Dielectric Loss in Integrated Microwave Circuits

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Dielectric loss is important in integrated microwave and millimeter wave circuits which require small attenuation. Such circuits are usually built with microstrip or suspended microstrip transmission lines. This paper shows that the dielectric loss, the filling factor of the microstrip, and the stored field energy in the dielectric substrate can be computed from the partial derivative $\partial U/\partial \epsilon_r$ *where* U *is the total electric field energy and* ϵ_r *the relative dielectric constant of the substrate. It also shows that the effective loss tangent is determined by the partial derivative* $\partial \epsilon_{eff}/\partial \epsilon_r$ *where* ϵ_{eff} *is the effective dielectric constant of the microstrip. Useful design formulas for computing the dielectric loss are given for the most important cases.*

I. INTRODUCTION

The dielectric loss in microstrip or suspended microstrip transmission lines is an important parameter in the design of hybrid integrated circuits which require small attenuation. This loss can be calculated if one knows the loss tangent of the dielectric substrate and the electric field distribution inside the substrate. Electric field computations are usually complicated and not practical for design purposes. It is therefore important to find a simple and accurate method for calculating the dielectric loss from other well known properties of the microstrip transmission line.

The results of dielectric loss computations for microstrips, which have been made by other authors, are quoted in many recent papers on hybrid integrated circuit design.[1–6] It can be shown that these results are applicable only if the boundary between the dielectric substrate and air is parallel to an electric field line. This paper presents general design equations valid for all microstrip transmission lines provided that the propagating mode can be approximated by a TEM mode.

II. EFFECTIVE DIELECTRIC CONSTANT AND FILLING FACTOR OF MICROSTRIP LINES

The effective dielectric constant of a microstrip line partially filled with dielectric material is defined by

$$\epsilon_{eff} = \left(\frac{\lambda_o}{\lambda}\right)^2, \tag{1}$$

where λ_o is the vacuum wavelength and λ the wavelength of the propagating mode on the microstrip. If the propagating mode can be approximated by a TEM mode one can also define ϵ_{eff} by

$$\epsilon_{eff} = \frac{C}{C_o}, \tag{2}$$

where C is the capacitance per unit length with partial dielectric filling and C_o the capacitance per unit length without dielectric material.

The filling factor q of a microstrip is defined by

$$q = \frac{U_1}{U} \tag{3}$$

where U_1 is the electric field energy stored in the dielectric and U the total electric field energy of the microstrip. Notice that some authors do not use the same definition for q. Poole and Von Hippel use the ratio given by equation (3).[7,8] This definition is useful because it simplifies the loss calculation.

III. PARTIAL DERIVATIVES OF FIELD ENERGY AND EFFECTIVE DIELELCTRIC CONSTANT

If one computes the partial derivative of the total electric field energy U with respect to the relative dielectric constant ϵ_1 of the substrate, one obtains the basic result

$$\frac{\partial U}{\partial \epsilon_1} = \frac{U_1}{\epsilon_1}. \tag{4}$$

The Appendix gives the derivation of this equation. We assume that the conductor configuration remains the same and that the potential difference between the conductors is constant. From equations (2) and (4), and from $U = CV^2/2$ we obtain

$$\frac{\partial \epsilon_{eff}}{\partial \epsilon_1} = \frac{\epsilon_{eff}}{\epsilon_1} \frac{U_1}{U} \tag{5}$$

The filling factor q is now given by

$$q = \frac{\epsilon_1}{\epsilon_{eff}} \frac{\partial \epsilon_{eff}}{\partial \epsilon_1}, \tag{6}$$

and the effective loss tangent of the microstrip is

$$(\tan \delta)_{eff} = \frac{\epsilon_1}{\epsilon_{eff}} \frac{\partial \epsilon_{eff}}{\partial \epsilon_1} \tan \delta \tag{7}$$

with $\tan \delta$ being the loss tangent of the dielectric substrate. One can show that the effective loss tangent of microstrips with more than one single lossy substrate is given by

$$(\tan \delta)_{eff} = \frac{1}{\epsilon_{eff}} \sum_{n=1}^{N} \epsilon_n \frac{\partial \epsilon_{eff}}{\partial \epsilon_n} \tan \delta_n \tag{8}$$

where ϵ_n and $\tan \delta_n$ are the relative dielectric constants and loss tangents of each substrate respectively and N the total number of lossy dielectric materials in the microstrip.

