Synchronizing Digital Networks

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It appears that stable synchronization of large digital transmission networks should be easy, granted accurate clocks, buffers which accept pulses at the incoming rate and deliver them at the local clock rate, and adequate delay for making frames coincide. An electric network analog of a simple linear system in which the clock frequency depends on the fullness of buffers and the departure of frequency setting from midsetting makes it obvious that the system is stable. System frequency should be made to depend strongly on accurate or master clocks; criteria are given for choosing parameters to achieve this. Strategies are given for periodic infrequent adjustments to compensate for changes in transmission time, and for adding new clocks to the network. The practical realization of a synchronized network calls for more information concerning variations of transmission time and for adequate components, particularly, buffers and adjustable delay devices.

The synchronization of digital networks has been studied theoretically and experimentally.[1–9] This paper does not purport to review excellent previous work, some of which has been highly theoretically oriented, through some results of earlier work are referred to. Rather, it discusses some problems of synchronization and illustrates them by means of a simple analysis of a simple example. We reach the following conclusions: that with good clocks, buffers and adequate delay both to compensate for changes in transmission time and to make frames coincide, there should be no trouble in stably synchronizing a nationwide network. This is in accord with earlier analysis and experiment.[6]

I. SYNCHRONIZING FRAMES OR BITS

In some papers, synchronization has been discussed in terms of synchronizing frames, that is, successive groups of bits identified by some framing signal present in each group.[1, 6] In this paper, synchronization will be discussed in terms of synchronization of bit streams.

The choice of the bit stream as the signal to be synchronized is partly arbitrary. However, there may be advantages in performing, at a terminal, as many operations as possible on an accurately timed binary bit stream.

For some purposes, and especially for time division switching, it seems essential to synchronize frames. This can be done by passing the bit stream through a suitable adjustable delay device. The delay measured in pulses, which is needed to make frames coincide, is not small. The frame time for a digital system designed for speech transmission is commonly 1/8000 second. If a transmission system runs at the rate of 5×10^8 pulses per second, the frame time includes about 70,000 pulses. This may be an akwardly large number of pulses to store.

II. CHANGES IN TRANSMISSION TIME AND CLOCK RATE

A scheme of synchronization must take into account both errors in clock frequency and changes in transmission time. These pose rather different problems. Both for this reason, and because it makes the presentation simpler, changes in transmission time are disregarded in the earlier portions of this paper, and treated separately later on.

III. THE SYSTEM CONSIDERED

Various approaches to synchronization are possible. One solution would be to transmit synchronizing signals from a central master source. Few people seem to like this method because of problems of reliability.

The system considered here is composed of a lot of centers (the dots of Fig. 1) with highly stable clocks, interconnected by two-way digital circuits (the lines of Fig. 1). The common frequency of operation will be determined by the characteristics of the clocks and of the transmission circuits.

We assume that each clock is equipped with a frequency control, so that its frequency can be adjusted to be above or below its central, "correct" value. We also assume that each receiver is equipped with a buffer which will accept a bit stream from a terminal at some received rate and emit bits at the rate or frequency determined by the local clock.

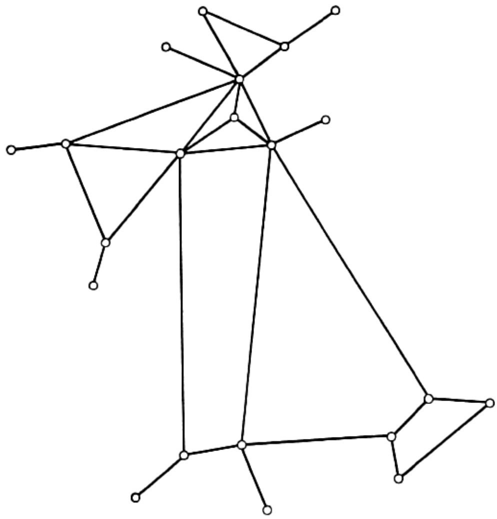

Fig. 1 — A network of centers to be synchronized.

We might assume a system in which each center knew the state of every other center's buffers and clock settings. In Section XI we discuss how such knowledge may be used in dealing with changes in transmission time. Initially, we will consider the case in which each center knows only the state of its buffers and the setting of its clock frequency. Thus, any adjustments of the clock frequency will be based on the frequency of each clock with respect to its center value, and on the state of the buffers, each of which reflects both the clock frequency at the other end of a transmission circuit relative to the local clock frequency, and any changes in the transmission time.

The case considered is illustrated schematically in Fig. 2. The elements concerned with automatic adjustment of the clock frequency are enclosed by a dashed line; they consist of the clock, buffers, and a network whose inputs are buffer readings and (optionally) the reading of the clock setting and whose output is a signal which adjusts the frequency setting of the clock. Other elements shown in Fig. 2 are adjustable delay for framing the bit stream from the buffer output, a decoder for going from received pulses to bit stream, and an adjustable delay before or after the decoder which can be used to compensate for changes in transmission time.

