Some Considerations of Broadband Noise Performance of Optical Heterodyne Receivers

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We derive an explicit expression in this paper for the spot noise factor of a perfectly aligned optical heterodyne receiver consisting of a semiconductor photodiode followed by an IF amplifier. We show that this noise factor F_R *, which is a function of the admittance of the diode, varies as a function of the modulation frequency. We obtain constraints imposed by the photodiode on the broadband noise performance of the optical receiver for any arbitrary lossless interstage network. The integral form of the constraint shows that the noise factor* F_R *cannot be made equal to its optimum value* F_{RO} *over any nonzero band of frequencies. We give explicit expressions for the amount of tolerance of broadband noise performance obtained with lossless interstage networks. We show that for certain types of approximations, and for a certain transistor IF amplifier usually used in practice, the interstage network which achieves broadband signal performance for the receiver also obtains broadband noise performance. The theory of broadband noise performance we present for optical heterodyne receivers can also be applied to the study of broadband noise performance of other linear systems normally encountered in practice.*

I. INTRODUCTION

Semiconductor photodiodes like Schottky barrier diodes or conventional p-n or p-i-n diodes are increasingly being used for detection in optical heterodyne (double detection) receivers.[1–12] They normally are fast and efficient, converting up to 70 per cent of the photons of the light beam into photoelectric current.[11] Because of the intensity of the light beam, almost all photodiodes used in optical detection give an output proportional to the intensity of light.[1] However, this output is normally so small that further amplification is required, and so any practical receiver consists of a photodiode followed by a high-gain low-noise IF amplifier.

We have already considered in detail the signal performance of such receivers.[13] In this paper we shall deal only with the noise performance of the receiver. The output of the diode is usually corrupted by noise generated within the diode and elsewhere in the system. We discuss briefly the characteristics of the photodiode in Section II and give its equivalent circuit. In Section III we discuss the noise performance of the IF amplifier and show that its noise factor is a function of its source admittance.[14] We show that the noise factor F_R of the optical receiver is a function of frequency in spite of the fact that the IF amplifier has a broadband noise performance characteristic.

In Section IV we discuss the role of the lossless interstage network in achieving broadband noise performance of the optical receiver and show that it is impossible to make the noise factor F_R equal to its optimum value F_{RO} over any nonzero band of frequencies.

Section V shows that Butterworth and Chebyshev approximations to F_R are realizable, and we obtain the tolerance of broadband noise performance for these approximations. We show that, for the photodiodes normally encountered in practice, this tolerance ϵ^2 is a monotonically increasing function of the complexity of the interstage network but decreasing for Chebyshev approximations.

We show in Section VI that to obtain broadband signal and noise performance characteristics from the optical receiver two separate lossless interstage networks are necessary. However, we also show that for certain types of approximations and for a certain transistor IF amplifier, the interstage network which achieves broadband signal performance also obtains broadband noise performance for the optical receiver.

The theory of broadband noise performance presented in this paper for an optical heterodyne receiver can also be applied to obtain broadband noise performance of other linear systems normally encountered in practice.

II. AVAILABLE SIGNAL AND NOISE OUTPUT POWERS

Fig. 1 shows the optical heterodyne receiver that we discuss. It consists of a photodiode followed by a lossless interstage network, and an IF amplifier of center frequency Ω_o and a semibandwidth W.† The

† The amplifier may, depending on the frequency of modulation, use vacuum tubes, transistors, masers, parametric amplifiers, tunnel diodes, or other active devices.

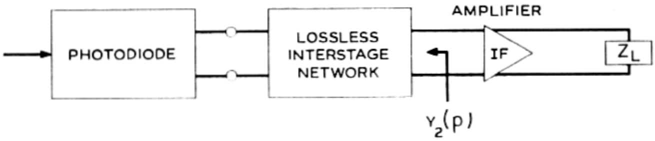

Fig. 1 — A double detection optical receiver. The input to the photodetector is the sum of the local oscillator beam and the incoming signal beam.

geometrical center frequency ω_o of the IF amplifier is defined as

$$\omega_o = \{(\Omega_o - W)(\Omega_o + W)\}^{1/2}. \tag{1}$$

The available gain G_o of the IF amplifier and its optimum noise factor F_o are assumed to be independent of the frequency ω for $\Omega_o - W \leqq \omega \leqq \Omega_o + W$.‡

In this paper we shall not consider any effects on the noise performance of the optical receiver of beam misalignment, nonuniformity of the surface of the diode, or distortion from transmission through a nonhomogeneous atmosphere.[20] Because background noise has been shown to be of almost negligible consideration, we shall assume that this noise has no effect on the broadband noise performance of the optical receiver.[21, 22]

The diode is normally so arranged that the junction or portions of it close to the junction are illuminated by the sum of the local oscillator beam and the incoming signal beam. The electron-hole pairs thus created by the incoming photons give rise to a small signal current.[1] In operation, a reverse bias V_o is put on the diode, and the characteristics of the device[12] for small excursions around V_o are that of a signal current generator I_s, and a direct current generator I_o, in parallel with a capacitance $C(V_o)$, and this combination in series with a parasitic series resistance $R(V_o)$.

The time-average current I_o is caused both by the time-average illumination and by the electrons and holes that are generated at or near the junction. The signal current I_s is caused by that portion of the illumination at or near the signal frequency of interest. In general, $C(V_o)$ and $R(V_o)$ are functions of the bias voltage. Assuming that, in practical cases, the excursions around the bias point are small, we shall henceforth assume that $C(V_o)$ is a constant capacitance C, and the series resistance $R(V_o)$ is a constant resistance R.

‡ Since G_o and F_o are real and even functions of ω, we shall only consider $\omega \geq 0$. The case in which G_{of} and B_{of} are functions of frequency ω is very complicated and will not be discussed in this paper.

To account for any thermal noise generated in the photodiode, we use an equivalent noise-voltage generator e_n with mean-squared value†

$$\overline{|e_n|^2} = 4kT_dR\,\Delta f, \tag{2}$$

where T_d is the temperature of the diode, k is Boltzmann's constant, and Δf is the spot frequency band about which we are concerned. In addition, there is another source of noise, shot noise, present in the photodiode. This can be accounted for[12] by placing in parallel with I_s and I_o a shot noise current generator i_n with mean-squared value

$$\overline{|i_n|^2} = 2qI_o\,|f(\omega\tau)|^2\,\Delta f, \tag{3}$$

where q is the electronic charge, and $f(\omega\tau)$ is a transit time reduction factor, τ being some effective transit time. We assume in this paper that $f(\omega\tau)$ which always satisfies the inequality

$$|f(\omega\tau)| \leqq 1, \tag{4}$$

can be considered independent of ω in $\Omega_o - W \leqq \omega \leqq \Omega_o + W$, and that

$$|f(\omega\tau)| = 1. \tag{5}$$

The equivalent circuit of the photodiode which describes its terminal signal and noise characteristics is shown in Fig. 2. This equivalent

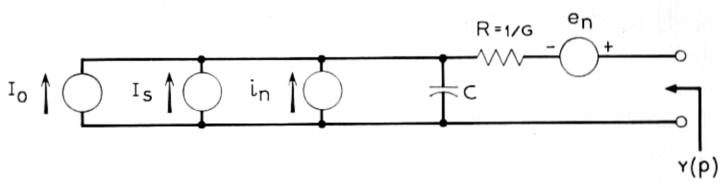

Fig. 2—Equivalent circuit of photodiode. The physical sources of noise that are present in the diode are also shown. The conductance G_p which appears in parallel with C is usually so small (around 10^{-7} mho) that it can be neglected for all practical purposes if $\omega \gg 0$.

circuit very well describes the behavior of the diode provided the lowest frequency of the signal occurring in the system is very far from zero, or $\omega \gg 0$.

The peak photoelectric current I_s, and dc current I_o for a double detection optical receiver can be shown to be given by[23,24]

$$I_s = \frac{2\eta q}{h\nu}\sqrt{P_oP_s}\,, \tag{6}$$

† The horizontal bar denotes an average.

and

$$I_o = \frac{\eta q}{h\nu}(P_o + P_s), \tag{7}$$

where η is the quantum efficiency of the photodiode,† h is Planck's constant, ν is the optical frequency, P_s is the signal power, and P_o is the local oscillator power.‡

The available signal and noise powers are easily determined from Fig. 2. The signal available power S_{pd} and noise available power N_{pd} can be written as§

$$S_{pd} = \frac{|I_s|^2}{8\omega^2 C^2 R}, \tag{8}$$

and

$$N_{pd} = \frac{\overline{|e_n|^2}}{4R} + \frac{\overline{|i_n|^2}}{4\omega^2 C^2 R}. \tag{9}$$

An important quantity that characterizes the noise performance of the photodiode is its signal-to-noise ratio S_{pd}/N_{pd}. According to equations 2, 3, and 5 through 9, this is given by¶

$$\frac{S_{pd}}{N_{pd}} = \left(\frac{\eta q}{h\nu}\right)^2 \frac{P_o P_s}{2kT_d G\left(\frac{\omega}{\omega_c}\right)^2 \Delta f + q\left(\frac{\eta q}{h\nu}\right) P_o \, \Delta f}, \tag{10}$$

where

$$\omega_c = \frac{1}{RC}. \tag{11}$$

The signal-to-noise ratio at the input to the diode will be defined as the best possible signal-to-noise ratio which an ideal detector could

† A quantum efficiency of greater than 70% has been obtained [11] for Schottky barrier photodiodes.

