The Stream Guide, a Simple, Low-Loss Optical Guiding Medium

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Heat generated by viscous friction in a high speed gas flow in a pipe causes a transverse refractive index variation that can continuously focus a light beam passing through this pipe. The comparatively simple construction, together with the low losses inherent in gaseous guiding media, makes the stream guide attractive as an optical beam waveguide. This article presents theoretical and experimental results.

In thermal gas lenses, temperature gradients and ensuing refractive index variations are caused by heat transfer from an external heat source.[1] The tubular gas lens power is supplied by resistive heating; the figure of merit, that is, the focusing strength per unit of applied power is a criterion for its efficiency.[2] The efficiency may be improved by using Peltier heat as it has recently been suggested in connection with the proposal for an alternating gradient, thermo-electric beam waveguide.[3] In still another way, heat is generated by viscous dissipation which exists in a high-speed gas stream in a pipe. The temperature rise creates a refractive index variation capable of continuously focusing a light beam passing through this pipe.

The adiabatic flow of a compressible fluid is treated in the literature.[4, 5] The axial density variation resulting from frictional pressure loss and the transverse density variation caused by viscous dissipation, together with the assumption of a parabolic transverse velocity and temperature profile, lead, after simplification, to the following expression for the refractive index $n(r, z)$:

$$n(r, z) = 1 - \frac{r^2}{a^2}(n_0 - 1)\frac{\kappa - 1}{2} R \frac{p_1}{p_0} M_1 M(z). \tag{1}$$

In this equation, a is the radius of the pipe, n_0 the refractive index at atmospheric pressure p_0, κ the ratio of the specific heats (c_p/c_v), R ($\simeq$0.90) the so-called recovery factor,* p_1, M_1 and $M(z)$ the pressure

* See p. 211 of Ref. 4.

and the Mach number at the beginning and along the pipe, respectively.

Provided that the absolute velocity is small compared with the velocity of sound, and that simultaneously the relative change of the velocity along the guide also is kept small, it can be shown that the eikonal equation yields as ray path $r(z)$:

$$r(z) = (1 + \beta z)^{\frac{1}{2}}\left[C_1 J_{\frac{1}{3}}\left(\frac{2}{3}\frac{\alpha}{\beta}(1 + \beta z)^{\frac{3}{2}}\right) + C_2 N_{\frac{1}{3}}\left(\frac{2}{3}\frac{\alpha}{\beta}(1 + \beta z)^{\frac{3}{2}}\right)\right] \qquad (2)$$

with

$$\alpha = \frac{M_1}{a}\left[(n_0 - 1)(\kappa - 1)R\frac{p_1}{p_0}\right]^{\frac{1}{2}} \quad \text{and} \quad \beta = \frac{f\kappa}{a} M_1^2$$

whereby $J_{1/3}(x)$ and $N_{1/3}(x)$ are the Bessel and Neumann functions of fractional order 1/3, C_1 and C_2 are integration constants and f represents the average friction coefficient. Equation 2 describes an undulating trace with steadily decreasing amplitude and period as the result of the ever increasing velocity and, consequently, focusing strength downstream.

If one considers the flow parameters to be locally independent of z one obtains for the period of the beam size variation $L_b(z)$ (with $R = 0.90$; see Ref. 6):

$$L_b(z) = 3.3a\left[(n_0 - 1)(\kappa - 1)M_1 M(z)\frac{p_1}{p_0}\right]^{-\frac{1}{2}}. \qquad (3)$$

The $1/e$ half width of the beam which does not change its size in this pseudouniform guide is given by $w_o = (\lambda L_b(z))^{1/2}/\pi$ (λ = light wavelength). One notices that small spot sizes can be achieved by choosing a monatomic gas with high refractive index, by selecting a small pipe diameter and by increasing either the initial velocity or the pressure, or both. However, a characteristic feature of frictional flow is a comparatively large axial velocity gradient for high initial velocities, a fact which directly affects the total pipe length employable for a specified exit speed.

The experimental setup is shown in Fig. 1. The gas was injected into the pipe through a large number of radial jets. The gas flow and pressure were adjusted in such a way that a light beam which entered the guide collimated ($w_{1/e} \simeq 0.9$ mm) left it either collimated or with a focal point coinciding with the end of the pipe. Thus the total pipe length represented an even or odd multiple of $L_b/2$, respectively. For a pipe length of 3.07m and at air flow rates between 40

