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"Mental Holography": Stereograms Portraying Ambiguously Perceivable Surfaces

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An algorithm has been devised that can generate the same stereogram for two (or more) selected surfaces. Prior to this development the only known ambiguous stereograms have been periodic grid patterns that could be perceived at various parallel depth planes. The new algorithm, however, can portray two (or more) selected surfaces of general shapes and the observer can perceive each of these surfaces, but only one at a time. The technique is an extension of random-dot stereograms and it is shown that in most cases adequate degrees of freedom remain for coloring the random-dot texture. Since these ambiguously perceivable stereograms permit the portrayal of both the visible and the hidden surfaces of objects, they are analogous to holograms. However, in the case of holograms the observer has to inspect them from various positions, while for ambiguous stereograms it is the mind of the observer that wanders around.

I. INTRODUCTION

There are many limitations in portraying our three-dimensional environment on a two-dimensional surface. One of the primary shortcomings is the difficulty of effectively representing the hidden surfaces of objects together with the visible ones. Perspective drawings and even stereoscopic images did not alleviate this limitation. The cubist artists tried to place the hidden surfaces of their objects side by side with the visible ones, but the results were rather confusing. The usual representation by multiple projections solves the problem geometrically yet it is very difficult to combine these projections into a unified spatial percept. The invention of panoramagrams using lenticular screens by Ives[1] and multiple lens arrays (fly's eye) by Lippmann[2] portrayed the three-dimensional objects within a wide angle, but in order to inspect some hidden surfaces the observer still has to move around the panoramagram. Furthermore, certain hidden surfaces (such as the boundaries of inside cavities of opaque objects) stayed invisible. Since holograms (invented by Gabor[3] and improved by Leith and Upatnieks)[4] are similar to panoramagrams only more simply made, some change in the geometry of the optical rays has to be initiated in order to obtain various stored organizations. This is usually achieved by the observer when inspecting the hologram from various angles.

This article describes a method of generating ambiguously perceivable stereograms. These stereograms contain several predetermined surfaces, out of which only one can be perceived at a time. Moreover, some surfaces might be "internal," hidden from every angle. In order to inspect the various surfaces the viewer can sit still; it is his mind that wanders around the object.

II. AMBIGUOUS STEREOGRAMS

Random-dot stereograms, introduced in this journal in 1960, have shifted interest to the problem of how the visual system resolves ambiguities.[5] Indeed, in random-dot stereograms, hundreds of dots are presented on a horizontal line in the left and right eye's views, and the observer is confronted with ambiguities as of which of the many dots in the left retinal projection corresponds to a given point in the right retinal projection. This problem is further amplified for random-dot stereograms, since no monocular familiarity or depth cues are provided that would aid perception. Findings with random-dot stereograms have shown that the visual system selects that organization which yields dense surfaces, and all other possible organizations pass unnoticed.[5–7]

Would it be possible to create random-dot stereograms which portray simultaneously more than one dense surface? If possible, which organization would be preferentially perceived? That there are ambiguous stereograms is well known in psychology. Grid-like structures containing vertical bars of constant periodicity (such as wallpapers and old-fashioned radiators) can be fused at multiple depth levels since the binocular disparity can be any integer multiple of the horizontal periodicity. In Ref. 7, such a periodic random-dot stereogram has been demonstrated which could be perceived as a plane in front of or behind the real plane of the printed page. These periodic random-dot patterns have been successfully used in studies of perception[7, 8] yet are limited in their scope, since only parallel planar surfaces can be portrayed by this method.

This report describes a general algorithm which generates a single stereogram portraying two (or more) specified surfaces. That such stereograms can exist is based on the fact that certain areas are seen by one eye only and thus can be freely selected for one surface. Also segments in which the two (or more) surfaces coincide add to the degrees of freedom, since these surfaces can be covered by any random texture at will. In general, there is no restriction on the surfaces to be portrayed simultaneously. Provided that the number of surfaces is restricted to two and the resolution is fine (the number of samples is large) there is enough freedom in choosing the texture elements that the formation of monocularly perceivable short periodicities can be prevented.

Before the general algorithm is discussed in detail, a brief illustration is given of the basic idea in Fig. 1. Fig. 1 (solid line shows how points P_i^1 and P_i^2 (belonging to surface B) have to be colored identically to P_i (the original point of fixation, belonging to surface A), in order to obtain the same retinal projections for both surfaces. Fig. 1 (dotted line) shows how this requirement forces P_i^3 and P_i^4 to be colored the same. Fig. 1 (dashed line) shows the next step P_i^5 and P_i^6 . As this procedure is continued the algorithm assigns the color of P_i to a gamut of points belonging to the two surfaces.

For a new point of fixation on either surface A or B, then there are two possibilities. Either this point of fixation already has been assigned a color by the previous algorithm or can be colored freely some brightness value. In the latter case the above described algorithm is continued which assigns this chosen color to another set of points belonging to the two surfaces. This procedure continues until each point on either surface has been colored. It is interesting to notice that the color of any point depends on some *global* relationship between surfaces A and B.