‡ In practice, a fraction $k \leqq 1$ of incident photons are absorbed in the active region of the diode. To account for this effect, I_s and I_o are usually multiplied by a factor k. We assume that $k = 1$. I_s is also usually multiplied by a reduction factor similar to the shot-noise function, $f(\omega\tau)$, determined by the signal frequency, optical wavelength, and device construction. We assume that this factor is unity.

To account for any mismatch between the local oscillator beam and signal beam, I_s and I_o are also multiplied by a beam matching factor β where $\beta \leqq 1$. We assume that $\beta = 1$.

§ We can argue from the physics of the diode that the shot noise source and thermal noise source shown in Fig. 2 are uncorrelated.

¶ We can assume that $P_o \gg P_s$, so that $P_o + P_s \approx P_o$.

achieve. This can be shown to be given by†

$$(S/N)_{in} = \frac{P_s}{h\nu \, \Delta f}. \tag{12}$$

III. IF AMPLIFIER NOISE FACTOR

The terminal characteristics of any IF amplifier used in the optical receiver (see Figs. 3 and 4) can normally be described by

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} + \begin{bmatrix} n_{i1} \\ n_{i2} \end{bmatrix} \tag{13}$$

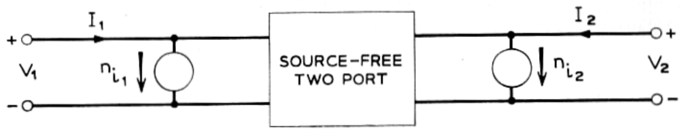

Fig. 3 — Separation of twoport with internal noise sources into a source-free twoport.

or

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} + \begin{bmatrix} n_v \\ n_i \end{bmatrix} \tag{14}$$

where n_{i1} and n_{i2}, or n_v and n_i characterize all physical sources of noise present in the IF amplifier.[14] we assume that the IF amplifier has ideal broadband signal and noise performance characteristics and that the amplifier remains stable for all linear passive input and output terminations.[26–28]

By definition, the spot noise factor at a specified frequency of any linear twoport network (such as an IF amplifier) is given by the ratio of the total output noise power per unit bandwidth exchangeable‡ at the output port to that portion of that power which is engendered by the input termination at the standard temperature T_o.[14] To derive

† Ref. 25 shows that it is impossible to measure amplitude and phase of an incoming optical signal with a better signal-to-noise ratio than given by equation 12.

‡ Exchangeable power, exchangeable gain, etc. coincide[29] with available power and available gain when the output impedance of the amplifier is positive-real. They are the logical generalizations of the available power and available gain when the output impedance has a negative-real part. Since the amplifier is assumed to be absolutely stable, we may substitute the word "available" for the word "exchangeable" wherever it appears in this paper.

the noise factor of the IF amplifier we are considering, let us connect the amplifier to a statistical source comprising an internal admittance Y_s, and a noise current generator I_{ns} (see Fig. 5). It can be shown easily that the noise factor F is given by

$$F = 1 + \frac{\overline{| n_i + n_v Y_s |^2}}{\overline{| I_{ns} |^2}}. \tag{15}$$

We notice that the mean-square source noise current is related to the source conductance G_s by the Nyquist formula

$$\overline{| I_{ns} |^2} = 4kT_o G_s \, \Delta f. \tag{16}$$

Also, we can express the noise voltage fluctuation $\overline{| n_v |^2}$ in terms of an equivalent noise resistance R_n as

$$\overline{| n_v |^2} = 4kT_o R_n \, \Delta f \tag{17}$$

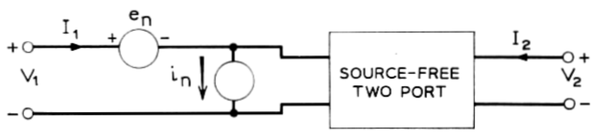

Fig. 4 — The Rothe-Dahlke noise model for a linear twoport network.

and the noise current fluctuation $\overline{| n_i |^2}$ can be expressed in terms of an equivalent noise conductance G_u where

$$\overline{| n_i |^2} = 4kT_o G_u \, \Delta f. \tag{18}$$

Let us also write

$$\overline{n_v n_i^*} = 4kT_o \rho \sqrt{R_n G_u} \, \Delta f, \tag{19}$$

where ρ is a complex number. It can be shown that

$$| \rho | \leqq 1. \tag{20}$$

From equations 15 through 19, the formula for the noise factor becomes§

$$F = 1 + \frac{G_u + 2\sqrt{R_n G_u} \, \{G_s \operatorname{Re} \rho - B_s \operatorname{Im} \rho\} + (G_s^2 + B_s^2) R_n}{G_s}. \tag{21}$$

As we can see from equation 21, the noise factor F is a function of the source conductance G_s and also of its susceptance B_s. We can

§ Re a and Im a denote the real and imaginary parts, respectively, of the complex number a.

show that F attains its optimum value¶

$$F_o = 1 + 2\sqrt{R_n G_u}\,\{\mathrm{Re}\,\rho + [1 - (\mathrm{Im}\,\rho)^2]^{\frac{1}{2}}\} \tag{22}$$

for a certain source admittance $Y_{of} = G_{of} + jB_{of}$ where

$$G_{of} = \frac{\sqrt{R_n G_u}}{R_n}\,\{1 - (\mathrm{Im}\,\rho)^2\}^{\frac{1}{2}} \tag{23}$$

and

$$B_{of} = \frac{\sqrt{R_n G_u}}{R_n}\,\mathrm{Im}\,\rho. \tag{24}$$

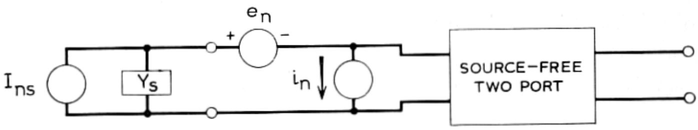

Fig. 5 — Network for noise factor computation.

Using equations 21 through 24 (see Ref. 14), we can show that F can be written in the form†

$$F = F_o + \frac{R_n}{G_s}\,|\,Y_s - Y_{of}\,|^2, \tag{25}$$

where the value of R_n is as given in equation 17.

Let $Y_2(p)$ be the admittance of the photodiode as seen by the IF amplifier (see Fig. 1). We shall now compute the over-all signal-to-noise ratio for the optical receiver, and its overall noise factor F_R. In defining F_R, we shall use the concept of noise factor as originally introduced by Friis and Fränz.[31] This noise factor F_R is defined as

$$F_R = \frac{(S/N)_{\mathrm{in}}}{(S/N)_{\mathrm{out}}}, \tag{26}$$

where $(S/N)_{\mathrm{in}}$ and $(S/N)_{\mathrm{out}}$ are the input and output exchangeable signal-to-noise power ratios of the optical receiver. From equations 10, 12, and 26 we can write

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¶ It can be shown that $\mathrm{Re}\,\rho + [1-(\mathrm{Im}\,\rho)^2]^{1/2}$ is always a nonnegative quantity.

† It can be shown (see Ref. 30) that equation 25 can also be written in the form $(G_s - G_{or})^2 + (B_s - B_{or})^2 = F^2{}_p$ where G_{or}, B_{or}, and F_p can be determined from Equations 21-25.

$$(S/N)_{\text{out}} = \frac{\eta P_s}{h\nu\,\Delta f} \frac{1}{1 + \frac{2kT_o}{qI_o}\frac{1}{R}\left(\frac{\omega}{\omega_c}\right)^2 \left\{F_o + \frac{R_n}{\text{Re}\,Y_2}\,|\,Y_2 - Y_{of}\,|^2 + \frac{T_d - T_o}{T_o}\right\}}, \tag{27}$$

and

$$F_R = \frac{1}{\eta}\left[1 + \frac{2kT_o}{qI_o}\frac{1}{R}\left(\frac{\omega}{\omega_c}\right)^2 \cdot \left\{F_o + \frac{R_n}{\text{Re}\,Y_2}\,|\,Y_2 - Y_{of}\,|^2 + \frac{T_d - T_o}{T_o}\right\}\right]. \tag{28}$$

We now note from equations **27** and **28** that $(S/N)_{\text{out}}$ and F_R are functions of the frequency of the detected signal, and for $Y_2 = Y_{of}$, $(S/N)_{\text{out}}$, and F_R attain their optimum values $(S/N)_O$ and F_{RO} where

$$(S/N)_O = \frac{\eta P_s}{h\nu\,\Delta f} \frac{1}{1 + \frac{2kT_o}{qI_o}\frac{1}{R}\left(\frac{\omega}{\omega_c}\right)^2 \left\{F_o + \frac{T_d - T_o}{T_o}\right\}}, \tag{29}$$

and

$$F_{RO} = \frac{1}{\eta}\left[1 + \frac{2kT_o}{qI_o}\frac{1}{R}\left(\frac{\omega}{\omega_c}\right)^2 \left\{F_o + \frac{T_d - T_o}{T_o}\right\}\right]. \tag{30}$$

If optimum noise performance of the heterodyne receiver at a finite number of signal frequencies is desired, it can be shown[16] that suitable lossless interstage networks can be designed so that $(S/N)_O$ in equation **29** and F_{RO} in Equation **30** can be realized at the respective signal frequencies. If the band of frequencies of interest is continuous and nonzero, it can be shown that we cannot make F_R equal to F_{RO} over the whole band.[15, 16] The question arises whether there are any constraints to be satisfied by F_R, imposed by the diode or any other components of the system, and what lossless interstage networks must be used to make F_R as close to F_{RO} as possible. We shall discuss these two topics in the rest of this paper.

