Injection-Locked-Oscillator FM Receiver Analysis

By C. L. RUTHROFF

(Manuscript received April 18, 1968)

The major components of an injection-locked-oscillator FM receiver are a linear mixer and a van der Pol type of negative resistance oscillator in a phase-locked configuration. In this paper the nonlinear differential equation describing the receiver output is solved and explicit expressions obtained for the output signal, noise, and controlling second and third order distortions. The input signal carrier is both amplitude and frequency modulated. Signal-to-distortion ratios have been computed and are presented for the case of a noise modulated FM input signal. The results indicate that excellent performance may be expected of such a receiver.

I. INTRODUCTION

It is well known that a conventional phase-locked loop can be used as a frequency modulation receiver.[1–3] It is perhaps less well known that the locking performance of the phase-locked loop and the injection-locked oscillator are described by the same differential equation.[4] These two facts suggest that an FM receiver using an injection-locked oscillator is possible. It is. And such a receiver is described here.

The principle of operation of the two receivers is the same but there are important practical differences. The baseband bandwidth of the phase-locked-loop receiver is limited by delay in the feedback loop to frequencies of about 1 MHz. The baseband bandwidth of the injection-locked-oscillator FM receiver can be as large as half the locking bandwidth of the injection-locked oscillator. With existing solid state oscillators such as the tunnel diode, locking has been achieved with bandwidths in excess of 200 MHz,[5] indicating that operation with basebands of about 100 MHz is possible.

This type of receiver is not used in present day systems and there has been little or no interest in it for about 20 years.[6–10] Woodyard[7] is credited by Edson[10] with the first explicit receiver operating on the locking principle but the beginnings go back to Appleton in 1922.[6] Beers[8] and Bradley[9] reported excellent measured performance in configurations using vacuum tube oscillators; in the light of these results it is surprising that interest has flagged in recent years.

The existence of solid state oscillating devices such as the tunnel diode, the avalanche diode, and the Gunn diode is sufficient cause for renewed interest in injection-locked-oscillator FM receivers. They are especially attractive at the higher microwave and millimeter wave frequencies, where conventional receivers are difficult to build. This paper describes the principle of operation of a receiver configuration suitable for dominant mode transmission lines. Applications at optical frequencies are also of interest.

The distortion analysis which follows requires a mathematical description of the locking behavior of an oscillator. For this purpose the van der Pol sine wave oscillator model is used; there is abundant evidence in the literature that all oscillators which have nearly sinusoidal outputs are adequately described by the van der Pol model. The receiver output is derived in the form of a nonlinear differential equation. The solution of the equation gives the output signal and distortions explicitly in terms of the frequency and amplitude modulation on the receiver input carrier.

An example of receiver performance is computed in some detail for the case of a carrier modulated with a band of gaussian noise. Such a modulation signal is often used to simulate the output of a multichannel telephone multiplex terminal. The receiver input signal is corrupted by additive noise and the distortions resulting from the effects of envelope noise are computed in addition to those caused by signal-sensitive nonlinearities inherent in the demodulation process.

II. DESCRIPTION AND ANALYSIS

The receiver is shown in block form in Fig. 1. This circuit configuration is convenient for description and is suitable for the microwave frequency range. Many other configurations are, of course, possible.

Let the input signal be a carrier, modulated in amplitude and frequency:

$$i_1(t) = I(t) \sin [pt + \theta(t)]. \tag{1}$$

The frequency modulation is $d\theta(t)/dt$; the envelope $I(t)$ is usually nearly constant with a small variable part representing noise or other

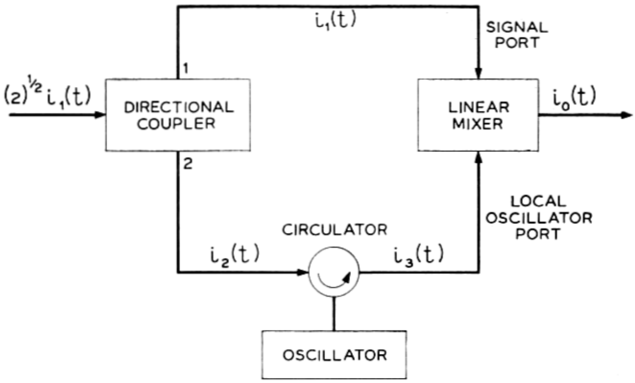

Fig. 1 — Injection-locked-oscillator FM receiver.

