Performance Degradation by Postdetector Nonlinearities

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This article gives the performance degradation found by using a computer program to calculate the effects of various processing techniques applied after envelope detection of narrowband Gaussian noise plus a CW signal. Nonlinear processes that we studied are: suppression of low output levels, hard limiting of high output levels, restricted dynamic range of output level, and quantizing the detector output.

By modifying the program, we produced performance curves for systems in which a steady CW component was present in addition to possible signals. We also adapted the program to generate performance curves for systems using a square-law detector so that a comparison could be made with the results obtained for an envelope detector. We found that the presence of a CW component, and all the nonlinear processes applied after envelope detection, caused a loss of sensitivity in detecting small CW signals. Quantizing even in as few as 8 levels, however, caused very little additional degradation.

I. INTRODUCTION

A computer program was developed that would draw receiver operating characteristic curves for a system using an envelope detector to search for CW signals in narrowband Gaussian noise.[1] The program computed the probability that the detector output would exceed a chosen threshold under two circumstances:

(*i*) No signals were present; this result gives the probability of false alarm, P_{FA}.

(*ii*) Signals at various S/N were present; this result gives the probability of detection, P_D.

S/N is the ratio of the CW signal power to the noise power ac-

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companying it in the specified narrow band. These probabilities were calculated using formulas which described the appropriate distributions of the detector output.

The program also averaged independent samples of the detector output, and produced appropriate receiver operating characteristic curves by deriving a Gram-Charlier A series for the distribution of the sample average.

When the program was adapted to take into account the modifications of the detector output caused by various subsequent nonlinear processes, receiver operating characteristic curves were produced for systems in which such processes occurred. The performance degradation was determined as the change in S/N that would be required to make the detectability of a signal, for given probability of false alarm, the same as without the nonlinear process.

II. DISCUSSION

Since some types of nonlinear processing greatly reduce the labor of handling large amounts of data, and others are unavoidable, or their effects are costly to minimize, it is useful to know the penalties on performance that are incurred by their presence.

The types of nonlinear process studied are: suppressing low output levels, hard limiting of high output levels, restricting dynamic range of the output level, and quantizing the output of the detector.

Suppressing low output levels occurs in some kinds of recording equipment where the output must reach a minimum level before a record can be made. High output limiting always occurs because the power handling capacity of physical equipment is limited. Dynamic range is restricted when the first two processes occur together; this also represents a commonly used technique in which a fluctuating output is converted to a binary waveform with respect to a chosen threshold. Quantizing is particularly useful when a digital computer is used to implement the statistical analysis. With this application in mind quantizing was evaluated in combination with restricted dynamic range.

For simplicity in programming, we assumed that the unit used to process the output of the envelope detector had an appropriate transfer characteristic from the set shown in Fig. 1. It can be seen that over part of its operating range this unit has a linear characteristic. Quantizing would cause the slope of the linear region in Fig. 1(c) to be replaced by a staircase.

In addition to the changes needed to allow investigation of the

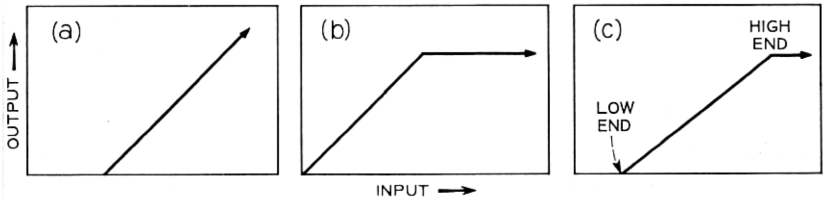


Fig. 1 — Transfer characteristics of post detector processor. (a) Suppression of low levels, (b) hard limiting of high levels, (c) restricted dynamic range.

above processing techniques, we made alterations in the program so that it would produce receiver operating characteristic curves for systems using a square-law detector, and for systems in which an unwanted CW component occurred in the specified narrow band in addition to the one representing a signal.

The latter case is typical of situations where a large, relatively stable, CW component tends to mask the presence of a smaller stable signal in the resolved band of a spectrum analyser. The results show how effective in such cases is a decision process similar to that described in Ref. 1, for detecting a small signal.

