided by proper terminations. For some simple terminations, which are usually used in practice, sufficient conditions, that assure total stability of the multipliers, are explicitly given.

I. INTRODUCTION

One of the principal limitations to efficient wideband harmonic generation with varactor diodes is the generation of spurious signals.[1, 2, 3] The origin of these signals is usually thought[1] to be due to a parametric "pumping up" of some signal in the multiplier passband, or to a parametric up-conversion process,[1] or a variation in the average capacitance of the diode at input frequency.[3] A multiplier which contains these spurious signals is considered to be unstable,[4] and it is this type of instability that is investigated in this paper.

At the present time, much is not known about the stability of harmonic generators, even though it is a widely-known experimental fact that this is a serious problem in high-efficiency varactor multipliers.[2, 4] Very little is also known about the conditions imposed by stability on the available circuit configurations. Consequently, present design procedures leave the problem to be solved experimentally, and this is often done at the expense of efficiency. Very often isolators are used to connect a chain of multipliers which are individually stable in order to guarantee stability of the chain.[4] The isolators used in the chain always lower the overall efficiency.

A start on this problem of stability in multipliers has been made by Ref. 4 which considers the stability conditions of multipliers of order 2^n with minimum number of idlers. Some simple conditions on the terminations have been obtained[4] in order to ensure stability of the multipliers. This paper extends this analysis to harmonic generators of arbitrary order and also obtains refinements to the conditions obtained in Ref. 4.

Varactor harmonic generators come under the general class of pumped nonlinear systems, which are systems driven periodically by a pump or a local oscillator at a frequency ω_0.[5] For such systems, a general method can be used[5] to obtain the scattering parameters which relate the small-signal fluctuations present at various points in the system. In particular, Ref. 5 obtains these scattering relations for lossless abrupt-junction varactor multipliers of order 2^n, 3^s, and 2^n3^s, n and s integers, with the least number of idlers.

These scattering relations for pumped systems have been obtained in Ref. 5 when the difference frequency ω is real and small. The concept of analytic continuation has been used to obtain these scattering parameters when this difference frequency is complex, and is still small in magnitude.

Stability conditions for pumped systems are then expressed in terms of the scattering matrix of the system and a certain characteristic equation is obtained which determines the stability of the system. For the system to be stable it is necessary and sufficient that the roots of this characteristic equation must lie external to a region R of the complex frequency plane. Proper terminations that guarantee stability of the system can be determined for the pumped system from this equation.

We then discuss AM-to-PM and PM-to-AM conversion properties of a set of lossless interstage networks usually used with multipliers.

Stability conditions of lossless abrupt-junction varactor multipliers, most frequently encountered in practice, are then considered. It has been shown that if the bias circuit is properly designed[6] so that there are no currents flowing in the vicinity of dc the characteristic equation‡ of the multiplier can be expressed as a product of an AM characteristic

‡ This condition can be achieved in practice by having a bias source with infinite internal impedance.

equation and a PM characteristic equation. If any root of the AM characteristic equation lies in the closed right-half§ of the complex plane there will not be a finite upper bound to the AM fluctuations originating at some point in the system. Such a system is defined to be unstable with respect to its AM fluctuations. Similarly, the PM fluctuations will be finite if and only if all zeros of the PM characteristic equation lie in the open left-half plane. For total stability of the multiplier no zero of its AM and PM characteristic equations should lie in the closed right-half plane.¶

It has been shown for multipliers of order 2^n that all roots of the AM characteristic equation always lie in the left-half plane for arbitrary values of input, output, and idler terminations.|| It has also been proved for these multipliers that PM stability is not achievable with arbitrary terminal impedances.

We then specifically consider PM stability of a 1-2 doubler, 1-2-4 quadrupler, and 1-2-4-8 octupler when their terminations are single-tuned series circuits.‡ Simple restrictions to be satisfied by these terminations are obtained to guarantee PM stability of the multipliers.

Stability of a 1-2-3 tripler for an arbitrary passive idler termination is the subject of discussion of the next section. We show that a tripler is potentially unstable for arbitrary input and output terminations. It has also been proven that a tripler is stable with respect to both AM and PM fluctuations if its terminations are single-tuned series circuits.

We next assume that the bias source impedance Z_0 can be a finite number. We then show that the stability characterization of a multiplier having finite bias source impedance is the same as that of a multiplier having infinite bias source impedance.

For a multiplier of any order, a general method of obtaining the conditions on available circuit configurations imposed by the condition of stability has also been presented.

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§ The closed right-half of the complex plane is the region of λ-plane where Re $\lambda \geqq 0$. The open left-half plane contains all the points of the λ-plane for which Re $\lambda < 0$.

¶ For total stability of systems whose characteristic equation $F(\lambda)$ cannot be expressed as a product of AM and PM characteristic equations, it is necessary and sufficient that no zero of $F(\lambda)$ lies in the closed right-half plane.

|| All terminations considered in this paper are assumed to be linear and passive.

‡ It can be shown that a single-tuned series circuit is a first-order approximation to any circuit usually used in practice, since the average elastance S_0 of the varactor diode is almost always nonzero.[7]

II. SCATTERING RELATIONS IN LOSSLESS VARACTOR FREQUENCY MULTIPLIERS

For a pumped nonlinear system a general method can be used[5] in order to obtain the scattering parameters which relate the small-signal fluctuations present at various points in the system.§ Such a method has been applied[5] in order to obtain scattering relations for lossless abrupt-junction varactor multipliers of order 2^n3^s, n and s integers, with minimum number of idlers. The scattering matrix $\underset{\sim}{S}$ is given by

$$\underset{\sim}{S} = \left[\begin{array}{c|c} \underset{\sim}{S}_{aa} & \underset{\sim}{S}_{ap} \\ \hline \underset{\sim}{S}_{pa} & \underset{\sim}{S}_{pp} \end{array}\right] \tag{1}$$

$$= \left[\begin{array}{cc|cc} \frac{1}{3} - \frac{(-1)^n}{3} 2^{-n} & (-1)^n 2^{-n} & & \\ & & \multicolumn{2}{c}{0} \\ 1 & 0 & & \\ \hline & & 0 & (-1)^n 3^{-s} \\ \multicolumn{2}{c|}{0} & & \\ & & 2^n 3^s & \frac{1}{3} - \frac{(-1)^n}{3} 2^n \end{array}\right]. \tag{2}$$

It is assumed that the bias circuit is properly designed and that $\omega/\omega_0 \ll 1$.¶

In order to discuss the stability of the multipliers it is necessary to include the effect of the external circuits on the scattering matrix $\underset{\sim}{S}$ of the multiplier. This can easily be done as is shown in the succeeding sections of this paper. It is also assumed in Ref. 5 that the difference frequency ω is real and small in deriving (2). Since we shall discuss stability of multipliers in this paper it is convenient to have a complex value for this difference frequency. The small-signal terminal voltage $\delta v_k(t)$ in the vicinity of the carrier frequency $\pm k\omega_0$ is represented in Ref. 5 as

$$\delta v_k(t) = 2\,\mathrm{Re}\,[V_{\alpha k} \exp{(jk\omega_0 + j\omega)t} + V_{\beta k} \exp{(-jk\omega_0 + j\omega)t}]. \tag{3}$$

Let the difference frequency have a complex value $\lambda = \sigma + j\omega$, σ and ω real. The terminal behavior of a pumped nonlinear system can be de-

§ Since the notation used in this paper is identical to that used in Ref. 5, details of these notations are not given in this paper for the sake of brevity. The assumptions under which these scattering relations can be obtained are also given in Ref. 5.