IV. DERIVATION OF INTEGRAL CONSTRAINTS†

We shall use the results obtained in the theory of broadband matching of linear systems[13, 13–19] to derive the expressions which relate

† The methods of derivation of most of the results in this section are very similar to those in Ref. 13.

the noise factor of the optical heterodyne receiver to other parameters of the system. The integral relation shows that it is impossible to make F_R equal to F_{RO} over any nonzero band of frequencies. Since the IF amplifier is assumed to have ideal broadband signal and noise performance characteristics, it follows that F_o and Y_{of} are independent of frequency ω for $\Omega_o - W \leqq \omega \leqq \Omega_o + W$.‡ It also follows that $B_{of} = 0$.§ Without any loss of generality we shall normalize all admittances with respect to G_{of}.

Let us look at Fig. 6 and define two reflection coefficients $\rho_1(p)$ and

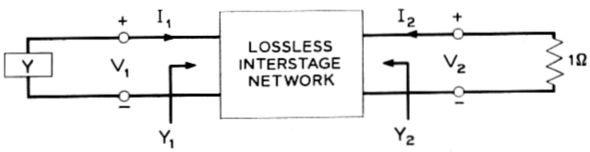

Fig. 6 — Lossless interstage network used in equalizing the noise performance of the optical receiver. $Y(p)$ is the admittance of the photodiode as shown in Fig. 2.

$\rho_2(p)$, and an all-pass function $\beta(p)$ where

$$\rho_1(p) = \frac{Y_1(p) - Y(-p)}{Y_1(p) + Y(p)}, \tag{31}$$

$$\rho_2(p) = \frac{Y_2(p) - 1}{Y_2(p) + 1}, \tag{32}$$

$$\beta(p) = \prod_{r=1}^{m} \frac{p - \alpha_r}{p + \alpha_r^*}, \tag{33}$$

and $p = \sigma + j\omega$ is the complex frequency variable. $Y(p)$ is the admittance of the photodiode as seen by the lossless interstage network, and $\alpha_1, \alpha_2, \cdots, \alpha_m$ are the poles of $Y(-p)$ in Re $p > 0$. Since the interstage network is lossless,[33] it can be shown that

$$| \rho_1(j\omega) | = | \rho_2(j\omega) |. \tag{34}$$

Also from Equations 25, 32, and 34

$$\frac{1}{| \rho_1(j\omega) |^2} = \frac{1}{| \rho_2(j\omega) |^2} = 1 + \frac{4R_n G_{of}}{F - F_o}. \tag{35}$$

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‡ The study of the case in which F_o and Y_{of} may be functions of frequency is very complicated and beyond the scope of this paper.

§ For any realizable admittance Y_{of}, $B_{of}(\omega)$ must be an odd function of ω.

From equations 31 and 33, we can also write

$$\beta(p)\{1 - \rho_1(p)\} = \beta(p)\frac{Y(p) + Y(-p)}{Y_1(p) + Y(p)}. \tag{36}$$

Equation 36 shows that regardless of the lossless interstage network used in the receiver, every zero of $\gamma(p) = \frac{1}{2}[1+Y(-p)/Y(p)]$ in $\mathrm{Re}\, p \geq 0$ must also be a zero of

$$\zeta(p) = \beta(p)\{1 - \rho_1(p)\}. \tag{37}$$

A zero p_o of $\gamma(p)$ in $\mathrm{Re}\, p \geq 0$ of multiplicity k is said to be a zero of transmission of the admittance $Y(p)$ of order k. Youla distinguishes four kinds of such zeros.[17] Class 1 contains all those in the strict right-half plane. Class 2 contains all those on the real-frequency axis which are simultaneously zeros of $Y(p)$. Class 3 contains all those on the real-frequency axis for which $0 < |Y(p_o)| < \infty$, and class 4 contains all those for which $|Y(p_o)| = \infty$. The restrictions imposed on $\rho_1(p)$ through equation 36 are formulated[13, 15–19] most compactly in terms of coefficients of the power series expansions of the following quantities:

$$s(p) = \beta(p)\rho_1(p) = \sum_{k=0}^{\infty} S_k(p - p_o)^k \tag{38}$$

$$\ln s(p) = \sum_{k=0}^{\infty} s_k(p - p_o)^k \tag{39}$$

$$\beta(p) = \sum_{k=0}^{\infty} B_k(p - p_o)^k \tag{40}$$

$$\ln \beta(p) = \sum_{k=0}^{\infty} b_k(p - p_o)^k \tag{41}$$

$$F(p) = \beta(p)[Y(p) + Y(-p)] = \sum_{k=0}^{\infty} F_k(p - p_o)^k \tag{42}$$

$$\frac{1}{\pi}\frac{p}{p^2 + \omega^2} = \sum_{k=0}^{\infty} f_k(p - p_o)^k \tag{43}$$

$$g(p) = \frac{F(p)}{2\beta(p)} = \sum_{k=0}^{\infty} \theta_k(p - p_o)^k. \tag{44}$$

Also, let $\eta(p)$ be a regular all-pass network such that

$$\eta(p) = \prod_{l=1}^{\nu} \frac{p - \mu_l}{p + \mu_l^*}, \tag{45}$$

and

$$\ln \eta(p) = \sum_{k=0}^{\infty} \eta_k (p - p_o)^k. \tag{46}$$

μ_l's are a set of points in the right half of the complex plane.

Let

$$\frac{1}{s_o(p)s_o(-p)} = 1 + \frac{4R_n G_{of}}{F - F_o}, \tag{47}$$

and let $s_o(p)$ be such that all its zeros and poles are in the left-half plane.† It is now clear‡ that if

$$\beta(p)\rho_1(p) = s(p) = \eta(p)s_o(p), \tag{48}$$

$$\frac{1}{|s(j\omega)|^2} = \frac{1}{|\rho_1(j\omega)|^2} = \frac{1}{|s_o(j\omega)|^2} = 1 + \frac{4R_n G_{of}}{F - F_o}. \tag{49}$$

We may now show[17-19] that $Y_1(p)$ is a positive-real admittance if and only if:

(i) At every class 1 transmission zero p_o of order k,

$$S_r = B_r, \qquad 0 \leqq r \leqq k - 1; \tag{50}$$

or

$$\begin{aligned} b_o &= \epsilon\pi j + \eta_o - \int_0^\infty f_o \ln\left\{1 + \frac{4R_n G_{of}}{F - F_o}\right\} d\omega, \\ \epsilon &= 0, \quad \text{if} \quad \frac{d}{d\omega}\left\{1 + \frac{4R_n G_{of}}{F - F_o}\right\} \neq 0, \\ \epsilon &= 1, \quad \text{if} \quad \frac{d}{d\omega}\left\{1 + \frac{4R_n G_{of}}{F - F_o}\right\} = 0, \end{aligned} \tag{51}$$

and

$$b_r = \eta_r - \int_0^\infty f_r \ln\left\{1 + \frac{4R_n G_{of}}{F - F_o}\right\} d\omega, \qquad 1 \leqq r \leqq k - 1. \tag{52}$$

† It will be recognized that $s_o(p)$ is a minimum-phase function.[34]

‡ Multiplication of $\rho_1(p)$ by $\beta(p)$ is necessary to make it analytic in the right-half plane. This multiplication makes $s(p)$ a bounded real scattering coefficient.[17] Multiplication of $s_o(p)$ by $\eta(p)$ introduces right-half plane zeroes. This is sometimes necessary[16] and is done so that $s(p)$ can satisfy all the constraints imposed by $Y(p)$.

