# B. S. T. J. BRIEF

## An Image Band Interpretation of Optical Heterodyne Noise

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### I. INTRODUCTION

Optical heterodyning and homodyning (heterodyning to baseband) have been studied by various authors.[1–3] The intermediate frequency or baseband signal that is obtained by ideal optical heterodyning or homodyning consists of the signal that would be expected from classical analysis plus an added gaussian quantum noise. The variance of this quantum noise is twice as large in the heterodyning case as it is in the homodyning case. Since classically heterodyning need not have a noise disadvantage over homodyning, this result at first seems puzzling. We shall show below that the disadvantage of heterodyning can be interpreted as an unavoidable "image band" quantum noise. We shall argue this both heuristically and formally using quantum field theory.

### II. HEURISTIC ARGUMENTS

Consider the classical heterodyne system. Two radio frequency (RF) bands can contribute to the intermediate frequency (IF) output. One contains the desired signal plus associated noises occupying the signal band. The other is the image band containing image band noises. Classically, to avoid the noises at the IF due to the image band, we filter out the RF image band before it can enter the receiver front end. In the quantum case, we have quantum noise at the IF due to the signal band and the image band. Unlike the classical case, we cannot eliminate the image band quantum noise. Thus heterodyning has twice the quantum noise of homodyning (which has no image band).

### III. RIGOROUS RESULTS

We shall now show that the "noisy" quantum heterodyne measurement of the signal on a mode of a quantum field often described in terms of an overcomplete set of measurement states,[4] can be described in terms of a compatible pair of measurements on the product space of that mode and an extraneous "image mode."

Suppose that the density operator for a single "signal" mode of the optical field in a bounded region of space is (using the notation of R. J. Glauber[5])

$$\rho_s^{(\beta)} = \int \frac{1}{\pi N} \exp \{|(\alpha_s - \beta)|^2/N\} \; |\alpha_s > < \alpha_s| \; d^2\alpha_s$$

where $N$ is proportional to the thermal noise variance, $\alpha_s$ and $\beta$ are complex numbers ($\beta$ represents signal) and $| \alpha_s >$ is an eigenvector of the annihilation operator $a_s$ which along with its adjoint $a_s^+$ satisfies

$$[a_s , a_s^+] = 1.$$

We could estimate the real or imaginary part of $\beta$ by measuring

$$M_1 = \frac{a_s + a_s^+}{2}$$

or

$$M_2 = \frac{a_s - a_s^+}{2j} \; ;$$

but they do not commute and cannot be measured simultaneously. Consider next another field mode as "image mode" which does not depend on $\beta$ and which has arbitrarily small thermal noise, i.e., its density operator is

$$\rho_I = \int \frac{1}{\pi M} \exp \{|\alpha_I|^2/M\} \; |\alpha_I > < \alpha_I| \; d^2\alpha_I$$

where $| \alpha_I >$ is an eigenvector of the image mode annihilation operator $a_I$

$$[a_I , a_I^+] = 1.$$

Since the annihilation and creation operators of the signal mode commute with the annihilation and creation operators of the image mode, the following two operators defined on the product space of the two modes commute

$$L_1 = \tfrac{1}{2}(a_s + a_s^+) + \tfrac{1}{2}(a_I + a_I^+),$$

$$L_2 = \frac{1}{2j}(a_s - a_s^+) - \frac{1}{2j}(a_I - a_I^+).$$

The density operator of the product space is simply $\rho_S^{(\beta)}\rho_I$ the product of the density operators.

The joint moment generating function of the random variable which results from simultaneous measurement of $L_1$ and $L_2$ is

$$M_{12}(u, v) = E\{\exp[uL_1(\text{outcome})] \exp[vL_2(\text{outcome})]\}$$
$$= TR[\rho_s^{(\beta)}\rho_I \exp(uL_1) \exp(vL_2)]$$

using the following properties[6]

$$\exp(A) \exp(B) = \exp(A + B) \exp[\tfrac{1}{2}[A, B]]$$

if

$$[A, [A, B]] = 0$$

and

$$[B, [A, B]] = 0$$

we obtain

$$TR[\rho_s(\beta)\rho_I(\beta) \exp(uL_1) \exp(vL_2)] = M_{12}(u, v)$$
$$= \exp[uRl(\beta)] \exp[v \operatorname{Im}(\beta)] \exp[\{(u^2 + v^2)/2\}(N/2 + M/2 + \tfrac{1}{2})]$$
$$= \exp[uRl(\beta)] \exp[v \operatorname{Im}(\beta)] \exp[\{(u^2 + v^2)/2\}(N/2 + \tfrac{1}{2})]$$
$$\text{for} \quad M \ll N$$

which is the result obtained by physical heterodyning[1–3] (note that $M \ll N$ corresponds to filtering image band noise).

If we had measured $M_1 = (a^+ + a)/2$ alone, the outcome random variable would have moment generating function

$$M_3(u) = E \exp[uM_1(\text{outcome})]$$
$$= TR\rho_s(\beta) \exp(uM_1) = \exp(uRl(\beta)) \exp\left[\frac{u^2}{2}(N/2 + \tfrac{1}{4})\right]$$

which is the result obtained by physical homodyning.[1–3]

We see that the homodyne measurement has half the gaussian quantum noise added to the real part of $\beta$ (which is the outcome mean) as the heterodyne measurement has. However, if we must know both the real and imaginary parts of $\beta$, we must heterodyne, since homodyning can only yield one part or the other (unless we split the received signal in half and homodyne each part).

## IV. CONCLUSIONS

We see from the above that the fact that heterodyning results in twice the quantum noise of homodyning is not strange at all. The added noise is heuristically and rigorously associated with an image band which can contribute extra noise classically as well.

## REFERENCES

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