In this algorithm, as long as only two surfaces have to be portrayed each, iterative step assigns one new constraint. In the case of three surfaces, each iterative step generates two new constraints which in

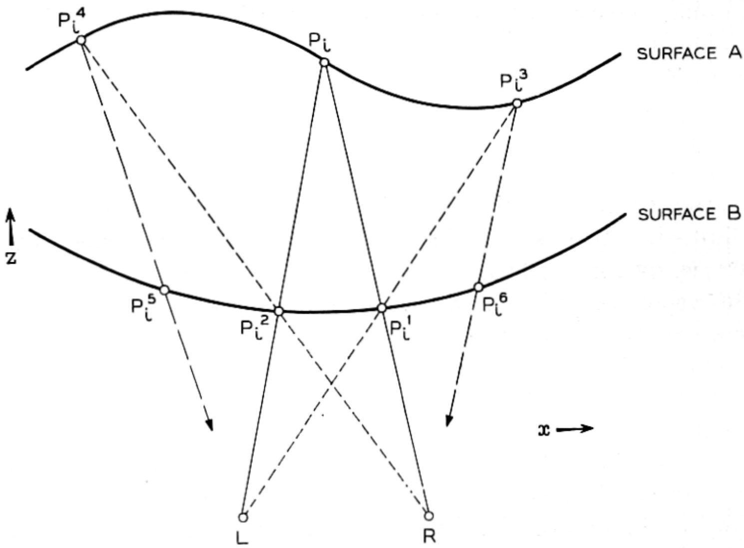


Fig. 1 — Iterative steps in the algorithm that generates the same stereoscopic projections for surfaces A and B.

turn generate 4, 8, 16, . . . new constraints. This exponential proliferation of constraints drastically reduces the degrees of freedom for three or more surfaces. Therefore, in the forthcoming examples we restrict ourselves to two surfaces to be portrayed.

The study of how such ambiguous stereograms are perceived can be of considerable scientific value but also permits some useful applications. It is now possible to portray hidden surfaces of objects together with the visible ones. Since in these ambiguous stereograms only one surface can be seen at a time, multivalued functions of two variables can be portrayed. For instance, if one surface is chosen to be the front view of an object (such as a statue or a machine part) and the other surface is chosen to be the rear view, one can obtain an entire 360 degree impression of an object, since the perception of the two surfaces may be alternated at will.

III. THE GENERAL ALGORITHM

The algorithm is an extension of the technique of random-dot stereograms.[5, 9] The following simple example will give an insight into the workings of the general algorithm. The two surfaces, A and B, to be portrayed by the stereogram are given in the x-z plane in Fig. 2 and for simplicity are selected as cylindrical; (that is, $z = f_A(x)$ and $z = f_B(x)$, independent of y). Since stereoscopic vision operates on corresponding single rows in the two views, the algorithm given here applies to any surface (not only to cylindrical ones).

In order to construct the stereogram we must specify the textures $T_L(x)$ and $T_R(x)$ for the left and right images, respectively. The right image of the stereogram is selected as the perpendicular projection of Fig. 2, while the left image is viewed from an angle of 45 degrees.

Examine for a moment the case where we have just one figure, $z = f(x)$. We may pick the texture $T_R(x)$ at random; the texture $T_L(x)$ is now basically chosen as follows:

$$T_L[x + f(x)] = T_R(x). \tag{1}$$

This just expresses the fact that a point x seen in the left image is displaced horizontally by a distance $f(x)$ when viewed in the right image.

There are two necessary qualifications to the above rule:

(*i*) If $x + f(x)$ and $x' + f(x')$ are equal for x unequal to x', in fact

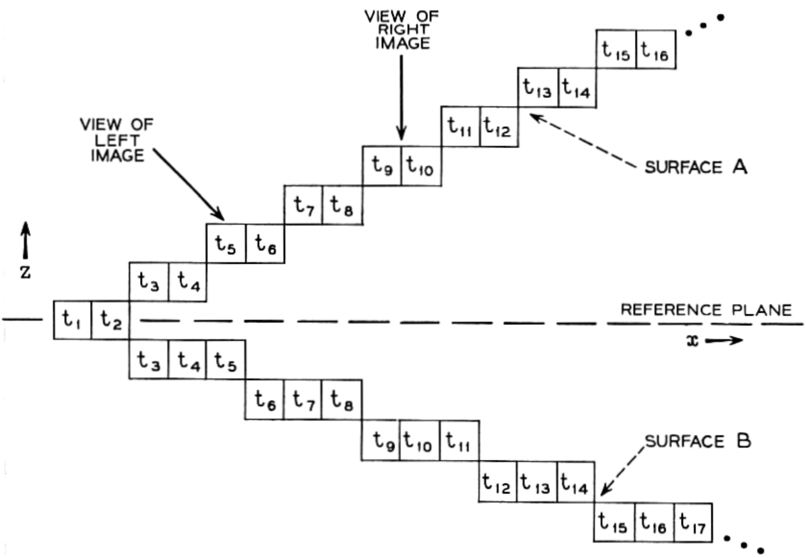


Fig. 2 — A simple example of two surfaces to be portrayed. (The cross section is indicated in the x-z plane.)

