On Permutation Switching Networks

By A. E. JOEL, JR.

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Rearrangeable switching networks are considered as permutation generators. Two-by-two network arrays may be implemented in these networks using a β reversing element. Nested tree arrays of these elements may be used to synthesize rearrangeable switching networks which appear minimal. Multistage network arrays of these elements may be implemented within a single coordinate device.

I. INTRODUCTION

Two-sided communication switching networks with an equal number of input and output terminals (N) have been looked upon as permutation generators of order N.[1,2] Alteration of the permutation being produced by any state of the network configuration may be achieved by rearranging or changing the state of the switching network. By limiting the switches in each stage to binary action devices certain economies have been achieved.[3] This paper describes additional savings, particularly in the number of required operating elements. Some further savings are achieved by introducing break contacts into switching networks.

II. THE β ELEMENT

A small space division switching network is a two-by-two array of crosspoints (see Fig. 1). Each of these crosspoints is generally considered as either electromechanical make-contacts or electronic two-state devices. Confining our attention to make-contacts, they may be actuated by a coordinate mechanism, as in a crossbar switch, or as individual relays.

Usually in switching networks the crosspoints are controlled so that only one may be operated in any column and row. Such use of 2×2 arrays confine the maximum number of states from the 16 possible states to those useful in switching networks. The 16 states

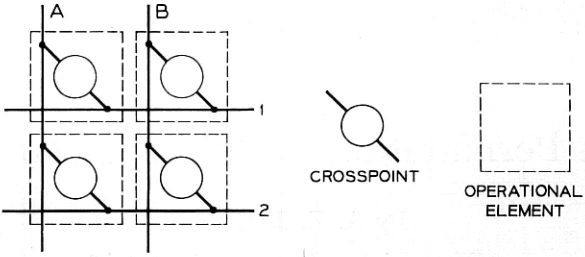

Fig. 1 — 2 × 2 crosspoint array.

include double connections (two operated in the same column or row) and the zero state. The zero state is useful if the crosspoint devices require holding power. When this state occurs it introduces a vacuous partition into the permutation of the network. Actually the only useful states, particularly when viewed from the standpoint of rearrangeable networks, are the two shown in Fig. 1: (A→1, B→2), and (A→2, B→1). In these rearrangeable networks the vacuous states are eliminated.

It is proposed that the equivalent of 2 × 2 arrays of four make-contacts and associated electromagnets be achieved by the simple expedient of two transfer contacts connected in the so-called reversing configuration (see Fig. 2a). Using this two-state or binary (β) element only two moving contacts and one electromagnet are required. The equivalent of β elements may be achieved in many other ways without break contacts. Figs. 2b and 2c show a β array made from the array of Fig. 1 using only 2 operating elements.

III. THE USE OF β ELEMENTS IN SYMMETRICAL REARRANGEABLE NETWORKS

Methods have been described by V. E. Beneš[3] for using 2 × 2 arrays to form rearrangeable link networks. Improvements result when applying β elements to these networks. For example, the equivalent of a 4 × 4 rearrangeable switching network can be made of three stages of 2 × 2 switches or a total of six β elements (Fig. 3). Here 6 β elements replace 16 crosspoints and their operating magnets. An 8 × 8 switch of 64 crosspoints may be replaced by 20 β arrays in 5 stages (see Fig. 4).

For extending to larger networks, the method taught by Beneš may be applied, replacing the middle 4 × 4 stage by the three symmetric stages of β elements. A rearrangeable network with N terminals

