Some Considerations of Stability in Lossy Varactor Harmonic Generators

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Explicit expressions are derived for the scattering parameters which relate small-signal fluctuations in a lossy varactor harmonic generator of order $N = 2^n$, *n an integer. The effect of losses on the stability of the mutiplier is then studied. The very important particular case is then examined in which all the losses occur in the series resistance of the varactor diode, and it is shown that absolute stability is obtained provided the efficiency* η_t *of the multiplier* $\ll N^{-1}$, *because of the particular distribution of the losses at various carrier frequencies. Therefore, the conclusion is reached that in most cases of practical interest restrictions have to be placed on the available circuit configurations to prevent instability of the multiplier.*

I. INTRODUCTION

A serious limitation to efficient wideband harmonic generation with varactor diodes is that instability in the multiplier might cause the generation of spurious tones.[1] It is the purpose of this paper to study the effect of losses on stability of abrupt-junction varactor frequency multipliers of order $N = 2^n = 2, 4$, and so on, with the minimum number of idlers.

The type of instability considered here is the one discussed in Refs. 2, 3, and 4. It produces undesired low-frequency fluctuations in the amplitude and phase of the output harmonic and is caused by the time-varying elastance of the varactor, which is potentially unstable with respect to phase perturbations.

The stability conditions of lossless abrupt-junction varactor multipliers have already been extensively discussed elsewhere in Refs. 3 and 4. More precisely, these works have shown that, in the absence of any losses in the varactor diode, the frequency characteristics of the input, output, and idler circuits must satisfy certain restrictions in order that the multiplier be stable. The main objective of this paper is to determine the amount of loss that the multiplier must have in order to be absolutely stable, that is, stable for arbitrary linear passive input, output, and idler circuits.

First we show that the over-all multiplier efficiency η_t can be expressed as the product of the efficiencies of the input, output, and idler circuits; that is

$$\eta_t = \eta_1 \times \eta_2 \times \cdots \eta_N ,$$

where η_t represents the ratio of the power P_L delivered to the output at carrier frequency $N\omega_o$ to the power supplied by the input pump at carrier frequency ω_o. The partial efficiency η_r is the efficiency of the circuit at the carrier frequency $r\omega_o$, or $1 - \eta_r$ represents the ratio of the power lost at $r\omega_o$ to the sum of P_L and of the total power lost at the frequencies $r\omega_o$, $2r\omega_o$, $\cdots$, $N\omega_o$.

Next we show that the behavior of the multiplier with respect to small amplitude and phase fluctuations is related in a simple way to the efficiencies η_1, η_2, and so on. For instance, in the case of very slow fluctuations, the *PM* scattering parameters of a doubler are given by the matrix,

$$\begin{bmatrix} 0 & -\eta_1\eta_2 \\ 2 & 1 - 2\eta_2(1 - \eta_1) \end{bmatrix}.$$

In the last two sections we examine the conditions of absolute stability and show that the multiplier may become unstable for some circuit conditions if

$$\eta_t > 1/N.$$

If, on the other hand,

$$\eta_t < 1/N,$$

then the multiplier is absolutely stable if and only if

$$\eta_r < 50\%, \quad \text{for} \quad r = 1, \cdots, N/2.$$

Finally, the important particular case is considered in which all the losses of the multiplier occur in the series resistance of the varactor. It is found that in this case absolute stability is obtained if and only if

$$\frac{\omega_o}{\omega_c} > 0.06, \qquad N = 2,$$

$$\frac{\omega_o}{\omega_c} > 0.1, \qquad N > 2,$$

where ω_c is the cutoff frequency of the varactor. If these conditions are satisfied, then the efficiency of the multiplier is found to be so low that the conclusion is reached that in most cases of practical interest restrictions must be placed on the available circuit configuration in order to obtain stability.

II. SCATTERING RELATIONS

Nominally driven abrupt-junction varactor frequency multipliers of order 2^n come under the general class of pumped nonlinear systems, and the general method presented in Ref. 5 can be used for such systems to obtain the scattering parameters which relate small-signal fluctuations that may be present at various points in the system.* These small-signal fluctuations are assumed to be small and they are at frequencies close to the carriers.