only the point corresponding to min (x, x') is seen by the left eye, the other being "in the shadow." The constraint (1) thus does not hold for the larger of x and x'; we say the larger is obscured in this case.
(*ii*) In the event that, after applying all the constraints, there are some values of $T_L(x)$ not determined by T_R, these values may be chosen at random.

Julesz and Miller developed these ideas extensively.[9]

Now to color two figures, $z = f_A(x)$ and $z = f_B(x)$, we apply a similar method; the main difference is that there are more constraints, so that the right image can no longer be chosen at random. The images must represent both A and B; thus, if T_L and T_R represent the left and right textures as above we have basically

$$T_L[x + f_A(x)] = T_R(x)$$
$$T_L[x + f_B(x)] = T_R(x) \tag{2}$$

for all x. Qualification *i* is still valid, and may serve to eliminate one or both of the above constraints for certain values of x. From this it is also seen that if x and x' are distinct, and neither x nor x' is obscured, then

$$f_A(x) + x = f_B(x') + x' \text{ implies } T_R(x) = T_R(x') = T_L[x + f_A(x)]. \tag{3}$$

These are also easily seen to be the only constraints on T_R.

Once again, qualification *ii* is valid; anything not explicitly constrained may be chosen at random.

A simple example of the computational ease of this algorithm is given in Table I. Notice that x, $f_A(x)$, and $f_B(x)$ are given in the first three rows. The rows labeled L_A and L_B are formed by putting the integer x into positions $x + f_A(x)$ and $x + f_B(x)$, respectively, subject to qualification *i*. A * indicates a position that has been "uncovered" by the shifting process—the texture here may be chosen at random if not otherwise constrained.

We may read off our constraints directly from Table I; if $L_A(x) \neq L_B(x)$, and neither $L_A(x)$ or $L_B(x)$ is a *, then

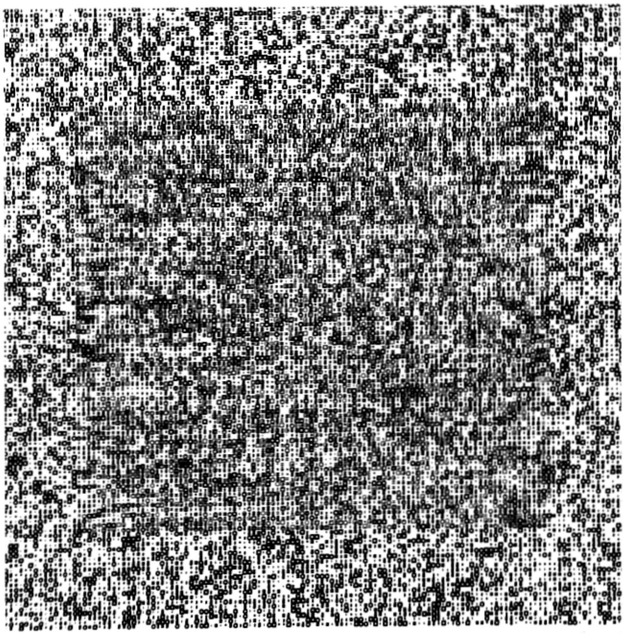
$$T_R[L_A(x)] = T_R[L_B(x)]. \tag{4}$$

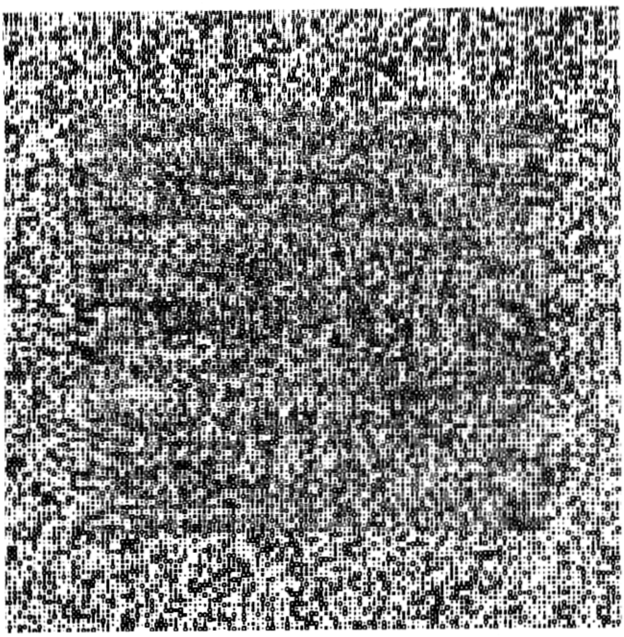
After all such constraints have been applied to the right image, the left image can be generated by reference to equation (2) and qualification *ii*. Table I gives final values for T_R and T_L in terms of randomly chosen texture values $t_1, t_2, t_3, \ldots$.