IV. DIELECTRLC ATTENUATION AND UNLOADED Q

The unloaded dielectric quality factor Q_D of the microstrip is

$$Q_D = \frac{1}{(\tan \delta)_{eff}} = \frac{1}{q \tan \delta}, \tag{9}$$

and the dielectric attenuation in dB per unit length is

$$\alpha_D = \frac{20\pi}{\ln 10} \frac{q \tan \delta}{\lambda} = 27.3 \frac{(\tan \delta)_{eff}}{\lambda}, \tag{10}$$

with λ being the microstrip wavelength $\lambda = \lambda_o/(\epsilon_{eff})^{1/2}$.

The effective dielectric constant for the standard microstrip of Fig. 1a is known and can be approximated by[9]

$$\epsilon_{eff} = \frac{\epsilon_1 + 1}{2} + \frac{\epsilon_1 - 1}{2} \left(1 + 10 \frac{h}{w}\right)^{-\frac{1}{2}}. \tag{11}$$

By introducing $F(w, h) = (1 + 10\, h/w)^{1/2}$ we obtain, from equation (6), the filling factor

$$q = \frac{1}{1 + \dfrac{F - 1}{\epsilon_1(F + 1)}}. \tag{12}$$

Figure 2 is a graph of the filling factor for the standard microstrip as a function of w/h with ϵ_1 as a parameter.

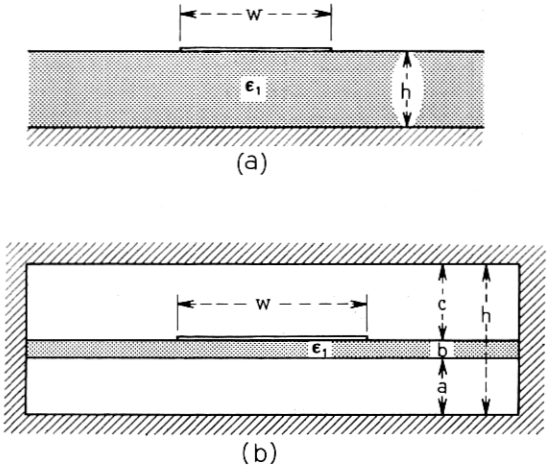

Fig. 1 — (a) Standard microstrip transmission line and (b) suspended microstrip transmission line.

Computation of q for the suspended microstrip shown in Fig. 1b is more difficult. An approximate value can be obtained if $w \gg h$, which means the fringe field contributions are small. The effective dielectric constant is

$$\epsilon_{eff} = \frac{a + b}{a + b + c}\left(1 + \frac{c\epsilon_1}{a\epsilon_1 + b}\right) \qquad w \gg h, \tag{13}$$

and the filling factor becomes

$$q = \frac{bc\epsilon_1}{(a\epsilon_1 + b)(a\epsilon_1 + b + c\epsilon_1)}. \tag{14}$$

A different approach is necessary if the fringe field contribution cannot be neglected. Figure 3 shows a suspended microstrip which has been used in circuits built by Engelbrecht and Kurokawa, Saunders and Stark, and Tatsuguchi and Aslaksen.[10–12] The effective dielectric constant of the configuration with the dimensions given in Fig. 3 has been computed by Brenner.[13] It is possible to approximate the result by Brenner by the simple rational function

$$\epsilon_{eff} = 1 + \frac{\epsilon_1 - 1}{0.38\epsilon_1 + 7.70}. \tag{15}$$

From equation (6) we obtain

$$q = \frac{\epsilon_1}{6.38 + 1.63\epsilon_1 + 0.065\epsilon_1^2}. \tag{16}$$

Figure 3 is a graph of this filling factor as a function of the relative dielectric constant ϵ_1. The filling factor reaches a broad maximum for relative dielectric constants between 6 and 12. This maximum is obtained for structures with substantial fringe field contributions. If one neglects the fringe field the filling factor is substantially reduced and decreases if ϵ_r is increased.