¶ Only lowest order terms in ω/ω_0 are retained in deriving (2). Since frequency selective circuits are always used in a multiplier and since the average elastance S_0 of the varactor can always be included with these external circuits for purposes of analysis, (2) is a first-order approximation to $\underset{\sim}{S}$ in the vicinity of the carrier.

scribed[5] by an equation of the form

$$\mathbf{V} = \underline{Z}_{\alpha-\beta}\mathbf{I} \tag{4}$$

where **V** and **I** are the terminal voltage and current column matrices and $\underline{Z}$ is an impedance matrix. We shall now utilize the principle of analytic continuation[8] to obtain $\underline{Z}$ (and other parameters) of the pumped nonlinear system when the difference frequency is complex. This can be done by the simple expedient of replacing the variable $j\omega$ by the complex variable $\lambda = \sigma + j\omega$ wherever it occurs[8] in (4).§ The truth of this statement, expressing a property of functions known as their permanence of form, follows directly from the identity theorem, since $\underline{Z}$ and its continuation obviously coincide on the $j\omega$-axis.[8]

We can, therefore, obtain scattering parameters of all pumped nonlinear systems (including those of lossless abrupt-junction varactor multipliers) when the difference frequency λ is complex.

III. STABILITY OF PUMPED NONLINEAR SYSTEMS

We shall first begin with a discussion of stability of pumped nonlinear systems in which small-signal fluctuations may be present at various points in the system. Since lossless varactor harmonic generators are specific pumped nonlinear systems all these results and remarks also apply to these harmonic multipliers.

A small-signal fluctuation originating at some point in the system is propagated, in general, throughout the system. We shall define a pumped nonlinear system to be stable if and only if the amplitude of small-signal fluctuations at any point in the system is finite for a finite small-signal fluctuation originating at some point in the system.

We shall make use of some of the results obtained in the study of stability of linear n-port systems.[9,10,11,12,13,14] The stability of a linear n-port system is usually described by the statement that the roots of a certain characteristic equation $F(\lambda)$ of the system must be external to a region R of the complex frequency plane, that is, $F(\lambda) \neq 0$ in region R, where $\lambda = \sigma + j\omega$ is the complex frequency variable. Some set of stability criteria can also be obtained[9,10,11,12,13] for a general class of linear reciprocal and nonreciprocal n-ports. For a reciprocal twoport, a well-known result by Gewertz[10] states that it is stable under all passive terminations if and only if it is passive. This theorem has been generalized by Youla[12] to the reciprocal n-port. Very little, how-

§ In order that $\delta v_k(t)$ is small compared to the carrier at frequency $k\omega_o$ for all time t, it is required that $\sigma \leqq 0$.

ever, is known[13,14] about the stability of linear nonreciprocal n-ports, when $n \geqq 3$.

It is shown in Section II that the terminal small-signal behavior of noise-free pumped nonlinear system can be described by‡

$$\mathbf{V} = \underline{Z}_{\alpha-\beta}\mathbf{I} \tag{4}$$

where $\underline{Z}_{\alpha-\beta}$ is a function of $j\omega_0$ and $\lambda = \sigma + j\omega$.

We shall restrict ourselves in this paper to the consideration of stability of pumped nonlinear systems having only two (physical) accessible ports. It can be noted, however, that most of the concepts developed for the system having two accessible ports can be extended in a straightforward manner if the system possesses more than two accessible terminal pairs. This will be evident to the reader when we discuss stability of a tripler elsewhere in this paper.

If $l\omega_0$ and $s\omega_0$ are the input and out put carrier frequencies, it can be shown[5] that the AM and PM fluctuations at different points in the system can be related through a scattering matrix $\underline{S}$:

$$\begin{bmatrix} (m_r)_l \\ (m_r)_s \\ (\theta_r)_l \\ (\theta_r)_s \end{bmatrix} = \left[\begin{array}{c|c} \underline{S}_{aa} & \underline{S}_{ap} \\ \hline \underline{S}_{pa} & \underline{S}_{pp} \end{array}\right] \begin{bmatrix} (m_i)_l \\ (m_i)_s \\ (\theta_i)_l \\ (\theta_i)_s \end{bmatrix}, \tag{5}$$

where m and θ are the AM and PM indexes of the system, $\underline{S}_{aa}$ is the AM scattering matrix, etc. We shall write (5) as

$$\mathbf{b} = \underline{S}\mathbf{a}. \tag{6}$$

Let the system be terminated in linear passive impedances (see Fig. 1) z_1, z_2, z_3, and z_4 with reflection coefficients ρ_1, ρ_2, ρ_3, and ρ_4.§ Let us define a matrix $\underline{\rho}$ where

$$\rho = \text{dia. } [\rho_1, \rho_2, \rho_3, \rho_4]. \tag{7}$$

Since z_i's are assumed passive, we have

‡ Let $\underline{A}$ be an arbitrary matrix. Then $\underline{A}^t$, $\underline{A}^*$, $\underline{A}^\dagger$, and $\Delta\underline{A}$ stand for the transpose, the complex conjugate, the complex conjugate transpose, and the determinant of $\underline{A}$, respectively. Column vectors are denoted by **V**, **I**, etc. A diagonal matrix $[\mu_i\delta_{ij}]$ $\{\delta_{ij} = 1, i = j;\ \delta_{ij} = 0, i \neq j\}$, is denoted as dia. $[\mu_1, \mu_2, \cdots, \mu_n]$. $\underline{1}_n$ is the unit matrix of order n.

§ The linear impedances z_1, z_2, z_3, and z_4 are normalized with respect to "characteristic impedances" at corresponding carrier frequencies. Characteristic impedance at input port is the "input impedance"[7] and that at the output port is the "load impedance".[7]

$$| \rho_i | \leqq 1, \qquad 1 \leqq i \leqq 4, \tag{8}$$

for Re $\lambda \geqq 0$.

From (6), we can show that the system is stable if and only if§

$$\Delta\{\underline{1}_4 - \underline{S}\underline{\rho}\} \neq 0, \quad \text{for} \quad \text{Re}\,\lambda \geqq 0. \tag{9}$$

We can, therefore, state that the characteristic equation of the system is given by

$$F(\lambda) = \Delta\{\underline{1}_4 - \underline{S}\underline{\rho}\} = 0; \tag{10}$$

and for stability of the system it is necessary and sufficient that no root of $F(\lambda)$ lies in the closed right-half plane.¶

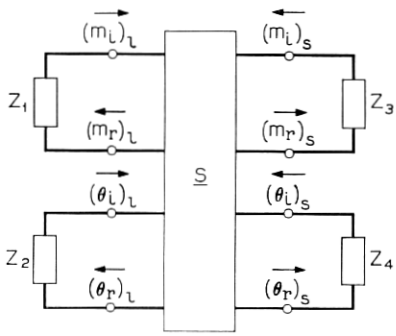

Fig. 1 — Pumped nonlinear system, in amplitude-phase representation, terminated in linear passive impedances.

Theorem 1: We shall now show[13] that two systems described by scattering matrices $\underline{S}_1$ and $\underline{S}_2$ possess identical stability characterizations if $\underline{S}_1$ and $\underline{S}_2$ possess identical principal minors[15] of all order.

The characteristic equation $F(\lambda)$ of a system described by scattering matrix $\underline{S}$ for a certain termination described by matrix $\underline{\rho}$ is given by (10). If $\underline{S}$ is nonsingular, we can write (10) as

$$\Delta\{\underline{S}^{-1} - \underline{\rho}\} = 0. \tag{11}$$

§ The constraints imposed on $\underline{S}$ for a twoport system may be found in Ref. 14. These constraints, if satisfied, guarantee stability of the system independent of the terminations.