TABLE I PROJECTIONS*

x	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$f_A(x)$	0	0	1	1	2	2	3	3	4	4	5	5	6	6
$f_B(x)$	0	0	−1	−1	−1	−2	−2	−2	−3	−3	−3	−4	−4	−4
$L_A(x)$	1	2	*	3	4	*	5	6	*	7	8	*	9	10
$L_B(x)$	1	2	4	5	7	8	10	11	13	14	16	17	19	20
$T_L(x)$	t_1	t_2	t_4	t_3	t_4	t_6	t_3	t_5	t_9	t_4	t_6	t_{10}	t_7	t_3
$T_R(x)$	t_1	t_2	t_3	t_4	t_3	t_5	t_4	t_6	t_7	t_3	t_5	t_8	t_9	t_4

* The left and right projections of the surfaces given in Fig. 2 before and after the constraints.





← Fig. 3 — Stereogram of unambiguous pyramidal staircase in front of the printed page with a 128 × 128 picture element resolution. There are nine steps, altogether. Use the viewers fastened inside the back cover of this issue to see stereoptical effect. Place the red filter over your left eye.

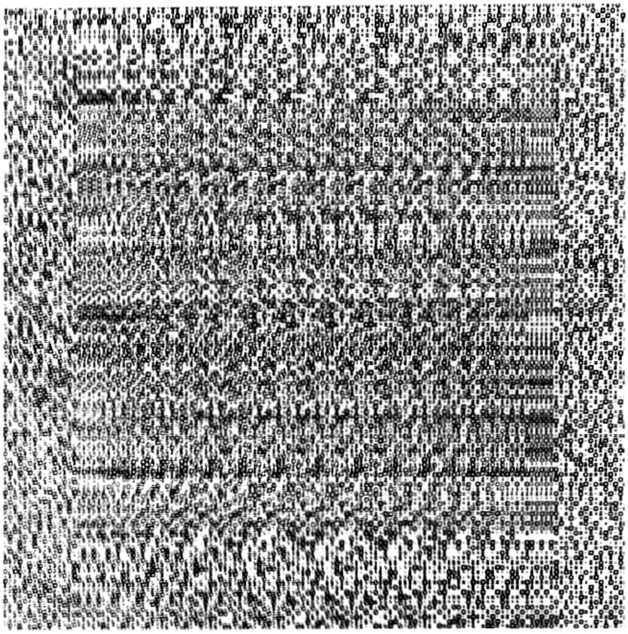
IV. CONCRETE EXAMPLES

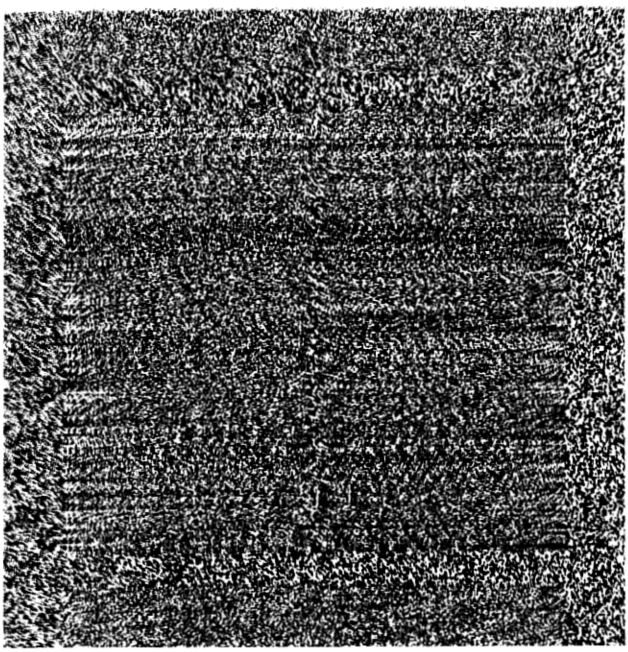
The demonstration will be quite general and the 1000 × 1000 dot resolution permits the portrayal of surfaces having complex shapes. The only restriction on the surfaces will be the use of cylindrical shapes. For this case, the algorithm determines the *same* constraints *for each row* but of course within the degrees of freedom each row is independently colored by a random process. For general surfaces each row would have to be computed separately, which would increase the computation time (now about two minutes on a GE 645 computer) nearly a thousand times. The cylindrical surfaces have another advantage; they permit us to use two unambiguous surfaces at the top and bottom margins of the stereogram respectively, to facilitate perceptual reversals for the unexperienced observer.