The varactor model that we use is shown in Fig. 1. It is a variable

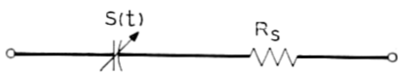

Fig. 1 — Varactor model.

capacitance in series with a resistance R_s. The multiplier has the minimum number of idlers. The linear passive circuits used in the multiplier as input, output, and idler terminations are assumed to produce no amplitude to phase or phase to amplitude conversion.† If input, output, and all idler circuits are tuned,‡ it can be shown[4–6] that the small-signal terminal relations of a harmonic generator can be expressed in the form (see Fig. 2)

$$\begin{bmatrix} (m_r)_1 \\ (m_r)_{2^n} \\ (\theta_r)_1 \\ (\theta_r)_{2^n} \end{bmatrix} = \left[\begin{array}{c|c} \underline{S}_{aa} & \underline{0} \\ \hline \underline{S}_{pa} & \underline{S}_{pp} \end{array}\right] \begin{bmatrix} (m_i)_1 \\ (m_i)_{2^n} \\ (\theta_i)_1 \\ (\theta_i)_{2^n} \end{bmatrix} \tag{1}$$

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* Notation in this paper is identical to that in Refs. 4 and 5. Details of these notations are not given in this paper for the sake of brevity.

† This condition is satisfied by circuits usually used with multipliers.[4]

‡ Tuning of idlers, and input and output circuits usually gives near optimum efficiency for the multipliers. (See Refs. 7, 8, and 9.)

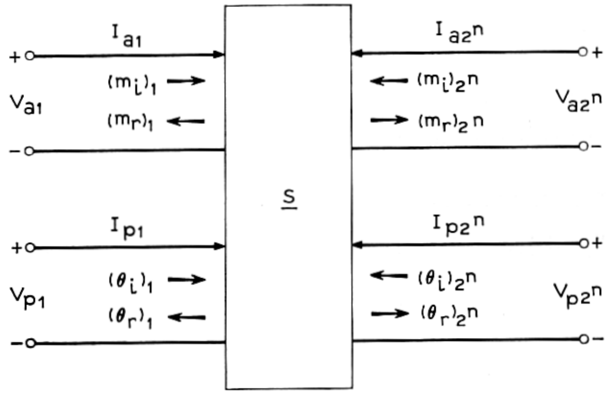

Fig. 2 — Small-signal terminal behavior of a harmonic generator of order 2^n. m is the AM index and θ is the PM index of the multiplier.

or*

$$\mathbf{b} = \underset{\sim}{S}\mathbf{a} \tag{2}$$

where $\underset{\sim}{S}$ is the scattering matrix of the multiplier and $(m_i)_j$ is the incident AM index at carrier frequency $j\omega_o$, $(\theta_r)_k$ is the reflected PM index at carrier frequency $k\omega_o$, and so on. The small-signal fluctuations in the vicinity of carrier frequency $k\omega_o$ are assumed to be at $k\omega_o \pm \omega$, $\omega < \omega_o/2$.

It can also be shown[4] that the AM scattering matrix $\underset{\sim}{S}_{aa}$ and the PM scattering matrix $\underset{\sim}{S}_{pp}$ are independent of the bias source impedance Z_o, and that the stability of the multiplier is completely determined by $\underset{\sim}{S}_{aa}$ and $\underset{\sim}{S}_{pp}$. It can also be shown[4] that a multiplier of order 2^n is stable with respect to its AM fluctuations for all input, output, and idler terminations. In this paper we shall, therefore, obtain an expression for $\underset{\sim}{S}_{pp}$ for a varactor harmonic generator of order 2^n with the minimum number of idlers† and consider its PM stability.

An abrupt-junction varactor multiplier of order 2^n with the least number of idlers can be shown[3,5] to be completely equivalent to a cascade of n lossless doublers‡ as shown in Fig. 3. Z_{2^k}, $0 \leqq k \leqq n$, is the termination impedance in the vicinity of carrier frequency $2^k\omega_o$.

* A column matrix is written in the form **a**, a matrix which is square is written as $\underset{\sim}{A}$, and a unit matrix of order n is written as $\underset{\sim}{1}_n$.

† Methods given in Ref. 5 can, in all cases, be used to obtain $\underset{\sim}{S}$ in equation (2).

‡ The conditions under which a multiplier of order $M_1 \times M_2$ is completely equivalent to a cascade of two multipliers of order M_1 and M_2 are given in Ref. 5.