¶ The reader will recognize that $F(\lambda) = 0$ gives the natural frequencies of the system. For stability of a system, simple zeros of $F(\lambda)$ on the $j\omega$-axis are usually allowed, since this just leads to sustained response of finite amplitude. However, multiple order zeros on the $j\omega$-axis lead to instability of the system.

Now $\Delta\{\underset{\sim}{S}^{-1} - \underset{\sim}{\rho}\}$ can be expanded in terms of the elements of $\underset{\sim}{\rho}$ as follows:

$$\Delta\{\underset{\sim}{S}^{-1} - \underset{\sim}{\rho}\} = \Delta\underset{\sim}{S}^{-1} - \sum_{k=1}^{4} \rho_k B_k + \sum_{k<r}^{4} \rho_k \rho_r B_{k,r} , \tag{12}$$

where B_k is the principal minor of $\underset{\sim}{S}^{-1}$ obtained by striking out the kth row and column, $B_{k,r}$ is the principal minor obtained by deleting the kth and rth rows and the kth and rth columns. It, therefore, follows that two systems described by scattering matrices $\underset{\sim}{S}_1$ and $\underset{\sim}{S}_2$ have identical stability characterizations if $\underset{\sim}{S}_1^{-1}$ and $\underset{\sim}{S}_2^{-1}$ have identical principal minors of all order. We know that $\underset{\sim}{S}_1^{-1}$ and $\underset{\sim}{S}_2^{-1}$ possess identical principal minors of all order if and only if $\underset{\sim}{S}_1$ and $\underset{\sim}{S}_2$ possess identical principal minors of all order. This proves the theorem.

If $F(\lambda) \neq 0$ for Re $\lambda \geqq 0$ for all allowable values of $\underset{\sim}{\rho}$, we shall say that the pumped system is absolutely stable. If there is only a set of $\underset{\sim}{\rho}$ which meets this requirement the system will be considered to be conditionally (or potentially) stable. It can be observed that if one port of the system is terminated in a linear passive impedance z_t , and if the real part of the impedance across any other pair of terminals is negative for Re $\lambda \geqq 0$, the system cannot be absolutely stable. This is one of the methods to investigate absolute stability of a system.

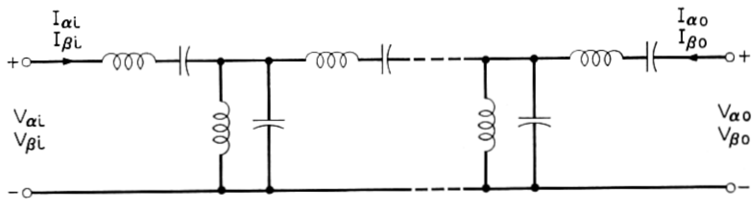

Fig. 2 — Typical interstage network used in a multiplier. All series and shunt arms are resonant at frequency $k\omega_0$.

IV. SOME PROPERTIES OF A CLASS OF LOSSLESS INTERSTAGE NETWORKS

Frequency separation is obtained in harmonic generators by using linear bandpass‡ filters. A typical example of a class of filters most commonly used in harmonic generators is shown in Fig. 2. This filter has a passband centered around carrier frequency $\pm k\omega_0$. Such filters with proper terminations are connected at accessible ports of a multiplier so as to obtain the desired frequency separations and proper impedance

‡ This can be a low-pass filter at the lowest carrier frequency present in the multiplier and a high-pass filter at the highest carrier frequency.[4]

terminations at different carrier frequencies present in the multiplier.§ A multiplier with input frequency ω_0, output frequency $n\omega_0$, and interstage networks $N_1, N_2, \cdots, N_k, \cdots, N_n$ is shown in Fig. 3.¶

For such interstage networks it will be shown that the scattering parameters|| are given by

$$\underline{S} = \left[\begin{array}{c|c} \underline{S}_{aa} & \underline{0} \\ \hline \underline{0} & \underline{S}_{pp} \end{array}\right] \tag{13}$$

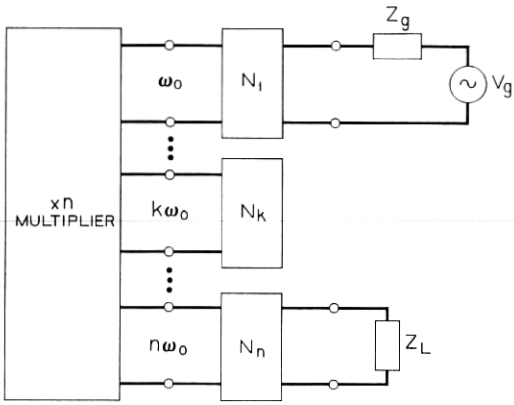

Fig. 3 — Lossless interstage networks as used in a frequency multiplier.

so that these networks do not produce AM-to-PM or PM-to-AM conversion.

Since the series arms are resonant at frequency $k\omega_0$, and the antiresonant frequency of the shunt arms is also $k\omega_0$, if $\omega/\omega_0 \ll 1$, we can write

$$\begin{bmatrix} V_{\alpha i} \\ V_{\beta i} \\ V_{\alpha 0} \\ V_{\beta 0} \end{bmatrix} = \begin{bmatrix} z_{ii} & 0 & z_{i0} & 0 \\ 0 & z_{ii} & 0 & z_{i0} \\ z_{0i} & 0 & z_{00} & 0 \\ 0 & z_{0i} & 0 & z_{00} \end{bmatrix} \begin{bmatrix} I_{\alpha i} \\ I_{\beta i} \\ I_{\alpha 0} \\ I_{\beta 0} \end{bmatrix}. \tag{14}$$

§ For example, this filter should also act as a matching filter at the input carrier frequency ω_0.

¶ It is assumed that all idler terminations are lossless.

|| Even though N_k is a two-port network we must obtain 4x4 scattering matrix of this network since amplitude and phase transmission characteristics of the pumped nonlinear system with which N_k may be used are not necessarily the same.[6] See Ref. 5 for the definitions of amplitude and phase transmission characteristics as used in this paper.

We shall now assume that large signal voltage at carrier frequency $k\omega_0$ is in phase with the large signal current.‡
We can now write

$$\begin{bmatrix} V_{ai} \\ V_{pi} \\ V_{a0} \\ V_{p0} \end{bmatrix} = \begin{bmatrix} z_{ii} & 0 & z_{i0} & 0 \\ 0 & z_{ii} & 0 & z_{i0} \\ z_{0i} & 0 & z_{00} & 0 \\ 0 & z_{0i} & 0 & z_{00} \end{bmatrix} \begin{bmatrix} I_{ai} \\ I_{pi} \\ I_{a0} \\ I_{p0} \end{bmatrix}. \tag{15}$$

Equations (14) and (15) show that the scattering parameters of a lossless interstage network are given by (13). This shows that if such interstage networks are used in multipliers which are characterized by uncoupled§ scattering matrices the resultant scattering matrix is also uncoupled.

V. STABILITY OF LOSSLESS ABRUPT-JUNCTION VARACTOR MULTIPLIERS

The general analysis of the stability conditions presented in the earlier sections will be applied to investigate stability of frequency multipliers of order 2^n3^s, n and s integers, when lossless interstage networks of the form discussed in Section IV are used with these multipliers. It will be shown that these multipliers are potentially unstable and we shall obtain some circuit configurations which guarantee their conditional stability.