Besides the 1000 × 1000 dot resolution there are a few stereograms composed of 128 × 128 picture elements. In the 1000 × 1000 dot array each dot (picture element) can take three different brightness values; for the coarser array, each picture element can take eight brightness values. This is done by using three (eight) characters (blank, period sign, degree sign, asterisk, and so on) of the General Dynamics (Stromberg Carlson) 4060 microfilm printer. For the 128 × 128 array the probability of each of the eight characters is equal (1/8). For the 1000 × 1000 array the probability of using the light and heavy period signs is 0.05 while the probability for the blank is 0.9. Thus the average number of portrayed dots in these stereograms is 10^5, which is within the resolution capabilities of the printing process.

Figure 3 shows an unambiguous pyramidal staircase in front of the real plane of the printed page with a 128 × 128 picture element resolution. In Fig. 4 the left and right images of Fig. 3 have been interchanged and the pyramidal staircase is descending behind the printed plane. To view these illustrations use the anaglyphoscopes fastened inside the back cover of this issue. Put the red filter over your left eye.

← Fig. 4 — Stereogram, identical to Fig. 3, except that the left and right images have been interchanged. The unambiguous pyramidal staircase is descending behind the page.





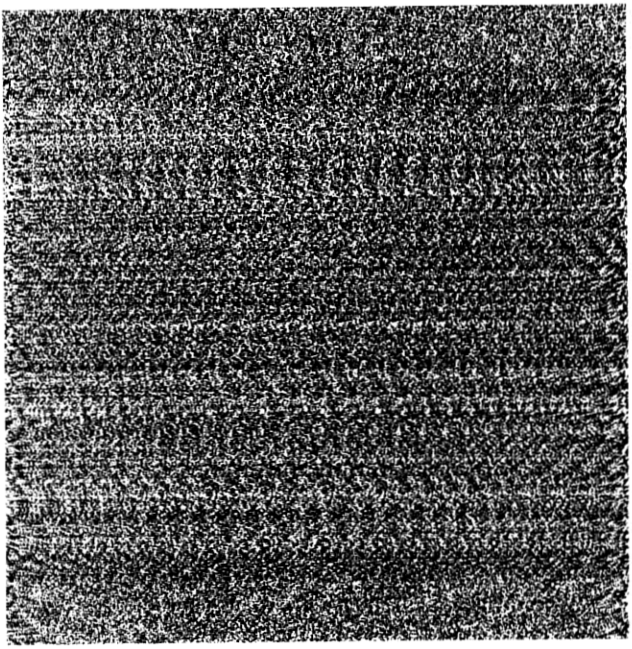
← Fig. 5 — Ambiguous stereogram, that contains both the pyramidal staircases in front of and behind the plane of the printed page as given in Figs. 3 and 4. Both of these organizations can be obtained when sterosopically viewing Fig. 5, but only one at a time.

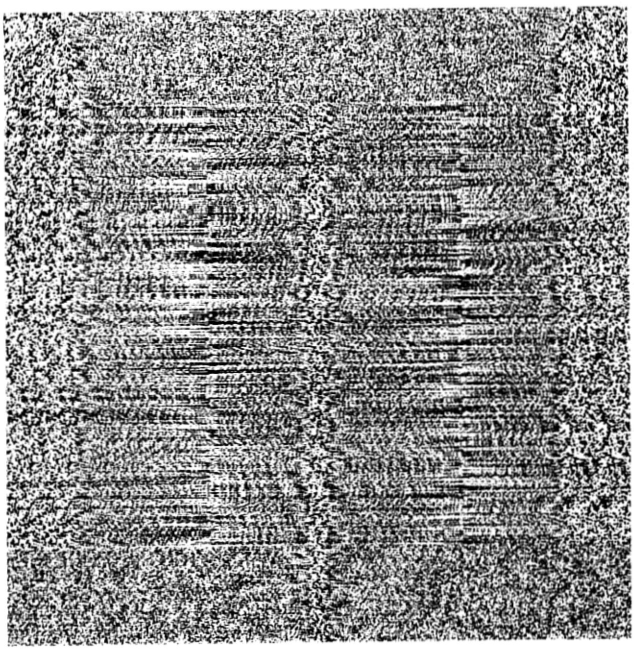
If you have good stereopsis, the depth should become apparent to you within several seconds.