undesired amplitude modulation. Let

$$I(t) = I_0 + I_1(t), \qquad I_1(t) \ll I_0 . \tag{2}$$

The function of the directional coupler in Fig. 1 is to divide the signal into two parts which are 90° out of phase. Letting the amplitudes be equal, the output of port 1 is given by (1), and the output of port 2 is

$$i_2(t) = I(t) \cos [pt + \theta(t)]. \tag{3}$$

The output of the injection-locked oscillator is

$$i_3(t) = I_3(t) \cos [pt + \theta(t) - \varphi(t)], \tag{4}$$

where $\varphi(t)$ is the phase tracking error. The receiver output is contained in $\varphi(t)$ and is discussed in detail later. The envelope variations in (3) are reduced in the passage of $i_2(t)$ through the oscillator. The output of the oscillator is used as the local oscillator for the linear mixer of Fig. 1, and since small envelope variations on the local oscillator port do not appear in the output, the envelope of (4) can be regarded constant, that is,

$$i_3(t) = I_3 \cos [pt + \theta(t) - \varphi(t)]. \tag{5}$$

The envelope variations on the input to the linear mixer signal port are not suppressed however, and the low frequency output of the mixer, from (1) and (5), is

$$i_0(t) = MI(t) \sin \varphi(t), \tag{6}$$

where M includes the linear mixer conversion constant. The receiver FM output is contained in the term $\sin \varphi(t)$.

The differential equation describing the locking behavior of the van der Pol sine wave oscillator has been derived in many places.[11, 12] With the input signal of (3) it has the following form.

$$\varphi'(t) = \theta'(t) + (p - \omega_0) - \Delta(t) \sin \varphi(t), \tag{7}$$

where: the prime indicates differentiation with respect to the argument,

$$\Delta(t) = \frac{I(t)}{I_3 \Phi'(\omega_0)},$$

ω_0 is the natural resonant frequency of the oscillator,

$| p - \omega_0 | \ll \omega_0$, and

$\Phi'(\omega_0)$ is the phase slope of the passive oscillator circuit at the resonant frequency.

Now, the injection-locked oscillator locking bandwidth is $2\Delta_0$ where $\Delta_0 = I_0/[I_3\Phi'(\omega_0)]$, so the expression for $\Delta(t)$ becomes

$$\Delta(t) = \frac{\Delta_0}{I_0} I(t). \tag{8}$$

A solution of (7) has been found by an iteration procedure and substituted into (6) to obtain the receiver output.

$$\begin{aligned} i_0(t) = \frac{MI_0}{\Delta_0} \Bigg\{ &[\theta'(t) + p - \omega_0] - \frac{\theta''(t)}{\Delta(t)} \\ &+ \frac{[\theta'''(t) + \Delta'(t)(\theta'(t) + p - \omega_0)]}{\Delta(t)^2} \\ &- \frac{[\Delta''(t)(\theta'(t) + p - \omega_0) + \theta''(t)(\theta'(t) + p - \omega_0)^2/2]}{\Delta(t)^3} \\ &+ \frac{[\theta''''(t) + 3\theta''(t)\, \Delta'(t)]}{\Delta(t)^3} + \cdots \Bigg\}. \end{aligned} \tag{9}$$

When the locking bandwidth $2\Delta_0$ is much larger than the baseband bandwidth, that is, when $\Delta_0 \gg \theta'(t)_{\max}$, the distortion will be small and the series (9) will converge rapidly. For this case the first few distortion terms will dominate. Expression (9) can be rearranged to identify these terms. Let $p = \omega_0$, and let

$$\Delta(t) = \Delta_0 + \Delta_1(t). \tag{10}$$

From (2) and (8) it follows that $\Delta_1(t) \ll \Delta_0$. The following relations will also be used.