The distortions produced by some of these processes on the detector output distributions are so radical that many samples must be averaged to get an adequate Gram-Charlier representation even using up to 31 moments, the limit available in the program. Consequently a large number (8192) of samples were averaged before deriving the Gram-Charlier series from which the receiver operating characteristic curves could be produced. We made some checks where it was possible to produce curves for fewer samples averaged. We marked these on the figures; they show only slight deviations from the results obtained by averaging 8192 samples.

Since the normal distribution becomes a satisfactory approximation as the number of samples averaged grows large it may be inferred that the degradation caused by nonlinear processes could be estimated by averaging enough samples and using the normal approximation. For some of the processes described here the receiver operating characteristic curves for averaging 8192 samples showed perceptible variation from the results obtained by a normal approximation at P_{FA} values less than 10^{-4}. Thus, to ensure the validity of a normal approximation it might be necessary to average a very large number of samples indeed.

When the results for one of the simulated nonlinear processes were plotted on the appropriate chart of Ref. 1, the variation of the degradation from the average was never found greater than 10 per cent and usually within a few per cent. Consequently the average degradation shown in the accompanying figures represents a good approximation within the range covered by the appropriate chart of Ref. 1.

Fig. 2 shows the degradation in performance when low output levels are suppressed, and when high levels are hard-limited. The position at which the levels are truncated is given along the abscissa as a multiple of the mean value of the ideal detector output when only noise is present (Rayleigh mean). The ordinate gives the change in S/N required with the distorting process to give the same performance as can be achieved without it.

Fig. 3 shows the degradation sustained when the dynamic range factor of the detector output is 10 (solid line) and 2 (broken line). The dynamic range factor is the ratio of the high end of the linear range to the low end for a transfer characteristic like that in Fig. 1(c). Fig. 3 also gives the degradation sustained when the output is virtually converted to a binary signal by limiting the dynamic range factor to 1.001 (dotted line). The lower truncation point is shown on the abscissa and the ordinate gives the degradation in dB.

III. QUANTIZING

When a digital computer is used to implement the statistical analysis of the detector output, it is important to know the maximum number of bits that need to be used to encode the output level. Three

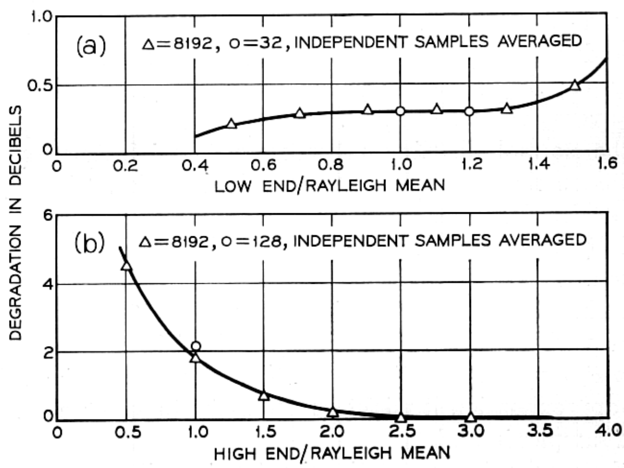


Fig. 2 — Degradation caused by (a) suppressing low output levels and (b) hard limiting high output levels.

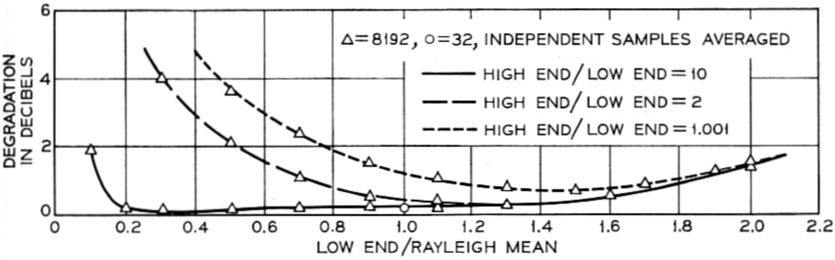


Fig. 3 — Degradation resulting from limited dynamic range.

bits allows coding into 8 levels, and four bits allows coding into 16 levels, and so on. If three bits are used, for example, the range of possible detector outputs must be divided into 8 regions. The highest region will include all levels that would equal or exceed the high end of a characteristic such as that in Fig. 1c, and the lowest region will include all levels below the low end. The remaining six regions will be between the two truncation points. The computer simulation was carried out by dividing the detector output in this way and assigning the value of the lower limit of any region to all the values in it less than the upper limit.