V. DISCUSSION

There are several types of substrates which are useful for building integrated circuits. These substrates are

(*i*) borosilicate glasses and other commercial glasses with loss tangents of the order of 10^{-2} at microwave and millimeter wave frequencies,[14]
(*ii*) semiconductor substrates such as Si and GaAs with loss tangents determined by $\tan \delta = \sigma/\omega\epsilon_o\epsilon_1$ where σ is the substrate conductivity in mho per centimeter, ϵ_o the free space permittivity $\epsilon_o = 8.85 \cdot 10^{-14}$ F per cm, and ϵ_1 the relative dielectric constant of the semi-conductor,
(*iii*) ceramics such as alumina, beryllia, and rutile with loss tangents of about 10^{-4} at microwave and millimeter wave frequencies, and
(*iv*) fused silica with $\tan \delta = 10^{-4}$ in the same frequency range.

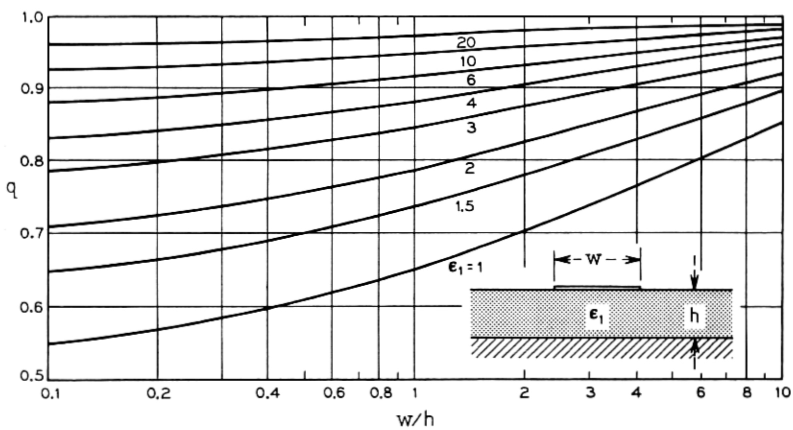

Fig. 2 — Filling factor q for standard microstrip transmission line as a function of the ratio w/h with relative dielectric constant ϵ_1 as parameter.

