Excitation of the Dominant Mode of a Round Fiber by a Gaussian Beam

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The excitation of the dominant HE_{11} mode of a round optical fiber by a gaussian beam has been calculated. The calculation is based on the assumption that reflected waves can be neglected. It is thus applicable only to fibers with low index difference between core and cladding.

It is found that optimum excitation of the HE_{11} mode is achieved for loosely guided beams if the product of the beam half-width w times the radial decay constant γ of the HE_{11} mode outside of the guide is unity, $\gamma w = 1$. For tightly coupled modes $2^{\frac{1}{2}}w$ must be equal to the core radius in order to achieve optimum excitation. As much as 99 percent of the power can be transferred to the HE_{11} mode.

Also investigated are the effects of an off-set or tilted beam on the mode excitation. The mode excitation drops to 36 percent if the amount of off-set equals the beam half-width. The effect of tilts depends on the parameter kd, free space propagation constant times core radius of the fiber. For small values of kd or loosely guided modes, the mode excitation is very sensitive to tilts of the gaussian beam. As long as the HE_{11} mode is the only mode that can propagate, increasing values of kd lead to less sensitivity with respect to tilts. For multimode operation of the fiber, the sensitivity to tilts increases with increasing values of kd. The minimum of tilt sensitivity coincides with the minimum spot size of the guided mode.

I. INTRODUCTION

Communication by means of optical fibers requires that light energy can be coupled into the fiber in an efficient way. Of the different methods of exciting an optical fiber, the simplest consists of shining a beam of laser light on the end of the fiber. It is the purpose of this paper to investigate the power loss that results at the transition from a laser beam propagating in free space to the lowest order HE_{11} mode of a round optical fiber.

The geometry of the problem is sketched in Fig. 1. It is assumed that the fiber core is embedded in an infinite material, its cladding. For simplicity it is assumed that the value of the refractive index outside of the core is unity. The theory is manageable only if reflections from the end of the fiber are neglected. The transmission coefficients are calculated by matching only the transverse component of the electric or of the magnetic field at $z = 0$. Finally, an average of these two values is taken.

The incident beam is assumed to have a field distribution of the form

$$E_x = A \exp\left[-\left(\frac{r}{w}\right)^2\right] \exp(-ikz) \quad \text{for} \quad z \approx 0 \tag{1}$$

and

$$H_y = \left(\frac{\epsilon_0}{\mu_0}\right)^{1/2} E_x \tag{2}$$

with

$$k = \frac{2\pi}{\lambda_0} = \omega(\epsilon_0\mu_0)^{1/2}. \tag{3}$$

Since the field components of the fiber modes are conveniently expressed in cylindrical polar coordinates r, ϕ and z, it is advantageous to transform the incident field to these coordinates.

$$E_r = E_x \cos\phi; \qquad E_\phi = -E_x \sin\phi; \tag{4}$$

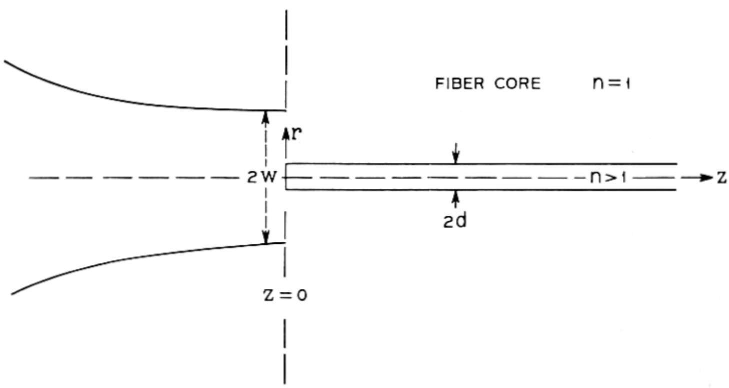

Fig. 1—Gaussian beam incident on the core of a dielectric fiber with refractive index n.