(*ii*) At every class 2 transmission zero $j\omega_o$ of order k,

$$S_r = B_r\,, \qquad 0 \leqq r \leqq k - 1, \tag{53}$$

and

$$\frac{S_k - B_k}{F_{k+1}} \leqq 0; \tag{54}$$

or

$$b_r = \eta_r - \int_0^\infty f_r \ln\left\{1 + \frac{4R_nG_{of}}{F - F_o}\right\} d\omega, \qquad 0 \leqq r \leqq k - 1, \tag{55}$$

and§

$$\frac{b_k - s_k}{\theta_{k+1}} \geqq 0. \tag{56}$$

If $|\,\omega_o\,| = 0$, or ∞, equation 56 may be replaced by

$$\frac{b_k - \eta_k + \int_0^\infty f_k \ln\left\{1 + \frac{4R_nG_{of}}{F - F_o}\right\} d\omega}{\theta_{k+1}} \geqq 0. \tag{57}$$

(*iii*) At every class 3 transmission zero of order k,

$$S_r = B_r\,, \qquad 0 \leqq r \leqq k - 2, \tag{58}$$

and

$$\frac{S_{k-1} - B_{k-1}}{F_k} \leqq 0; \tag{59}$$

or

$$b_r = \eta_r - \int_0^\infty f_r \ln\left\{1 + \frac{4R_nG_{of}}{F - F_o}\right\} d\omega, \qquad 0 \leqq r \leqq k - 1, \tag{60}$$

and

$$\frac{b_{k-1} - s_{k-1}}{\theta_k} \geqq 0, \tag{61}$$

with equality if and only if the matching network is nondegenerate. If $|\,\omega_o\,| = 0$ or ∞, equation 61 may be replaced by

$$\frac{b_{k-1} - \eta_{k-1} + \int_0^\infty f_r \ln\left\{1 + \frac{4R_nG_{of}}{F - F_o}\right\} d\omega}{\theta_k} \geqq 0. \tag{62}$$

§ If the interstage network is nondegenerate[16,17], equation 56 becomes an equality. If $|\,Y(j\omega_o)\,| \neq \infty$, the network is said to be nondegenerate if and only if $Y_1(j\omega_o) + Y(j\omega_o) \neq 0$. If $|\,Y(j\omega_o)\,| = \infty$, the network is said to be nondegenerate if and only if $|\,Y_1(j\omega_o)\,| \neq \infty$.

(*iv*) At every class 4 zero of order k,

$$S_r = B_r, \qquad 0 \leqq r \leqq k - 1, \tag{63}$$

and

$$\frac{F_{k-1}}{S_k - B_k} \leqq a_{-1} ; \tag{64}$$

where a_{-1} is the residue of $Y(p)$ at $p_o = j\omega_o$;
or

$$b_r = \eta_r - \int_0^\infty f_r \ln \left\{1 + \frac{4R_n G_{of}}{F - F_o}\right\} d\omega, \qquad 0 \leqq r \leqq k - 1, \tag{65}$$

and

$$\frac{2\theta_{k-1}}{b_k - s_k} \geqq a_{-1} , \tag{66}$$

with equality if and only if the matching network is nondegenerate. If $| \omega_0 | = 0$ or ∞, equation 66 may be replaced by

$$\frac{2\theta_{k-1}}{b_k - \eta_k + \int_0^\infty f_k \ln \left\{1 + \frac{4R_n G_{of}}{F - F_o}\right\} d\omega} \geqq a_{-1} . \tag{67}$$

If equations 50 through 67 are satisfied, $Y_1(p)$ is a positive-real admittance. If $Y_1(p)$ is positive-real, the Darlington method can be used to obtain the lossless twoport interstage network needed in the receiver.

Let us now use the theory of broadband noise performance that we have presented to derive the constraints imposed by the photodiode on the noise performance of the optical receiver. The normalized admittance $Y(p)$ of the photodiode shown in Fig. 2 can be written as†

$$Y(p) = \frac{1}{RG_{of}} \frac{p}{p + \omega_c}. \tag{68}$$

From equation 36 we can show that the only transmission zero of $Y(p)$ lies at $p_o = 0$, and is of order 1. Also from equation 33,

$$\beta(p) = \frac{p - \omega_c}{p + \omega_c} \tag{69}$$

$$= -1 + \frac{2p}{\omega_c} - \frac{2p^2}{\omega_c^2} + \cdots . \tag{70}$$

† The equivalent circuit shown in Fig. 2 for the photodiode is valid for frequencies $\omega \gg 0$. Also, without any loss of generality, we normalize all admittances with respect to G_{of} .

From equation 42 we can write

$$F(p) = \frac{1}{G_{of}} \frac{2p^2C}{\omega_c(1 + p/\omega_c)^2} \tag{71}$$

$$= \frac{1}{G_{of}} \left[\frac{2Cp^2}{\omega_c} - \frac{4Cp^3}{\omega_c^2} + \cdots \right]. \tag{72}$$

Since the transmission zero is of class 2, we can write from equations 53 and 54 that

$$S_o = -1, \tag{73}$$

and

$$S_1 \leqq 2/\omega_c \ , \tag{74}$$

where

$$s(p) = \pm\eta(p)s_o(p), \tag{48}$$

and

$$\frac{1}{s_o(p)s_o(-p)} = 1 + \frac{4R_nG_{of}}{F - F_o} \ . \tag{47}$$

Also from equations 55 through 57 it follows that

$$\frac{1}{\pi}\int_0^\infty \frac{1}{\omega^2} \ln\left\{1 + \frac{4R_nG_{of}}{F - F_o}\right\} d\omega \leqq \frac{2}{\omega_c} - \sum_{l=1}^{\nu} \frac{\mu_l}{\mu_l^*}\left\{\frac{1}{\mu_l} + \frac{1}{\mu_l^*}\right\}, \tag{75}$$

where μ_l's are a set of points in the right-half plane. Since Re $(1/\mu_l) \geq 0$ for all l, we put $\eta\ (p) = 1$. We, therefore, have

$$\frac{1}{\pi}\int_0^\infty \frac{1}{\omega^2} \ln\left\{1 + \frac{4R_nG_{of}}{F - F_o}\right\} d\omega \leqq \frac{2}{\omega_c}. \tag{76}$$

Also, from equations 25, 28, and 30, we can write equation 76 as

$$\frac{1}{\pi}\int_0^\infty \frac{1}{\omega^2} \ln\left[1 + \frac{8kT_o}{\eta q I_o}\frac{R_nG_{of}}{R}\frac{(\omega/\omega_c)^2}{F_R - F_{RO}}\right] d\omega \leqq \frac{2}{\omega_c}. \tag{77}$$

We must notice that R_n and G_{of} are completely determined by the IF amplifier used in the system, and if we assume that the signal power P_s remains constant at all frequencies of interest, equation 77 shows that F_R cannot be made equal to F_{RO} over any nonzero band of frequencies in spite of the fact that any arbitrary linear lossless interstage network may be used in the receiver. This is one of the important results of this paper. We must notice that the equivalent circuit shown in Fig. 2 has been assumed in deriving equation 77. This equivalent circuit very well describes the behavior of the diode provided that the lowest frequency of the signal occurring in the system is very far from zero.[12] Also we must observe from equation 77 that F_R can be made equal to F_{RO} at a finite number of discrete frequencies.[16]

V. RATIONAL FUNCTION APPROXIMATIONS

We have shown that F_R cannot be equal to F_{RO} over any nonzero interval $\Omega_o - W \leqq \omega \leqq \Omega_o + W$ of the frequency spectrum.† We shall, therefore, make some rational function approximations to a flat noise performance characteristic of the optical receiver. If these rational function approximations satisfy all the constraints of Section IV, a finite linear lumped lossless network can be found which realizes this kind of noise factor for the optical receiver.[34] A complete treatment of this problem is beyond the scope of this paper; but let us consider certain kinds of approximations widely used in network theory.

5.1 *Butterworth Approximations*

The problem at hand is to approximate F_R as close to F_{RO} as possible over the range $\Omega_o - W \leqq \omega \leqq \Omega_o + W$. A set of polynomials which can be used for this purpose are Butterworth polynomials.[34,35] Let

$$F_R = \frac{1}{\eta} + \left(F_{RO} - \frac{1}{\eta}\right)\left\{1 + \epsilon^2\left(\frac{\omega^2 - \omega_o^2}{2\omega W}\right)^{2n}\right\}, \tag{78}$$

where n is the order of complexity of the interstage network to be used in obtaining broadband performance from the optical receiver, and n is also the order of the Butterworth polynomial. It may be verified that F_R approximates F_{RO} in a maximally flat manner. The behavior of F_R as a function of ω is shown in Fig. 7. Since it can be shown that F_R in equation 78 can be made to satisfy equations 73 through 77 by properly choosing ϵ^2 for all values of n, the approximation of equation 78 is realizable.

From equation 47,

$$s_o(p)s_o(-p) = \frac{1}{1 + \dfrac{4R_nG_{of}M_o}{\epsilon^2\left(\dfrac{\omega^2 - \omega_o^2}{2\omega W}\right)^{2n}}}, \tag{79}$$

† Since the noise factor is a real and even function of ω, we shall only consider the behavior of F_R for $\omega \geqq 0$.