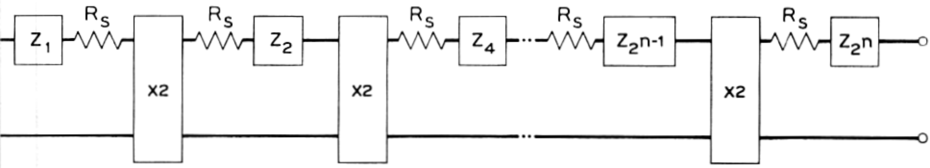

Fig. 3 — Equivalence of an abrupt-junction varactor multiplier of order 2^n to a chain of n doublers. R_s is the series resistance of the varactor diode.

Input and output circuits which are not shown in Fig. 4 can be any arbitrary linear passive circuits. For $\omega/\omega_o \ll 1$, it can be shown that the AM scattering matrix $\underset{\sim}{S}_{aa}$ and PM scattering matrix $\underset{\sim}{S}_{pp}$ of the k^{th} lossless doubler are given[5] by

$$\underset{\sim}{S}_{aa} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ 1 & 0 \end{bmatrix} \tag{3}$$

and

$$\underset{\sim}{S}_{pp} = \begin{bmatrix} 0 & -1 \\ 2 & 1 \end{bmatrix}. \tag{4}$$

Now let us consider the k^{th} lossless doubler. The "input impedance" $(R_{ok})_{\text{in}}$ and the "load impedance" $(R_{ok})_{\text{out}}$ of the lossless doubler are given by[5, 7]

$$(R_{ok})_{\text{in}} = \frac{|S_{2^k}|}{2^{k-1}\omega_o}, \qquad 1 \leqq k \leqq n \tag{5}$$

and

$$(R_{ok})_{\text{out}} = \frac{|S_{2^{k-1}}|^2}{2^{k+1}|S_{2^k}|\omega_o}, \qquad 1 \leqq k \leqq n. \tag{6}$$

Since all impedances are purely resistive, we can define partial efficiencies η_{2^i}'s by the relations

$$\eta_{2^k} = \frac{[R_{o(k+1)}]_{\text{in}}}{Z_{2^k} + R_s + [R_{o(k+1)}]_{\text{in}}}, \qquad 0 \leqq k \leqq n-1 \tag{7}$$

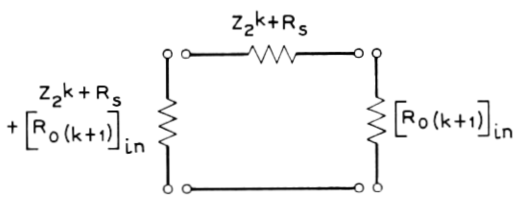

Fig. 4 — An interstage network used with the multiplier.

and

$$\eta_{2^n} = \frac{Z_L}{Z_{2^n} + R_s + Z_L} \tag{8}$$

where Z_L is the load resistance connected to the multiplier. We notice that η_{2^k}, $1 \leqq k \leqq n - 1$ is equal to the ratio of carrier power flowing into the input port of $(k + 1)^{\text{th}}$ doubler to that supplied by the k^{th} doubler, η_1 the ratio of carrier power supplied to the first doubler to the power supplied by the pump, and that η_{2^n} is the ratio of power dissipated in the load resistor Z_L to that supplied by the n^{th} doubler. The over-all efficiency η_t of the multiplier can, therefore, be written as

$$\eta_t = \prod_{r=0}^{n} \eta_{2^r} . \tag{9}$$

Consider Fig. 4. The scattering matrix of the $(k + 1)^{\text{th}}$ interstage network can be shown to be[5, 6, 10]

$$\begin{bmatrix} 0 & \eta_{2^k} \\ 1 & 1 - \eta_{2^k} \end{bmatrix} . \tag{10}$$

If $\omega/\omega_0 \ll 1$, we can then show from equations (4) and (10) that the PM scattering matrix $\underset{\sim}{S}_{pp}$ for the multiplier shown in Fig. 3 can be written as

$$\underset{\sim}{S}_{pp} = \begin{bmatrix} 0 & (-1)^n \eta_t \\ 2^n & 1 + \sum_{r=0}^{n-1} (-2)^{r+1} \eta_{2^n} \eta_{2^{n-1}} \cdots \eta_{2^{n-r}} - (-2)^n \eta_t \end{bmatrix} . \tag{11}$$