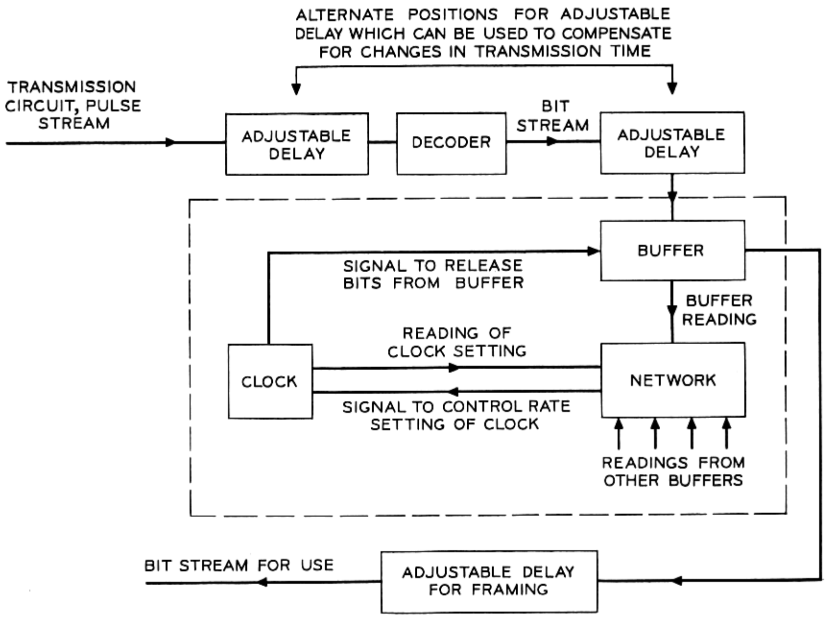

Fig. 2 — Block diagram showing components used in synchronization.

IV. STABILITY OF CLOCKS

It is clear that both the stability of the clocks and the pulse rate are overwhelmingly important. If the clocks were perfectly stable (if they could be exactly synchronized at the factory and if they maintained their frequency exactly after being shipped to the centers), then the buffers would merely have to take care of fluctuations in transmission time. As the transmission time will not change without bound, finite, realizable buffers would insure the satisfactory operation of the system.

The clocks cannot be synchronized perfectly, but will differ in rate by some fraction, d, which may be 10^{-8} for a good crystal oscillator or perhaps as good as 10^{-12} for an atomic clock. Comparatively inexpensive ($1,800) commercial frequency standards are now available which have a short-term d of 10^{-11} and a long-term value of 2×10^{-11} (see Ref. 10).

Consider two centers interchanging pulses at a rate r per second. If no effort is made to synchronize the clocks, the number N of pulses the buffer must absorb per day would be roughly $N = 86{,}400\ rd$.

Let us consider some cases:

r	d	N
5×10^7	10^{-7}	4.32×10^5
5×10^7	2×10^{-11}	864
5×10^7	10^{-12}	43.2

This is instructive. If the clocks were stable enough, the buffers could be dumped and the delays readjusted at strategic times, with resulting errors through loss of message bits. This might be feasible with the Hewlett-Packard clock.[10] If very stable clocks are used, any adjustments can be very slow or very infrequent.

V. TRANSMISSION TIME

The time delay in transmission is a complication in any analysis of synchronization. This makes the idea of infrequent periodic adjustments or very slow continuous adjustments attractive. If the time intervals between adjustments or the time constants involved in the adjustment process are long compared with the transmission time, then a sort of kinetic model, in which transmission time can be disregarded, will apply.

Transcontinental transmission time via coaxial cable is of the order of 0.02 second, and via microwave radio somewhat less. For a clock stability $d = 10^{-8}$ and a pulse rate 5×10^8, around five pulses would accumulate in the buffers each second, and an adjustment once a second seems reasonable. For a $d = 10^{-10}$ and the same pulse rate, 5 pulses would accumulate in 100 seconds, a time long compared with the 0.3 second transmission time for a synchronous satellite.

Thus, it appears from the outset that very slow adjustments of clock frequency are permissible. We will see later that very slow adjustments are desirable as well. There is every reason to believe that we can disregard transmission time in considering the stability and performance of the synchronizing scheme which we discuss in Section VIII.*

VI. THE BUFFERS

We have noticed that at each center we may make use of the departure from the center frequency setting of the clock frequency

* A criterion for stability which is independent of transmission time has been available for some years.[3] In work to be published, I. W. Sandberg gives a less stringent criterion which is dependent on transmission time, and which is in accord with the qualitative statements made here.

adjustment and of the loading of the buffers, which we call b_1, b_2, and so on. These loadings may, with respect to their design or "normal" values, be positive or negative numbers.

