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A Unified View of Synchronous Data Transmission System Design

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This paper illustrates the basic equivalence of many of the linear data transmission design techniques. It shows the unifying feature of these techniques to be a generalization of Nyquist's original ideas relating time samples and frequency domain constraints. It examines pulse amplitude (with and without constraints on the input data) and pulse shape modulation systems, and shows their relationships. It uses a number of previously-described systems to illustrate the range of possibilities of the very general design approach. This paper presents some new results on noise and channel parameter monitoring and on spectrum shifting by constraining the input data.

I. INTRODUCTION

Over the years, many seemingly different techniques have been proposed for synchronous data transmission. Unfortunately, the literature devoted to these techniques tends to expand the differences between a specific system and all other systems. It is our purpose to show the basic equivalence of the various techniques; hence, to show a unified view of the field. In doing this we examine some well known and some relatively unknown transmission systems in a new light and propose some new techniques.

The unified design view that we take here is basically a generalization of Nyquist's ideas[1] which have recently been expanded upon by

Gibby and Smith.[2] This really is the thread which ties together virtually all of the literature on synchronous data transmission. Section II summarizes the basic ideas of these two papers.

Section III describes the model of the general, linear, data transmission system to be considered. The system uses M channels and is described by means of an input data vector rather than by any statements about the transmitter characteristics. Thus, one type of data vector implies a pulse amplitude modulated system (PAM) while another type of data vector implies pulse shape modulation (PSM) such as frequency shift keying (fsk) or pulse position modulation (PPM.)

In Section IV, we use Nyquist's approach to find the design constraints for distortionless transmission (no intersymbol or interchannel interference). This section shows that the conditions for distortionless transmission depend upon the input data vector description; hence, different design constraints result from PAM or PSM transmission.

Section V illustrates the application of the constraints to some special cases. These include:

(*i*) Linear precoding and decoding for PAM transmission.

(*ii*) The use of band-limited orthogonal signals for multichannel PAM transmission.

(*iii*) Noise monitoring in PAM systems.

(*iv*) Binary PSM transmission (including the specific case of Sunde's FM model with a linear receiver instead of a phase derivative receiver).

(*v*) Parameter monitoring in PSM systems.

(*vi*) Zero stuffing techniques for shifting spectrum location. (The section shows this to have some promise for voice channel transmission without carrier modulation.)

II. THE UNIFYING VIEW

In designing a data system, one usually starts with a desired time response for the total system. Because it is only necessary to examine the output signal at fixed times (for example, at T second intervals where $1/T$ is the rate at which independent symbols are being transmitted), one needs to specify the over-all response at those times (for example, $t = kT$, all k). Since the total response of the transmitting filter, the channel, and the receiving filter is easier to determine in the frequency domain than in the time domain, one must relate the time response constraints to frequency domain constraints.

This briefly is the basic problem attacked by Nyquist. Here is a summary of his results, as expanded by Gibby and Smith.

Any time function $r(t)$ with Fourier transform $R(\omega)$

$$r(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R(\omega) e^{i\omega t} \, d\omega \tag{1}$$

has sample values at multiples of T seconds which may be written

$$r(qT) = r_q = \frac{1}{2\pi} \int_{-\infty}^{\infty} R(\omega) e^{i\omega qT} \, d\omega \tag{2a}$$

$$r_q = \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} \int_{(2n-1)\pi/T}^{(2n+1)\pi/T} R(\omega) e^{i\omega qT} \, d\omega. \tag{2b}$$

Or, changing the variable,

$$r_q = \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} \int_{-\pi/T}^{\pi/T} R(u + 2n\pi/T) e^{iuqT} \, du. \tag{2c}$$

Assuming that

$$\sum_n R(u + 2n\pi/T)$$

is a uniformly convergent series, one obtains

$$r_q = \frac{1}{2\pi} \int_{-\pi/T}^{\pi/T} \sum_{n=-\infty}^{\infty} R(u + 2n\pi/T) e^{iuqT} \, du. \tag{2d}$$

Noting that r_q is just the q^{th} coefficient of an exponential Fourier series expansion of

$$\frac{1}{T} \sum_{n=-\infty}^{\infty} R(u + 2n\pi/T) \qquad -\pi/T \leqq u \leqq \pi/T,$$

one obtains

$$\frac{1}{T} \sum_{n=-\infty}^{\infty} R(u + 2n\pi/T) = \sum_{q=-\infty}^{\infty} r_q e^{-iuqT} \qquad -\pi/T \leqq u \leqq \pi/T \tag{3}$$

which is very closely related to the Poisson sum formula.[3] Throughout this paper the reader should keep in mind the interval $-\pi/T \leqq u \leqq \pi/T$; we will not be repeating it explicitly.

Equation 3 relates a function of the frequency domain characteristic (namely the sum of the values at frequency intervals $2\pi/T$) to the time response constraints r_q which will be chosen for a particular transmission scheme. Fig. 1 illustrates several frequency characteristics which satisfy the time response constraint that $r_q = 0$ for $q \neq 0$. When $r_0 \neq 0$,

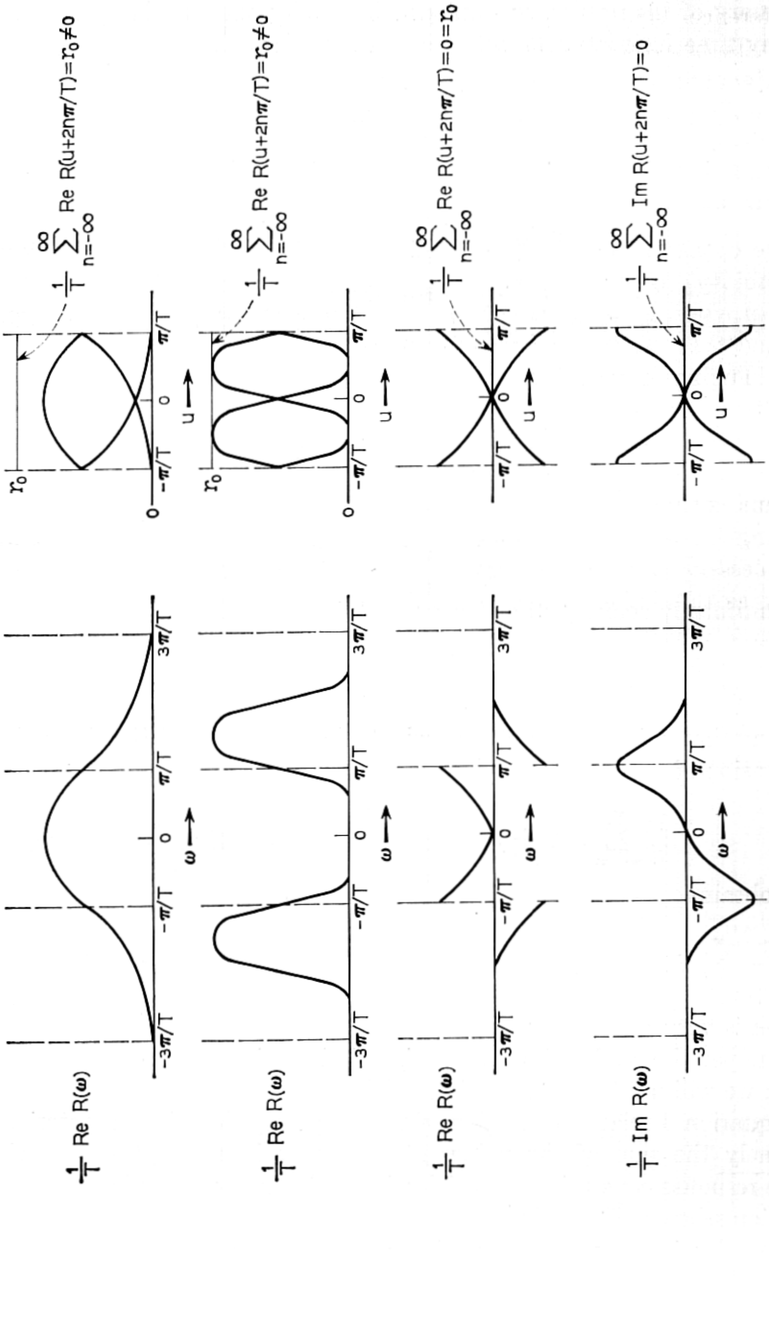

Fig. 1 — Such constraints on $R(\omega)$ that $r(qT) = 0, q \neq 0$.

we have the usual design for PAM transmission without intersymbol interference. When $r_0 = 0$, we have a response which is useful when crosstalk between channels is to be eliminated. That is, the output of a signaling path whose time response has $r_q = 0$ for all q contains no information about the input data at the sampling times. For further discussion of equation 3 see Ref. 2.