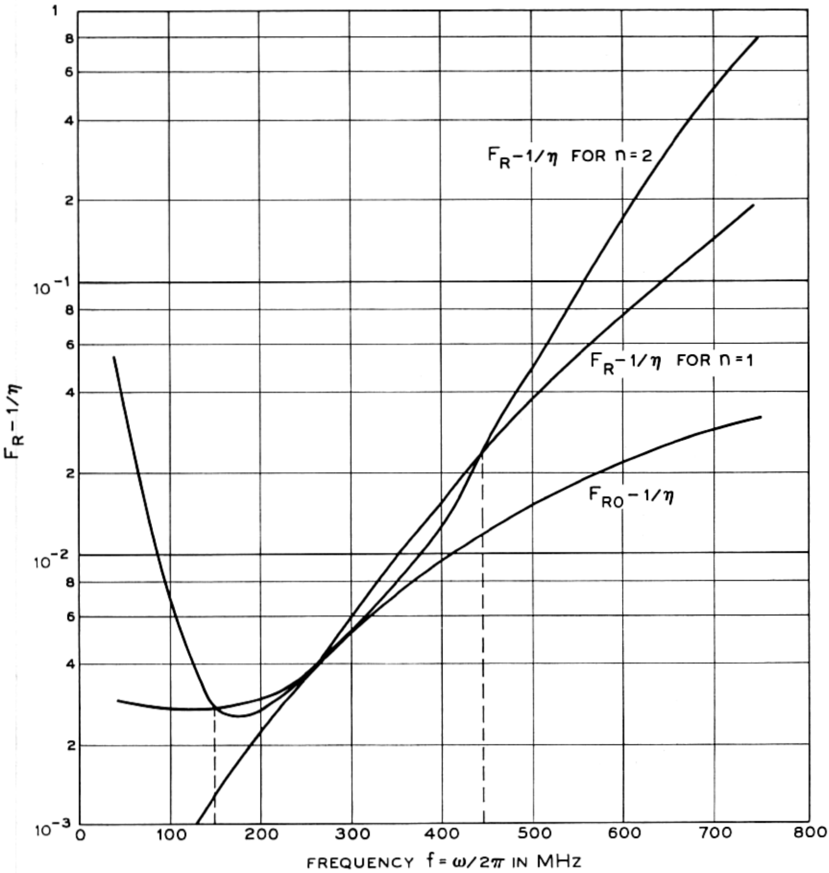

Fig. 7 — Butterworth approximations of order $n = 1,2$. It is assumed that $f_c = 31.83$ GHz, $\eta = 0.70$, $T_d = 290°$K, $I_o = 500\ \mu\alpha$, $F_o = 2.0833$, and $\epsilon^2 = 1$.

where

$$M_o = \frac{1}{F_o}. \tag{80}$$

It can be shown[35] that

$$s_o(p) = \pm \frac{(p^2 + \omega_o^2)^n}{(p^2 + \omega_o^2)^n + a_{n-1}(2pW)(p^2 + \omega_o^2)^{n-1} + \cdots + a_o(2pW)^n}, \tag{81}$$

where

$$a_{n-1} = \frac{\left(\frac{4R_n G_{of} M_o}{\epsilon^2}\right)^{1/2n}}{\sin \frac{\pi}{2n}}. \tag{82}$$

We can now expand equation 81 into a Taylor series‡ about $p = 0$. We have

$$s_o(p) = -1 + a_{n-1} \frac{2W}{\omega_o^2} p - \cdots . \tag{83}$$

Now

$$\begin{aligned} \eta(p) &= \prod_{l=1}^{\nu} \frac{p - \mu_l}{p + \mu_l^*} \\ &= (-1)^{\nu}\left[1 - p \sum_{l=1}^{\nu} \left\{\frac{1}{\mu_l} + \frac{1}{\mu_l^*}\right\} \frac{\mu_l}{\mu_l^*} + \cdots \right]. \end{aligned} \tag{84}$$

From equations 48, 74, 83, and 84, we have

$$a_{n-1} \frac{2W}{\omega_o^2} + \sum_{l=1}^{\nu} \frac{\mu_l}{\mu_l^*} \left\{\frac{1}{\mu_l} + \frac{1}{\mu_l^*}\right\} \leqq \frac{2}{\omega_c}. \tag{85}$$

Since $\{(1/\mu_l) + (1/\mu_l^*)\} \geqq 0$ for all l, let us put $\eta(p) = 1$. We can then write from equations 82 and 85 that§

$$\epsilon^2 \geqq \frac{4R_n G_{of} M_o}{\left(\frac{\omega_o}{\omega_c} \frac{\omega_o}{W} \sin \frac{\pi}{2n}\right)^{2n}}. \tag{86}$$

A typical value of $f_c = \omega_c/2\pi$ for a photodiode is about 31.83 GHz.¶ For this value of f_c, $f_o = \Omega_o/2\pi = 300$ MHz, and $2W/\Omega_o = 100$ percent, we have plotted in Fig. 8 $\epsilon_{\min}^2/4R_n G_{of} M_o$ as a function of n. It may be seen from the plot that $\epsilon_{\min}^2$ is a monotonically increasing function of n. This behavior of $\epsilon_{\min}^2$ can be explained by the fact that Butterworth polynomials approximate the ideal broadband noise performance characteristic of the optical receiver in a maximally flat fashion[35].

Since no useful purpose is served by using higher values of n, we

‡ We choose negative sign for $s_o(p)$ to satisfy equation 73. This does not entail any loss in generality.[17]

§ Equation 86 can also be obtained by using equation 77.

¶ Typical values of R and C for a photodiode are $C = 1 \mu\mu$f, and $R = 5$ ohms.[36]

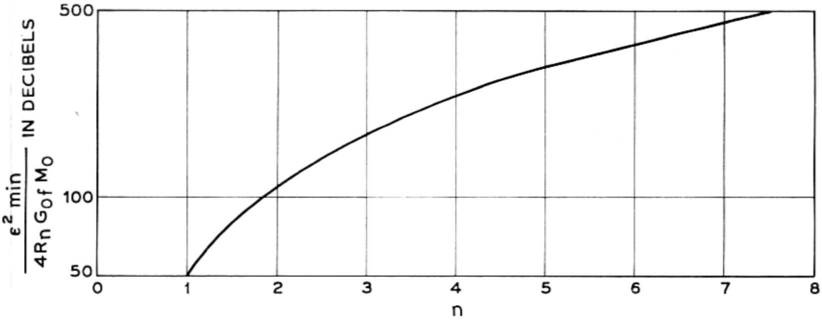

Fig. 8 — A typical plot when Butterworth polynomials are used to approximate the ideal noise performance characteristic. Even though n is a discrete variable the plot is given for all $n \geq 1$.

shall only consider the case $n = 1$.* For $n = 1$,

$$\epsilon_{\min}^2 = \frac{4R_n G_{of} M_o}{\left(\frac{\omega_o}{\omega_c} \frac{\omega_o}{W}\right)^2}, \tag{87}$$

and

$$s(p) = s_o(p) = -\frac{p^2 + \omega_o^2}{p^2 + 2p\frac{\omega_o^2}{\omega_c} + \omega_o^2}. \tag{88}$$

Now from equations 31, 48, and 88, it can be shown that

$$Y_1(p) = \frac{1}{RG_{of}} \frac{\frac{\omega_o^2}{\omega_c}}{p + \frac{\omega_o^2}{\omega_c}}. \tag{89}$$

Fig. 9 shows the circuit to realize $Y_1(p)$. Remember that ω_o^2 is the geometric mean of the band of frequencies of interest, and

$$L = \frac{1}{C(\Omega_o - W)(\Omega_o + W)}, \tag{90}$$

$$t = \sqrt{G_{of} R}. \tag{91}$$

This circuit agrees very well with our physical intuition.

* Since we are interested in minimum value of ϵ^2 we have used the equality sign in equation 77

5.2 *Chebyshev Approximations*

Of the various means of approximating a given function, the Chebyshev method is one of the most interesting and important. It can be shown that given a set of n parameters, a function $f(\omega^2)$ approximates $g(\omega^2)$ in the Chebyshev sense if the parameters are determined in such a way that the largest value of $| g(\omega^2) - f(\omega^2) |$ in a given interval is minimum.* Since for a given complexity of the structure the maximum amount of tolerance for a Chebyshev approximation is the same through the band, this type of approximation seems to be the most desirable in the broadband noise performance of optical receivers.

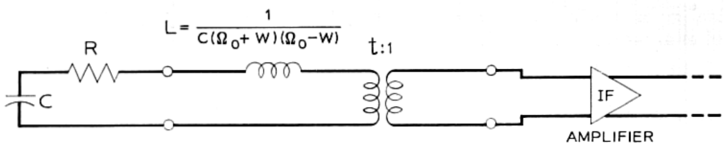

Fig. 9—Lossless interstage network for a Butterworth approximation of order $n = 1$. The ideal transformer ratio t is given by $t = \sqrt{RG_{of}}$.