Buffers will have finite capacity and hence will overflow when clock frequencies at two interconnected centers are persistently different. Here it is assumed that when a buffer overflows it remains completely full or empty until the sign of the difference in clock frequencies changes, at which time it again accepts and release pulses. It is also assumed that during the overflow condition the buffer reading b remains at some extreme value $\pm b_m$, and becomes smaller in magnitude once the sign of the difference in clock frequencies changes.

The bounds imposed on the b's by buffer overflow are valuable. The clock at the other end of a circuit may be in really bad trouble. In the extreme, no pulses may be coming in. Thus, we should not allow, or else we should disregard, buffer readings beyond some limiting extremes. As the buffers are finite, drastic malfunction will cause them to overflow and so limit the range of the buffer reading b.

VII. CLOCKS OF DIFFERING ACCURACY

In adjusting clock frequency, account should be taken of the accuracy of the clock. In a large system, it may be desirable to use very accurate master clocks at some important nodes and less accurate subsidiary clocks at other nodes. It is essential that some provision be made so that the final frequency of operation depends most strongly on the accurate master clocks and less on the less accurate subsidiary clocks. Thus, in adjusting the clocks, either the magnitude of the adjustments should be suitably smaller for the more accurate clocks than for the less accurate clocks, or else the adjustments of the more accurate clocks should be made less frequently or both.

VIII. MATHEMATICAL DESCRIPTION OF THE SYNCHRONIZING SCHEME

Let us now consider one particular node of the network of Fig. 1, at which the clock frequency is f_1. This node is shown as 1 in Fig. 3. Connected to it are nodes 2, 3, 4, . . . , n, where the frequencies are f_2, f_3, f_4, f_n. If the 1, n buffer is set to b_{1n0} at $t = 0$, the various buffer readings at 1 are

$$b_{1n} = b_{1n0} + \int_0^t (f_n - f_1)\, dt \tag{1}$$

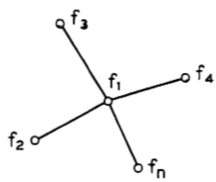

Fig. 3 — A node of the network of Fig. 1.

We will assume that b_{1n} can be positive or negative. If we let $b_{1n} = 0$ when the buffer is half full, then the buffer reading can have any value in some range from $-b_m$ to $+b_m$ where $2b_m$ is the size of the buffer.

Let the "center" clock frequency at 1 for "center setting" of the frequency control be f_{10}. The center settings are intended to adjust the clock to the desired system frequency. The frequency f_{n0} of a given clock at center setting differs from the intended frequency because of the error of the clock.

The buffer reading must be an integer. If the range of variation of this integer is great enough, it should be possible to treat the buffer reading as a continuous variable in a differential equation; this is what we will do.

A suitable linear strategy for adjusting the frequency f_1 was found to be

$$A \sum_n \int_0^t (f_n - f_1)\, dt - B_1(f_1 - f_{10}) - C \frac{df_1}{dt} + A \sum_n b_{1no} = 0. \quad (2)$$

We can regard the equation as applying either to control of the clock frequency or the rate of change of clock frequency. If we make $C = 0$, then the equation prescribes departure from center clock setting, $(f_1 - f_{10})$, in terms of buffer content. If $C \neq 0$, then the equation prescribes rate of change of clock setting, df_1/dt, in terms of buffer content and departure from center clock setting.

In either case B_1 should be larger for more accurate clocks and smaller for less accurate clocks. This will give the departure from center clock setting greater weight for more accurate clocks and less weight for less accurate clocks.

IX. AN ELECTRIC ANALOG

Let us now consider an electric analog, the circuit shown in Fig. 4. Node 1 is connected to ground through a capacitance C. Current

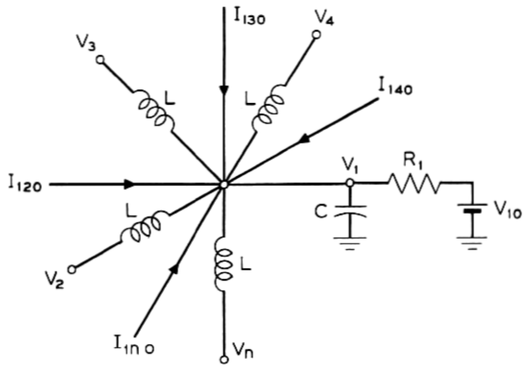

Fig. 4 — Electric analog of a node synchronization system considered.

flows to C through a resistance R_1 to which a bias voltage V_{10} is applied. Current also flows to node 1 because this node is connected to other nodes 2, 3, 4, . . . , n, at which the voltages are V_2, V_3, V_4, . . . , V_n, by inductances L. The differential equation for the voltage V_1 is

$$(1/L) \sum_n \int_0^t (V_n - V_1)\, dt - (1/R_1)(V_1 - V_{10}) - C \frac{dV_1}{dt} + \sum_n I_{1n0} = 0. \qquad (3)$$