III. THE GENERALIZED TRANSMISSION MODEL

The optimum theoretical method (in the sense of minimizing error probability) for transmitting data through a Gaussian channel consists of waiting until all data have been accumulated at the transmitter and then sending a single waveform to represent the entire message. The optimum receiver (in the presence of white noise) consists of filters matched to each message waveform. The disadvantage of this form of communication lies in the fact that transmitter and receiver complexity grows exponentially with message length.

Thus, system designers usually restrict system complexity by not waiting for the entire message before transmitting. Short portions of the message can be encoded systematically and transmitted sequentially as they arrive using relatively simple terminal equipment.

Fig. 2 illustrates the general approach to transmission system design considered in this paper. The input data samples, a_{mk}, m = 1, 2, . . . , M are applied at $t = kT$ to the M signal generators $A_m(\omega)$. The sequence $\{a_{mk}\}$ can be considered to be a random sequence of impulses of weight a_{mk} (where a_{mk} is in general multilevel) and spaced T seconds apart. Since there are M signal generators, symbols are be-

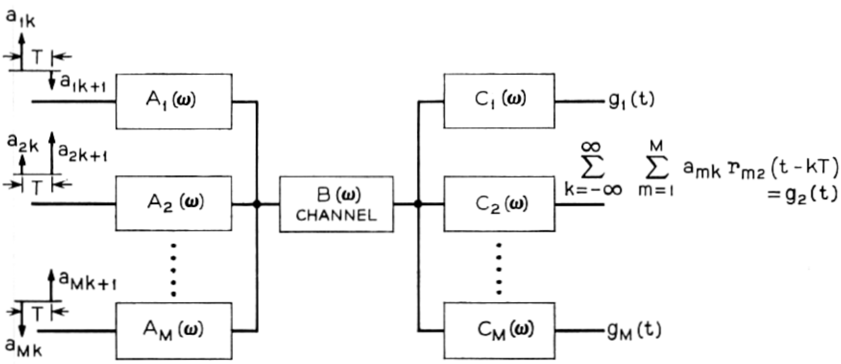

Fig. 2 — General transmission system.

ing sent at the rate M/T symbols per second. The receiver consists of M linear filters, $C_p(\omega)$, $p = 1, 2, \ldots, M$.

The channel input represents the sum of the transmitter outputs and may be written

$$\sum_{m=1}^{M} \sum_{k=-\infty}^{\infty} a_{mk} \mathfrak{a}_m(t - kT)$$

where

$$\mathfrak{a}_m(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} A_m(\omega) e^{i\omega t} \, d\omega .$$

Then, using

$$r_{mp}(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} A_m(\omega) B(\omega) C_p(\omega) e^{i\omega t} \, d\omega \tag{4}$$

the output of the p^{th} ($p = 1, 2, \ldots, M$) filter may be written

$$g_p(t) = \sum_{m=1}^{M} \sum_{k=-\infty}^{\infty} a_{mk} r_{mp}(t - kT). \tag{5a}$$

It will be assumed that any decisions will be made on the basis of the output waveform at integral multiples of T seconds. These output samples at the time $t=lT$,

$$g_{pl} = g_p(lT) = \sum_{m=1}^{M} \sum_{k=-\infty}^{\infty} a_{mk} r_{mp}(lT - kT) \tag{5b}$$

may also be written in vector notation as

$$g_{pl} = \sum_{k=-\infty}^{\infty} \vec{r}_p(lT - kT) \cdot \vec{a}_k \tag{5c}$$

where

$$\vec{r}_p(lT - kT) = [r_{1p}(lT - kT), r_{2p}(lT - kT), \cdots, r_{Mp}(lT - kT)] \tag{6}$$

and

$$\vec{a}_k = \begin{bmatrix} a_{1k} \\ a_{2k} \\ \vdots \\ a_{Mk} \end{bmatrix}. \tag{7}$$

The model described, then, represents a general pulse amplitude modulation system. For example, if the elements of $\vec{a}_k$ are random multilevel values and the transmitter and receiver are systematically chosen to be

$$A_m(\omega) = A_0(\omega - \omega_m) + A_0(-\omega - \omega_m) \tag{8a}$$

$$C_m(\omega) = C_0(\omega - \omega_m) + C_0(-\omega - \omega_m) \tag{8b}$$

one has a frequency division multiplex PAM system. Likewise, choosing

$$A_m(\omega) = A(\omega) \exp\left[-j\frac{(m-1)\omega T}{M}\right] \tag{9a}$$

$$C_m(\omega) = C(\omega) \exp\left[j\frac{(m-1)\omega T}{M}\right] \tag{9b}$$

leads to a time division multiplex PAM system.

For the model to be as general as possible, it should include the possibility of using multiple waveshapes to convey information. This can be accomplished by restricting the k^{th} data word to be

$$\vec{a}_k = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ \vdots \\ 0 \end{bmatrix} \cdots \text{ or } \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}. \tag{10}$$

Thus, the system transmits one of M possible waveforms in each time slot and includes such modulation techniques as FM, PM, pulse position or pulse duration modulation.

IV. DESIGN CRITERIA FOR THE GENERALIZED MODEL

The general design constraints for the two different interpretations of $\vec{a}_k$ are imposed by the requirement of distortionless transmission (that is, no intersymbol or interchannel interference). Each of the two cases leads to a different definition of distortionless transmission and hence to different design constraints.

4.1 *Pulse Amplitude Modulation*

(Elements of $\vec{a}_k$ are random and multilevel.)

The output of the p^{th} receiver filter at $t = lT$ is

$$g_{pl} = \sum_{k=-\infty}^{\infty} \vec{r}_p(lT - kT) \cdot \vec{a}_k . \tag{5c}$$

For distortionless transmission it is required that this output depend only upon the input value a_{pl}; that is,

$$g_{pl} = a_{pl}K_p \tag{11}$$

where K_p is a constant that depends on the p^{th} channel $A_p(\omega)B(\omega)C_p(\omega)$. This requirement constrains the time response[4]

$$r_{mp}(lT - kT) = \delta_{mp}\,\delta_{lk}K_p \tag{12}$$

where

$$\begin{aligned} \delta_{mp} &= 0 \qquad m \neq p \\ &= 1 \qquad m = p. \end{aligned}$$

Using equations 3, 4, and 12, the time domain constraint becomes the frequency domain constraint

$$\frac{1}{T}\sum_{n=-\infty}^{\infty} A_m\left(u + \frac{2n\pi}{T}\right)B\left(u + \frac{2n\pi}{T}\right)C_p\left(u + \frac{2n\pi}{T}\right) = \delta_{mp}\,K_p\,. \tag{13}$$

This is a generalized Nyquist criterion which applies to all linear PAM systems.