Let us try to approximate F_R by

$$F_R = \frac{1}{\eta} + \left(F_{RO} - \frac{1}{\eta}\right)\left[1 + \epsilon^2 T_n^2\left(\frac{\omega^2 - \omega_o^2}{2\omega W}\right)\right], \tag{92}$$

where $T_n(x)$ is an n^{th} degree Chebyshev polynominal given by[34, 35, 37]

$$T_n(x) = \cos(n \cos^{-1} x). \tag{93}$$

The behavior of F_R as a function of ω for $n = 1, 2$ is shown in Fig. 10. The equiripple behavior of F_R is evident from equation 93. We can also show that the approximation of the type given in equation 92 can be made to satisfy equations 73 through 77. It can also be shown that

$$s_o(p) = -\frac{(p^2 + \omega_o^2)^n + b_{n-2}(2pW)^2(p^2 + \omega_o^2)^{n-2} + \cdots}{(p^2 + \omega_o^2)^n + a_{n-1}(2pW)(p^2 + \omega_o^2)^{n-1} + \cdots}, \tag{94}$$

where

$$a_{n-1} = \frac{\sinh\left[\frac{1}{n}\sinh^{-1}\frac{2\sqrt{R_n G_{of} M_o}}{\epsilon}\right]}{\sin \pi/2n}. \tag{95}$$

---

* The Chebyshev approximating function has the equiripple property.[34, 35, 37]

If we expand $s_o(p)$ about $p = 0$, we can show that

$$s_o(p) = -1 + pa_{n-1}\frac{2W}{\omega_o^2} + \cdots . \tag{96}$$

We again put $\eta(p) = 1$ to obtain minimum ϵ^2. From equations 74 and 96

$$\epsilon_{\min}^2 = \frac{4R_n G_{of} M_o}{\sinh^2\left[n \sinh^{-1}\left\{\left(\frac{\omega_o}{\omega_c}\right)\left(\frac{\omega_o}{W}\right)\sin\frac{\pi}{2n}\right\}\right]}. \tag{97}$$

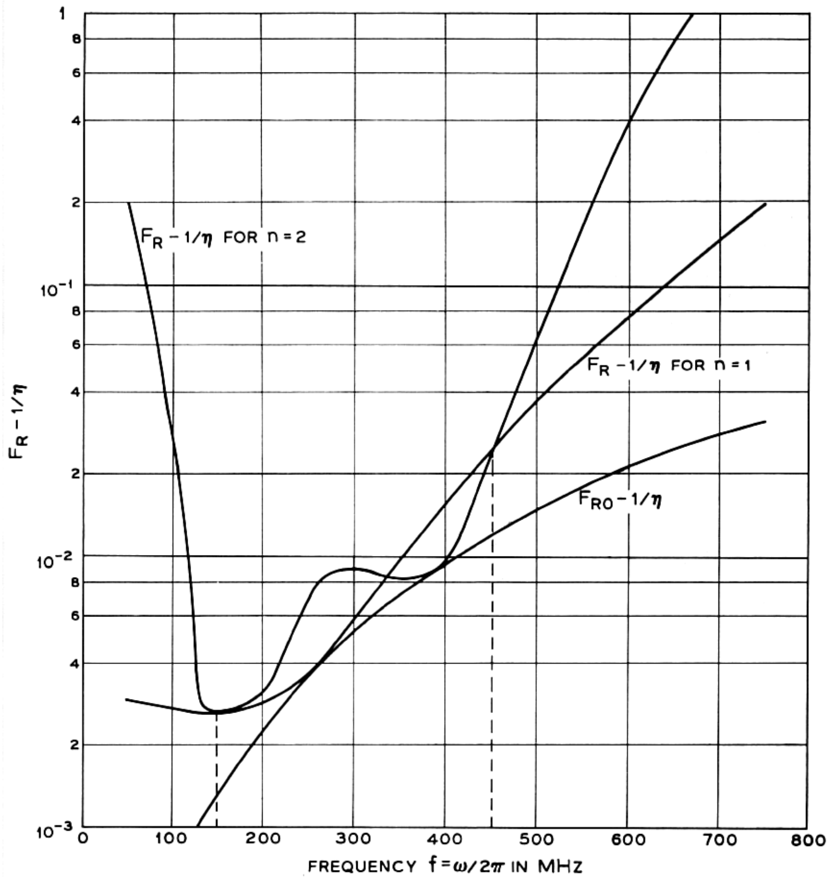

Fig. 10 — Noise factor F_R as a function of ω for Chebyshev approximations of order $n = 1,2$. It is assumed that $f_c = 31.83$ GHz, $\eta = 0.70$, $T_o = 290°$K, $I_o = 500\ \mu a$, $F_o = 2.0833$, and $\epsilon^2 = 1$.

In practical cases, assuming that $\omega_0/\omega_c \ll 1/\sqrt{15}$, $(\omega_0/\omega_c)(\omega_0/W) \sin \pi/2n \ll 1$, for $\omega_0/W \geqq \sqrt{15}$.† We can then write

$$\epsilon_{\min}^2 = \frac{16 R_n G_{of} M_o}{\pi^2 \left(\frac{\omega_o}{\omega_c}\right)^2 \left(\frac{\omega_o}{W}\right)^2 \left(\frac{\sin \pi/2n}{\pi/2n}\right)^2}. \tag{98}$$

A normalized plot of $\epsilon_{\min}^2$ for $f_o = \Omega_o/2\pi = 300$ MHz, $f_c = 31.83$ GHz, and $2W/\Omega_o = 100$ percent is given in Fig. 11. We notice that

$$\frac{[\epsilon_{\min}^2]_{n=1}}{[\epsilon_{\min}^2]_{n=\infty}} = \frac{\pi^2}{4} \approx 2.5, \tag{99}$$

and

$$\frac{[\epsilon_{\min}^2]_{n=2}}{[\epsilon_{\min}^2]_{n=\infty}} = \frac{\pi^2}{8} \approx 1.25. \tag{100}$$

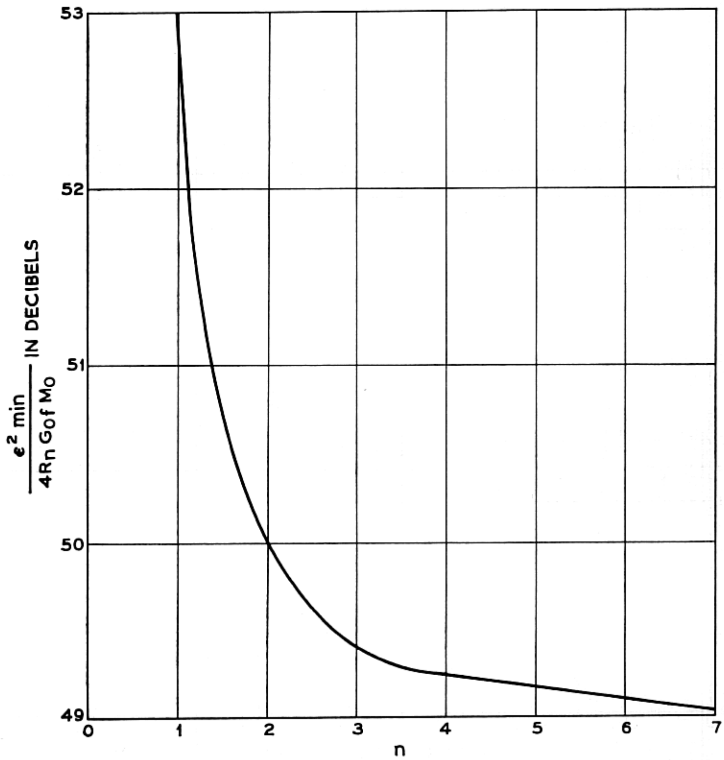

Fig. 11 — A typical plot when Chebyshev polynomials are used to approximate the ideal noise performance characteristic. Even though n is a discrete variable the plot is given for all $n \geq 1$.

§ It can be shown from equation 1 that $\omega_o/W \leqq \sqrt{15}$ for $2W/\Omega_o \geqq \frac{1}{2}$.

From equations 99 and 100 and Fig. 11, we conclude that $\epsilon_{\min}^2$ is a monotonically decreasing function of n, but that no great improvement in the value of $\epsilon_{\min}^2$ is obtained by using very high values of n. Since the complexity of the network increases with n, we shall only consider the cases $n = 1, 2$. For $n = 2$, $\epsilon_{\min}^2$ attains 1.25 times the minimum possible value. For $n = 1$ the Butterworth and Chebyshev approximations are the same. For $n = 2$, from equation 98

$$\epsilon_{\min}^2 = 2R_n G_{of} M_o \left(\frac{\omega_c}{\omega_o}\right)^2 \left(\frac{W}{\omega_o}\right)^2, \tag{101}$$

and from equations 94, 95, and 101, we can write

$$s_o(p) = -\frac{(p^2 + \omega_o^2)^2 + \frac{1}{2}(2pW)^2}{(p^2 + \omega_o^2)^2 + \left(\frac{\omega_o}{\omega_c}\right)\left(\frac{\omega_o}{W}\right)(2pW)(p^2 + \omega_o^2) + \frac{1}{2}(2pW)^2}. \tag{102}$$

Equations 31, 48, and 102 show that

$$Y_1(p) = \frac{1}{RG_{of}} \frac{\omega_o^2}{\omega_c} \frac{p^2 + \omega_o^2}{p^3 + p^2 \frac{\omega_o^2}{\omega_c} + p(\omega_o^2 + 2W^2) + \frac{\omega_o^4}{\omega_c}}. \tag{103}$$

The lossless interstage network realizing $Y_1(p)$ in equation 103 is shown in Fig. 12.

Similar methods can be used to determine the lossless interstage networks when $n > 2$. We have shown however that no great improvement can be obtained by using very high values of n.