III. DERIVATION OF THE ABSOLUTE STABILITY CONDITIONS

First, consider the case of a doubler. The scattering matrix of a stage consisting of an ideal doubler with two resistances $R_{s,1}$ and $R_{s,2}$ connected* in series to the input and output ports, respectively, is:†

$$\begin{bmatrix} 0 & -\eta_1 \eta_2 \\ 2 & 1 - 2\eta_2(1 - \eta_1) \end{bmatrix} . \tag{12}$$

By means of standard techniques,[4] one obtains that absolute stability requires that‡

$$2\eta_1\eta_2 + | 1 - 2\eta_2 + 2\eta_1\eta_2 | \leqq 1 \tag{13}$$

* Notice that $R_{s,1} = Z_1 + R_s$ and $R_{s,2} = Z_2 + R_s$.

† Put $n = 1$, $N = 2$, $\eta_t = \eta_1\eta_2$ in equation (11). See also Appendix A for an alternate derivation of absolute stability conditions.

‡ Condition (13) requires that the magnitude of the largest output reflection that can be obtained when the termination of the input port is passive be less than unity.

which is satisfied if and only if

$$\eta_1 < 0.5. \tag{14}$$

It is important to notice that (14) shows that the output circuit losses do not have any effect on the absolute stability conditions of a doubler. This property will be used in the following discussion of the absolute stability of a multiplier of order $N > 2$.

Consider a multplier with $n > 1$. It can be shown that in this case it is necessary and sufficient that

$$\eta_1 < 0.5, \ \eta_2 < 0.5, \ \cdots, \ \eta_{N/2} < 0.5. \tag{15}$$

The fact that (15) guarantees absolute stability follows directly from (14) and the fact that a chain of absolutely stable stages is stable.

In order to show the necessity of (15), consider the k^{th} ideal doubler of Fig. 5, and the impedances presented to its input and output ports by the remaining part of the circuit. The impedance presented to the input port is given by

$$Z_{1,k} = Z_{2^{k-1}} + R_s + Z_{o,(k-1)} \,. \tag{16}$$

Since $Z_{o,(k-1)}$ approaches zero as the magnitude of $Z_{2^{k-2}}$ approaches infinity, $Z_{1,k}$ can have all complex values with nonnegative real part. Furthermore, the impedance $Z_{2,k}$ terminating the output port of the k^{th} ideal doubler has arbitrary imaginary part, because of the presence of Z_{2^k}. Therefore, since (14) shows that the absolute stability of a doubler does not depend on the real part of the output impedance, one concludes that it is necessary that

$$\eta_{2^k} < 0.5, \qquad 0 \leqq k \leqq n - 1, \tag{17}$$

if the chain is to be stable for all allowable values of $Z_{2^{k-1}}$, Z_{2^k}, and $Z_{2^{k+1}}$.

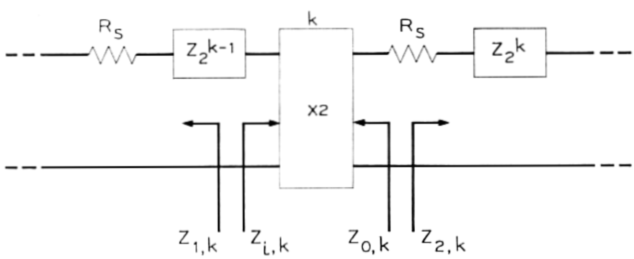

Fig. 5 — Lossless k^{th} doubler.

IV. ABSOLUTE STABILITY CONDITIONS

Equation (15) shows that a multiplier of order $N = 2^n$ of the type considered in this paper will become unstable for some frequency characteristics of the input, output, and idler circuits, if the efficiency η_t is greater than N^{-1}, that is, if

$$\eta_t > 1/N. \tag{18}$$

Therefore, if equation (17) is satisfied then the circuit must satisfy certain conditions such as those derived in Refs. 2, 3, and 4, in order that the multiplier be stable. If, on the other hand,

$$\eta_t < 1/N, \tag{19}$$

then (15) shows that the multiplier will be stable for all circuit conditions if and only if the efficiencies of the input and idler circuits are all less than 50 per cent.