Figure 5 demonstrates an ambiguous stereogram that contains both the pyramidal staircases above and below the printed plane as given in Figs. 3 and 4. Both of the organizations can be obtained when stereoscopically viewing Fig. 5, but only one at a time. In a brief study Fig. 3 or Fig. 4 have been shown to 21 subjects who have never seen these stimuli before. After viewing one of these unambiguous stimuli for a minute the ambiguous stimulus of Fig. 5 has been shown. Ten subjects perceived that organization in the ambiguous stereogram which corresponded to the previous unambiguous organization. Nine subjects perceived the ambiguous stereogram always as the descending staircase, while two subjects always as the ascending staircase. There was no attempt on our part to train these subjects to learn to reverse the organization. On the other hand, the reader can learn easily the reversal if he alternates between Figs. 3 or 4 prior to viewing Fig. 5. Convergence movements of the eyes can influence the reversals, but it might require careful studies to determine whether reversal could be obtained while the eyes are immobilized. In Fig. 5, the maximum disparity is 8 picture elements. The degrees of freedom are 46 (out of 128).

The degrees of freedom can be greatly increased and the staircasing greatly reduced by increasing the resolution to 1000×1000 dots. Such an ambiguous stereogram is shown in Fig. 6, portraying a single wedge behind the printed plane and two wedges in front of the printed plane. The maximum depth is ± 60 picture elements (dots), and to facilitate perceptual reversal a 150 picture element wide margin in the upper and lower portions of the images contains the unambiguous surfaces A and B, respectively. A 50 picture element wide gap at zero depth level separates the unambiguous surfaces from the ambiguous organiza-

← Fig. 6 — Ambiguous stereogram of 1000×1000 picture element resolution with unambiguous margins in the upper and lower areas. Surface A is a wedge behind the plane of the printed page, while surface B is a double wedge in front of the page. Either one of the two surfaces can be perceived at will when stereosopically viewed, yet reversal can be aided by viewing the upper or lower margins, respectively.





← Fig. 7 — Ambiguous stereogram with unambiguous margins in the upper and lower areas. Surface A is a horizontal plane in front of the printed plane, while surface B is a wedge behind the printed plane. For viewing instructions see Fig. 6.

tion. When looking at the upper portion of the fused stereogram the unambiguous ascending wedges usually carry with them the percept of the ambiguous wedges, while when looking down the ambiguous organization reverses. Of course, these unambiguous margins serve only as an aid, and the ambiguous organization can be reversed at will when the margins are covered up. This stereogram is similar to Fig. 5, yet because of increased resolution the degrees of freedom are 359 (out of 1000 samples). This is adequate to portray images without excessive formation of perceivable periodic stripes.

Particularly interesting is Fig. 7, which has 1000 × 1000 resolution. Here the upper margin contains the unambiguous surface A which is a plane with 40 picture element disparity, while the lower margin contains the unambiguous surface B which portrays a descending wedge having a maximum disparity of -60 picture elements. In this example there is no gap between the ambiguous and unambiguous surfaces. Organization A is very strong and our everyday experience would suggest that when each dot of a front plane is seen by both eyes without any hidden areas present, then this plane should be the only percept. Yet, as Fig. 7 demonstrates, it is relatively easy to obtain the other organization too. Here the degrees of freedom are only 99 (out of 1000), yet in spite of this low degree of freedom the image quality is very good.

When we try to portray more than two surfaces, the degrees of freedom rapidly diminish. On the other hand, some of the stereograms with two surfaces yield some additional percepts. For instance, in Fig. 7 after the front plane is perceived, sometimes the percept of an ascending wedge above the plane can be obtained too.

Figure 8 shows a stereogram that can be perceived in many different ways as illustrated in Fig. 9. The unambiguous margins correspond to the two shapes in Fig. 9a, but the reader might obtain the other percepts as well. Obviously the strongest constraints are obtained for

← Fig. 8 — Ambiguous stereogram with unambiguous margins in the upper and lower areas. Two slanted planes (above the plane of the printed page) that interest each other are portrayed. Fig. 9 shows the various percepts which can be obtained.

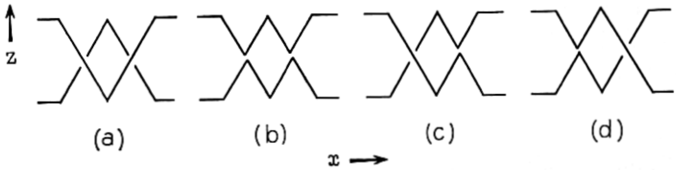


Fig. 9 — Schematic illustration of the various ways Fig. 8 can be perceived. The cross sections in the x-z plane are indicated.

parallel surfaces A and B with a few dots separation in depth. This occurs near the intersection of the two surfaces, yielding visible clusters of dots having the same brightness values.