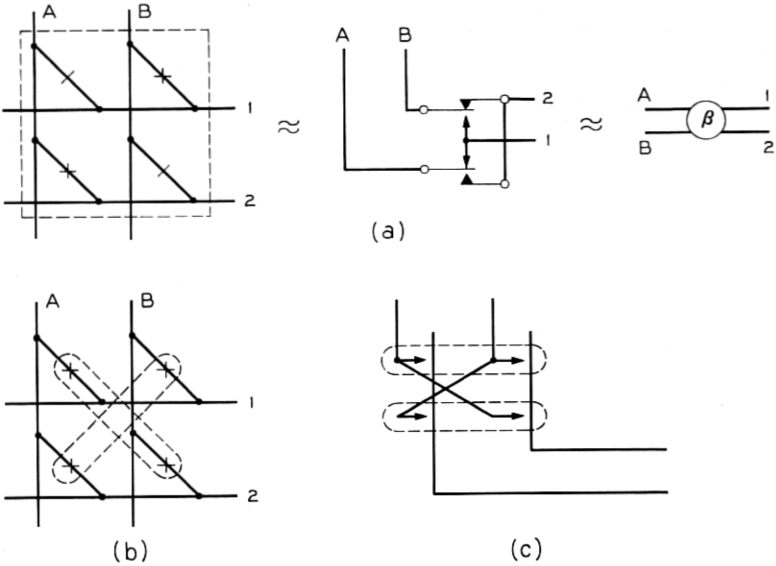

Fig. 2 — Binary β element. (a) One operator. (b) Two operators.

on each side will comprise S stages of these elements with $N/2$ elements per stage where

$$S = 2 \log_2 N - 1.$$

The total β elements per input terminal is

$$\frac{N}{2}(2 \log_2 N - 1)\frac{1}{N} = \log_2 N - \tfrac{1}{2}.$$

For a network with $N = 1024$ terminals per side (see Fig. 5), then 9.5 β elements are used in the network per input terminal. This is considerably less than the 32 coordinate array crosspoints per input required in the network described by Beneš.

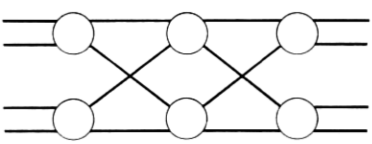

Fig. 3 — Three stage 4×4 network of β elements.

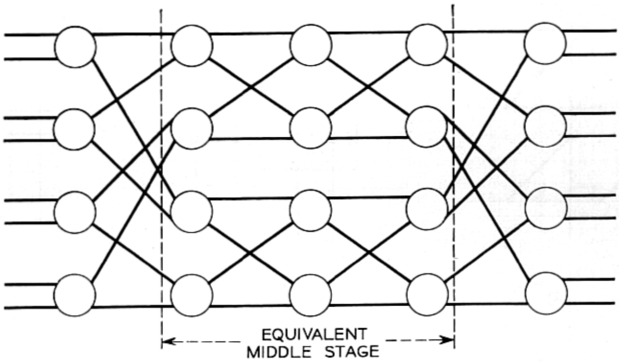

Fig. 4 — Five stage 8 × 8 Beneš type network of β elements.

IV. NESTED TREE TYPE PERMUTATION NETWORKS

The 4 × 4 symmetric network may be further simplified by omitting one β element to obtain a network (see Fig. 6), where each input reaches each output through one or more selecting trees of β elements. The β elements in the first stage is used by inputs 2 and 3 to select one of two trees in stages 2 and 3, either of which can reach a given output depending upon the state of the β element to which it is connected.

This 4 × 4 network (Fig. 6) is a trivial case of the "nested trees" concept. The stages 2 and 3 are two superimposed trees with stage 1 a one-stage selecting tree.

The concept is better illustrated with higher order networks. The 8 × 8 array in Fig. 7 uses 17 β elements with 1, 2, and 3 stage trees in a total of 6 stages. This compares very favorably with a single stage 8 × 8 array requiring only about one quarter of the number of operating magnets, about one half of the number of moving contacts, and only 4 more contacts. A 16 × 16 array with 49 β elements would have 1, 2, 3, and 4 stage trees in a total of 10 stages.

In general with nested trees the first stage connected with the outputs (or inputs) has

$$s_1 = \log_2 N$$

stages of $N/2$ β elements and is the highest order tree group. Then the remaining stages form lower order trees, which might be con-