It has been shown[5] that a multiplier of order 2^n3^s with any input, output, and idler terminations can be considered as a chain of n doublers, s triplers, and $n + 2s + 1$ interstage networks (see Fig. 4). All these interstage networks¶ will be assumed to be of the form presented in Section IV. A lossless abrupt-junction varactor tripler with an arbitrary lossless idler termination is shown in Fig. 5. It is assumed that the tripler is tuned at the idler frequency, $Z_2(2\omega_0) = 0$, and that $\omega/\omega_0 \ll 1$. By the techniques of Ref. 5 we can show that the scattering parameters of a tripler can be represented as

‡ This condition usually leads to optimum efficiency of multipliers and is usually satisfied in practice.[7]

§ The scattering matrix is defined by us to be an uncoupled scattering matrix if $S_{ap} = S_{pa} = 0$.

¶ The average elastance S_0 of the varactor diode is considered as a part of the interstage networks usd in the multipliers.

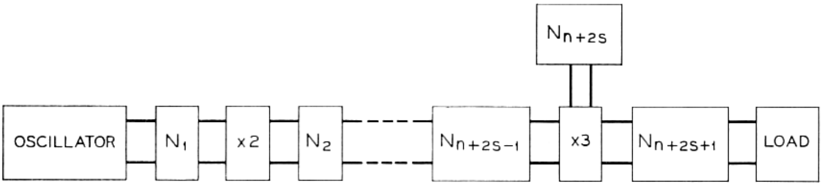

Fig. 4 — Lossless abrupt-junction varactor multiplier of order $2^n 3^s$. N_i is an interstage network of the form shown in Fig. 2.

$$\underline{S} = \left[\begin{array}{cc|cc} 0 & \dfrac{\mu - 1/2}{\mu + 3/2} & \multicolumn{2}{c}{0} \\ 1 & \dfrac{-1}{\mu + 3/2} & & \\ \hline \multicolumn{2}{c|}{0} & \dfrac{-1}{\mu + 1/2} & \dfrac{1}{3}\dfrac{\mu - 3/2}{\mu + 1/2} \\ & & 3 & 0 \end{array}\right], \tag{16}$$

where

$$\mu = \frac{R_{02}}{Z_2}, \tag{17}$$

$$R_{02} = \frac{3 \mid S_1 \mid^2}{8 \mid S_2 \mid \omega_0}. \tag{18}$$

For a tripler, we can hence write

$$\underline{S}_{aa} = \begin{bmatrix} 0 & \dfrac{\mu - 1/2}{\mu + 3/2} \\ 1 & \dfrac{-1}{\mu + 3/2} \end{bmatrix} \tag{19}$$

$$\underline{S}_{pp} = \begin{bmatrix} \dfrac{-1}{\mu + 1/2} & \dfrac{1}{3}\dfrac{\mu - 3/2}{\mu + 1/2} \\ 3 & 0 \end{bmatrix} \tag{20}$$

and

$$\underline{S}_{ap} = \underline{S}_{pa} = \underline{0}. \tag{21}$$

Since a doubler,[5] a tripler, and all interstage networks used in the multiplier have uncoupled scattering matrices it follows that general

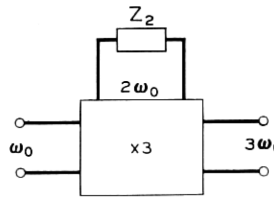

Fig. 5 — Lossless abrupt-junction varactor tripler with an arbitrary lossless idler termination Z_2.

scattering parameters of multipliers of order $2^n 3^s$ are given by the following equation:

$$\underline{S} = \left[\begin{array}{c|c} \underline{S}_{aa} & \underline{0} \\ \hline \underline{0} & \underline{S}_{pp} \end{array}\right]. \tag{22}$$

If such a multiplier is terminated in passive impedances as shown in Fig. 6, the characteristic equation of the system according to (10) can be written as

$$F(\lambda) = \Delta\{\underline{1}_4 - \underline{S}\underline{\rho}\} = 0, \tag{23}$$

where

$$\underline{\rho} = \text{dia.}\,[\rho_{m1}\,,\,\rho_{m2}\,,\,\rho_{\theta 1}\,,\,\rho_{\theta 2}] = \left[\begin{array}{c|c} \underline{\rho_m} & \underline{0} \\ \hline \underline{0} & \underline{\rho_\theta} \end{array}\right]. \tag{24}$$

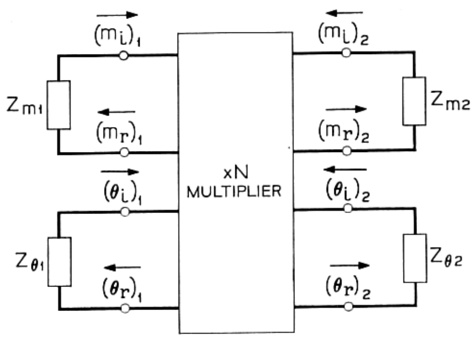

Fig. 6 — Multiplier of order N. AM and PM ports of the multiplier are terminated in linear passive impedances.

From (22) through (24), we can write

$$F(\lambda) = \Delta\{\underline{1}_2 - \underline{S}_{aa}\underline{\rho}_m\}\Delta\{\underline{1}_2 - \underline{S}_{pp}\underline{\rho}_\theta\} \tag{25}$$

$$= F_a(\lambda)F_p(\lambda), \tag{26}$$

where

$$F_a(\lambda) = \Delta\{\underline{1}_2 - \underline{S}_{aa}\underline{\rho}_m\} \tag{27}$$

and

$$F_p(\lambda) = \Delta\{\underline{1}_2 - \underline{S}_{pp}\underline{\rho}_\theta\}. \tag{28}$$

For stability of the multiplier it is necessary and sufficient that the zeros of $F_a(\lambda)$ and $F_p(\lambda)$ lie in a region external to the closed right-half plane. $F_a(\lambda)$ and $F_p(\lambda)$ will be called the AM and PM characteristic equations of the multiplier respectively. It must be borne in mind that the uncoupled nature of the scattering matrix of the multiplier with a properly designed bias circuit enables us to express $F(\lambda)$ as a product of $F_a(\lambda)$ and $F_p(\lambda)$. If this cannot be done we will not be able to investigate the nature of roots of $F(\lambda)$ by studying only the roots of $F_a(\lambda)$ and $F_p(\lambda)$.

For multipliers for which we can express $F(\lambda)$ as the product of $F_a(\lambda)$ and $F_p(\lambda)$ we can define AM and PM stability independently. If no zeros of $F_a(\lambda)$ lie in the closed right-half plane we shall say that the multiplier is AM stable. A multiplier is PM stable if all roots of $F_p(\lambda)$ lie in the open left-half plane. For total stability of the multiplier it must be both AM and PM stable.

5.1 *AM Stability of Multipliers of Order 2^n*

The AM stability of lossless abrupt-junction varactor multipliers of order 2^n wth minimum number of idlers will be considered in this section. It has been shown[5] that a multiplier of order 2^n is equivalent to a cascade of n doublers as shown in Fig. 7. It will be assumed that interstage networks are passive, do not produce AM to PM or PM to AM conversion, and that the load z_n is a linear passive impedance. Since

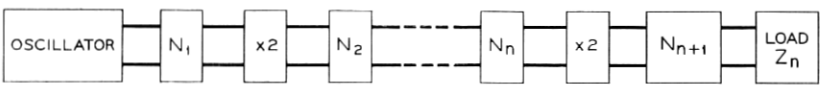

Fig. 7 — Lossless abrupt-junction varactor multiplier of order 2^n. Only AM (or PM) ports of the doubler and interstage networks are shown in the figure.

N_{n+1} is a passive interstage network it follows that the amplitude terminal impedance for the nth doubler is also passive.