Notice that equation 13 represents M^2 equations which must be satisfied by

$$MT/\pi \times \text{positive frequency range of nonzero } B(\omega)$$

independent variables. Therefore, $B(\omega)$ must have a radian bandwidth of at least $M\pi/T$ for M channel distortionless transmission.

4.2 *Pulse Shape Modulation*

($\vec{a}_k$ given by equation 10.)

The definition of distortionless transmission of the previous part (equation 11) could also be applied here. However, it is possible to use a different definition with quite interesting results because of the constraint upon $\vec{a}_k$. Here, distortionless transmission will require that the output of the p^{th} filter at $t = lT$ be

$$g_{pl} = a_{pl}K_{p1} + K_{p2} \tag{14}$$

where K_{p1} and K_{p2} are constants. That is, the output of the p^{th} receiver takes on one of two values, at $t = lT$, $K_{p1} + K_{p2}$ or K_{p2} depending upon the value of a_{pl}.

This definition of distortionless transmission eases the constraints upon the various time responses. Examining

$$g_{pl} = \sum_{k=-\infty}^{\infty} \vec{r}_p(lT - kT) \cdot \vec{a}_k \tag{5c}$$

it is seen that all elements of $\vec{r}_p(lT - kT)$ must be identical but not necessarily zero for $k \neq l$; that is,

$$r_{mp}(lT - kT) = r_{qp}(lT - kT) \quad \text{all} \quad m, q. \tag{15}$$

With this condition satisfied, $g_p(lT)$ is independent of $\vec{a}_k$ for $k \neq l$ (that is, the information transmitted at times other than lT). Next it is required that all elements of $\vec{r}_p(0)$ be identical except $r_{pp}(0)$ (that is, $r_{mp}(0) = r_{qp}(0)$ $m, q \neq p$). Thus, $g_p(lT)$ will take on one value if $a_{pl} = 1$ and a different value if any other $a_{ql} = 1$ $q \neq p$. These statements may be summarized by the equation

$$r_{mp}(lT - kT) = F_{p,l-k} + \delta_{mp}\, \delta_{lk} G_p \tag{16}$$

where $F_{p,\,l-k}$ and G_p are constants which depend only on the subscripts and are independent of m.

Using (3), (4), and (16), the time domain constraints become the frequency domain constraints

$$\frac{1}{T} \sum_{n=-\infty}^{\infty} A_m\left(u + \frac{2n\pi}{T}\right) B\left(u + \frac{2n\pi}{T}\right) C_p\left(u + \frac{2n\pi}{T}\right)$$

$$= \delta_{mp}\, G_p + \sum_{l,k=-\infty}^{\infty} F_{p,l-k} e^{-ju(l-k)T} \tag{17a}$$

$$= \delta_{mp}\, G_p + F_p(u) \tag{17b}$$

recognizing that the last term is really a Fourier series expansion. Notice that there is a good bit of freedom in the design because $F_p(u)$ can be chosen arbitrarily. Alternatively, this means that the time domain response samples, $F_{p,\,l-k}$, can be arbitrarily chosen but, these samples must be the same for the response to each transmitter. Thus, because the input data has been restricted, the definition of distortionless transmission can be relaxed.

V. DESIGN CRITERIA APPLICATIONS

Let us apply the general design criteria derived in the previous section to some special cases to illustrate the principles involved. These examples include PAM, PSM, and systems in which the data vector is partly independent multilevel and partly constrained (that is, where some of the components of the data vector are unconstrained and the rest are forced to be zero). The examples clearly bring out the relationship between transmitting information with amplitude or waveform variation.

5.1 *Pulse Amplitude Modulation Systems*

5.1.1 *Linear Precoding and Decoding*

Pierce[5] has suggested the use of linear precoding and decoding matrices for data systems to improve performance in the presence of impulse noise. Fig. 3 shows a system using this concept. It differs from normal smear-desmear techniques in that there are M channels instead of just one (that is, the input data are block-encoded).

The customer data, now labeled α_{nk}, $n = 1, \ldots, N$, are applied to the precoder at $t = kT$. The transformed data a_{mk} are then applied to the input of the m^{th} transmitter. In terms of the input data, one has

$$\vec{a}_k = \mathbf{P}\vec{\alpha}_k \tag{18}$$

where $\mathbf{P}$ is the N by M $(M \geqq N)$ precoder matrix and the input data is

$$\vec{\alpha}_k = \begin{bmatrix} \alpha_{1k} \\ \alpha_{2k} \\ \vdots \\ \alpha_{Nk} \end{bmatrix}. \tag{19}$$

Similarly, the output data may be written

$$\vec{\gamma}_k = \mathbf{D}\vec{g}_k \tag{20}$$

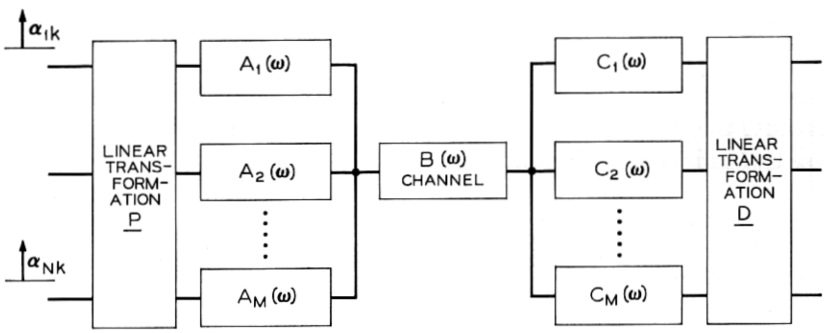

Fig. 3 — Transmission system using linear precoding and decoding transformations.

where **D** is the decoding matrix and

$$\vec{\gamma}_k = \begin{bmatrix} \gamma_{1k} \\ \gamma_{2k} \\ \vdots \\ \gamma_{Nk} \end{bmatrix} \tag{21}$$

and

$$\vec{g}_k = \begin{bmatrix} g_{1k} \\ g_{2k} \\ \vdots \\ g_{Mk} \end{bmatrix}. \tag{22}$$

It may be noted that the transmitted signal may be written

$$\sum_{m=1}^{M} \sum_{k=-\infty}^{\infty} a_{mk} \mathfrak{a}_m(t - kT)$$

or in terms of the input signal

$$\sum_{m=1}^{M} \sum_{j=1}^{N} \sum_{k=-\infty}^{\infty} P_{mj} \alpha_{jk} \mathfrak{a}_m(t - kT)$$

or

$$\sum_{k=-\infty}^{\infty} \sum_{j=1}^{N} \alpha_{jk} \sum_{m=1}^{M} P_{mj} \mathfrak{a}_m(t - kT)$$

or

$$\sum_{k=-\infty}^{\infty} \sum_{j=1}^{N} \alpha_{jk} \mathfrak{a}_j'(t - kT)$$

where

$$a_j'(t) = \sum_{m=1}^{M} P_{mj} \mathfrak{a}_m(t). \tag{23}$$

Thus, using a linear coder merely corresponds mathematically to using a different set of signal generators with no coder. It might be desirable in some cases to treat the precoder separately,[6] because it could be an easily modified device (that is, one consisting only of gain or delay variables) which could be used to combat noise, change the data rate or shift the spectrum of the signals on the channel.

As an example, consider the two precoding matrices

$$\mathbf{P}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{P}_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

used with time division multiplex transmitters (that is, serial transmission

$$A_m(\omega) = A(\omega) \exp \left[-j \frac{(m-1)\omega T}{M} \right] \Bigg).$$

Matrix $\mathbf{P}_1$ corresponds to no precoding while matrix $\mathbf{P}_2$ represents interleaving which might be effective in combating burst noise if the input data is redundant (that is, digitally encoded into blocks of length 3, in this case. Notice that interleaving is basically a digital technique for error control in burst noise. This is amply illustrated by the presence of identical values as the single nonzero element in each row and column of the matrix. For analog error control (smearing or spreading the information over several symbols) in burst noise, the elements of $\mathbf{P}$ can be any real values.