$$1/\Delta(t) = \frac{1 - \Delta_1(t)/\Delta_0 + (\Delta_1(t)/\Delta_0)^2 + \cdots}{\Delta_0} \tag{11}$$

$$\theta'(t - 1/\Delta_0) = \theta'(t) - \frac{\theta''(t)}{\Delta_0} + \frac{\theta'''(t)}{2!\,\Delta_0^2} - \frac{\theta''''(t)}{3!\,\Delta_0^3} + \cdots . \tag{12}$$

Substitution of (10), (11), and (12) into (9) transforms the receiver output to

$$i_0(t) = MI_0/\Delta_0\left\{\theta'\left(t - \frac{1}{\Delta_0}\right) + \frac{\theta'''(t)}{2\,\Delta_0^2} - \frac{5\theta''''(t)}{6\,\Delta_0^3} + \cdots + \frac{[\Delta_1(t)\theta'(t)]'}{\Delta_0^2} + \cdots - \frac{[\theta'^3(t)]'}{6\,\Delta_0^3} + \cdots\right\}. \tag{13}$$

The first term is the frequency modulation on the input signal, delayed by $1/\Delta_0$ seconds. The second and third terms are linear distortion and correspond to video roll-off. The fourth and fifth terms are second and third order nonlinear distortions, respectively. Notice that the second order distortion requires both amplitude and frequency modulation—if there is no AM there is no second order distortion. The third order distortion is independent of any AM on the input signal.

III. EXAMPLE—NOISE MODULATION

Let the modulating signal, $\theta_s'(t)$, be a flat band of gaussian noise of bandwidth $O - W$ with a two-sided spectral density

$$S_{\theta' s}(f) = \sigma^2/2W \text{ watts per cycle of bandwidth.} \qquad -W \leqq f \leqq W. \tag{14}$$

Assuming an FM transmitter sensitivity of 1 Hz per volt, the parameter σ is also the rms frequency deviation. The notation $S_h(f)$ is used to indicate spectral density of $h(t)$ and is, of course, a function of frequency.

Normally the input signal to a receiver is contaminated by noise. Let this noise have a constant spectral density of n_0 watts per cycle of bandwidth. The input signal is $(2)^{\frac{1}{2}}\, i_1(t)$ where

$$i_1(t) = (2C)^{\frac{1}{2}} \sin[\omega_0 t + \theta_s(t)] + n(t), \tag{15}$$

and C is the input carrier power. The additive noise, $n(t)$, is narrowband gaussian noise and can be represented as follows.[13]

$$n(t) = x_c(t) \sin [\omega_0 t + \theta_s(t)] - x_s(t) \cos [\omega_0 t + \theta_s(t)] \tag{16}$$

where $x_c(t)$ and $x_s(t)$ are gaussian random variables with zero mean and variances equal to the variance of $n(t)$. Using (16) and assuming a large carrier-to-noise ratio, that is, $(2C)^{\frac{1}{2}} \gg n(t)$, (15) becomes

$$i(t) \doteq [(2C)^{\frac{1}{2}} + x_c(t)] \sin [\omega_0 t + \theta_s(t) - x_s(t)/(2C)^{\frac{1}{2}}]. \tag{17}$$

From (1), (2), and (8) we have

$$\theta'(t) = \theta_s'(t) - x_s'(t)/(2C)^{\frac{1}{2}}, \tag{18}$$

and

$$\Delta_1(t) = [\Delta_0/(2C)^{\frac{1}{2}}]x_c(t). \tag{19}$$

Expression (18) contains the desired output signal $\theta_s'(t)$ and a noise term which represents the frequency modulation caused by the additive noise. The signal-to-noise relationships of FM receivers have been discussed widely in the literature;[14] since the noise is small relative to the signal, that is, $x_s'(t)/(2C)^{\frac{1}{2}} \ll \theta_s'(t)$, it will have a negligible effect on the distortion and need not be considered further.