Here I_{1n0} is a current flowing into node 1, not from but associated with node n, at time $t = 0$. We see that equation (3) is identical with equation (2) if we let

$$\left.\begin{aligned} V_n &= f_n \\ V_{10} &= f_{10} \\ (1/L) &= A \\ (1/R_1) &= B_1 \\ I_{1n0} &= Ab_{1n0} \\ C &= C \end{aligned}\right\} \qquad (4)$$

The behavior of the frequency of a network of oscillators adjusted according to the strategy of equation (2) will be the same as the behavior of the voltage in an L, R, C network of the form shown in Fig. 5. We should notice that it is perfectly permissible to make

$C = 0$ in (2) or (3). A network analog for this case has been given by Brilliant.[5]

In a network such as that of Fig. 5, no matter how extensive or how interconnected, all the voltages settle down to some final value because of the damping (energy loss) of the resistors R_n. What is the final voltage? It must be such that the total of the currents flowing to all nodes is zero. This means that

$$0 = \sum_m \sum_n I_{nm0} + \sum_n (V_{n0} - V)/R_n \,. \tag{5}$$

The double summation is in each case over all nodes. Not all nodes are connected to one another, and I_{nm0} will be zero for $n = m$ and for all nodes n not connected to m.

The analogous expression for final frequency is

$$0 = A \sum_m \sum_n b_{nm0} + \sum_n B_n(f_{n0} - f). \tag{6}$$

Again, b_{nm0} will be zero for $n = m$ and for all nodes n not connected to m.

We will see that the final frequency depends on the center frequencies of the oscillators, on the A and B coefficients, and on the

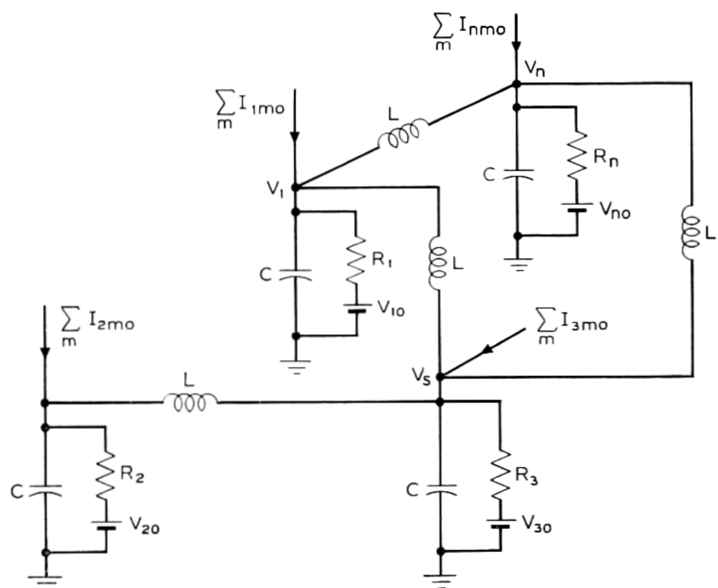

Fig. 5 — Interconnected nodes of electric analog.

initial buffer settings. If the initial buffer settings b_{nm0} are all zero, the final frequency depends only on the f_{n0}'s and the B_n's. If the buffers overflow, this in effect resets them.

We can see from Fig. 5 that the system will be stable in the case of certain nonlinearities.

For example, all the R_n's can be nonlinear as long as no resistance is ever negative at any current. Thus, if in the equations of the oscillator system the second term on the right of (2) is replaced by a nonlinear function of $(f_1 - f_{10})$, the system will be stable as long as the term decreases monotonically with increasing f_1. In the case $C = 0$, this corresponds to a nonlinear control of clock setting as a function of buffer content.

It is easy to show that in a special case buffer overflow cannot result in instability. This is the case in which the buffer readings at two ends of a transmission circuit are complementary, that is, their sum is zero. Disregarding changes in transmission time, if the buffers are both set to zero or to complementary values at the same time, or if both buffers overflow and then recover, the readings will be complementary.

Appendix A shows that for this case, in the electric analog of Figs. 4 and 5 buffer overflow results in the dissipation of energy, and this convinces the writer that in this case overflowing buffers cannot make the network unstable. The writer is mortified that he is unable to demonstrate this for the case of non-complementary readings, but he suspects that buffer overflow will not result in instability in this case, either.

Buffer overflow would affect the final frequency of operation. Further, all the buffers at a given node can conceivably overflow permanently (if the clock center rate shifts drastically, for example). In such a case, the node will operate out of synchronism with the rest of the network. This will cause buffer overflows at nodes connected to the asynchronous node. Such overflows can affect the frequency of operation of the rest of the network, but need not prevent its synchronous operation.