Let us now assume that the terminal impedance of the jth doubler is z_j where z_j is passive. We shall now show that the input impedance $(z_{in})_j$ of the jth doubler (see Fig. 8) is passive, $1 \leqq j \leqq n$. Since the generator impedance is assumed to be passive, no AM instability can arise in the multiplier.

The AM scattering matrix of a doubler is given by

$$\underset{\sim}{S}_{aa} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ 1 & 0 \end{bmatrix}. \tag{29}$$

Let the reflection coefficient of z_j normalized to some convenient number be ρ_j. It can be shown[16] that

$$| \rho_j | \leqq 1, \quad \text{for} \quad \text{Re}\, \lambda \geqq 0. \tag{30}$$

From (29), we have,[16]

$$(\rho_{in})_j = \tfrac{1}{2}\{1 - \rho_j\}. \tag{31}$$

From (30) and (31), it follows that

$$| (\rho_{in})_j | \leqq 1, \quad \text{for} \quad \text{Re}\, \lambda \geqq 0. \tag{32}$$

Equation (32) proves the desired result that if z_j is passive, $(z_{in})_j$ is also passive.

This shows that if input, output, and all idler terminations of a multiplier of order 2^n are passive, the impedance measured at any accessible pair of terminals is also passive. This result leads to the conclusion[13] that a multiplier of order 2^n is absolutely stable with respect to its AM fluctuations.

5.2 *PM Stability of Multipliers of Order* 2^n

The phase terminal behavior of a multiplier of order 2^n has also been shown[5] to be equivalent to a chain of n doublers as shown in Fig. 7.

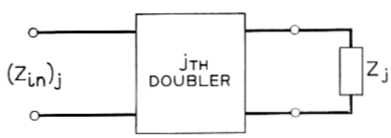

Fig. 8 — jth doubler.

The PM scattering matrix of a doubler is given by

$$\underline{S}_{pp} = \begin{bmatrix} 0 & -1 \\ 2 & 1 \end{bmatrix}. \tag{33}$$

If the phase terminal impedance of jth doubler has a reflection coefficient $(\rho_p)_j$, we have

$$\{(\rho_p)_{\text{in}}\}_j = \frac{-2(\rho_p)_j}{1 - (\rho_p)_j}. \tag{34}$$

For $(\rho_p)_j = \frac{1}{2}$, $\{(\rho_p)_{\text{in}}\}_j = -2$. This shows that the phase input impedance of jth doubler is not necessarily passive if its phase terminal impedance is passive. A doubler is, therefore, potentially unstable with respect to its PM fluctuations if its phase port is terminated in an arbitrary passive impedance. For this reason, we conclude that a multiplier of order 2^n, $n \geqq 1$, can become unstable with respect to its PM fluctuations for some values of its input, output, and idler terminations.

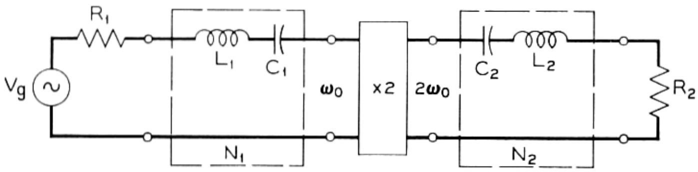

Fig. 9 — Lossless abrupt-junction varactor doubler. Interstage networks N_1 and N_2 are assumed to be single-tuned series circuits.

The PM stability of a doubler, a quadrupler, and an octupler when interstage networks are single-tuned series circuits is studied next. Since the average elastance of a varactor diode is always nonzero, these circuits are always a first-order approximation to any circuits usually used in practice. For any other set of interstage networks used in the multiplier recourse can be had to Section V to obtain the constraints imposed by the condition of PM stability.

5.3 *PM Stability of a Doubler*

A lossless abrupt-junction varactor doubler with single-tuned series circuits for its generator and load impedances is shown in Fig. 9. R_1 and R_2 are the real parts of generator and load impedances of the multiplier.‡ These are given[5] by

‡ It is assumed that the generator is matched to the varactor diode at carrier frequency ω_0.

$$R_1 = \frac{|S_2|}{\omega_0}, \tag{35}$$

and

$$R_2 = \frac{|S_1|^2}{4|S_2|\omega_0}. \tag{36}$$

The bandwidths B_i's for the single-tuned series circuits are defined as

$$B_i = \frac{R_{0i}}{L_i}, \qquad 1 \leqq i \leqq 2, \tag{37}$$

where R_{0i} is the normalizing number for the ith termination. It is assumed for the doubler that

$$R_{0i} = R_i, \qquad 1 \leqq i \leqq 2. \tag{38}$$

From (28), (33), and (37), we can show that the PM characteristic equation $F_p(\lambda)$ of the doubler can be represented as

$$F_p(\lambda) = 2\lambda^2 + B_2\lambda + B_1B_2 = 0. \tag{39}$$

We can observe from (39) that a doubler is PM stable for any finite nonzero values of B_1 and B_2. Therefore, it follows that a doubler is conditionally stable with respect to its AM and PM fluctuations if single-tuned series circuits are used for its input and output terminations.

5.4 *PM Stability of a quadrupler*

Before we discuss PM stability of a quadrupler we shall present in this section a systematic method to obtain the characteristic equation of a multiplier of any order which is equivalent to a chain of multipliers.[5] Let us say that a multiplier of order $M_1 \times M_2$ is equivalent‡ to a multiplier of order M_1 cascaded with a multiplier of order M_2 as shown in Fig. 10. It is assumed that the 2×2 scattering matrices of M_1, M_2, and the linear interstage network N are known. The impedance $Z_{M_1M_2}$ is assumed to be normalized with respect to its port number.[16] The reflection coefficient $\rho_{M_1M_2}$ of the load termination $Z_{M_1M_2}$ is given by

$$\rho_{M_1M_2} = \frac{Z_{M_1M_2} - 1}{Z_{M_1M_2} + 1}. \tag{40}$$

‡ The conditions under which this is true are given in Ref. 5.

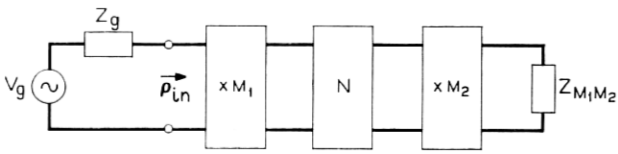

Fig. 10 — Multiplier of order $M_1 \times M_2$.

Since the scattering matrices of M_1, M_2, and N are known, reflection coefficient ρ_{in} can be calculated. If the generator reflection coefficient ρ_g is given by

$$\rho_g = \frac{Z_g - 1}{Z_g + 1}, \tag{41}$$

the characteristic equation of the multiplier is given by

$$1 - \rho_g \rho_{\text{in}} = 0. \tag{42}$$

Let us now consider PM stability of a quadrupler. A lossless abrupt-junction varactor quadrupler is equivalent to a cascade of two doublers. We shall now investigate its PM stability when its input, output, and idler terminations are single-tuned series circuits as shown in Fig. 11. The normalizing impedance for the idler port is assumed to be

$$R_{02} = \frac{|S_1|^2}{4\,|S_2|\,\omega_0}. \tag{43}$$

It can be noted that R_{02} is the "input impedance" of the second doubler. The bandwidths B_i's are defined as in the earlier section.