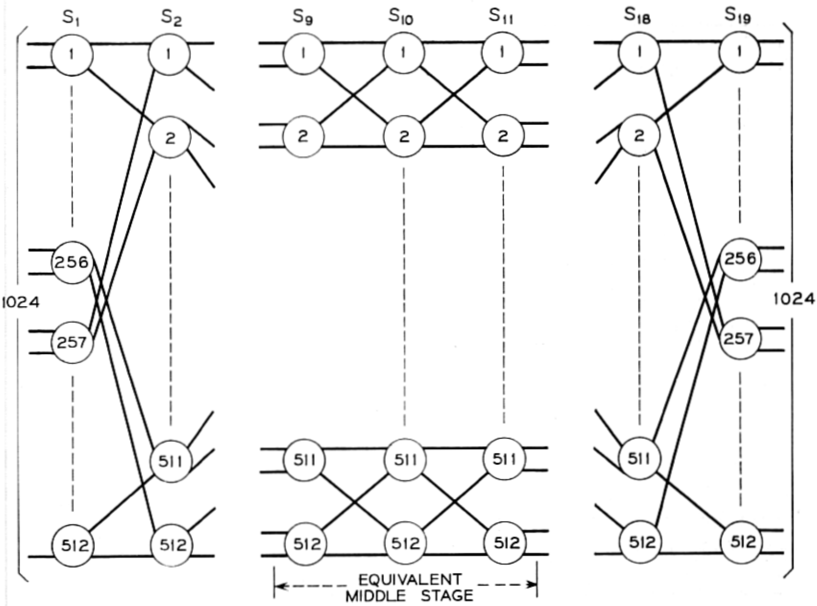

Fig. 5 — Beneš type 1024 terminal network of β elements.

sidered in different spatial dimensions or disjunctive nested parts of the network.

The total β elements in such a network is

$$\sum_{s=1}^{\log_2 N} \frac{N}{2^s} \log_2 \frac{N}{2^{s-1}}.$$

This function simplifies to

$$N \log_2 N - N + 1.$$

Derived in another manner this same formula has been obtained by Goldstein and Leibholz[4]

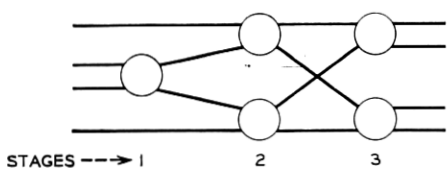

Fig. 6 — Three stage "tree type" 4 × 4 network of β elements.

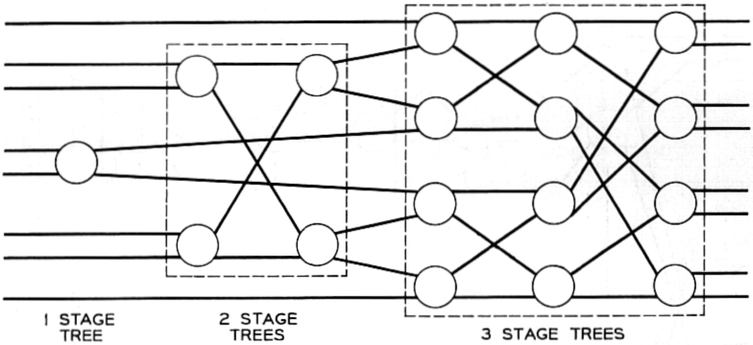

Fig. 7 — Six stage "nested tree" type 8 × 8 network of β elements.

Figure 8 shows the derivation of a nested tree network for $N = 32$ where the different order nested trees are shown as boxes to illustrate the various parts of the network. Figure 9 illustrates the spatial concept for the same network.

The total number of stages in such networks may be defined as the total number of stages in all trees. Thus,

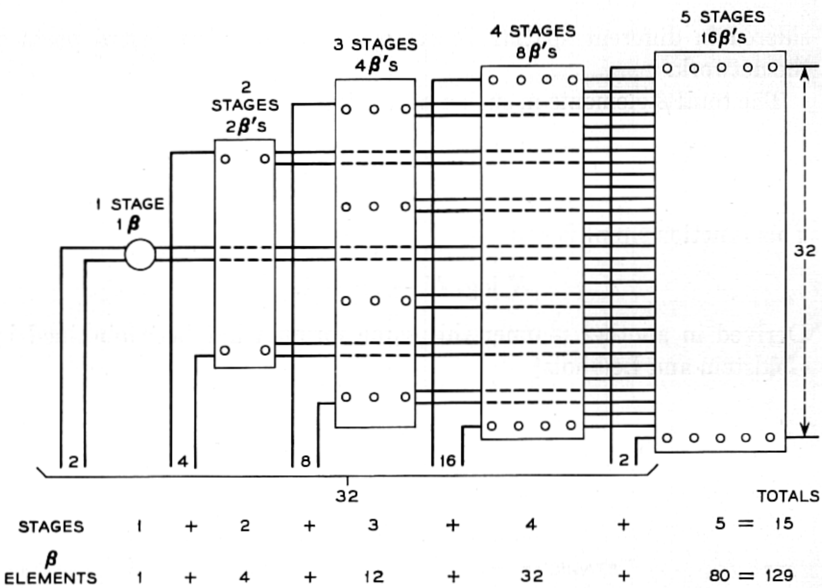

Fig. 8 — Schematic of "nested tree" type 32 × 32 network of β elements.