The choice of a particular precoding matrix would presumably be based upon some knowledge of the noise characteristics. The decoder can then be designed for distortionless transmission by solving $\mathbf{DP} = \mathbf{I}_{NN}$ if it is assumed that

$$\frac{1}{T} \sum_{n=-\infty}^{\infty} A_m\left(u + \frac{2n\pi}{T}\right) B\left(u + \frac{2n\pi}{T}\right) C_p\left(u + \frac{2n\pi}{T}\right) = \delta_{mp} K_p . \tag{13}$$

Similarly, $\mathbf{D}$ could be obtained by considering the transformed transmitter and receiver and solving

$$\frac{1}{T} \sum_{n=-\infty}^{\infty} A'_j\left(u + \frac{2n\pi}{T}\right) B\left(u + \frac{2n\pi}{T}\right) C'_i\left(u + \frac{2n\pi}{T}\right) = \delta_{ij} K'_i . \tag{24}$$

The transformed receiver $C'_i(\omega)$ is just

$$C'_i(\omega) = \sum_{i=1}^{N} D_{ip} C_p(\omega). \tag{25}$$

The two approaches are equivalent.

5.1.2 *A Two-Channel PAM System*

Fig. 4 shows a two-channel PAM system. All of the main features of the design constraints can be easily shown by means of this example. The four equations which must be satisfied are

$$\frac{1}{T}\sum_{n=-\infty}^{\infty} A_m\left(u + \frac{2n\pi}{T}\right)B\left(u + \frac{2n\pi}{T}\right)C_p\left(u + \frac{2n\pi}{T}\right) = \delta_{mp} K_p$$

$$m, p = 1, 2. \qquad (13)$$

It is apparent that $A_m[u + (2n\pi)/T]$ and $C_p[u + (2n\pi)/T]$ must have nonzero values for at least two values of n (two intervals of π/T bandwidth or two intervals of width $2\pi/T$ when both positive and negative frequencies are considered). Hence, the total bandwidth must be $2\pi/T$ for distortionless transmission. If the bandwidth is greater than $2\pi/T$, an infinite number of designs are possible.

A clearer idea of the implications of equation 13 can be gained by examining the impulse responses in the time domain which are shown in Fig. 5. Notice that $r_{11}(t)$ and $r_{22}(t)$ are the usual pulses required for data transmission. The crosstalk waveforms $r_{12}(t)$ and $r_{21}(t)$ are required to be zero at all $t = kT$ so that the output of either channel at $t = kT$ does not depend upon the input to the other channel. This does not mean, however, that there can be no frequency overlap between the transmitter of one channel and the receiver of the other. It does mean that the characteristics must be chosen so that

$$\frac{1}{T}\sum_{n=-\infty}^{\infty} A_m\left(u + \frac{2n\pi}{T}\right)B\left(u + \frac{2n\pi}{T}\right)C_p\left(u + \frac{2n\pi}{T}\right) = 0 \qquad m \neq p. \qquad (26)$$

Fig. 1 shows one such characteristic and Fig. 6 gives one possible design for the two-channel system which anticipates the next example.

Notice that if $A_2(\omega)$ were zero (that is, a single channel system), the second receiver could be used for a noise monitor.[7] By taking the

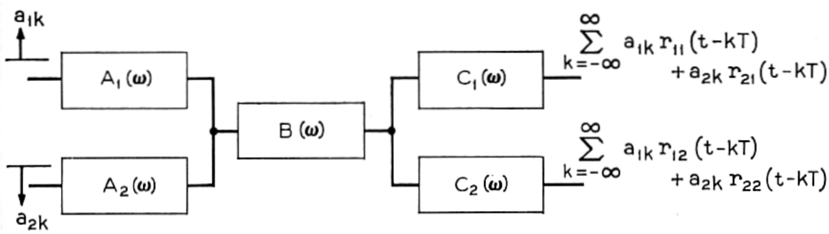

Fig. 4 — Two-channel PAM system.

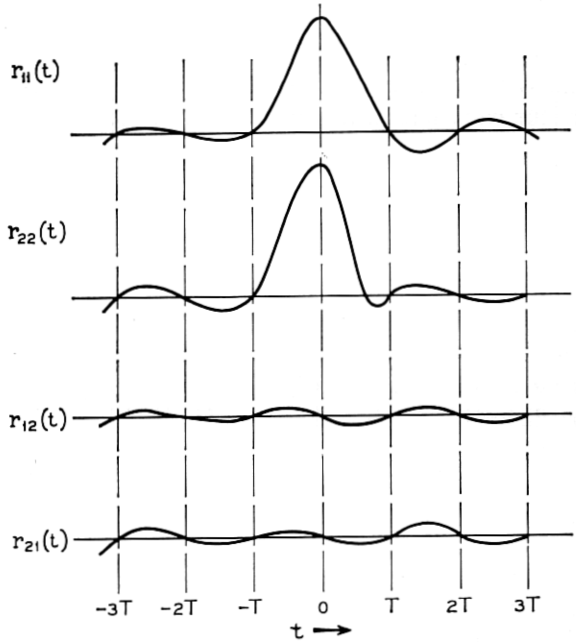

Fig. 5 — Required time domain responses for distortionless transmission.

output of this filter at kT seconds (when the noiseless component is zero), squaring, and averaging, one can get an estimate of the variance of the channel noise.

5.1.3 *Band-Limited Orthogonal Signals for Multichannel Transmission*[8]

Thus far, we have discussed only general design constraints without regard for the specific choices that a designer must make if he has available more than the minimum bandwidth (as he must). In other words, if only the minimum bandwidth were available, the designer would have no choice but to match the M^2 equations with the M^2 variables (a slight choice does arise between serial and parallel formats). However, arbitrary choices can be made when one has more than M^2 variables (bandwidth $> M\pi/T$).

Chang[8] has considered one such possibility; namely, a frequency division multiplex system in which signals at the channel output, $A_m(\omega)B(\omega)$, are orthogonal. In other words, taking $B(\omega) = 1$ for notational simplicity, Chang's signals are chosen to satisfy the time domain requirement

$$\int_{-\infty}^{\infty} \mathrm{a}_m(t)\mathrm{a}_p(t - kT)\, dt = \delta_{mp}\, \delta_{k0}\, K_p \,. \tag{27}$$

In the frequency domain this requirement becomes

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} A_m(\omega) A_p^*(\omega) e^{j\omega kT}\, d\omega = \delta_{mp}\, \delta_{k0}\, K_p \,. \tag{28a}$$

Using the technique of equations 1 through 2d, we obtain

$$\frac{1}{2\pi} \int_{-\pi/T}^{\pi/T} \sum_{n=-\infty}^{\infty} A_m\left(u + \frac{2n\pi}{T}\right) A_p^*\left(u + \frac{2n\pi}{T}\right) e^{jukT}\, du = \delta_{mp}\, \delta_{k0}\, K_p \tag{28b}$$