We can now show that the PM characteristic equation of a quadrupler can be written as

$$F_p(\lambda) = 4\lambda^3 + 2\lambda^2(B_4 - B_2) + \lambda(2B_1B_2 + B_2B_4) + B_1B_2B_4 = 0. \tag{44}$$

In order that a quadrupler is PM stable it is necessary and sufficient that no zero of (44) lies in the closed right-half plane. The Routh-

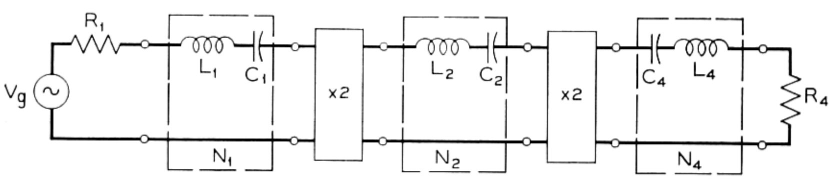

Fig. 11 — Lossless abrupt-junction varactor quadrupler. Interstage networks N_1, N_2, and N_4 are single-tuned series circuits.

Hurwitz[17] criteria can be used to obtain the constraints on the coefficients so that the quadrupler is PM stable. It can be shown from this criterion that if

$$\frac{B_4}{B_2} > 2\frac{B_1}{B_4} + 1 \tag{45}$$

all the zeros of (44) lie in the open left-half plane and the quadrupler is PM stable. Hence, we conclude that a quadrupler can be made conditionally stable‡ if (45) is satisfied.

Let us now assume that

$$\frac{B_4}{B_2} = \frac{B_2}{B_1} = \gamma. \tag{46}$$

The minimum value of γ which guarantees PM stability of the multiplier can be obtained from (45). We can show that (45) is satisfied if and only if

$$\gamma > 1.629. \tag{47}$$

Specifically, we would like to note here that a quadrupler becomes unstable with respect to its PM fluctuations if $B_2 \to \infty$.
Also, we note that it is PM stable if simple bandwidth restrictions given by (45) or (47) are satisfied.

5.5 *PM Stability of an Octupler*

The AM stability of an octupler has been proved earlier in this section. The PM characteristic equation of an octupler with single-tuned series circuits for its input, output, and idler terminations can be shown to be given by the following equation:

$$\begin{aligned} F_p(\lambda) = {} & 8\lambda^4 + 4\lambda^3(B_8 - B_4 - B_2) \\ & + 2\lambda^2(2B_1B_2 + 3B_2B_4 - B_2B_8 + B_4B_8) \\ & + \lambda(2B_1B_2B_8 + B_2B_4B_8 - 2B_1B_2B_4) + B_1B_2B_4B_8 = 0. \end{aligned} \tag{48}$$

B_i is the bandwidth of the multiplier at carrier frequency $i\omega_0$.

The Routh-Hurwitz criterion can again be used to get the constraints on B_i's so that the octupler is PM stable. These constraints can be shown to be

‡ We have shown earlier in this section that a quadrupler is AM stable for all passive terminations.

$$\frac{B_8}{B_4} > \frac{B_2}{B_4} + 1 \tag{49}$$

$$2\frac{B_1}{B_8} + 3\frac{B_4}{B_8} + \frac{B_4}{B_2} > 1 \tag{50}$$

and

$$\begin{aligned} 10\left(\frac{B_8}{B_1}\right) &+ 2\left(\frac{B_4}{B_1}\right)\left(\frac{B_8}{B_1}\right) - 14\left(\frac{B_4}{B_1}\right) - 2\left(\frac{B_8}{B_4}\right)\left(\frac{B_8}{B_1}\right) \\ &- \left(\frac{B_8}{B_1}\right)^2 + \left(\frac{B_4}{B_1}\right)\left(\frac{B_8}{B_1}\right)\left(\frac{B_8}{B_2}\right) - 3\left(\frac{B_4}{B_1}\right)^2 + 6\left(\frac{B_4}{B_8}\right)\left(\frac{B_4}{B_1}\right) \\ &- \left(\frac{B_4}{B_1}\right)^2\left(\frac{B_8}{B_2}\right) - 4\left(\frac{B_2}{B_4}\right) - 12\left(\frac{B_2}{B_1}\right) + 4\left(\frac{B_2}{B_8}\right) \\ &- 3\left(\frac{B_2}{B_1}\right)\left(\frac{B_4}{B_1}\right) + 6\left(\frac{B_2}{B_1}\right)\left(\frac{B_4}{B_8}\right) + 2\left(\frac{B_2}{B_1}\right)\left(\frac{B_8}{B_4}\right) \\ &+ \left(\frac{B_2}{B_1}\right)\left(\frac{B_8}{B_1}\right) > 0. \end{aligned} \tag{51}$$

If we can choose B_i's so that we can satisfy (49) through (51), the multiplier will be PM stable. Let us now choose

$$\frac{B_8}{B_4} = \frac{B_4}{B_2} = \frac{B_2}{B_1} = x; \tag{52}$$

and see whether there exists a value of x which satisfies (49) through (51) simultaneously. The answer is in the affirmative and we can prove that if

$$x > 1.992 \tag{53}$$

the multiplier is PM stable. This shows that an octupler can be made conditionally stable by using single-tuned series circuits which satisfy certain bandwidth restrictions.

5.6 *PM Stability of Multipliers of Order 2^n*

Methods presented in earlier sections can be used to investigate PM stability of multipliers of order 2^n, $n \geqq 4$. It is our conjecture based on earlier discussions and results that a multiplier of order 2^n with single-tuned series circuits as interstage networks is PM stable if bandwidths B_{2^i}'s, $0 \leqq i \leqq n$ satisfy the following equation:

$$\frac{B_{2^i}}{B_{2^{i+1}}} \ll 1. \tag{54}$$

VI. STABILITY OF A TRIPLER

The scattering relations for a tripler are given in (16). Even if the idler termination for the tripler is lossless it is evident from examining (19) and (20) that a tripler is not AM or PM stable‡ for arbitrary input, and output terminations.

Hence, we shall assume that single-tuned series circuits are used for input, output, and idler terminations of the tripler as shown in Fig. 12. Bandwidths B_1 and B_3 are defined as usual. B_2 is defined as

$$B_2 = \frac{R_{02}}{L_2}, \tag{55}$$

where R_{02} is given in (18).

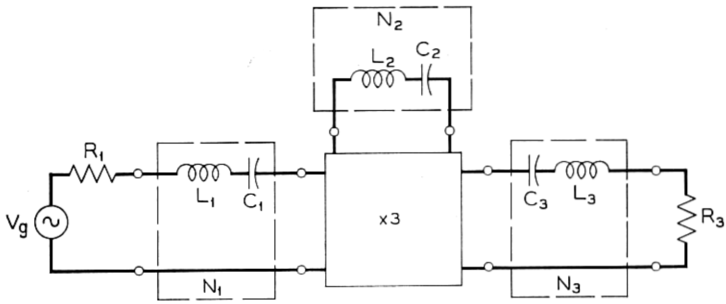

Fig. 12 — Lossless abrupt-junction varactor tripler. N_1, N_2, and N_3 are single-tuned series circuits.

We can now obtain $F_a(\lambda)$ and $F_p(\lambda)$ for the tripler from (19) and (20). These can be shown to be given by

$$\begin{aligned} F_a(\lambda) &= 6\lambda^3 + \lambda^2(5B_1 + 3B_3) \\ &\qquad + \lambda(B_1B_2 + B_2B_3 + 3B_3B_1) + B_1B_2B_3 \\ &= 0 \end{aligned} \tag{56}$$

and

$$\begin{aligned} F_p(\lambda) &= 6\lambda^3 + \lambda^2(B_1 + 3B_3) + \lambda(B_1B_2 + B_2B_3 + B_3B_1) + B_1B_2B_3 \\ &= 0. \end{aligned} \tag{57}$$

‡ One of the reflection coefficients in $\underline{S}_{pp}$ can be made in magnitude larger than unity by arbitrarily choosing μ. Also $\underline{S}_{aa}$ does not satisfy the criterion given in Ref. 14 for the absolute AM stability of the system.