$$H_r = H_y \sin\phi; \qquad H_\phi = H_y \cos\phi. \tag{5}$$

The amplitude coefficient c_t of the HE_{11} mode is approximately determined by the equation

$$c_t = \frac{(I_1 I_2)^{1/2}}{2P} \tag{6}$$

with

$$I_1 = \int (E_r \mathcal{H}_\phi^* - E_\phi \mathcal{H}_r^*) r\, dr\, d\phi \tag{7}$$

and

$$I_2 = \int (\mathcal{E}_r^* H_\phi - \mathcal{E}_\phi^* H_r) r\, dr\, d\phi. \tag{8}$$

P is the power carried by the incident gaussian mode. The script letters indicate the field components of the guided HE_{11} mode,[1] while the other field components belong to the incident gaussian mode.

The r integrations must be carried out numerically while the ϕ integrations can be done analytically even in the more complicated cases of an off-set incident field distribution shown in Fig. 2 or a tilted incident field distribution shown in Fig. 3.

The field components of the guided modes are described by cylinder functions. The arguments of these functions inside of the fiber core at $r < d$ are κr with the radial propagation constant κ determined by

$$\kappa^2 = n^2 k^2 - \beta^2 \tag{9}$$

where β is the propagation constant of the guided mode in z direction. On the outside, $r > d$, the argument of the cylinder functions is γr with

$$\gamma^2 = \beta^2 - k^2. \tag{10}$$

The decay constant γ determines the rate at which the field intensity of the guided mode decays outside of the fiber core. For large values of r the fields behave like

$$\exp(-\gamma r). \tag{11}$$

Equation (6) for the amplitude transmission coefficient is not exact. It was derived under the assumption that reflections at $z = 0$ are negligible. The power transmission coefficient T follows from

$$T = |c_t|^2. \tag{12}$$

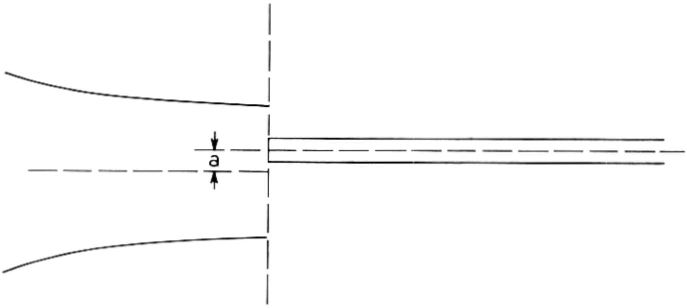

Fig. 2—Fiber excited by a gaussian beam off-set with respect to the fiber axis by an amount a.

II. NUMERICAL RESULTS

We begin the discussion of the dependence of the transmission coefficient T from the incident gaussian field to the guided HE_{11} mode with the simplest case shown in Fig. 1 for a refractive index $n = 1.01$. The gaussian beam is perfectly aligned with its beam waist being coincident with the end of the fiber core at $z = 0$. The transmission coefficient as a function of the product γw is shown in Fig. 4. Each curve belongs to a different value of kd. The normalization of the curves with respect to the radial decay constant γ is convenient since it compresses the dependence of the curve on the horizontal axis. The position of the peaks would differ by two

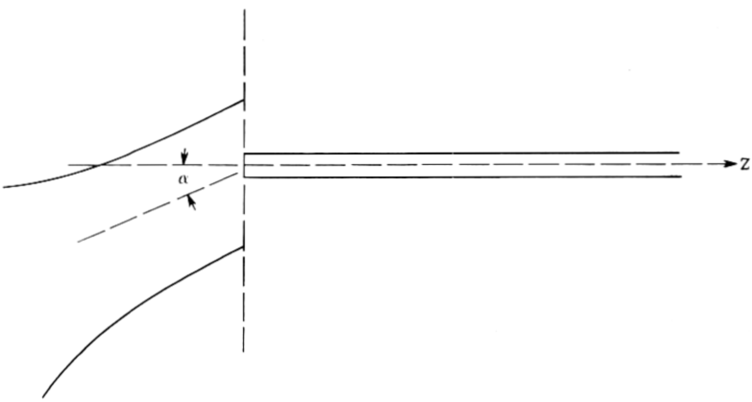

Fig. 3—Fiber excited by a tilted gaussian beam.