At this point the particular case

$$Z_{2^k} = 0, \qquad 0 \leqq k \leqq n \tag{20}$$

deserves special attention. This represents in fact the important case in which all the losses occur in the series resistance R_s of the varactor. It will be assumed that the output load has the particular value which gives maximum efficiency.[7]

For the absolute PM stability of a doubler, equation (15) requires that

$$\eta_1 < 0.5. \tag{21}$$

We can show* that this condition can only be satisfied if and only if the overall efficiency

$$\eta_t < 36\%. \tag{22}$$

In the case of a quadrupler, the condition of absolute stability requires that

$$\eta_1 < 0.5 \tag{23}$$

and

$$\eta_2 < 0.5. \tag{24}$$

We can show† that equations (23) and (24) can be satisfied if and

* From Ref. 7, p. 331, $\eta_1 < 0.5$ for $\omega_o/\omega_c > 0.06$. For this value of ω_o/ω_c, $\eta_t <$ 36 percent.

† See Ref. 7, pp. 364–365.

only if

$$\eta_t < 0.7\%. \tag{25}$$

It therefore follows that for absolute stability of multipliers of order 2^n, it is necessary that

$$\eta_t \ll 2^{-n}. \tag{26}$$

Thus, if (15) is satisfied then the multiplier is so inefficient that it becomes of little practical interest. Therefore one concludes that, if all the losses occur in the series resistance of the varactor, in most cases of practical interest the question of stability cannot be neglected and the frequency characteristics of the input, output, and idler circuits have to satisfy certain restrictions (such as those given in Refs. 2, 3, and 4) in order to guarantee stability of the multiplier.

V. RESULTS AND CONCLUSIONS

Scattering relations for lossy abrupt-junction varactor harmonic generators are presented in this paper. Explicit expressions have been given for the PM scattering parameters of the multiplier in terms of partial efficiencies defined for the multiplier.

Absolute PM stability of 2^n multipliers is then considered. It is shown that the multiplier is stable if and only if

$$\eta_{2^j} < 0.5, \qquad 0 \leqq j \leqq n - 1. \tag{15}$$

We have also shown that a multiplier of order 2^n and having all the losses occur in the series resistance R_s of the varactor diode is absolutely stable if its efficiency is much lower than 2^{-n}, the inverse of order of multiplication of the multiplier.

The problem of stability is then of major importance in all high efficiency varactor multipliers and proper circuits should always be designed to assure at least the conditional stability of these multipliers.[2–4]

APPENDIX

PM Stability of 2^n Multipliers

Let us investigate by an alternate method absolute PM stability of 2^n multipliers for $n \geq 1$. Let us consider the k^{th} lossless doubler (see Fig. 5) in the equivalent circuit shown in Fig. 3.

If $Z_{i,k}$ and $Z_{o,k}$ are the phase terminating impedances of the k^{th} lossless doubler (see Fig. 5), we can derive from equations (4) through (6) that

$$Z_{o,k} = R_s \left\{ 1 + \frac{1}{2} \frac{\frac{m_{2^{k-1}}^2}{2^{2(k-1)}} \left(\frac{\omega_c}{\omega_o} \right)^2}{[Z_{o,(k-1)}/R_s] + [Z_{2^{k-1}}/R_s] + 1 - \frac{1}{2^{k-1}} m_{2^k} \frac{\omega_c}{\omega_o}} \right\}$$

$$1 \leqq k \leqq n; \qquad (27)$$

and

$$Z_{i,k} = R_s \left\{ 1 - \frac{1}{2^{k-1}} m_{2^k} \frac{\omega_c}{\omega_o} + \frac{1}{2} \frac{\frac{m_{2^{k-1}}^2}{2^{2(k-1)}} \left(\frac{\omega_c}{\omega_o} \right)^2}{[Z_{i,(k+1)}/R_s] + [Z_{2^k}/R_s] + 1} \right\},$$

$$1 \leqq k \leqq n \qquad (28)$$

where m_k is the modulation ratio of the varactor at carrier frequency $k\omega_o$.

Since Z_{2^k}'s are all linear passive impedances, it is seen from eqs. (27) and (28) that the multiplier is absolutely stable with respect to PM fluctuations if and only if

$$\frac{m_{2^k}}{2^{k-1}} \left(\frac{\omega_c}{\omega_o} \right) < 1, \qquad 1 \leqq k \leqq n. \qquad (29)$$

If any of these conditions are not satisfied, the multiplier will become unstable for a certain set of Z_{2^k}'s.

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