By Routh-Hurwitz criterion, it is necessary and sufficient that

$$5B_1(B_1B_2 + 3B_3B_1) + 3B_3(B_2B_3 + 3B_3B_1) + 2B_1B_2B_3 > 0 \quad (58)$$

so that no zero of $F_a(\lambda)$ lies in the closed right-half plane.

Similarly, for PM stability of the tripler, it is necessary and sufficient that

$$2\frac{B_3}{B_1} + \frac{B_1}{B_2} + 3\frac{B_3}{B_2} + \left\{\frac{B_1}{B_3} + \frac{B_3}{B_1} - 2\right\} > 0. \quad (59)$$

Since $(B_1/B_3) + (B_3/B_1) - 2 \geqq 0$ for all positive values of B_1 and B_3, it follows that a tripler is both AM and PM stable when single-tuned series circuits are used for its terminations. There are no bandwidth restrictions imposed by the condition of stability.

This does not mean that a tripler can be connected with another circuit (for example a stable doubler) without affecting the total stability of the system. We can indeed show that a 1-2-4-6 multiplier which is equivalent to a cascade of a doubler and a tripler imposes certain bandwidth restrictions on its external circuits so as to be assured of its stability.

VII. BIAS CIRCUIT AND ITS INFLUENCE ON THE STABILITY OF HARMONIC GENERATORS

It was assumed all along that the bias circuit in lossless abrupt-junction varactor multipliers is designed properly so that there are no currents flowing at sideband frequencies $\pm\omega$. We shall now assume that the varactor harmonic generator has a finite impedance at frequencies $\pm\omega$ so that there are currents flowing at those sideband frequencies. It will be our purpose in this section to investigate how this assumption affects the stability of the multiplier. The study of the influence of the bias circuit on the output signal-to-noise ratio of harmonic generators and other related results are reserved for a future publication in which we shall discuss noise performance of harmonic generators.

We shall also restrict ourselves in this section to the consideration of lossless abrupt-junction varactor harmonic generators which satisfy the following condition. If we choose the time origin so that carrier current I_1 is real and positive, all carrier currents I_k's, $2 \leqq k \leqq n$, of the nth order harmonic generator are all real. We shall also assume that the multiplier is tuned at all carrier frequencies so that carrier voltages are in phase or out of phase with the respective carrier currents.

There are a large number of multipliers which by design satisfy these conditions.[7, 18] We known that the multipliers of order 2^n3^s discussed in this paper come under this category. We can also show[7] that the 1-2-4-5 quintupler can be designed to satisfy this condition.

Tuning circuits‡ for the multiplier are considered part of the terminations as shown in Fig. 13. We shall also assume that all idler terminations are lossless. The small-signal voltages $V_{\alpha k}$ and $V_{\beta k}$ at sideband frequencies $\pm k\omega_0 + \omega$ can be written as

$$V_{\alpha k} = \sum \frac{S_{k-l}}{j(l\omega_0 + \omega)} I_{\alpha l} + \sum \frac{S_{k+m}}{j(-m\omega_0 + \omega)} I_{\beta m} + \frac{S_k}{j\omega} I_{\alpha 0} \tag{60}$$

$$V_{\beta k} = \sum \frac{S_{-k+l}}{j(-l\omega_0 + \omega)} I_{\beta l} + \sum \frac{S_{-k-m}}{j(m\omega_0 + \omega)} I_{\alpha m} + \frac{S_{-k}}{j\omega} I_{\alpha 0} \tag{61}$$

$$V_{\alpha 0} = \sum \frac{S_{-l}}{j(l\omega_0 + \omega)} I_{\alpha l} + \sum \frac{S_m}{j(-m\omega_0 + \omega)} I_{\beta m} . \tag{62}$$

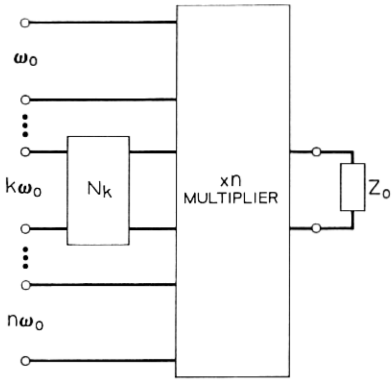

Fig. 13 — Lossless abrupt-junction varactor harmonic generator of order n.

With the assumption that $\omega/\omega_0 \ll 1$, and using amplitude-phase representation, we can write (60) through (62) as§

$$V_{ak} = \sum \pm \left| \frac{S_{k-l}}{l\omega_0} \right| I_{al} + \sum \pm \left| \frac{S_{k+m}}{m\omega_0} \right| I_{am} \tag{63}$$

$$V_{pk} = \sum \pm \left| \frac{S_{k-l}}{l\omega_0} \right| I_{pl} + \sum \pm \left| \frac{S_{k+m}}{m\omega_0} \right| I_{pm} \pm \left[\frac{S_k - S_k^*}{2\omega} \right] I_{\alpha 0} \tag{64}$$

‡ Average elastance S_0 of the varactor diode is included in these terminations.
§ Note that S_k's are all pure imaginary because of our assumptions about I_k's.

and

$$V_{\alpha 0} = \sum \pm 2 \left| \frac{S_l}{l\omega_0} \right| I_{al} \ . \tag{65}$$

Let us now assume that all idler and bias terminations are such that¶

$$V_{\alpha 0} = -Z_0 I_{\alpha 0} \tag{66}$$

$$V_{ak} = -Z_{ak} I_{ak} \ , \qquad 2 \leqq k \leqq n - 1 \tag{67}$$

and

$$V_{pk} = -Z_{pk} I_{pk} \ , \qquad 2 \leqq k \leqq n - 1. \tag{68}$$

From (63) through (68), we can write

$$\begin{bmatrix} V_{a1} \\ V_{an} \\ V_{p1} \\ V_{pn} \end{bmatrix} = \begin{bmatrix} z_{a1a1} & z_{a1an} & 0 & 0 \\ z_{ana1} & z_{anan} & 0 & 0 \\ z_{p1a1} & z_{p1an} & z_{p1p1} & z_{p1pn} \\ z_{pna1} & z_{pnan} & z_{pnp1} & z_{pnpn} \end{bmatrix}. \tag{69}$$

The scattering parameters of a lossless abrupt-junction varactor harmonic generator hence can be described by

$$\underline{S} = \left[\begin{array}{c|c} \underline{S}_{aa} & \underline{0} \\ \hline \underline{S}_{pa} & \underline{S}_{pp} \end{array} \right]. \tag{70}$$

It follows from (62) through (68) that $\underline{S}_{aa}$ and $\underline{S}_{pp}$ in (69) are the same as those that can be obtained by assuming $Z_0 = \infty$. For example, the scattering matrix of a doubler with finite bias source impedance Z_0 is given by[6]

$$\underline{S} = \left[\begin{array}{cc|cc} \frac{1}{2} & -\frac{1}{2} & \multicolumn{2}{c}{\underline{0}} \\ 1 & 0 & & \\ \hline \multicolumn{2}{c|}{\underline{S}_{pa}} & 0 & -1 \\ & & 2 & 1 \end{array} \right]. \tag{71}$$

The characteristic equation of a harmonic generator with finite bias source impedance Z_0 can, according to (10), be represented as

$$F(\lambda) = \Delta\{\underline{1}_4 - \underline{S}\underline{\rho}\} = 0, \tag{10}$$

¶ See Section IV.

where $\underline{\rho}$ is defined in Section III. From (10), (24), (25), and (70), we can write

$$F(\lambda) = \Delta\{\underline{1}_2 - \underline{S}_{aa}\underline{\rho}_m\}\Delta\{\underline{1}_2 - \underline{S}_{pp}\underline{\rho}_\theta\} \tag{72}$$

$$= F_a(\lambda)F_p(\lambda). \tag{73}$$

Equations (70) and (72) show that stability of a harmonic generator is not affected by the finite bias source impedance present in the multiplier even though it increases the output fluctuations of a harmonic generator.[6] If a harmonic generator is stable for certain generator and load impedances for $Z_0 = \infty$, it is also stable when Z_0 is finite. This is one of the important results of this paper.