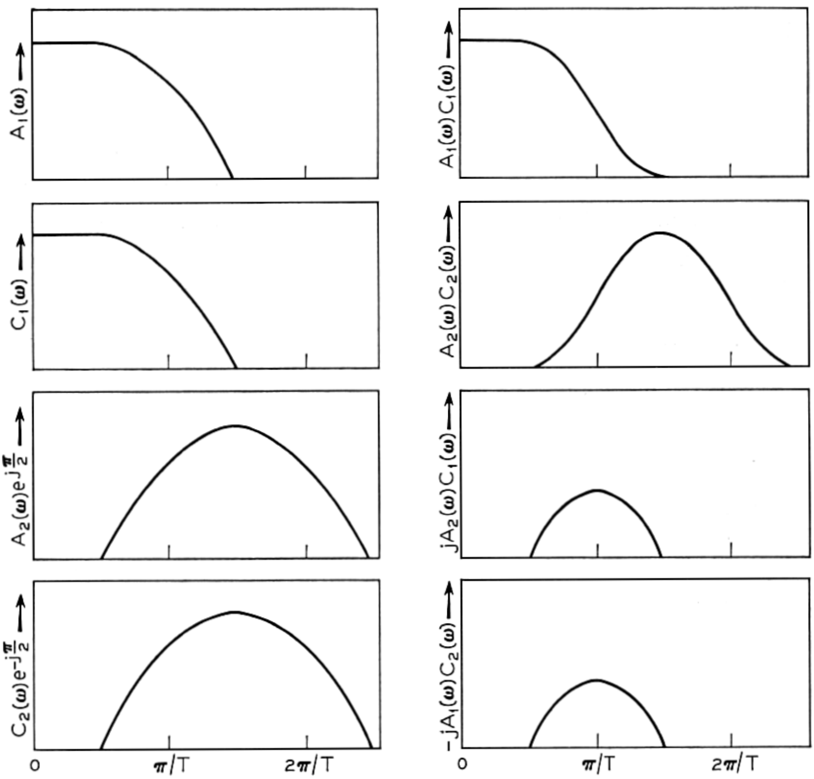

Fig. 6 — A possible two-channel system assuming $B(\omega) = 1$.

or

$$\frac{1}{T}\sum_{n=-\infty}^{\infty} A_m\left(u + \frac{2n\pi}{T}\right)A_p^*\left(u + \frac{2n\pi}{T}\right) = \delta_{mp} K_p \tag{29a}$$

for the frequency domain representation. This equation is identical to equation 13 if $C_p(\omega) = A_p^*(\omega)$ (which is best in the presence of white noise) assuming $B(\omega) = 1$. [Nonideal $B(\omega)$ can be considered by assuming that $A_m(\omega)$ is the channel output rather than transmitter output]. Thus, it is seen that the requirement of orthogonality is a special case of the general design criteria with the additional constraint that $C_p(\omega) = A_p^*(\omega)$.

In addition to this constraint, Chang chose a frequency division multiplex format with overlapping signal spectra such that

$$| A_m(\omega) | \neq 0$$

only for

$$\left(m - \frac{3}{2}\right)\frac{\pi}{T} < | \omega | < \left(m + \frac{1}{2}\right)\frac{\pi}{T}.$$

One can insert these assumptions into equation 29a and arrive at at the design conditions. It is, however, more enlightening to examine the system in the light of the previous discussion of a two-channel system. Fig. 7a shows the spectra of the three transmitters which affect the output of the m^{th} channel under the assumptions outlined above. (For concreteness of the discussion, m is even; odd m would proceed similarly.) No intersymbol interference in the m^{th} channel requires

$$\frac{1}{T}\sum_{n=-\infty}^{\infty} A_m\left(u + \frac{2n\pi}{T}\right)A_m^*\left(u + \frac{2n\pi}{T}\right) = K_m\ . \tag{29b}$$

In other words, the characteristic $|A_m(\omega)|^2$ must have vestigial symmetry about $\omega = m\pi/T$ and $(m - 1)\pi/T$.

Let us turn now to the crosstalk terms. The equations which must be satisfied for distortionless transmission are

$$\sum_{n=-\infty}^{\infty} A_{m-1}\left(u + \frac{2n\pi}{T}\right)A_m^*\left(u + \frac{2n\pi}{T}\right) = 0 \tag{30a}$$

and

$$\sum_{n=-\infty}^{\infty} A_{m+1}\left(u + \frac{2n\pi}{T}\right)A_m^*\left(u + \frac{2n\pi}{T}\right) = 0. \tag{30b}$$

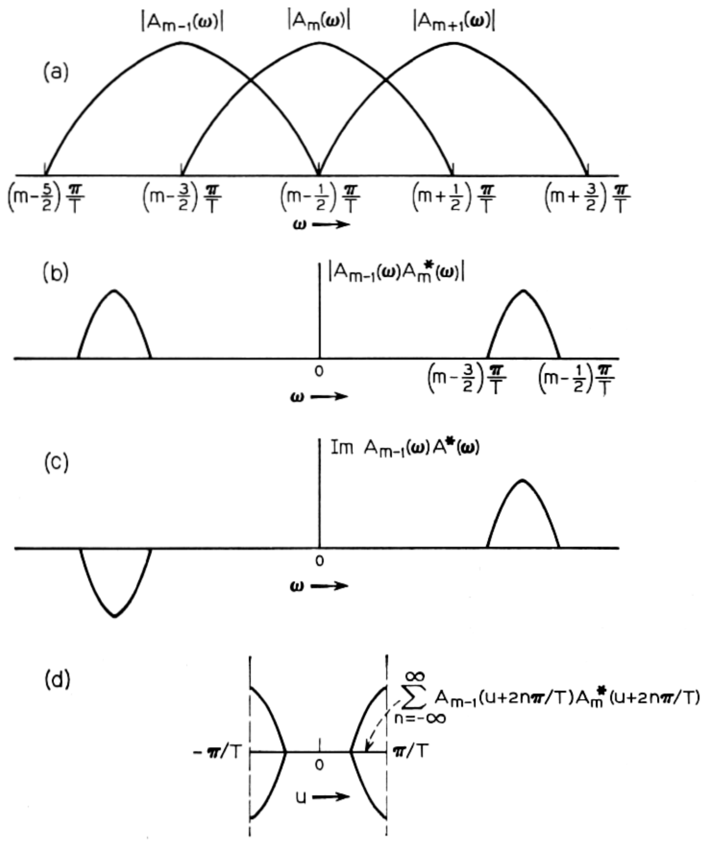

Fig. 7 — (a) Spectra of transmitters which affect m^{th} output. (b) Magnitude of $A_{m-1}(\omega)A_m{}^*(\omega)$. (c) Required $A_{m-1}(\omega)A_m{}^*(\omega)$. (d) Demonstration that above $A_{m-1}(\omega)A_m{}^*(\omega)$ satisfies constraint for periodic zeros.

Fig. 7b shows the magnitude of $A_{m-1}(\omega)A_m^*(\omega)$ which is symmetric about $|\omega| = (m - 1)\pi/T$. The only way these components can sum to zero following equation 30a is if they are imaginary as shown in Fig. 7c (with the sum given in Fig. 7d). The same argument applies to the $A_{m+1}(\omega)A_m^*(\omega)$ product and is illustrated in Fig. 8. In other words, $A_m^*(\omega)$ must be ± 90 degrees out of phase with $A_{m+1}(\omega)$ and $A_{m-1}(\omega)$ in the regions of overlap of the functions.