$$S = \log_2 N\left(\frac{\log_2 N + 1}{2}\right).$$

The number of β elements per input is, for large numbers, $\log_2 N - 1$. This is an improvement of 0.5 β element per input compared with the symmetric network approach described in the previous section.

To double the size of a nested network requires the addition of new higher order trees and doubling the input capacity of the previous network by converting the former output trees to input trees. Increasing the size by this means is much easier than the reassignment of links required when symmetric type networks grow.

V. SERIAL PERMUTATION NETWORKS

The nested tree type link network is nonsymmetric. Other nonsymmetric arrangements of reversing devices have been proposed to

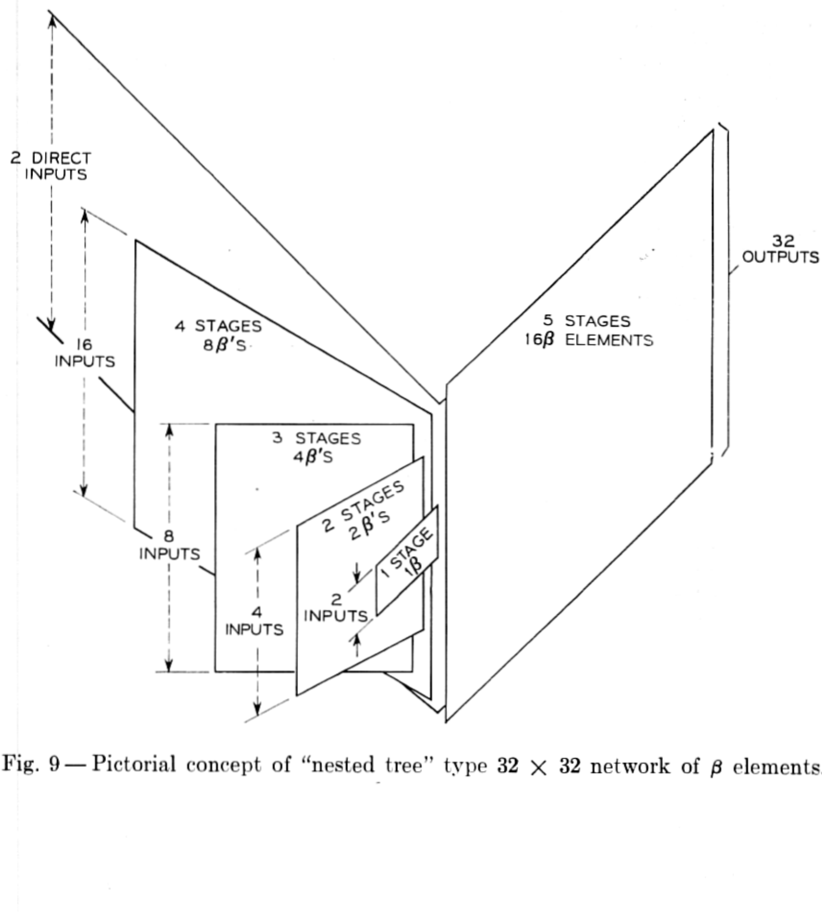

Fig. 9 — Pictorial concept of "nested tree" type 32 × 32 network of β elements.

form permutation networks.[5] These networks are serial in character as contrasted to the link type.

As shown in Fig. 10, assuming a permutation generator of P_{N-1}, then $(N - 1)$ β elements will insert the N^{th} lead into the permutation to produce P_N. The total β arrays per input for N inputs would be

$$\frac{N-1}{2}.$$

For $N = 4$ a complete network of 3 stages derived in this manner is shown in Fig. 11. For $N > 4$, serial networks of this type require more β elements than the link type. However, these networks are more readily adaptable to networks where N is small, where N is not a power of 2, and to obtain network growth in one dimension. However, growth is expensive since when N inputs are present the addition of another input requires $(N - 1)$ β elements.