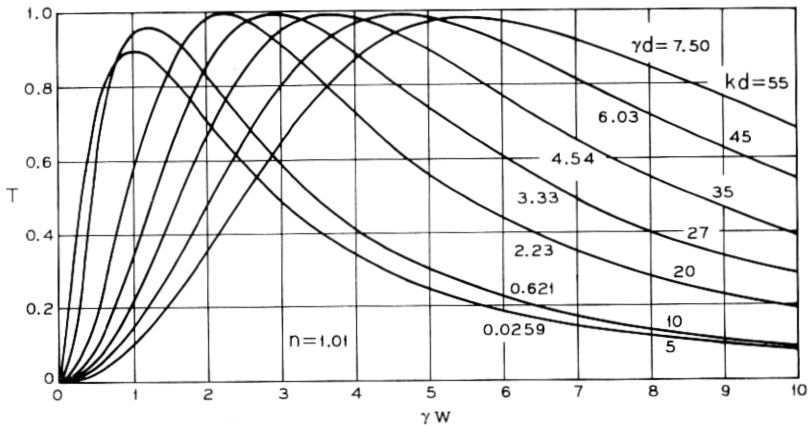

Fig. 4—Transmission coefficient T as a function of γw for several values of kd and $n = 1.01$.

orders of magnitude if the curves were drawn simply as functions of w.

Two remarkable properties can be deduced from Fig. 4. The transmission coefficient approaches extremely close to 100 percent. The dependence of the transmission peaks as a function of kd is shown in more detail in Fig. 5. According to this figure, the transmis-

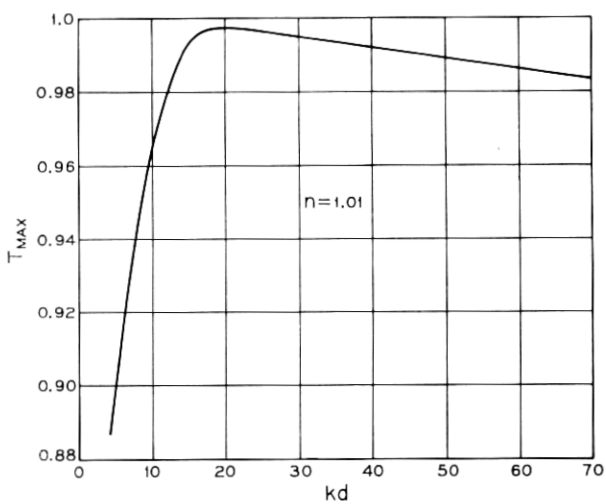

Fig. 5—The peak values of the transmission coefficient as a function of kd.

sion coefficient can be as high as 99.7 percent. These values are probably slightly optimistic as we shall see shortly.

The position of the transmission peaks can be predicted for two regions of operation. For small values of kd the guided mode is only loosely supported by the fiber core. Most of the field is on the outside decaying according to equation (11). In this case the transmission curves peak at

$$\gamma w = 1. \tag{13}$$

This means that the $1/e$ point of the exponential decay of the mode field coincides with the corresponding point of the gaussian curve. For $\lambda = 1\mu$ and $kd = 5$, we have $d = 0.8\mu$ so that for this example $1/\gamma = w = 31\mu$; $kd = 10$ correspond to $1/\gamma = w = 2.6\mu$.