It is seen that the amplitude characteristic design is based upon the condition of no intersymbol interference in each channel and is based upon the usual Nyquist design. The remaining freedom in choosing

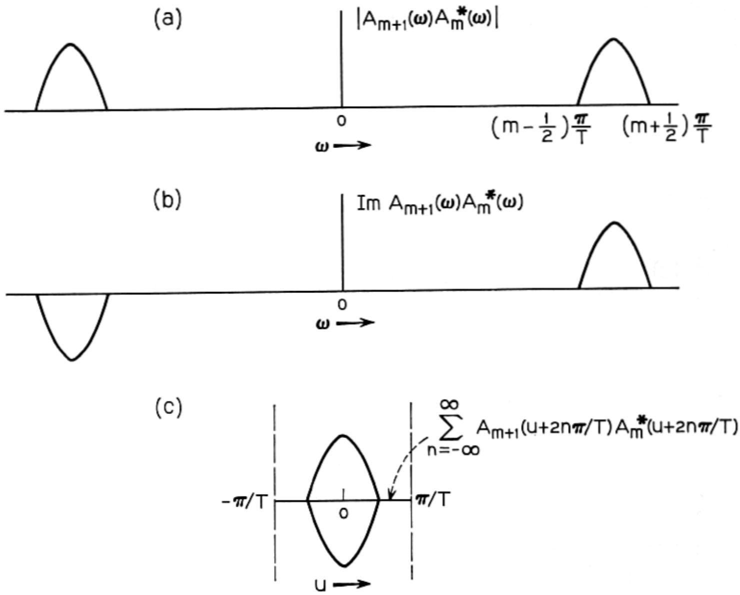

Fig. 8 — (a) Magnitude of, and (b) required $A_{m+1}(\omega)A_m^*(\omega)$. (c) Demonstration that above $A_{m+1}(\omega)A_m^*(\omega)$ satisfies constraint for periodic zeros.

the phase characteristic is then used to eliminate interchannel interference with the requirement being

$$\text{phase of } A_m(\omega) = \text{phase of } A_{m-1}(\omega) \pm 90°. \tag{31}$$

5.1.4 *Noise Monitoring*[7]

The noise monitoring feature mentioned previously can be generalized to the M channel case. The minimum bandwidth of $M\pi/T$ must be exceeded by the practical system. The bandwidth redundancy can be used for noise monitoring by adding another filter $C_{M+1}(\omega)$ at the receiver. This receiver must satisfy the equations

$$\frac{1}{T}\sum_{n=-\infty}^{\infty} A_m\left(u + \frac{2n\pi}{T}\right)B\left(u + \frac{2n\pi}{T}\right)C_{M+1}\left(u + \frac{2n\pi}{T}\right) = 0$$

$$\text{for} \quad 1 \leqq m \leqq M \tag{32}$$

and a nontrivial solution can result because of the bandwidth redundancy. Then, the impulse responses $r_{mM+1}(t)$ go through zero at all $t = kT$ and the noiseless output of the $M+1^{\text{th}}$ filter

$$\sum_{m=1}^{M} \sum_{k=-\infty}^{\infty} a_{mk} r_{mM+1}(t - kT)$$

is zero periodically, independent of the input data.

The filter output at $t = lT$ can be squared and averaged to obtain an estimate of the noise level and hence an estimate of the transmission performance. If the shape of the noise power spectrum is known, one gets a quantitative estimate of the noise power. Timing errors or poor knowledge of $B(\omega)$ can lead to the noiseless output of the $M+1^{\text{th}}$ filter being nonzero at the sample time. The indicated noise variance would be greater than the correct value, thus indicating poorer performance than the noise alone. However, timing or channel characterization errors actually do lead to poor system performance so that the monitor indication is in the right direction. Notice that this monitoring scheme is not tied to any particular choice of transmitter or receiver and is perfectly general.

5.2 *Pulse Shape Modulation Systems*

5.2.1 *A Binary PSM System*

Insight into pulse shape modulation system design constraints can, perhaps, best be gained by examining a binary system such as that shown in Fig. 9a. The equations that must be satisfied are

$$\frac{1}{T} \sum_{n=-\infty}^{\infty} A_m\left(u + \frac{2n\pi}{T}\right) B\left(u + \frac{2n\pi}{T}\right) C_p\left(u + \frac{2n\pi}{T}\right) = \delta_{mp}\, G_p + F_p(u) \tag{17b}$$

for $m, p = 1, 2$. Because it is a binary system, the receiver can be just a single filter

$$C(\omega) = C_2(\omega) - C_1(\omega) \tag{33}$$

$$\frac{1}{T} \sum_{n=-\infty}^{\infty} A_1\left(u + \frac{2n\pi}{T}\right) B\left(u + \frac{2n\pi}{T}\right) C\left(u + \frac{2n\pi}{T}\right) = -G + F(u) \tag{34a}$$

$$\frac{1}{T} \sum_{n=-\infty}^{\infty} A_2\left(u + \frac{2n\pi}{T}\right) B\left(u + \frac{2n\pi}{T}\right) C\left(u + \frac{2n\pi}{T}\right) = G + F(u) \tag{34b}$$

where it has been assumed without loss of generality that

$$G = G_1 = G_2$$

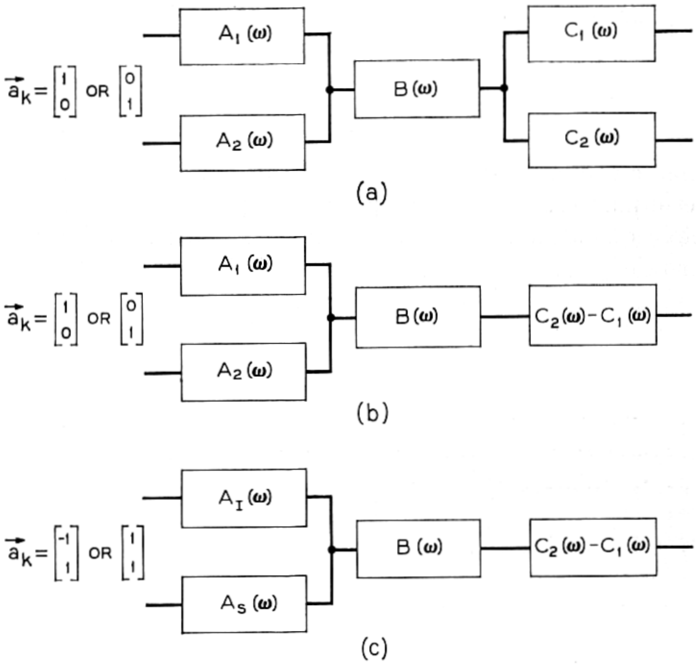

Fig. 9 — (a) Binary PSM system. (b) Modified binary PSM system. (c) Equivalent binary PSM system or PAM system with data constraints.

and where $F(u)$ is an arbitrary function of frequency

$$F(u) = F_2(u) - F_1(u) = \sum_{q=-\infty}^{\infty} f_q e^{-juqT}. \tag{35}$$

This modified system described above is shown in Fig. 9b.

Fig. 10 shows two possible time domain responses which satisfy equations 34a and b. Notice that the responses differ only at $t = 0$ and are identical to all other $t = kT$. This is the time domain implication of equations 34a and b. This corresponds to the case where two signals are chosen to produce the same intersymbol interference which was discussed briefly by Simon and Kurz.[9]

An alternate way of viewing equations 34a and b is to notice that the two transmitters can each be decomposed into two components. The information component $A_I(\omega)$ of each satisfies

$$\frac{1}{T} \sum_{n=-\infty}^{\infty} A_I\left(u + \frac{2n\pi}{T}\right) B\left(u + \frac{2n\pi}{T}\right) C\left(u + \frac{2n\pi}{T}\right) = G \tag{36}$$

or the usual Nyquist criterion, and is transmitted with an amplitude of ± 1. The steady component $A_s(\omega)$ satisfies

$$\frac{1}{T}\sum_{n=-\infty}^{\infty} A_s\left(u + \frac{2n\pi}{T}\right)B\left(u + \frac{2n\pi}{T}\right)C\left(u + \frac{2n\pi}{T}\right) = F(u) \tag{37}$$

and is sent with an amplitude of one regardless of the data stream. The transmitted waveform $A_s(\omega)$ can be anything because $F(u)$ is arbitrary. Fig. 9c shows this system, which is equivalent to the original. The corresponding data vectors are

$$\vec{a}_k = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \text{ or } \begin{bmatrix} 1 \\ 1 \end{bmatrix}. \tag{38}$$

The basic equivalence of the PAM and PSM systems is thus made explicit. The difference in the two systems is basically a noninformation bearing signal which represents an inefficient (theoretically) use of power. This point has been brought out by Bennett and Davey[10] in discussing the Sunde[11] model of a synchronous FM system.