The quantity of interest in broadband radio systems is the ratio of signal spectral density to distortion spectral density. From (13) the second and third order distortions are

$$D_2(t) = \frac{[\Delta_1(t)\theta_s'(t)]'}{\Delta_0^2}, \tag{20}$$

and

$$D_3(t) = \frac{[\theta_s'^3(t)]'}{6\,\Delta_0^3}. \tag{21}$$

The distortion spectra are

$$S_{D_2}(f) = \frac{1}{\Delta_0^4} S_{[\Delta_1\theta'_s]'}(f) = \frac{f^2}{2C\,\Delta_0^2} S_{x_c\theta'_s}(f), \tag{22}$$

and

$$S_{D_3}(f) = \frac{1}{36\,\Delta_0^6} S_{[\theta'^3_s]'}(f) = \frac{f^2}{36\,\Delta_0^6} S_{\theta'^3_s}(f). \tag{23}$$

To evaluate $S_{D_2}(f)$ notice that $x_c(t)$ and $\theta_s'(t)$ are statistically independent. The spectrum of the product can then be written

$$S_{x_c\theta'_s}(f) = S_{x_c}(f) * S_{\theta'_s}(f),$$

where the asterisk means convolution. Performing the convolution, and letting $B > 4W$, where B is the RF bandwidth of the FM signal, the output distortion spectral density in the baseband is

$$S_{D_2}(f) = \frac{f^2}{2C\,\Delta_0^2} S_{x_c}(f)*S_{\theta'_s}(f)$$

$$= \frac{f^2\sigma^2 n_0}{2\,\Delta_0^2 C}, \qquad 0 \leqq f \leqq W, \tag{24}$$

and the signal-to-distortion ratio is

$$\frac{S_{\theta'_s}(f)}{S_{D_2}(f)} = \left(\frac{C}{n_0 W}\right)\left(\frac{\Delta_0}{f}\right)^2, \qquad 0 \leqq f \leqq W. \tag{25}$$

The evaluation of $S_{D_3}(f)$ requires an expression for $S_{\theta'_s}$ where $\theta'_s(t)$ is the input signal (14). From Hatch,[15]

$$S_{\theta'_s}(f) = \frac{9\sigma^6}{4W}\left[1 - \frac{1}{3}\left(\frac{f}{W}\right)^2\right], \qquad 0 \leqq f \leqq W, \tag{26}$$

$$S_{D_3}(f) = \frac{f^2\sigma^6}{16W\,\Delta_0^6}\left[1 - \frac{1}{3}\left(\frac{f}{W}\right)^2\right], \qquad 0 \leqq f \leqq W, \tag{27}$$

and

$$\frac{S_{\theta'_s}(f)}{S_{D_3}(f)} = \frac{8\,\Delta_0^6}{\sigma^4 f^2\left[1 - \frac{1}{3}\left(\frac{f}{W}\right)^2\right]}, \qquad 0 \leqq f \leqq W. \tag{28}$$

For a locking bandwidth $2\Delta_0 = 200$ MHz, a baseband $W = 5$ MHz, and an rms frequency deviation $\sigma = 15$ MHz, the signal-to-distortion ratios in the worst message channel, that is, at $f = W$ are:

$$\left.\frac{S_{\theta'_s}(f)}{S_{D_2}(f)}\right|_{f=W} = 52.2 \text{ Db} \tag{29}$$

$$\left.\frac{S_{\theta'_s}(f)}{S_{D_3}(f)}\right|_{f=W} = 69.7 \text{ Db}. \tag{30}$$

In computing (29), the carrier-to-noise ratio in the RF band B was assumed to be near threshold, that is, $C/(n_0 B) = 16$ (12 dB) where

$$B = 2W\left(1 + \frac{4\sigma}{W}\right) \tag{31}$$

is the Carson bandwidth and 4σ is the peak frequency deviation. If this receiver were used in a radio system with a fading margin of 30 dB the signal-to-distortion ratio for second order distortion would be 82.2 dB during periods of normal propagation and the performance would be limited by the third order distortion.

IV. AUTOMATIC FREQUENCY CONTROL

If the natural resonant frequency of the oscillator, ω_0, is not equal to the input signal carrier frequency p, the receiver output contains a direct current component. From (9),

$$i_{DC} = \frac{MI_0}{\Delta_0}(p - \omega_0). \tag{32}$$

This current gives the direction and magnitude of the frequency difference and, by using it to control a suitable oscillator parameter, the oscillator can be kept tuned automatically to the input carrier frequency.