The conclusions arrived at in this section are applicable to harmonic generators of order 2^n3^s discussed earlier in this section.

VIII. REMARKS AND CONCLUSIONS

A general method has been presented in this paper to investigate the stability of pumped nonlinear systems, and to obtain the conditions imposed thereby on the available circuit configurations. The type of instability investigated is that which causes spurious tones to appear at any point in the system in the vicinity of a carrier.

It has been shown that the roots of a certain characteristic equation

$$F(\lambda) = \Delta\{\underline{1}_4 - \underline{S}\underline{\rho}\} = 0 \tag{10}$$

should lie in the open left-half plane for the system to be stable.

For lossless abrupt-junction varactor multipliers of order 2^n3^s in which a certain set of interstage networks are used it has been shown that there is no AM-to-PM and PM-to-AM conversion and the characteristic equation can be expressed as

$$F(\lambda) = \Delta\{\underline{1}_2 - \underline{S}_{aa}\underline{\rho}_m\}\Delta\{\underline{1}_2 - \underline{S}_{pp}\underline{\rho}_\theta\} \tag{25}$$

$$= F_a(\lambda)F_p(\lambda), \tag{26}$$

and that we can treat separately AM and PM stabilities of the system.

A multiplier of order 2^n has been shown to be AM stable for all passive terminations. However, it is not absolutely stable with respect to PM fluctuations.

The conditional stability of a 1-2 doubler, 1-2-4 quadrupler, and 1-2-4-8 octupler is investigated next. All these multipliers are shown to be PM stable if single-tuned series circuits are used as their terminations, and bandwidths B_i's of these terminations satisfy certain conditions.

The PM characteristic equation of a doubler is given by

$$F_p(\lambda) = 2\lambda^2 + B_2\lambda + B_1B_2 = 0. \tag{39}$$

It is PM stable for any finite B_1 and B_2.

A quadrupler has the following PM characteristic equation:

$$F_p(\lambda) = 4\lambda^3 + 2\lambda^2(B_4 - B_2) + \lambda(2B_1B_2 + B_2B_4) + B_1B_2B_4 = 0. \tag{44}$$

The quadrupler is PM stable if

$$\gamma > 1.629, \tag{47}$$

where

$$\frac{B_4}{B_2} = \frac{B_2}{B_1} = \gamma. \tag{46}$$

An octupler has also been shown to be PM stable if

$$x > 1.992, \tag{53}$$

where

$$\frac{B_8}{B_4} = \frac{B_4}{B_2} = \frac{B_2}{B_1} = x. \tag{52}$$

The scattering relations for a tripler when its idler termination is a passive impedance Z_2 are obtained. It has been shown that a tripler is not absolutely stable both with respect to its AM and PM fluctuations. However, it is stable when the interstage networks used in the tripler are single-tuned series circuits. The condition of stability does not impose any bandwidth restrictions.

Finally, it has been shown that the scattering matrix $\underset{\sim}{S}$ of a lossless abrupt-junction varactor harmonic generator with a finite bias source impedance Z_0 can be expressed as

$$\underset{\sim}{S} = \left[\begin{array}{c|c} \underset{\sim}{S}_{aa} & \underset{\sim}{0} \\ \hline \underset{\sim}{S}_{pa} & \underset{\sim}{S}_{pp} \end{array}\right], \tag{70}$$

where $\underset{\sim}{S}_{aa}$ and $\underset{\sim}{S}_{pp}$ are the same as those obtained by assuming $Z_0 = \infty$. It is then shown that stability characterization of a lossless varactor harmonic generator is not affected by finite bias source impedance.

The noise analysis of harmonic generators and other related results will be discussed in a future publication.

REFERENCES

1. Buck, D. C., Origin of Spurious Signals in a Varactor Tripler, Proc. IEEE, *53,* No. 10, October, 1965, p. 1677.
2. Hines, M. F., Bloidsell, A. A., Collins, F., and Priest, W., Special Problems in Microwave Harmonic Generator Chain, Digest of Technical Papers, 1962 International Solid-State Circuits Conference, Philadelphia, Pa., 1962.
3. Burckhardt, C. B., Spurious Signals in Varactor Multipliers, Proc. IEEE, *53,* No. 4, April, 1965, pp. 389–390.
4. Dragone, C., Phase and Amplitude Modulation in High Efficiency Varactor Frequency Multipliers of Order $N = 2^n$—Stability and Noise, B.S.T.J., *46,* No. 4, April, 1967, pp. 797–834.
5. Dragone, C. and Prabhu, V. K., Scattering Relations in Lossless Varactor Frequency Multipliers, B.S.T.J., *46,* October, 1967, pp. 1699–1731.
6. Prabhu, V. K., Noise Performance of Abrupt-Junction Varactor Frequency Multipliers, Proc. IEEE, *54,* No. 2, February, 1966, pp. 285–287.
7. Penfield, P., Jr. and Rafuse, R. P., *Varactor Applications,* The M.I.T. Press, Cambridge, Mass., 1962.
8. Guillemin, E. A., *The Mathematics of Circuit Analysis,* The Technology Press, Cambridge, Mass., and John Wiley and Sons, Inc., New York, N. Y., 1959.
9. Llewellyn, F. B., Some Fundamental Properties of Transmission Systems, Proc. IRE, *40,* No. 3, March, 1952, pp. 271–283.
10. Gewertz, C. M., Synthesis of a Finite, Four-Terminal Network from its Prescribed Driving-Point Functions and Transfer Function, J. Math. Phys., *12,* 1933, pp. 1–257.
11. Folke Bolinder, E., Survey of Some Properties of Linear Networks, IRE Trans. Circuit Theor., *CT-4,* 1957, pp. 70–78.
12. Youla, D. C., A Stability Characterization of the Reciprocal Linear Passive *N*-port, Proc. IRE, *47,* 1959, pp. 1150–1151.
13. Youla, D. C., A Note on the Stability of Linear, Nonreciprocal *N*-ports, Proc. IRE, *48,* 1960, pp. 121–122.
14. Ku, W. H., Unilateral Gain and Stability Criterion of Active Two-Ports in Terms of Scattering Parameters, Proc. IEEE, *54,* No. 11, November, 1966, pp. 1617–1618.
15. Hohn, F. E., *Elementary Matrix Algebra,* The MacMillan Company, New York, N. Y., 1963.
16. Carlin, H. J. and Giordano, A. B., *Network Theory,* Prentice-Hall, Inc., Englewood Cliffs, N. J., 1964.
17. Truxal, J. G., *Control Engineers' Handbook,* McGraw-Hill Book Company, Inc., New York, N. Y., 1958.
18. Burckhardt, C. B., Analysis of Varactor Frequency Multipliers for Arbitrary Capacitance Variation and Drive Level, B.S.T.J., *44,* No. 4, April, 1965, pp. 675–692.