The HE_{11} mode is no longer the only possible guided mode for large values of kd. At the value

$$kd = \frac{2.405}{(n^2 - 1)^{1/2}} \tag{14}$$

the TE_{01} mode begins to propagate. For $n = 1.01$, this point appears for $kd = 17$. For tightly guided modes, most of the field energy is concentrated inside of the fiber core. In this case, the peak of the transmission coefficient occurs at

$$w = d/2^{\frac{1}{2}}. \tag{15}$$

For a very tightly guided mode, the propagation constant approaches $\beta = nk$ so that we obtain from equations (10) and (15)

$$\gamma w = (n^2 - 1)^{\frac{1}{2}} kd/2^{\frac{1}{2}}. \tag{16}$$

For $n = 1.01$, we thus have $\gamma w = 0.1\ kd$. This relationship is indeed apparent in Fig. 4.

For larger refractive indices of the core, our approximation becomes questionable. This breakdown of the approximation is apparent in Fig. 6 where $n = 1.432$. The curve with $kd = 3$ exceeds the value unity very slightly, violating the principle of conservation of power. This shows that our approximate values for T are slightly too large. However, for small values of $n - 1$, it can be expected that the approximation is good because back-scattering of power from the end of the fiber core becomes negligible. This expectation is confirmed by the fact that none of the curves in Fig. 4 exceeds the value unity. It is hard to predict the degree of accuracy of the approximation.

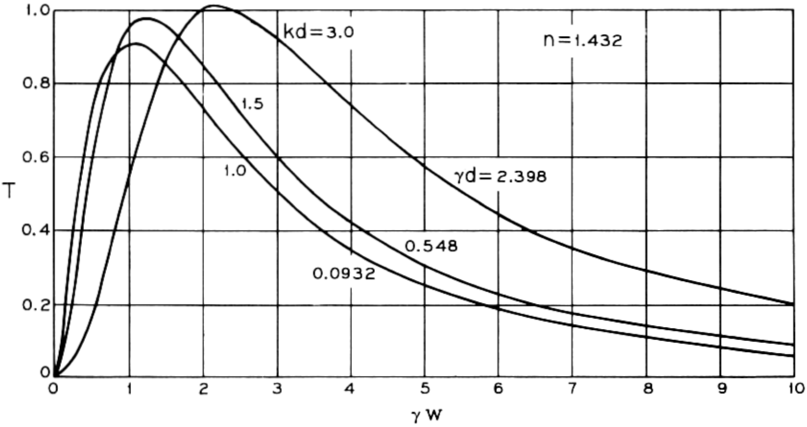

Fig. 6—Transmission coefficient T as a function of γw for $n = 1.432$. The curve with $kd = 3$ exceeds $T = 1$ indicating a breakdown of the approximation.

The values of Fig. 4 are perhaps slightly too high but it is clear that the power transmission from the gaussian mode to the guided HE_{11} mode is very efficient even if it does not quite reach 99.7 percent.

Since perfect beam alignment cannot be achieved, it is important to know how sensitive the transmission coefficient is to misalignments of the beam.

Fig. 7 shows data for the transmission coefficient T as a function of the amount of off-set "a" of the gaussian beam shown in Fig. 2. The independent variable of Fig. 7 is the product γa. Each curve was drawn for its optimum value of γw according to Fig. 4. Fig. 7 shows that the transmission coefficient decreases to 0.36 if $a = w$. This is a simple relationship that apparently holds for all values of kd. An off-set of the gaussian beam is thus not as critical as one might have feared. The direction in which the beam is off-set with respect to the polarization of the input field has been found to be unimportant. The same curves shown in Fig. 7 were obtained for any direction of the off-set.