V. DISCUSSION

One may question whether $\Phi'(\omega)$ is really constant in the band of interest. If the oscillator circuit is a single resonance then the locking bandwidth is equal to the 3 dB bandwidth of the passive circuit divided by the current gain of the locked oscillator. For a 20 dB gain the circuit phase changes $\pm 4.5°$ over the locking band, therefore $\Phi'(\omega)$ is reasonably constant over bandwidths less than the locking bandwidth. Since $\Phi'(\omega)$ is the slope of a passive circuit, it can be made arbitrarily close to constant over the necessary frequency range by increasing the complexity of the circuit.

It would be nice to compare the theoretical performance of the injection-locked-oscillator receiver with the conventional FM receiver consisting of a limiter and a balanced discriminator. There is, however, no theoretical analysis of the conventional receiver comparable with the analysis presented here. Such an analysis can probably be done with the aid of recent work by Bedrosian and Rice,[16] although it would be difficult to describe analytically the amplitude-to-phase conversion in a real limiter and to take into account the nonlinearity of the envelope detectors which are used in the discriminator.

The situation with the injection-locked-oscillator receiver is different. The amplitude-to-phase conversion has been accounted for in the present analysis and practical linear mixers are certainly linear.

A most important question about the practical realization of the receiver is the extent to which real oscillators can be described by the van der Pol model. The evidence in the literature[5] answers the question favorably in that the locking equation (7) does indeed describe the locking behavior of tunnel diode and avalanche diode oscillators.

REFERENCES

1. Jaffee, R., and Rechtin, E., "Design and Performance of Phase-Lock Circuits Capable of Near-Optimum Performance Over a Wide Range of Input Signal and Noise Levels," I.R.E. Trans. Inform. Theory, *IT-1*, No. 1 (March 1955), pp. 66–76.
2. Gilchriest, C. E., "Application of a Phase-Locked Loop to Telemetry as a Discriminator or Tracking Filter," I.R.E. Trans. Telemetry and Remote Control, *TRC-4*, No. 1 (June 1958), pp. 20–35.
3. Moschytz, G. S., "Miniaturized RC Filters Using Phase-Locked Loop," B.S.T.J., *44*, No. 5, (May–June 1965), pp. 823–870.
4. Ruthroff, C. L., and Stover, H. L., unpublished work.
5. Shaw, R. C., and Stover, H. L., "Phase-Locked Avalanche Diode Oscillators," Proc. I.E.E.E., *54*, No. 4, (April 1966), pp. 710–711.
6. Appleton, E. V., "The Automatic Synchronization of Triode Oscillators," Proc. Cambridge Phil. Soc., *21*, Part 2 (1922–1923), pp. 231–248.
7. Woodyard, J. R., "Application of the Auto-Synchronized Oscillator to Frequency Demodulation," Proc. I.R.E., *25*, No. 5 (May 1937), pp. 610–619.
8. Beers, G. L., "A Frequency-Dividing Locked-In-Oscillator Frequency-Modulation Receiver," Proc. I.R.E., *32*, No. 12 (December 1944), pp. 730–737.
9. Bradley, W. E., "Single Stage FM Detector," Elec., *19*, No. 10 (October 1946), pp. 88–91.
10. Edson, W. A., *Vacuum-Tube Oscillators*, New York: John Wiley & Sons, 1953.
11. Khokhlov, R. V., "A Method of Analysis in the Theory of Sinusoidal Self-Oscillations," I.R.E. Trans. Circuit Theory, *CT-7*, No. 4, (December 1960), pp. 398–413.
12. Adler, R., "A Study of Locking Phenomena in Oscillators," Proc. I.R.E., *34*, No. 6 (June 1946), pp. 351–357.
13. Rice, S. O., "Mathematical Analysis of Random Noise," B.S.T.J., *23*, No. 3, (July 1944), pp. 282–332, and *24*, No. 1 (January 1945), pp. 46–156.
14. Rice, S. O., "Statistical Properties of a Sine-Wave Plus Random Noise," B.S.T.J., *27*, No. 1 (January 1948), pp. 109–157.
15. Hatch, R. W., unpublished work.
16. Bedrosian, E., and Rice, S. O., "Distortion and Crosstalk of Linearly Filtered, Angle-Modulated Signals," Proc. I.E.E.E., *56*, No. 1 (January 1968), pp. 2–13.