If widespread buffer overflow is avoided, adjustment according to equation (2) will result in stable operation.

X. PARAMETERS AND TIME CONSTANTS

Let us consider equation (2) with $C = 0$. By using (1), this can be written

$$f_1 - f_{10} = (1/T_1) \sum_n b_{1n} \tag{7}$$

$$T_1 = B_1/A = L/R_1 \,. \tag{8}$$

We see that T_1 is a time constant. Equation (7) prescribes the departure of the clock frequency from center setting in terms of the sum of buffer readings b_{1n} for the various transmission circuits terminating at node 1. How shall we choose the parameter T_1?

The smallest amount by which the sum of the buffer readings can change is unity. The frequencies f_1 and f_{10} differ by a very small amount. Thus, the fractional change in frequency caused by unit change in the sum of the buffer readings can be written $1/T_1f_1$. If we are to take full advantage of the stability of the clock at the node, we should make this smallest change small compared with the clock stability expressed as a fraction, which we call d_1. Hence, we should choose

$$1/T_1f_1 < d_1 \,, \qquad T_1 > 1/d_1f_1 \,. \tag{9}$$

If this is not so, changes in buffer readings will cause sudden changes in frequency larger than the changes which would occur if the clock were not adjusted.

The buffer readings must, however, be able to change the frequency of the clock by several times its fractional stability d_1 if we are to be sure to bring all the clocks to the common intended frequency. Strictly, it might be possible to accomplish this if $(1/T_1f_1)\ b_m > d_1/n$, where b_m is the maximum buffer reading and n is the number of buffers. It would seem wise to choose b_m large enough so that this criterion is considerably exceeded. We might reasonably ask that

$$b_m/T_1f_1 > d_1 \,, \qquad b_m > T_1f_1d_1 \,. \tag{10}$$

As an example, let us consider a case in which

$$T_1 = 10/d_1f_1 \tag{11}$$

$$b_m = 10T_1f_1d_1 = 100. \tag{12}$$

Assume that $f_1 = 5 \times 10^8$. For various values of d_1, the computed values of T_1 are:

Fractional oscillator stability, d_1	*Stability (pulses per day)* $86{,}400\ d_1$	*Time constant,* T_1 *(seconds)*
10^{-8}	4.32×10^5	2
2×10^{-11}	864	1000 (17 minutes)
10^{-12}	43.2	20,000 (5.5 hours)

The time constants T_1 are large, implying that the time for the system to come to equilibrium is large. The time constants may not seem excessive, however, when we consider oscillator stability measured in pulses per day. It would be rash to try to adjust an exceedingly stable oscillator in too short a time. Further, time jitter in the received pulses (see Appendix D) and perhaps other quickly changing phenomena could cause undesirable changes in operating frequency if T_1 were made smaller.

XI. COPING WITH CHANGES IN TRANSMISSION TIME

We have not yet considered the effect of changes in transmission time. Changes in buffer reading have been ascribed to differences in frequency. But changes in transmission time can also cause changes in buffer reading.

Consider two interconnected nodes. If the clock at one speeds up, the buffer at the node will tend to empty and the buffer at the node to which the fast clock is connected will tend to fill. Thus, a change in clock rate will change the buffer readings at interconnected nodes in opposite sense.

Consider an increase in transmission time between nodes, for example, an increase in transmission time in both directions. Because of the increase in transmission time, more pulses will be stored in the lines and the buffer readings will decrease at both ends. Thus, changes in transmission time will change the buffer readings at interconnected nodes in the same senses.

If we can compare the buffer readings at two interconnected nodes, we can distinguish changes caused by changes in transmission time from changes caused by changes in clock frequencies. Once we identify a change in transmission time, we can correct for it by means of an adjustable delay between the transmission system and the buffer input. Such corrections have been provided in some synchronization schemes.[6] It is not clear, however, that an automatic system acting in the same way among all nodes is best in coping with changes in transmission time.

Another course would be to use no adjustable delay, and merely provide buffers large enough to accommodate changes in transmission time. Then, changes in transmission time would cause buffers to fill or empty. This would have some effect on system frequency. If the system included highly stable clocks, such changes would necessarily be very small.

What should be done about changes in transmission time depends on the magnitude of such changes and on how rapidly they occur. Unfortunately, adequate information is not available.

In cable and waveguide systems, transmission time can vary with temperature and with the gas pressure within the waveguide or cable. What is known is discussed in Appendix B. For a circuit 3,000 miles long, we might expect variations of more than a thousand pulses over the year. However, because cable is buried and waveguide would be, we would expect the transmission time to vary little during the day, or over a period of several days. Experience with the L4 system tends to confirm this.

If the short-term stability of transmission time of cable and waveguide is as good as would seem, it would be satisfactory to make compensating adjustments in delay at intervals of days or weeks; this would argue for a scheme of adjustment separate from that used for clock synchronization.