5.2.2 *Sunde's FM Model With a Linear Receiver*

In Sunde's[11] model of a synchronous FM system, one of two phase continuous signals

$$\begin{aligned} a_1(t) &= \cos\frac{2\pi q}{T}\,t + \theta & \\ & & -\frac{T}{2} < t < \frac{T}{2} \\ a_2(t) &= -\cos\frac{2\pi(q+1)t}{T} + \theta & \\ a_1(t) &= a_2(t) = 0 & \text{elsewhere} \end{aligned}$$

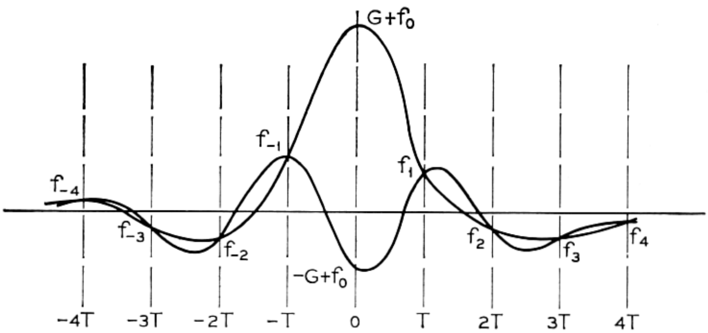

Fig. 10 — Possible time domain responses for distortionless PSM system.

is sent during each interval. The transmitter output may be written

$$\sum_{k=-\infty}^{\infty} a_{1k} \cos\left(\omega_c t - \frac{\pi}{T} t + \theta\right) - (1 - a_{1k}) \cos\left(\omega_c t + \frac{\pi}{T} + \theta\right)$$

or

$$\sin \frac{\pi}{T} t \sin (\omega_c t + \theta) + \sum_{k=-\infty}^{\infty} (2a_{1k} - 1) \cos \frac{\pi}{T} t \cos (\omega_c t + \theta)$$

where

$$\omega_c = \frac{(2q + 1)\pi}{T}.$$

This second form of the output is an explicit example of an information-bearing component (second term where $a_{1k} = 1, 0$) and a steady state component.

To achieve distortionless transmission (with a linear receiver) one must choose any linear filter which satisfies

$$\frac{1}{T} \sum_{n=-\infty}^{\infty} A_I\left(u + \frac{2n\pi}{T}\right) B\left(u + \frac{2n\pi}{T}\right) C\left(u + \frac{2n\pi}{T}\right) = G \tag{39}$$

where

$$A_I(\omega) = \int_{-T/2}^{T/2} \cos \frac{\pi}{T} t \cos (\omega_c t + \theta) e^{-i\omega t} \, dt \tag{40a}$$

$$= e^{i\theta} S(\omega - \omega_c) + e^{-i\theta} S(\omega + \omega_c) \tag{40b}$$

where

$$2S(\omega) = \frac{\sin (\omega - \pi/T)T/2}{\omega - \pi/T} + \frac{\sin (\omega + \pi/T)T/2}{\omega + \pi/T}. \tag{40c}$$

If one makes the assumption that $S(\omega + \omega_c)$ is negligible at positive frequencies, then

$$A_I(\omega) = \cos \frac{(\omega - \omega_c)T}{2} \left[\frac{-\pi/T}{(\omega - \omega_c)^2 - \pi^2/T^2}\right] e^{i\theta} \qquad \omega > 0. \tag{40d}$$

By substituting equation 40d into 39 the requirements on $B(\omega)C(\omega)$ can be found. The minimum bandwidth solutions are (neglecting constants)

$$B(\omega)C(\omega) = \frac{(\omega - \omega_c)^2 - \pi^2/T^2}{\cos (\omega - \omega_c)T/2} e^{-i\theta}$$

$$\omega_c + n\pi/T < \omega < \omega_c + (n + 1)\pi/T \quad n = -1, 0 \tag{41}$$

$$= 0 \qquad \text{elsewhere}$$

which are shown in Fig. 11a. Solutions in other regions (other values of n) are possible but require infinite gain.

Sunde's solution for the minimum bandwidth filtering before a phase derivative (nonlinear) detector is given in Fig. 11b for comparison. Notice that the linear receiver requires only half the bandwidth required by the phase derivative detector for distortionless transmission. The response of the linear filter to the steady state term is unimportant because it is deterministic and can be removed.

5.2.3 *Monitoring System Parameters*

If a second filter $C_g(\omega)$ is added to the system of Fig. 9c, one again can monitor some aspect of system performance if the equation

$$\frac{1}{T}\sum_{n=-\infty}^{\infty} A_I\left(u + \frac{2n\pi}{T}\right)B\left(u + \frac{2n\pi}{T}\right)C_g\left(u + \frac{2n\pi}{T}\right) = 0 \tag{42}$$

is satisfied. Thus, the output at the sample times will be independent of the input data. However, there will be a constant output value (excluding noise) of

$$\sum_{q=-\infty}^{\infty} f_{gq}$$

where

$$\frac{1}{T}\sum_{n=-\infty}^{\infty} A_s\left(n + \frac{2n\pi}{T}\right)B\left(u + \frac{2n\pi}{T}\right)C_g\left(u + \frac{2n\pi}{T}\right) = F_g(u) \tag{43a}$$

$$= \sum_{q=-\infty}^{\infty} f_{gq}e^{-juqT} \tag{43b}$$

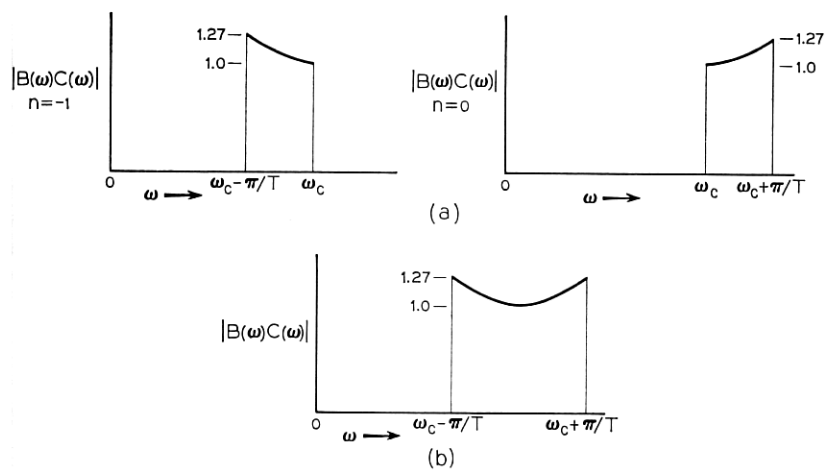

Fig. 11 — Minimum bandwidth filtering for FM system with (a) linear receiver and (b) phase derivative receiver.

because of the steady state transmitter. The total output of this filter is then a (generally) nonzero constant which depends upon the steady state transmitter and channel characteristics and the noise. It does not depend upon the data sequence or the receiver's estimate of the sequence. A change in this constant reflects a change in the transmitter or channel parameters (for example, phase or gain) and can be used to modify the receiver characteristics (such as, phase or threshold level). Thus, the noninformation part of the transmitted signal, in addition to perhaps simplifying implementation, also can be used to provide needed information to the receiver. A simple example is the reference tone for carrier recovery which in fact makes the PAM system into a PSM system.