From Fig. 3 it can be seen that if a dynamic range factor of 10 is used and the low end is placed at three tenths of the Rayleigh mean, very little degradation in performance is suffered. When this range was quantized into 6 equal steps it was found that the total degradation was only about 0.01 dB more than the value shown in Fig. 3.

Two other ways of dividing the range into 6 unequal steps were tried, but the results were poorer than that for equal steps. In the first of these the range between the low end and high end was divided into 6 intervals in such a way that the change in probability density was the same between each. The degradation in performance was about 0.1 dB greater than that using 6 equal steps. The other way of quantizing the range divided it into 6 intervals of constant change in cumulative probability. The degradation in this case was 0.39 dB greater than that using 6 equal steps.

IV. CW INTRUDER

Fig. 4 shows that the system performance falls off rapidly owing to the presence of a CW component as the level of such a component increases from −10 dB with respect to the noise in the narrow band. The distribution curves corresponding to the no-signal hypothesis in this case were those for Gaussian noise plus a CW component at the

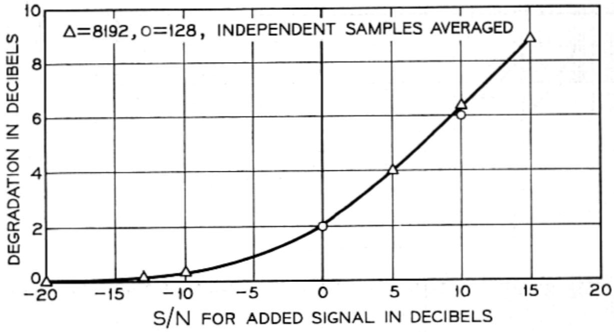
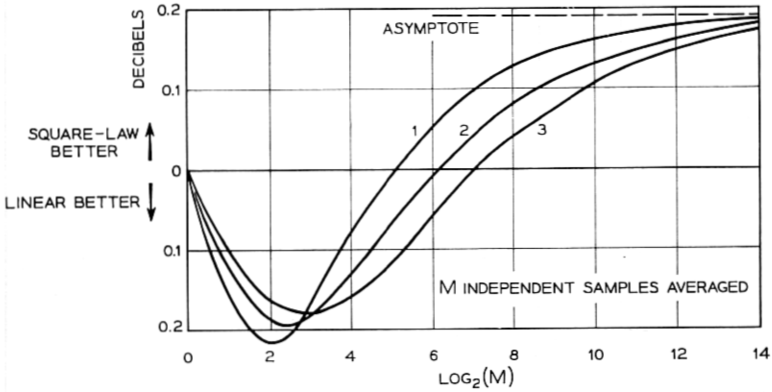


Fig. 4 — Degradation resulting from added CW signal.

S/N specified on the abscissa. The curves corresponding to the hypothesis that a signal was present were those for Gaussian noise plus a resultant sine wave component. The power of the resultant equalled the power of the small signal plus the power of a large CW component at the level specified on the abscissa. The results thus represent the average for all possible phase relationships between the CW component and small signal assuming that all phase values are equally likely, and many samples of the detector output are averaged.

V. SQUARE-LAW DETECTOR

It was found that satisfactory Gram-Charlier series approximations to the output distribution of a square-law detector could be produced only when a few hundred or more samples were averaged. Consequently subprograms were written to compute values for Chi-square and noncentral Chi-square distributions over the desired ranges of S/N and samples averaged. Using these and the results reported in Ref. 1 it was possible to produce the curves shown in Figs. 5, 6, and 7. which enable the receiver operating characteristic curves of Ref. 1 to be used to estimate the performance for a square-law detector with quite good accuracy. Fig. 5 compares the performance of linear and square-law detectors at a false-alarm probability of 10^{-6} and three different detection probabilities when the number of samples averaged ranges from 1 to 8192. The approximate S/N corresponding to a P_D, P_{FA} pair is also given over the range of samples averaged. Curve 1 in Figs. 6 and 7 compares the detector performance for the same S/N as curve 1 in Fig. 5, but when the P_{FA} is 5×10^{-4}, and 0.09, respectively, and curve 2 in Figs. 6 and 7 do the same for the S/N that applies to curve 2 in Fig. 5.