Variations in transmission time for microwave radio systems (see Appendix C) may be comparable to those for cable or waveguide systems, but changes may be more rapid. Diurnal changes might be taken care of by the buffers, and slower changes by daily delay adjustments.

A node may often be connected to the rest of the network by cable and/or waveguide circuits as well as by microwave circuits. In this case, it seems attractive to the writer to use the cable or waveguide circuits for adjusting clock rate. Then the buffers and delay adjustments associated with the microwave circuits could be used solely to compensate for changes in microwave transmission time.

Thus, a somewhat mixed strategy of adjustment may be called for. The following seems reasonable to the writer:

(*i*) If possible, avoid the use of microwave circuits for synchronization. Use parallel cable or waveguide circuits for synchronization, and use buffers or adjustments of delay to absorb changes in microwave transmission time.

(*ii*) The system will probably contain several master clocks which are more accurate than other subsidiary clocks. The parameters T_1 will be chosen [according to (9) or (11)] so that frequency is chiefly dependent on the center frequencies of the master clocks. Hence, the correct function of the buffer readings at the subsidiary clocks is to adjust these clocks to the (correct) operating frequency. At each node with a subsidiary clock, periodic adjustments of delay should be made, such as to make all buffer readings zero. At the same time a delay adjustment should be made to make any buffer at a master clock on a line from the subsidiary clock zero. Simultaneously, the subsidiary clock should be readjusted so that its center frequency f_{n0} is equal to the current operating frequency f_n. This adjusts for changes in transmission time by making buffer readings at the ends of links to and from subsidiary clocks complementary (and equal to zero). It may be desirable to make these adjustments at a common time at all nodes concerned. Notice that in making the adjustments at a node, no knowledge of buffer readings or clock settings at other nodes is needed. Such adjustments might be made once a day or once a week.

(*iii*) It is desirable that the network link all master clocks together by cable or waveguide, directly or indirectly, but with no intervening buffering and retiming by less accurate clocks. When this is so, it would seem desirable to make periodic adjustments of the delays in each transmission circuit connecting two master clocks, such as to render the buffer readings complementary at the two ends of the link. This compensates for changes in transmission time. It is undesirable to adjust the frequency at center setting, since we rely on the frequencies of the master clocks at center settings to determine the operating frequency. This adjustment of delays need be made only infrequently (once a day or once a week). To make it, we must know at each master clock the buffer readings at the other ends of the circuits connecting it to other master clocks.

(*iv*) In adding a subsidiary clock to a network, it should be adjusted so that its frequency at center setting is equal to the current operating frequency and the buffers at both ends of all circuits connecting it to the network should be set to zero. If a master clock is added, the adjustment of frequency at center setting should be omitted.

XII. CONCLUDING REMARKS

The behavior of a simple scheme of synchronization was investigated and found to be stable in the absence of buffer overload if clock adjustments are sufficiently slow compared with transmission time.

The criteria given for choice of parameters result in time constants very long in comparison with transmission time.

A strategy of infrequent periodic adjustment (once a day or once a week) has been suggested. This can compensate for changes in transmission time and correct the frequencies at center setting of subsidiary clocks. A strategy has been given for adding new clocks to the system. If these strategies are followed, and if parameters are chosen as prescribed, it seems likely that the buffers will not overload except in case of clock failure. Clock failure will cause loss of synchronization at a node.

It has been shown that in a special case, buffer overload from other causes will not result in instability; it seems plausible that this is so in general.

It appears that there are no inherent obstacles to the synchronization of large digital networks. In the practical realization of such networks it would be desirable to have more information concerning the variation of transmission time with time, and on the availability of suitable components, including:

(*i*) Adequate buffers which will accept pulses at one rate, emit them at another, and behave under overload in the fashion described earlier.

(*ii*) Adequate delay of the order of 10^5 pulses, to bring frames into coincidence.

XIII. ACKNOWLEDGMENTS

The writer is indebted to I. W. Sandberg, J. C. Candy, and J. R. Gray for valuable comments and suggestions, to S. L. Freeny and S. E. Miller for information in Appendix B, and to D. C. Hogg for most of the material in Appendix C.

APPENDIX A

Buffer Overflow

The purpose of this section is to consider the consequences of buffer overflow by studying the behavior of the electrical analog.