5.3 *Approximations with Greater than Optimum Noise Factor*

In the preceding parts of this section we used Butterworth and Chebyshev polynomials to approximate the ideal broadband noise performance characteristic of the IF amplifier in such a way that the

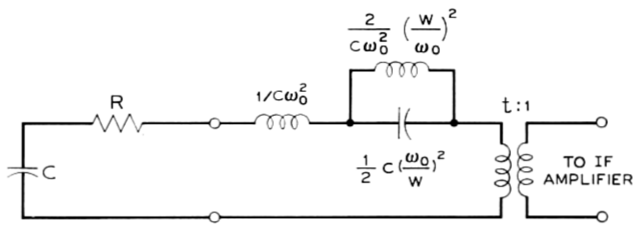

Fig. 12—Lossless interstage network for a Chebyshev approximation of order $n = 2$. The ideal transformer ratio t is given by $t = \sqrt{RG_{of}}$.

minimum passband noise factor is F_{RO}. These polynomials also can be used[17, 18] in a manner in which the minimum passband noise factor is KF_{RO} where

$$K \geqq 1. \tag{104}$$

Such approximations are given by

$$F_R = \frac{1}{\eta} + K\left(F_{RO} - \frac{1}{\eta}\right)\left[1 + \left(\frac{\omega^2 - \omega_o^2}{2\omega W}\right)^{2n}\right], \tag{105}$$

and

$$F_R = \frac{1}{\eta} + K\left(F_{RO} - \frac{1}{\eta}\right)\left[1 + \epsilon^2 T_n^2\left(\frac{\omega^2 - \omega_o^2}{2\omega W}\right)\right], \tag{106}$$

where $T_n(x)$ is an n^{th} degree Chebyshev polynomial.

We can seen from equations 105 and 106 that

$$[F_R]_{\min} = KF_{RO} - \frac{1}{\eta}(K - 1) \geqq F_{RO} . \tag{107}$$

If equations 73 through 77 are to be satisfied, it can be shown from equation 105 that

$$\left[1 - \frac{1}{K} + \frac{4R_n G_{of} M_o}{K}\right]^{1/2n} - \left[1 - \frac{1}{K}\right]^{1/2n} \leqq \left(\frac{\omega_o}{\omega_c}\right)\left(\frac{\omega_o}{W}\right) \sin\frac{\pi}{2n}. \tag{108}$$

Also, if F_R in equation 106 is to be realizable, it can be shown that the following constraint must be satisfied:

$$\sinh\left[\frac{1}{n}\sinh^{-1}\frac{\left(1 - \frac{1}{K} + 4R_n G_{of} M_o/K\right)^{\frac{1}{2}}}{\epsilon}\right] - \sinh\left[\frac{1}{n}\sinh^{-1}\frac{\left(1 - \frac{1}{K}\right)^{\frac{1}{2}}}{\epsilon}\right] \leqq \left(\frac{\omega_o}{\omega_c}\right)\left(\frac{\omega_o}{W}\right)\sin\frac{\pi}{2n}. \tag{109}$$

In general, for arbitrary n, equations 108 and 109 can only be solved numerically. The numerical solution of these two equations requires that the value of $R_n G_{of} M_o$, ω_o/ω_c, and ω_o/W be known. For any specific IF amplifier, the values of K and ϵ^2 can be determined from equations 108 and 109 and the interstage network can then be synthesized. Since we do not propose to go into the characteristics of the IF amplifier, we shall not consider these two equations any more in this paper.

Minimum average noise factor approximation, least-squares approximation,[23] and the like also can be used in the theory of broadband noise performance of the optical receiver. If these approximations satisfy the restrictions which are imposed by the photodiode, and which are given in equations 73 through 77, the methods given in Section IV can be used to obtain a positive-real $Y_1(p)$. This $Y_1(p)$ enables us to determine the lossless interstage network required in the broadband noise performance of the optical receiver.

VI. GAIN AND NOISE FACTOR

It has been shown for an optical heterodyne receiver[13] that the available output power P_{oa} must satisfy the constraint given by

$$\frac{1}{\pi}\int_0^\infty \frac{1}{\omega^2}\ln\left\{1+\left(\frac{h\nu}{\eta q}\right)^2\frac{8R_fG_{og}}{RP_oP_s}\frac{(\omega/\omega_c)^2}{\dfrac{1}{P_{oa}}-\dfrac{1}{P_{o\max}}}\right\}d\omega \leqq \frac{2}{\omega_c} \tag{110}$$

where†

$$R_f = \frac{\operatorname{Re} y_{22}}{|y_{21}|^2} \tag{111}$$

$$G_{og} = \frac{|y_{12}y_{21}|}{2\operatorname{Re} y_{22}}\sqrt{\lambda^2-1} \tag{112}$$

$$\lambda = \frac{2\operatorname{Re}(y_{11})\operatorname{Re}(y_{22}) - \operatorname{Re}(y_{12}y_{21})}{|y_{12}y_{21}|} \tag{113}$$

$$P_{o\max} = \varphi_o R\left(\frac{\omega_c}{\omega}\right)^2 \tag{114}$$

$$\varphi_o = \tfrac{1}{2}G_{a\max}\left(\frac{\eta q}{h\nu}\right)^2 P_oP_s \tag{115}$$

$$G_{a\max} = \left|\frac{y_{21}}{y_{12}}\right|\frac{1}{\lambda+\sqrt{\lambda^2-1}}. \tag{116}$$

Equation 110 is identical in form to equation 77, and it can be shown from Ref. 13 that obtaining the broadband signal and noise performance characteristics of the optical receiver are analogous problems. It can also be shown from Ref. 13 that if Butterworth and Chebyshev approximations of the form given by

$$P_{oa} = P_{o\max}\frac{K'}{1+\left(\dfrac{\omega^2-\omega_o^2}{2\omega W}\right)^{2n}}, \qquad 0 < K' \leqq 1 \tag{117}$$

† For an IF amplifier which is absolutely stable, it can be shown[26-28] that $\lambda \geq 1$.

and

$$P_{oa} = P_{o\,\max} \frac{K''}{1 + \epsilon^2 T_n^2 \left(\frac{\omega^2 - \omega_o^2}{2\omega W} \right)}, \qquad 0 < K'' \leqq 1 \tag{118}$$

are used for the available output power of the optical receiver† realizability by lossless interstage networks requires that

$$[1 - K' + 4K'R_f G_{og} G_{a\,\max}]^{1/2n} - [1 - K']^{1/2n} \leqq \left(\frac{\omega_o}{\omega_c}\right)\left(\frac{\omega_o}{W}\right) \sin \frac{\pi}{2n} \tag{119}$$

and

$$\sinh \left[\frac{1}{n} \sinh^{-1} \frac{(1 - K'' + 4K''R_f G_{og} G_{a\,\max})^{\frac{1}{2}}}{\epsilon} \right] - \sinh \left[\frac{1}{n} \sinh^{-1} \frac{(1 - K'')^{\frac{1}{2}}}{\epsilon} \right] \leqq \left(\frac{\omega_o}{\omega_c}\right)\left(\frac{\omega_o}{W}\right) \sin \frac{\pi}{2n}. \tag{120}$$

We now notice that equation 119 is similar in form to equation 108 and 120 is similar to 109. However, it can be shown that the element values of the lossless interstage network obtained by solving either equation 119 or 120 will not be identical to those obtained by solving either 108 or 109. This shows that the problem of broadband noise performance, in general, requires a network different from that required for obtaining the broadband signal performance of the optical receiver. But for $K = K' = K'' = 1$, and for $G_{og} = G_{of}$, it can be shown‡ from equations 108, 109, 119, and 120 that the network which achieves broadband signal performance for the optical receiver also achieves broadband noise performance.

For a single stage common emitter transistor IF amplifier (see Fig. 13), we can show that the source conductance G_{of} for minimum noise factor is approximately equal to the source conductance G_{og} for maximum available gain.§ We can then say that a common emitter transistor IF amplifier can be used with advantage in obtaining simultaneously broadband signal and noise performance from the optical receiver.

† We can compare equation 117 to equation 105 and equation 118 to equation 106.

‡ We have assumed $B_{og} = B_{of} = 0$ for the IF amplifier.

§ In fact it can be shown (see Ref. 39) that for reasonable transistor parameters and frequencies below $(1 - \alpha_o)f_\alpha$, G_{of} is always within a factor of $\sqrt{2}$ of the common emitter G_{og}. α_o is the low frequency alpha of the transistor and f_α is the alpha cutoff frequency.