CURVE 1	6.4	−2	−8	−14	*(0.01)
CURVE 2	11	2	−5	−11	*(0.5)
CURVE 3	14.6	5	−2.5	−9	*(0.995)

Fig. 5 — Comparison of linear and square-law detectors. S/N (dB) in filter bandwidth, for $P_D = (*)$, $P_{FA} = 10^{-6}$.

VI. CONCLUSION

We have given curves which show that the sensitivity for detecting small signals using an envelope detector is degraded when any of several common kinds of nonlinear processes occur between the detector output and the averager used prior to the decision threshold. When only one sample of the detector output is used to form a decision, the system will be oblivious to the presence of the nonlinear process as long as the decision threshold lies within the linear range.

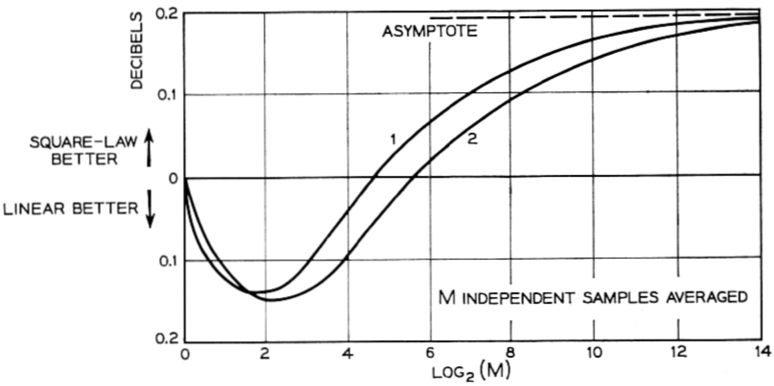


Fig. 6 — Comparison of linear and square-law detectors, $P_{FA} = 5.10^{-4}$.

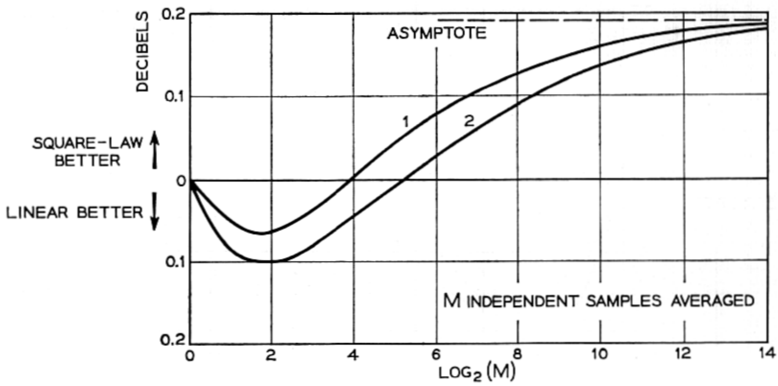


Fig. 7 — Comparison of linear and square-law detectors, $P_{FA} = 0.09$.

The system performance in such a case is thus restricted to threshold variations within this range.

It is interesting that J. V. Harrington has analyzed the detection of repeated signals in noise using binary integration.[2] The system he assumed corresponds closely to the one discussed here for which Fig. 3 shows the results. Harrington deduced that the optimum position for the quantizing threshold was about 1.44 times the Rayleigh mean, in which case binary integration would give results 0.77 dB poorer than if ideal processing with a linear detector had been used. These numbers are in excellent agreement with the minimum degradation condition shown in Fig. 3.

We have shown that the presence of an unwanted CW signal, even −10 dB with respect to the narrow-band noise, causes some degradation of the sensitivity for detecting small CW signals, and the degradation increases rapidly as the level of the unwanted component rises from there.

Computation shows that there is little performance difference to be expected from the use of envelope or square-law detectors. Envelope detectors would thus seem preferable in view of their greater simplicity, especially when a considerable dynamic range is to be covered.

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