Consider equation (1):

$$b_{1n} = b_{1n0} + \int_0^t (f_n - f_1)\, dt. \tag{1}$$

Because of the finite content of the buffer, if for $b_{1no} = 0$ the buffer is set half full at $t = 0$, we must have

$$-b_m < b_{1n} < b_m , \qquad | b_{1n} | < b_m \tag{13}$$

One satisfactory way to treat buffer overflow is to let b_{1n0} of (1) change during time intervals when the integral would otherwise cause an increasing violation of (13). For example, suppose that the integral is increasing with time. When b_{1n} reaches the value b_m, b_{1no} starts to decrease and decreases so as to make $b_{1n} = b_m$ for as long as the integral continues to increase. As soon as the integral starts to decrease, b_{1no} remains constant (until another buffer overflow) at whatever value it had when the integral started to decrease, and b_{1n} starts to decrease. A little thought shows that this results in just the behavior that an overflowing buffer of the type described in the paper would exhibit.

In equations (3) and (4) and in Fig. 4 we see that the exact analog of b_{1n} is LI_{1n} given by

$$LI_{in} = L\left[I_{1n0} + (1/L)\int_0^t (V_n - V_1)\,dt\right]. \tag{14}$$

In exploring the effect of buffer overload it is sufficient to consider a circuit element consisting of an inductor and two bias currents. For simplicity we will assume that these bias currents are equal and opposite, with magnitudes I_o, as shown in Fig. 6. The input and output currents are thus equal and of magnitude I. The input and output voltages are V_2 and V_1. The current I_L through the inductance L is

$$I_L = I + I_0 . \tag{15}$$

Assume the same buffer overload current at each end, of magnitude I_m. Thus, the magnitude of the current I cannot become greater than I_m.

Suppose that $I = 0$ at $t = 0$ and the voltages are such as to increase the magnitude of I. How much energy have we put into the circuit by the time $I = I_m$? This energy E_m is

$$E_m = \int_{I=0}^{I_m} (V_2 - V_1)I\,dt. \tag{16}$$

Now

$$V_2 - V_1 = L\frac{dI_L}{dt} = L\frac{d}{dt}(I + I_o) = L\frac{dI}{dt}. \tag{17}$$

Hence

$$E_m = L\int_0^{I_m} I\,dI = (1/2)LI_m^2 . \tag{18}$$

Notice that E_m does not depend on I_o and that it is recoverable, that is, we get it all back if we change the current from I_m to 0 in such a manner that the magnitude of I is always less than I_m.

The voltage difference $V_2 - V_1$ can cause the current I_L to change even after I has reached its limiting magnitude I_m. I_o must then change to keep the magnitude of I from exceeding I_m. In this regime of buffer overload,

$$I = I_m \tag{19}$$

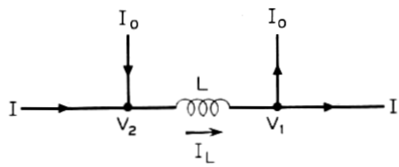

Fig. 6 — Electric analog of buffer, used in studying buffer overflow.

$$I_L = I_o + I_m \,. \tag{20}$$

What about the energy E that is supplied to the circuit during this period? This energy is

$$\begin{aligned} E &= \int_{t_1}^{t_2} (V_2 - V_1) I_m \, dt \\ &= \int_{t_1}^{t_2} L \frac{dI_L}{dt} I_m \, dt \\ &= LI_m(I_{L2} - I_{L1}) \\ &= LI_m(I_{o2} - I_{o1}). \end{aligned} \tag{21}$$

Overload operation persists only as long as I_o is increasing. If we come to a point where I_o would decrease, we hold I_o constant. The buffer is no longer overloaded and we return to the regime of a fixed bias current.

Thus,

$$I_{02} > I_{01} \tag{22}$$

$$E > 0. \tag{23}$$

In fixed bias operation, the recoverable energy is merely E_m as given by (18). The positive energy E has been dissipated. Hence, the effect of buffer overload is to dissipate energy, and buffer overload cannot result in instability.

APPENDIX B

Cable and Waveguide

The transmission time through cable or waveguide can change for several reasons. The gas pressure in a cable is controlled, and changes in gas pressure cause changes in transmission time, as could changes in gas temperature. While large changes in pressure could produce large effects (see Appendix C), it seems likely that another effect will dominate.

This is the linear expansion of the cable or waveguide because of changes in temperature. Structurally, waveguide would probably be largely steel; cable might be considered as copper. The thermal coefficient of expansion of steel is about 12×10^{-6}; for copper it is about 18×10^{-6}. For a change in temperature of 40°F or 22°C, the fractional change FC in length would be approximately

Material	*Fractional change in length, FC, for 22°C change in temperature*
Steel	260×10^{-6}
Copper	400×10^{-6}

During an experiment on the L-4 field installation in Dayton, Ohio, T. J. Pedersen observed phase shift versus time of a 12 MHz sine wave sent through an 84-mile loop of the L-4 system. During a 24-hour period he measured a peak-to-peak change of about 1.5°. The velocity of propagation is about 175,000 miles per second, so the total phase shift in the 84-mile loop was about 2×10^6 degrees. This is a fractional change of

$$0.75 \times 10^{-6}.$$

During the time that this change took place, the temperature of the cable was changing about 0.3°F per day. If this temperature change was indeed the source of the phase shift, the fractional change in transmission time for a 40°F change in temperature would be

$$\text{FC} = (0.75 \times 10^{-6})(40/0.3) = 100 \times 10^{-6}.$$

This value may not be accurate, but an estimate based on the thermal expansion of a metal may not be accurate either.