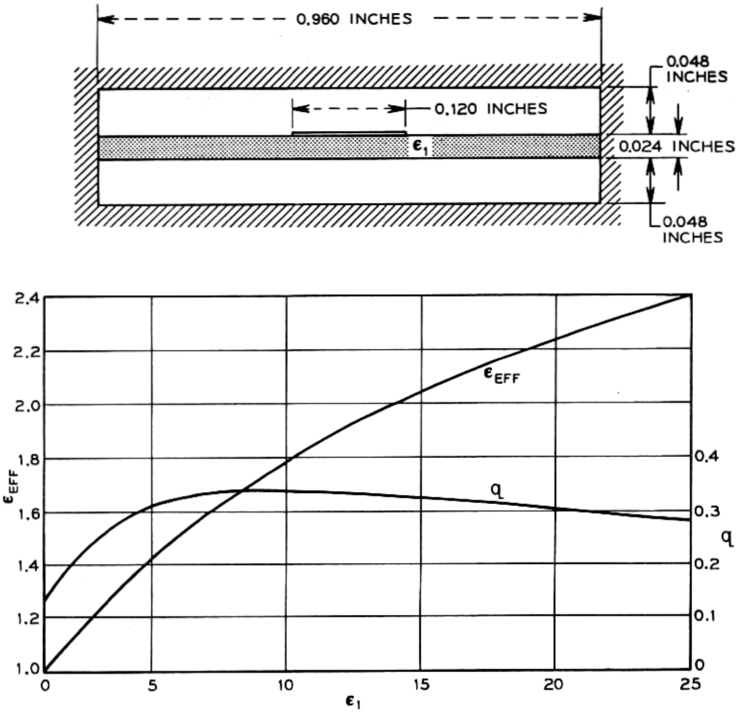

Fig. 3 — Effective dielectric constant ϵ_{eff} and filling factor q for suspended microstrip transmission line for $w = h = 0.120$ inch, $a = c = 0.048$ inch and $b = 0.024$ inch.

For glasses one can, therefore, expect an unloaded dielectric quality factor of $Q_D = 100/q$; with high quality ceramics and fused silica one obtains $Q_D = 10000/q$. However, loss tangents of many substrates above 30 GHz are presently not available.

The unloaded Q resulting from conductor loss alone is typically $Q_c = 100$ to 1000 for completely shielded microstrips at microwave and millimeter wave frequencies. The total unloaded Q is $Q_T = Q_D Q_c/(Q_D + Q_c)$. One concludes that the conductor loss is predominant for circuits built with high quality ceramics and quartz. For microstrips built on glass substrates and some semiconductor substrates, the filling factor is important for computing the total loss of the microstrip.

APPENDIX

Partial Derivative of Field Energy U

The total electric field energy U stored in a microstrip is given by the volume integral

$$U = \int_V \frac{D^2}{2\epsilon}\, dV, \tag{17}$$

where D is the displacement, $D = \epsilon E$, and $\epsilon = \epsilon_o \cdot \epsilon_1(x, y, z)$ is an isotropic dielectric constant. We make a small perturbation subject to boundary conditions which follow equation (20).

$$\delta U = \int_V \frac{D\,\delta D}{\epsilon}\, dV - \int_V \frac{D^2\,\delta\epsilon}{2\epsilon^2}\, dV. \tag{18}$$

By using $E = -\text{grad}\,\varphi$ and $\text{div}\, D = \rho$ one obtains, from $\text{div}\,(\varphi\,\delta D) = -E\,\delta D + \varphi\,\text{div}\,\delta D$,

$$\delta U = \int_V \varphi\,\delta\rho\, dV - \int_V \text{div}\,(\varphi\,\delta D)\, dV - \frac{1}{2}\int_V E^2\,\delta\epsilon\, dV, \tag{19}$$

and from the theorem by Gauss

$$\int_V \text{div}\,(\varphi\,\delta D)\, dV = \sum_{K=1}^{N} \varphi_K \int_{F_K} \delta D_n\, dF = \sum_{K=1}^{N} \varphi_K\,\delta Q_K\,, \tag{20}$$

where the surface integral is carried out over all conductor surfaces $K = 1, 2, \ldots, N$. We are interested in a perturbation subject to the following boundary conditions:

(*i*) The space charge is zero, $\delta\rho \equiv 0$.
(*ii*) The charge on each conductor remains constant, $\delta Q_K = 0$.
(*iii*) $\delta\epsilon$ is constant in the dielectric substrate, and $\delta\epsilon = 0$ outside the substrate.

If the dielectric constant of the substrate is ϵ from equation (18) we obtain

$$\delta U = -\delta\epsilon\, \frac{\displaystyle\int_{V_1} \frac{\epsilon}{2} E^2\, dV}{\epsilon}. \tag{21}$$

The volume integral is the electric field energy U_1 stored in the dielectric substrate. For two conductors and $\Delta\varphi = \varphi_1 - \varphi_2 =$ constant one has $\delta U = +\delta\epsilon \cdot U_1/\epsilon$ and consequently

$$\frac{\partial U}{\partial \epsilon_1} = \frac{U_1}{\epsilon_1}. \tag{22}$$

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