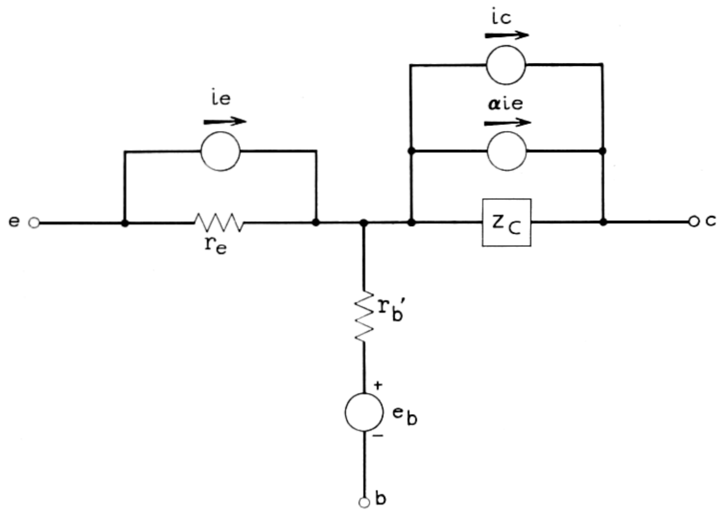

Fig. 13 — Simplified signal and noise equivalent circuit for the transistor.

VII. RESULTS AND CONCLUSIONS

A theory of obtaining broadband noise performance from an optical heterodyne receiver is presented in this paper. It is shown that the following constraint must be satisfied by any lossless interstage network used for obtaining broadband noise performance from the optical receiver:

$$\frac{1}{\pi}\int_0^\infty \frac{1}{\omega^2}\ln\left\{1+\frac{8kT_o}{\eta q I_o}\frac{R_n G_{of}}{R}\frac{(\omega/\omega_c)^2}{F_R - F_{RO}}\right\}d\omega \leqq \frac{2}{\omega_c}. \qquad (77)$$

This equation 77 shows that it is impossible to make F_R equal to F_{RO} for any nonzero band of frequencies and for any realizable lossless interstage networks.

We then consider certain types of rational function approximations to an ideal noise performance characteristic of the optical receiver. We show that Butterworth approximations to an ideal characteristic are realizable, but that the broadband noise performance of the receiver deteriorates with increasing values of n, the order of complexity of the interstage network. By approximating the ideal characteristic by Chebyshev polynomials, it can be shown that the performance improves with n, but no great improvement can be obtained by using very high values of n. We have shown that the performance for $n = 2$ is slightly worse than for $n = \infty$. Realizations of networks for $n = 1, 2$ are given.

We also consider the problem of obtaining simultaneously both signal and noise broadband performance from the optical heterodyne receiver and show that, in general, these two problems require two separate lossless interstage networks. We then show that for a common emitter transistor IF amplifier, and for certain types of Butterworth and Chebyshev approximations, these two networks turn out to be identical.

We give design methods and equations for any kind of rational function approximations to an ideal broadband noise performance characteristic of the optical receiver, and explicitly state the constraints to be satisfied by these approximations.

As is evident from Section IV, the theory of broadband noise performance presented in this paper for an optical heterodyne receiver can be applied to any other linear twoport network driven by a source whose internal admittance is a function of frequency.

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REFERENCES

1. Anderson, L. K., "Photodiode Detection," Proc. Symp. Opt. Masers, Polytechnic Press (April 16–19, 1963).
2. Saito, S., Kurokawa, K., Fujii, Y., Kamura, T., and Uno, Y., "Detection and Amplification of the Microwave Signal in Laser Light by a Parametric Diode," Proc. IRE, *50,* No. 11 (November 1962), pp. 2369–2370.
3. Sommers, H. S., Jr., "Demodulation of Low-Level Broad-Band Optical Signals with Semiconductors," Proc. IEEE, *51,* No. 1 (January 1963), pp. 140–146.
4. DiDomenico, M., Jr. and Svelto, O., "Solid-State Photodetection: A Comparison between Photodiodes and Photoconductors," Proc. IEEE, *52,* No. 2 (February 1964), pp. 136–144.
5. Anderson, L. K., "Measurement of the Microwave Modulation Frequency Response of Junction Photodiodes," Proc. IEEE, *51,* No. 5 (May 1963), pp. 846–847.
6. Riesz, R. P., "High-Speed Semiconductor Photodiodes," Rev. Sci. Inst., *33* (September 1962), pp. 994–998.
7. Inaba, H., and Siegman, A. E., "Microwave Photomixing of Optical Maser Outputs with a P-I-N Junction Photodiode," Proc. IRE, *50,* No. 8 (August 1962), p. 1823.
8. Kibler, L. U., "A High-Speed Point Contact Photodiode," Proc. IRE, *50,* No. 8 (August 1962), pp. 1834–1835.
9. Sharpless, W. M., "Cartridge-Type Point-Contact Photodiode," Proc. IEEE, *52,* No. 2 (February 1964), pp. 207–208.
10. DiDomenico, M., Jr., Sharpless, W. M., and McNicol, J. J., "High-Speed Photodetection in Germanium and Silicon Cartridge-Type Point-Contact Photodiodes," Appl. Opt., *4* (June 1965), pp. 677–682.

11. Schneider, M. V., "Schottky Barrier Photodiodes with Antireflection Coating," B.S.T.J., *45,* No. 9 (November 1966), pp. 1611–1638.
12. Penfield, P., Jr. and Sawyer, D. E., "Photoparametric Amplifier," Proc. IEEE, *53,* No. 4 (April 1965), pp. 340–347.
13. Prabhu, V. K., "A Theory of Broadband Matching for Optical Heterodyne Receivers," Scheduled for Appl. Opt.
14. Haus, H. A., et al., "Representation of Noise in Linear Twoports," Proc. IRE, *48,* No. 1 (January 1960), pp. 69–74.
15. Bode, H. W., Network Analysis and Feedback Amplifier Design, D. Van Nostrand and Co., Inc., Princeton, N. J., 1945.
16. Fano, R. M., "Theoretical Limitations on the Broadband Matching of Arbitrary Impedances," J. Franklin, Inst. 249 (January–February 1950), pp. 57–83, 139–154.
17. Youla, D. C., "A New Theory of Broad-Band Matching," IEEE Trans. Circuit Theory, *CT-11* (March 1964), pp. 33–50.
18. Chan, Y. T. and Kuh, E. S., "A General Matching Theory and Its Application to Tunnel Diode Amplifiers," IEEE Trans. Circuit Theory, *CT-13* (March 1966), pp. 6–17.
19. Prabhu, V. K., unpublished work.
20. Fried, D. L., "Optical Heterodyne Detection of an Atmospherically Distorted Signal Wavefront," Proc. IEEE, *55,* No. 1 (January 1967), pp. 57–67.
21. Fried, D. L. and Seidman, J. B., "Heterodyne and Photon-Counting Receivers for Optical Communications," Appl. Optics, *6,* No. 2 (February 1967), pp. 245–250.
22. Mandel, L., "Heterodyne Detection of a Weak Light Beam," J. Opt. Soc. Amer., *56,* No. 9 (September 1966), pp. 1200–1206.
23. Lucovsky, G., Lasser, M. E., and Emmons, R. B., "Coherent Light Detection in Solid-State Photodiodes," Proc. IEEE, *51,* No. 1 (January 1963), pp. 166–172.
24. Champagne, B. P., "Optimization of Optical Systems," Applied Optics, *5,* No. 11 (November 1966), pp. 1843–1845.
25. Oliver, B. M., "Thermal and Quantum Noise," Proc. IEEE, *53,* No. 5 (May 1965), pp. 436–454.
26. Llewellyn, F. B., "Some Fundamental Properties of Transmission Systems," Proc. IRE, *40,* No. 3 (March 1952), pp. 271–283.
27. Youla, D. C., "A Note on the Stability of Linear Nonreciprocal *N*-ports," Proc. IRE, *48* (1960), pp. 121–122.
28. Ku, W. H., "Unilateral Gain and Stability Criterion of Active Two-Ports in Terms of Scattering Parameters," Proc. IEEE, *54,* No. 11 (November 1966), pp. 1617–1618.
29. Haus, H. A. and Adler, R. B., *Circuit Theory of Linear Noisy Networks,* Cambridge, Mass.: The Technology Press, and New York: John Wiley and Sons, Inc., 1959.
30. Fukui, H., "Available Power Gain, Noise Figure, and Noise Measure of Two-Ports and Their Graphical Representation," IEEE Trans. Circuit Theory, *CT-13,* No. 2 (June 1966), pp. 137–141.
31. Friis, H. T., "Noise Figure of Radio Receivers," Proc. IRE, *32,* No. 7 (July 1944), pp. 419–422.
32. Fränz, K., "Messung der Empfängerempfindlichkeit bei kurzen elektrischen wellen," Z. Elect. Electroak, *59* (1942), pp. 105–111.
33. Carlin, H. J. and Giordano, A. B., *Network Theory,* Englewood Cliffs, N. J.: Prentice-Hall, Inc., 1964.
34. Guillemin, E. A., "Synthesis of Passive Networks," New York: John Wiley and Sons, Inc., 1962.
35. Weinberg, L., *Network Analysis and Synthesis,* New York: McGraw-Hill Book Co., Inc., 1962.
36. Schneider, M. V., private communication.
37. Balabanian, N., *Network Synthesis,* Englewood Cliffs, N. J.: Prentice-Hall, Inc., 1958.
38. Nielsen, E. G., "Behavior of Noise Figure in Junction Transistors," Proc. IRE, *45,* No. 7 (July 1957), pp. 957–963.
39. Prabhu, V. K., unpublished work.