Figure 10 portrays a cosine function and a cosine function of lesser amplitude and half periodicity. Since both surfaces are in front of the surround and close together, an interesting perceptual phenomenon can be experienced. For the previous demonstrations only one

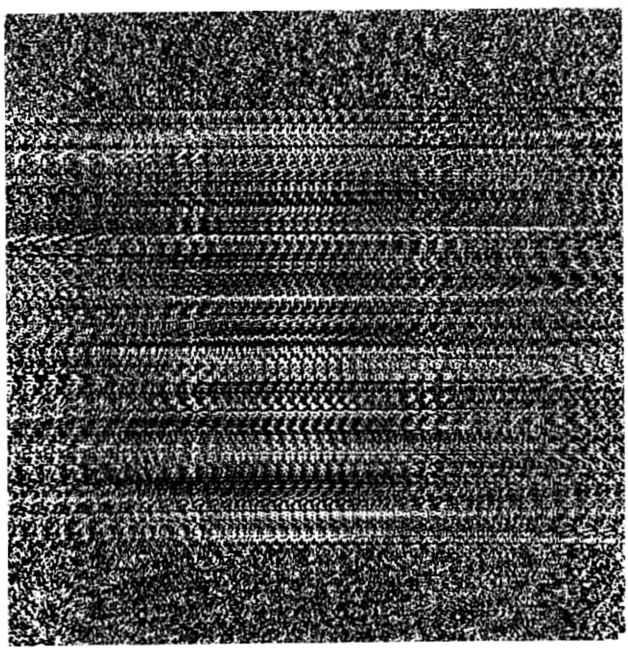


Fig. 10 — Ambiguous stereogram with unambiguous margins in the upper and lower areas. It portrays a cosine function and a cosine function of lesser amplitude (height) and half periodicity. Both surfaces appear in front of the printed plane.

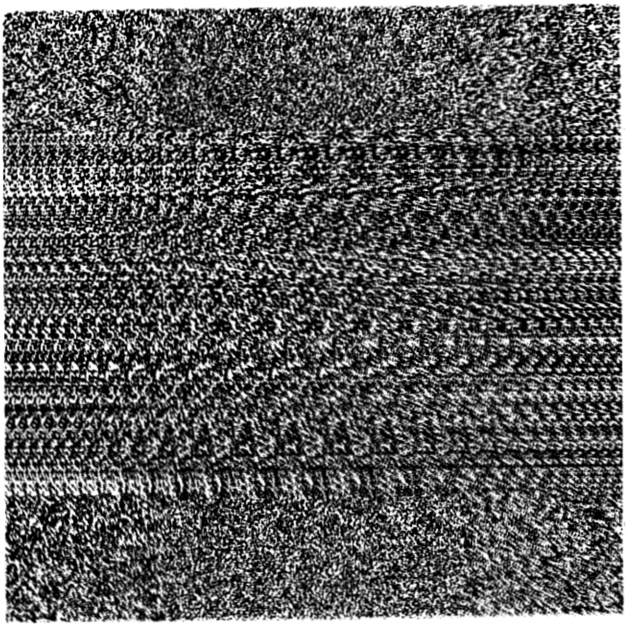
organization could be perceived at a time and considerable amounts of convergence movements had to be initiated in order to get rid of the prevailing organization and to bias the other organization. In Fig. 10 it is possible to retain one organization and meanwhile start to perceive the other organization. The perceptual effect is that of a transparent surface behind which another transparent surface is seen. However, this double perception is not a stable state and it is easier to perceive only one organization at a time. The degrees of freedom are 119 (out of 1000).

Finally Fig. 11 shows a case in which three surfaces are portrayed. Besides a cosine function in front and behind the surround there is a plane with zero disparity. Interestingly enough for this special case the degrees of freeedom are not additionally reduced. As long as the surfaces A and B are each other's mirror images and the third surface is a plane with zero disparity, it is possible to portray three surfaces without additional constraints. The degrees of freedom are 78 (out of 1000). An unambiguous rectangle at the top and bottom margin is presented at ± 70 picture element disparities in order to aid perceptual reversal.

V. CONCLUSIONS

From these demonstrations it is possible to see some of the limitations of the ambiguous random-dot stereograms. In order to avoid short periodicities the surfaces to be portrayed should not intersect or come closer than a few picture elements in the z direction. The worst case is if the two surfaces are one picture element apart. In this case the periodicity is one and the two surfaces are formed of horizontal lines having the *same* color. As we have discussed the two surfaces can coincide but after separation they have to separate in a discontinuous fashion having a jump in depth of several picture elements. In Fig. 12 special care has been taken to separate the two surfaces in depth. Therefore a cosine function (to be seen in front of the plane of the printed page) has been placed on a 10 picture element high pedestal. The other surface is a wedge (behind the printed plane). The degrees of freedom are 128 (out of 1000). Because of this pedestal the shortest periodicity is limited to ten and the resulting stereogram can be easily fused and reversed.