5.3 *Pulse Amplitude Modulation System With Zero Constraints on Certain Channels*

In section 5.2 we showed that PSM could be considered as PAM with constraints on the input to certain channels. In other words, the equivalent PSM system shown in Fig. 9c contained one channel whose input was constrained to be a one at all times. Now we will discuss a system in which certain channel inputs are constrained to be zero.

Consider a four-channel PAM system and assume it to be serial; that is,

$$A_m(\omega) = A(\omega) \exp\left[-j\frac{(m-1)\omega T}{4}\right] \tag{9a}$$

$$C_m(\omega) = C(\omega) \exp\left[j\frac{(m-1)\omega T}{4}\right]. \tag{9b}$$

If the input data vector is given by

$$\vec{a}_k = \begin{bmatrix} a_{1k} \\ a_{2k} \\ a_{3k} \\ a_{4k} \end{bmatrix} \tag{44}$$

then a bandwidth of $4\pi/T$ is required. Now, under certain circumstances it might be desirable to reduce this data rate by inserting zeroes for some of the a_{mk} (that is, not transmitting anything at certain times).

Chang[6] has considered this possibility for improving performance in the presence of severe noise. In this case, some a_{mk} can be made zero and the remaining data can be transmitted with increased power (maintaining a fixed average power) to improve the noise margin.

Another purpose of this zero stuffing technique might be to shift (as well as reduce the bandwidth of) the spectrum of the transmitted signals. As a trivial example, the data vector

$$\vec{a}_k = \begin{bmatrix} a_{1k} \\ 0 \\ a_{3k} \\ 0 \end{bmatrix} \tag{45}$$

could be transmitted with a flat spectrum over either the region

$$0 < |\omega| < 2\pi/T \quad \text{or} \quad 2\pi/T < |\omega| < 4\pi/T$$

and zero elsewhere. As a nontrivial example, consider a generalization of a signaling system, invented by Bennett and Feldman,[12] to prevent intersymbol and interchannel interference in multiplex transmission. The original system has been described very briefly by Sunde.[13] Here, the generalized system can be approached by writing the data vector

$$\vec{a}_k = \begin{bmatrix} a_{1k} \\ a_{2k} \\ 0 \\ 0 \end{bmatrix} \tag{46}$$

where only the outputs of the first two receivers must be examined. With the assumption (for simplicity) that $B(\omega) = 1$ the constraining equations become

$$\frac{1}{T} \sum_{n=-\infty}^{\infty} A\left(u + \frac{2n\pi}{T}\right) C\left(u + \frac{2n\pi}{T}\right) = r_0 \tag{47}$$

and

$$\frac{1}{T} \sum_{n=-\infty}^{\infty} A\left(u + \frac{2n\pi}{T}\right) C\left(u + \frac{2n\pi}{T}\right) \exp \pm j\left(u + \frac{2n\pi}{T}\right) \frac{T}{4} = 0. \tag{48a}$$

Recognizing that exp ($\pm ju\ T/4$) is a nonzero term which can be removed, 48a becomes

$$\frac{1}{T}\sum_{n=-\infty}^{\infty} A\left(u + \frac{2n\pi}{T}\right)C\left(u + \frac{2n\pi}{T}\right) \exp \pm j\frac{n\pi}{2} = 0. \qquad \text{(48b)}$$

Fig. 12a illustrates the type of characteristic $A(\omega)C(\omega)$ which satisfies the constraints. (There are others of larger bandwidth which also will satisfy the constraints.) If the characteristic is limited to $|\omega| < 4\pi/T$ and zero elsewhere, it has symmetry about $\omega = 2\pi/T$ and vestigial symmetry about $|\omega| = \pi/T$ and $3\pi/T$. It can easily be verified that the equations are satisfied when one uses the value of the multiplying factor exp $\pm jn\pi/2$ which is shown in each region.

Notice that a response which is flat from $\pi/T < |\omega| < 3\pi/T$ and zero elsewhere satisfies the equations and represents the minimum bandwidth approach to this scheme. The time response one obtains at the receiver using this technique is illustrated in Fig. 12b. It is constrained to be zero at all $t = kT$ except $k = 0$ and $\pm(4q - 2)$ for all q and can be used for transmission, as explained, without distortion.

The advantage of this particular scheme is that it represents a baseband technique for shifting the transmission spectrum without modulation merely by inserting zeros into the data stream. It appears particularly attractive for placement within a voice channel (for example, 200Hz-3KHz) as Fig. 13 shows. Here no modulation has been required, the energy is concentrated in the center of the band and

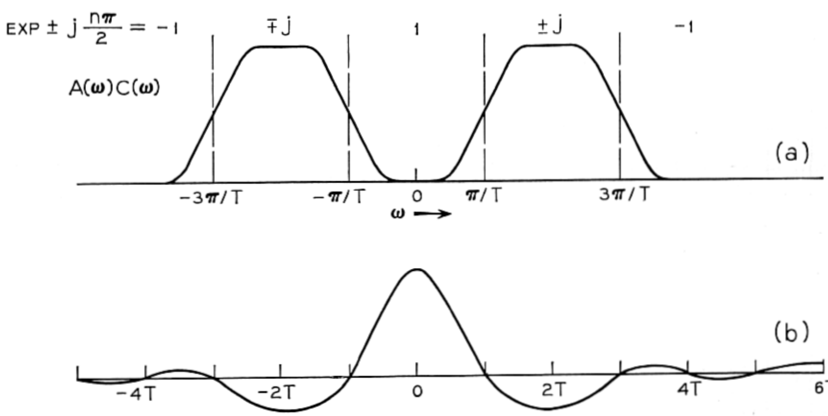

Fig. 12 — (a) Frequency characteristic, and (b) time response, for distortionless transmission with zero stuffing.

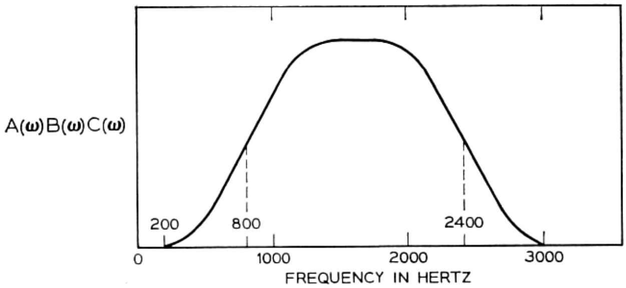

Fig. 13 — Zero stuffing spectrum for voiceband transmission.

one could obtain symbol rates of 3200 symbols per second with easily realized filtering. The primary disadvantage would be increased sensitivity to timing errors.

VI. CONCLUDING COMMENTS

The thesis of this paper has been that all linear data system designs are based on the modified Poisson sum formula

$$\frac{1}{T}\sum_{n=-\infty}^{\infty} R\left(u+\frac{2n\pi}{T}\right)=\sum_{q=-\infty}^{\infty} r_q e^{-juqT} \tag{3}$$

which relates the time domain samples to the frequency domain constraints. Various types of systems which a designer may choose require a variety of constraints on the time samples r_q. These values of r_q, which depend upon the type of system chosen, then specify the frequency domain requirements.

Section 5 gave a sampling of the range of systems which can be designed using equation 3. That section certainly does not exhaust the possibilities and we hope that it does not limit the reader's imagination. Most of the examples (as well as most real systems) assume systematic choice of transmitted signals; usually related by integral time or frequency shift. There may, however, be potential gain in considering nonsystematic transmitters and receivers. This may easily be done using equation 3. The last case examined, that of spectrum shifting by adding zeros, is just such a nonsystematic function when viewed from a serial transmission point of view.

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