The dependence of the transmission coefficient on tilts of the input field is shown in Fig. 8. Again w was chosen so that the maximum transmission coefficient is obtained in the absence of a tilt. The trend of these curves is interesting. The transmission coefficient is very sensitive to tilts for small values of kd. This is not surprising since the fields extend far from the fiber core so that a slight tilt causes the two wavefronts of the input field and the guided mode to become

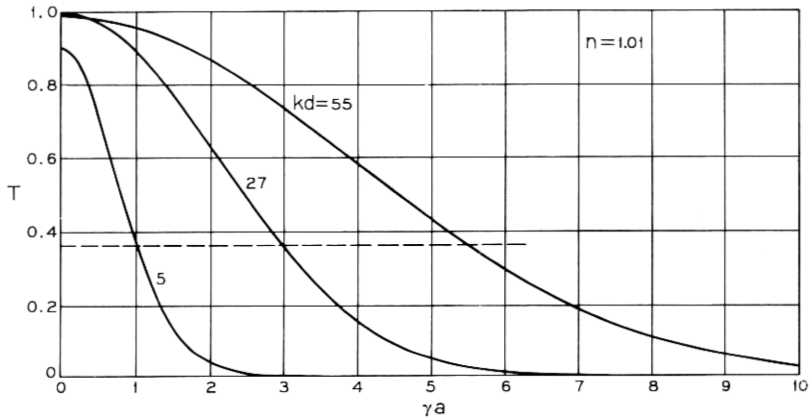

Fig. 7—Peak transmission coefficient T as a function of beam off-set.

seriously misaligned. As the guided mode (and since maximum transmission is assumed also the input field) contracts, the transmission coefficient is far less sensitive to tilts. The least sensitive curve appears for $kd = 20$ in Fig. 8. The next guided mode can be excited by the input field as soon as kd exceeds the value 17. As more and more guided modes appear, the transmission coefficient to the lowest order mode, the HE_{11} mode, becomes more sensitive to tilts. The best operating point as far as sensitivity to tilts is concerned is ap-

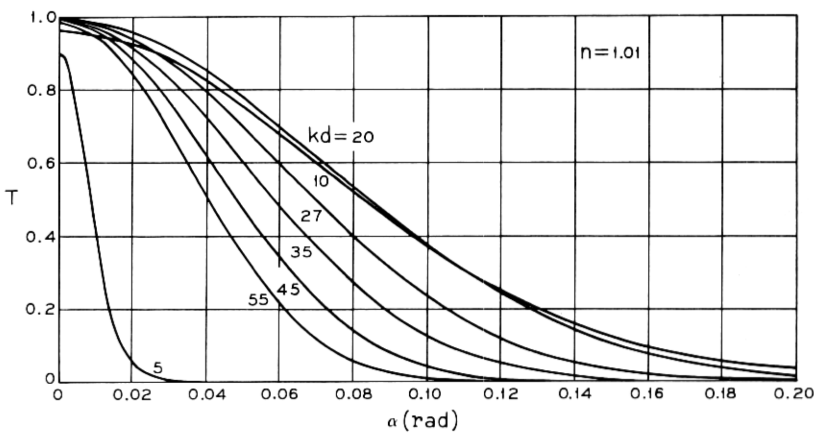

Fig. 8—Peak transmission coefficient as a function of tilt angle α.

parently close to the point where the next guided mode begins to propagate. This behavior can be explained as follows. If the wave length is kept constant and d is increased, the radial extension of the field decreases at first for increasing values of d. However, as d increases further, the field cross-section increases again. The least sensitivity to tilts occurs at the minimum field cross-section.

III. CONCLUSIONS

A numerical study of the excitation of the lowest order HE_{11} mode of the round optical fiber by an incident gaussian mode showed that the achievable transmission coefficient is very high. The predicted optimum value of 99.7 percent may be slightly overoptimistic because of the approximate nature of the calculation. However, Snyder[2] predicts transmission coefficients as high as 80 percent for the case of excitation by a truncated plane wave. The gaussian beam is far better matched to the HE_{11} mode so that a much higher transmission coefficient is not surprising.[3]

An off-set of the peak of the gaussian beam equal to its beam half-width w decreases the transmission coefficient to 36 percent. Tilts of the input field distribution are more serious for small values of the ratio of fiber core radius to wavelength. The least tilt sensitivity is obtained under conditions where the HE_{11} mode is operated close to the cut-off frequency of the TE_{01} mode. The beam cross-section assumes a minimum at this point.

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