Another limitation is the rapid decrease in the degrees of freedom as the number of surfaces is three or more. By increasing the resolution of the images the absolute degree of freedom increases as well,



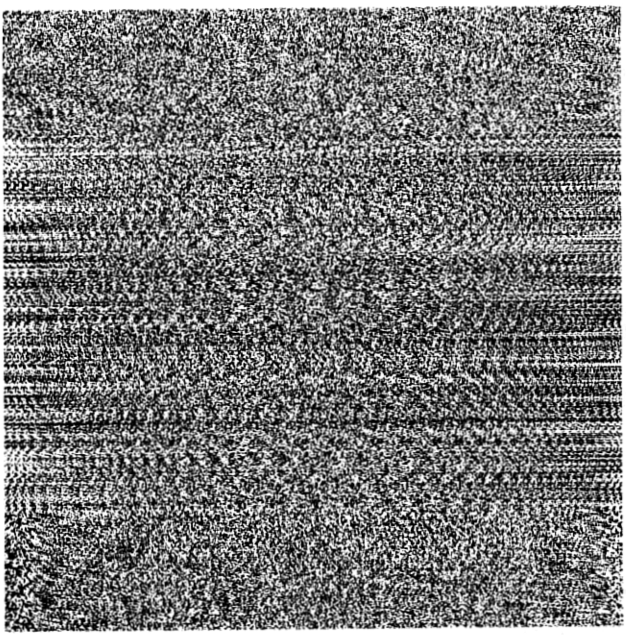


Fig. 11 — Ambiguous stereogram with two unambiguous rectangles in the upper and lower areas. It portrays three surfaces. A cosine function in front of the plane of the printed page, the same function behind the plane, and the printed plane, itself (a plane with zero disparity).

so that three or more surfaces could be adequately portrayed. Unfortunately, the resolving acuities of the eyes limit the size of individual picture elements to about 1 minute of arc. With finer image resolution more than one picture element (of different brightness levels) falls on a single receptor of the retinas and the image contrast rapidly decreases.

A third limitation of ambiguous stereograms is the unavoidable fact that because of the constraints there will be some other dense surfaces perceivable besides the selected two surfaces (as pointed out in the demonstration). Since most of these phantom surfaces are perceived at greater depth than the desired ones, there is a way to eliminate them. If the desired surfaces span the depth limits for fusion, then the phantom surfaces will be outside the region of maximum disparity for stereoscopic fusion.

As long as the surfaces to be portrayed are placed such that they stay separate in distance and the number of surfaces is two, the above technique gives satisfactory results. The obtained results are analogous to holography, but only superficially. After all, holograms contain a vast number of stereoscopic views, while ambiguous stereograms contain only a single one. Holography is based on the diffraction properties of coherent wave optics, while our technique uses plain geometrical optics. For holography the observer has to move around the hologram in order to inspect it from various angles, while for ambiguous stereograms the viewer can stand still. It is his mind that wanders around the object. Furthermore, ambiguous stereograms can portray any mathematical surface, including completely hidden ones, from any view, this could be obtained by computer-aided or computer-generated holograms as well, but would require more effort.

The advantages of holography or lenticular screen panoramagrams could be combined with ambiguous stereoscopy. It might be possible to generate stereograms by computer and place them behind a lentic-

Fig. 12 — Ambiguous stereogram with wide unambiguous margins. It portrays a cosine function in front of the printed plane and a wedge behind the plane.

ular screen such that each separate stereoscopic view is constrained by our algorithm. Since each view is independent from each other, no further reduction of the obtainable degrees of freedom will result. Unfortunately, the generation of ambiguous stereograms for the most general surfaces is at the limit of present computer economies and to compute hundreds of them for a single portrayal is certainly impractical. Yet, with next generation computers these and similar representations can be tried.

The emphasis of this article has been on the reporting of a new tool for pictorial representations and the obvious psychological implications have been only briefly mentioned. We can now study, for example, whether convergence motions of the eyes are necessary to destroy an existing perceptual organization in order to reverse to the other one. Furthermore, each of the ambiguous organizations can be biased by a few randomly introduced unambiguous picture elements in order to counteract natural bias. (This biasing technique has been successfully tried for periodic random-dot patterns in a study of perception time.[7, 8])

A pilot study was reported above which showed that if one of the organizations has been first presented as an unambiguous stereogram, then the perception of the ambiguous stereogram could be influenced accordingly. It remains to be seen whether prior auditory or tactile information would similarly influence perception.

Originally, random-dot stereograms were conceived to remove from the monocular images all the familiarity cues and Gestalt factors that influence perception in uncontrollable ways. It is therefore somewhat unexpected that this further development of the technique in the form of ambiguous random-dot stereograms seem to provide a powerful tool for the study of Gestalt factors. In this instance the shapes are the configurations of the surfaces in depth. Questions exemplified in Figs. 8 and 9 can be studied, such as whether good Gestalt or good continuation outweigh the reduction of disparity. Reversible figures have been frequently used in perceptual psychology such as Necker cubes (two-dimensional outline drawings of a cube), ambiguous staircases, and so on. However, these stimuli have been selected from a small repertoire and exploited certain ambiguities inherent in two-dimensional drawings. That ambiguities can be produced in three-dimensions without practical limitations on the organizations to be portrayed is a result which seems far from trivial.

VI. ACKNOWLEDGMENT

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