Since the $(N - 1)$ β elements are operated in only $(N - 1)$ combinations to achieve the desired output, then savings may be made by using combinatorial arrangements. Combinations of $\log_2 N - 1$ elements are achieved by trees of $N - 1$ stages where the several β elements of a stage may be actuated by the same magnet. Thus, for the third stage of Fig. 11, we have Fig. 12, where the two of the β elements are actuated by the same magnet.

For $N = 8$, although 33 β elements would be required (almost twice the number required for a link type network) one less magnet would be required, 17 instead of 18. As N increases the magnet savings become large, but generally it is impractical to actuate the large number of β elements required for the higher order stages.[6] For $N = 1024$, for example, the last stage would require 10 relays, 3 of which would actuate 128, 256, and 512 β elements, respectively.

The smaller the number of paths through an array or switch of a

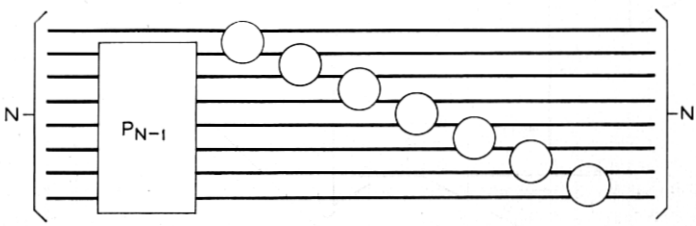

Fig. 10 — Serial permutation network extension.

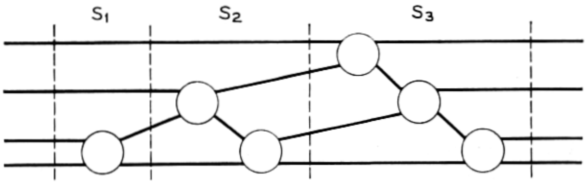

Fig. 11 — Three stage serial 4 × 4 network of β array.

switching stage, the fewer existing elements of a permutation are disturbed by rearranging. The smaller the switches, the greater the number of stages required. However, the number of stages does not increase as rapidly as the switch size decreases. The β array is ideal for rearrangeable switching networks, and the β array is a contribution to keeping the number of operating elements (magnets) small.

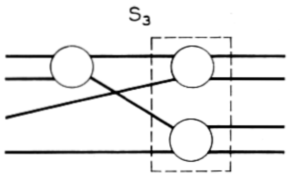

Fig. 12 — Stage 3 of serial 4 × 4 network combining β elements.

VI. A NETWORK DEVICE

The β elements in the various arrays just described may be thought of as crosspoints in a coordinate device. For example, the 8 × 8 network has 4 rows and 5 columns of β elements. A 4 × 5 coordinate device could be built which has 4 select and 5 hold magnets or operating elements, like a crossbar switch. First the select elements would be operated in combination to prepare to change the state of β elements in a particular column. Then the column operating element would reset the β elements in this combination. In five such steps the entire 8 × 8 network would be reconfigured. For such a large array as a 1024 × 1024 symmetrical network 19 steps would be required.

Crossbar switches with vertical multiples divided such as in Fig. 2c, could be used as network devices in this sense. The interstage link wiring would be between verticals. A 128 × 128 network requires 769 β elements. A 10 horizontal crossbar switch provides 5 β elements per vertical so that 16 10 × 10 switches would implement this size rearrangeable network. This compares with 30 10-vertical crossbar switches for the more familiar three-stage network of 10 × 10 switches for a 100 × 100 rearrangeable network.

Nonblocking networks without interruptions caused by rearrangeability can be made from rearrangable networks by extension of switch sizes to a full Clos network. For a 100 × 100 network, 20 10 × 19 and 19 10 × 10 switches are required. Nonblocking without interruption may also be achieved by alternately using two permutation networks connected in parallel. For the 128 × 128 network only 36 10 × 10 switches with divided vertical multiples would be required in this configuration.

VII. SUMMARY

The β element reduces the number of devices required to implement rearrangeable switching networks. In particular, networks consisting of nested trees of these elements produce minimal permutation switching networks. Crossbar switches or similar coordinate devices may be used to implement networks made of β arrays. Two permutation networks in parallel are equivalent of a nonblocking switching network without rearrangement.

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