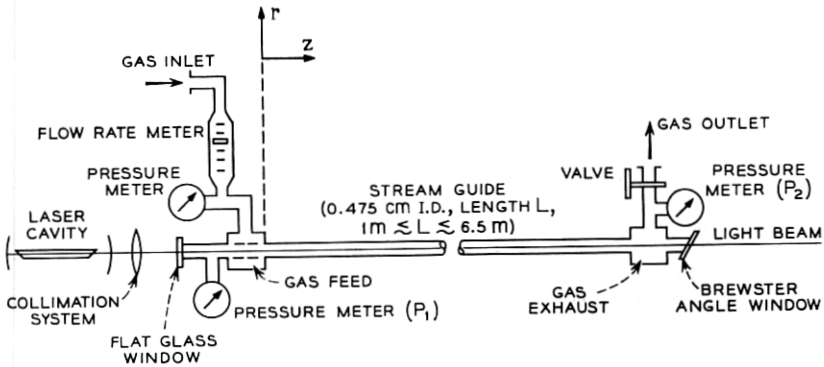

Fig. 1—Experimental setup for measurements with the stream guide.

to 55 liters per minute ($23.2 \leq p_1 \leq 64.7$ psi, $p_2 = 14.7$ psi) the averaged period of the beam size variation ranged between approximately 6.14 and 1.53m. Typical values for the Reynolds number were in the order of 40,000.

In Fig. 2 the pressure and flow rate is varied in such a fashion as to keep the averaged focusing strength constant (L_b = length of pipe L). For a given focusing strength, by increasing the pressure the required velocity decreases and the final velocity approaches more and more the initial velocity. The use of argon instead of air increased the focusing strength as expected. The corresponding theoretical periods of the beam size variation at the beginning and the end of the pipe as computed via the experimental pressure and flow rate from (3) are also shown (L_{b_1} and L_{b_2}, respectively). Their average value L_{av} is roughly constant and the results for both air and argon are in close agreement. However, for $L = 6.42$m, L_{av} was approximately 36 percent lower than the experimental value. In general, different pipe lengths and varying operating conditions resulted in values of L_{av} which varied around the experimental results.

The axial velocity distribution depends largely on the friction coefficient f. Its value may be determined theoretically, but owing to its involved dependence on various factors, an experimental evaluation is more reliable. From Fig. 2 follows that f is smaller for smaller absolute velocities. Thus, for comparable focusing action, an increase in pressure not only results in smaller velocity gradients because of a reduced initial velocity but also because of a reduced friction coefficient.

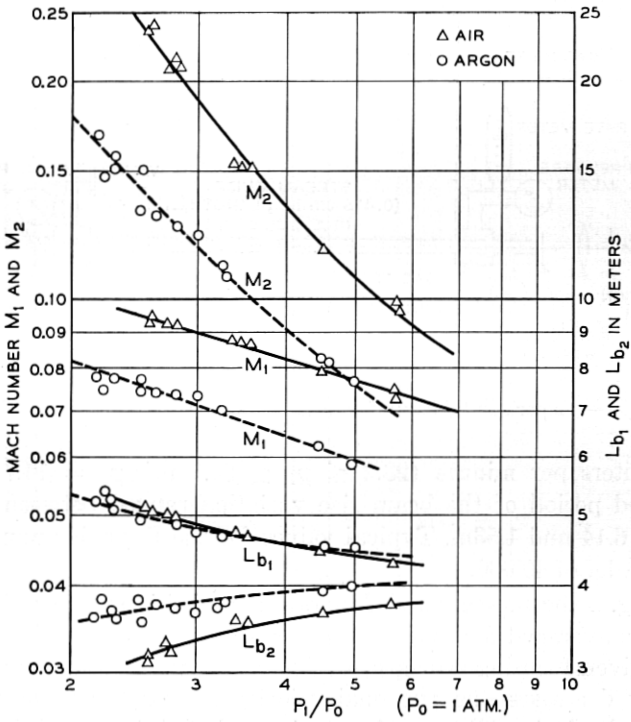

Fig. 2—Experimental Mach number and theoretical period of the beam size variation L_{b_1} and L_{b_2} (according to equation 3) at the beginning and the end of the pipe as a function of the initial pressure p_1/p_0 for air and argon (temperature 22°C). The experimental period of the beam size variation was constant and corresponded to the total length of the pipe (6.42m).

The eventual application of the stream guide for long distance communication requires that the pressure be reestablished periodically, whereby the period is expected to be in the range of a few meters. This can be accomplished by alternately injecting gas from a high pressure supply line and exhausting it into a lower pressure pipe running parallel to the guide.

Focusing properties which are close to those of a uniform guide can be achieved if the stream guide is operated at high absolute pressures together with small pressure drops along the guide. Rayleigh scattering losses impose an upper pressure limit of approximately 500 psi for a maximum transmission loss of 1 dB per km (for air) if losses resulting from turbulent scattering are neglected. Also, eventual mode conversion losses caused by a non-parabolic refractive index variation are not taken into account.

Whether the stream guide has actual applications as a simple, continuous guiding medium depends on the results of further studies of its optical properties, especially the effects of velocity gradients and turbulence on beam fluctuations and power loss.

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