Change in temperature increases the resistivity of copper, and this causes a change in the reactive as well as the resistive component of skin impedance. Further, the capacitance may change with temperature. Both calculations and measured values of cable parameters as a function of temperature indicate that such effects will change transmission time much less than linear expansion.

Change in diameter of a waveguide causes a change in group velocity. The change in transmission time with temperature which this would cause is considerably smaller than a change proportional to linear expansion.

It may be conjectured whether cable or waveguide is or need to be free to expand in length as temperature changes. We have no waveguide systems in operation at present, and variations of transmission time for coaxial cable systems have not been adequately measured. We can only conclude:

(*i*) Change in transmission time will be very slow, so that infrequent adjustments would be satisfactory.

(*ii*) Total fractional changes in transmission time may be from 100–400 parts per million.

For a path length of 3,000 miles, a velocity of 175,000 miles per second and a pulse rate of 5×10^8 pulses per second, the number of pulses stored in the line would be 10^7. For the fractional changes in transmission time quoted above, the changes in number of pulses stored in a 3,000 mile cable would be

Fractional change in transmission time	*Change in number of pulses stored*
0.75×10^{-6} (observed change in one day)	7.5
100×10^{-6} (estimated effect of 40°*F* change in temperature)	1000
220×10^{-6} (from expansion of steel caused by 40°F change in temperature)	2200
400×10^{-6} (from expansion of copper caused by 40°F change in temperature)	4000

APPENDIX C

Microwave Radio

The velocity of radio waves traveling through the atmosphere depends on temperature, pressure, humidity and probably on rain.

The effect of the first three factors is given in terms of N units. In terms of the index of refraction n (ratio of velocity in vacuo to velocity in the medium)

$$N = (n - 1)10^6.$$

A simple expression gives N quite accurately[11]

$$N = \frac{77.6}{T}\left(p + 4{,}810\,\frac{e}{T}\right)$$

p = total pressure in millibars
e = partial pressure of water vapor in millibars
T = absolute temperature °K = °C + 273.

Here we are concerned with rough estimates of changes in N over short and long periods.

The diurnal change in temperature may account for the most rapid fluctuations in N. For a constant pressure and disregarding water vapor, N is inversely proportional to absolute temperature. For a 20°C (36°F) change in temperature around 20°C (68°F), at a pressure of one bar the change in N would be about 18.

Data taken over a six-year period from forty-five U. S.[12] weather stations show that the monthly mean value of N at the earth's surface (N_s) varies from 230 to 400 over the country and through the year. In the U. S. the largest local variation in the monthly means is on the southeast and Gulf Coasts and amounts to a charge of N of 50 units.

During a given month, the variation in N_s (for one and ninety-nine percent probability) can be as much as 100 N units.

Concerning rain, it has been calculated that 150mm per hour rain over a 1 km path will introduce a phase shift of some 500 degrees at 30 GHz.[13] This is equivalent to a change in N of 14 units.

It is not easy to arrive at a reasonable estimate of short-term and long-term changes in the average value of N over a long transmission system. On the basis of the foregoing data, it appears to the writer that changes during the day would probably not exceed 20 units, while changes during the month might be as much as 100 units, and changes during the year might be several hundred units.

If we assume a 3000 mile path, the nominal transmission time is about 0.016 second, and at a pulse rate of 5×10^8 pps the number of pulses in the transmission system will be 8×10^6. The change in this number of pulses will be the number times N times 10^{-6}. For some values of change in N, the change in number of pulses will be

Change in N	*Change in number of pulses*
20	160
100	800
200	1600

APPENDIX D

Jitter Due to Regenerative Repeaters

In an experimental digital repeater line[1] the systemic jitter introduced by a single repeater had an rms value θ_1 of about 3.3°. The rms jitter after N repeaters, θ_N, is given by[14]

$$\theta_N = \theta_1 \sqrt{\frac{P(N)}{P(1)}} \tag{24}$$

$$P(N) = \frac{N}{2} - \frac{(2N - 1)!}{4^N[N - 1!]^2} \tag{25}$$

The function $P(N)$ has been tabulated:[15] $P(1) = 0.250$ and for $N > 100$, $P(N) = N/2$. Thus, for a large $N > 100$ number of repeaters,

$$\theta_n = \theta_1 \sqrt{2N} \tag{26}$$

Some computed values of θ_n are:

Number of repeaters	θ_n, *rms phase jitter, (degrees)*
1	3.3
100	47
300	81
1000	148
3000	256

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