Model for Computation of Interference to Radio-Relay Systems From Geostationary Satellites

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A statistical model is suggested for the computation of interference into terrestrial radio-relay systems from geostationary satellites.

The model is general. It incorporates radio-relay characteristics, satellite arrangements, and allowable satellite power flux densities. A program simulator can be used to calculate the satellite power flux density corresponding to a particular radio-relay interference objective.

Interference distributions are computed for AT&T and CCIR radio-relay models using the power flux density that was suggested for study at the 1969 CCIR Interim Meeting at Geneva, Switzerland.

I. INTRODUCTION

Terrestrial radio-relay systems operating in the 4-GHz and 6-GHz bands share these frequencies with communication satellite systems, and additional bands may be designated for shared usage. In the future, as the geostationary orbit occupancy increases, coexistence of these systems will be possible only if controls are imposed on both systems.

Presently, the International Radio Consultative Committee (CCIR) recommends[1] that radio-relay antennas, radiating frequencies normally received by satellites, maintain a specified angular separation with respect to the geostationary orbit or, where this is not practical, the application of power limitation to the terrestrial transmitters. They further recommend[2] that satellite power flux density be limited as a function of the angle of illumination at the earth's surface. The pointing restriction is intended to protect satellites from terrestrial systems. Methods have been devised[3] for calculating beam-orbit separation as a function of azimuth displacement of the antenna from intercept, and the recommendation is being complied with by the Bell System. The satellite power limitation has been challenged as being too restrictive and, in the interest of satellite system economy, an increase at the higher angles of arrival has been proposed.

The derivation of the present satellite power flux-density limitation assumes one radio-relay station in a 50-hop system will have a direct exposure to a geostationary satellite, and the remaining stations will have sufficient antenna discrimination that the additional interference is not significant. On this premise the flux-density limit for a tangential ray was established. A linear escalation for higher angles of arrival accounts only for a differential in transmission loss and satellite antenna gain between the horizon and the subsatellite point. A proposal to greatly increase the flux-density limit at all angles other than zero degrees requires a new evaluation.

At present the number of communication satellites is small and the effective radiated power is below existing limits. Consequently, an experiment with existing satellites interfering into the terrestrial network will not generate fruitful results in determining adequate protection for both systems. A laboratory experiment is more tractable but it is very difficult to simulate actual cases because the interference is a function of the spatial arrangement of the two systems, and the results should be applicable internationally. To place an experimental satellite in the geostationary orbit for the purpose of measuring interference into terrestrial systems is ludicrous for it would yield no more information than a laboratory experiment and would be vulnerable to the same limitations.

A new attempt to evaluate the satellite power flux density limitation originated in the U.S.A. in preparation for the 1969 CCIR meeting in Geneva, Switzerland. It was concluded that an analytical approach is the most promising.

In subsequent sections we present a description of the system model developed by the authors for studying satellite interference into terrestrial systems, along with the analysis, computer simulation, and some results.

II. SYSTEM MODEL

The satellite system is assumed deterministic with all satellites in the geostationary orbit. The spacing between satellites is assumed fixed with each satellite transmitting the same effective radiated power. Moreover, it is assumed the entire orbit is filled.

The assumption that all satellites are in the geostationary orbit is realistic because the number of medium-orbit systems is small, and the additional interference from medium orbit satellites can be accounted for by decreasing the interference allocation of the stationary satellites. Of course, the short-term interference contribution from medium orbit satellites should not be ignored, but this is not the subject of this discussion. Furthermore, it is assumed that if a satellite is visible to a radio-relay station, it will illuminate the area of the radio-relay station with the flux density permitted by the angle of arrival. Actually, this is not always the case. A satellite may be visible but it may use a highly directive antenna that does not illuminate all radio-relay stations with the maximum permissible flux density. Again, the effect of this assumption is not serious because the spot-beam satellites would reduce the interference and their effect can be adequately taken into account by increasing the effective satellite spacing.

The radio-relay system is assumed to be composed of a number of hops. The number of hops depends on the system—50 hops for a CCIR system and 140 hops for an AT&T system. Figure 1 introduces the system model concept.

The azimuth of each radio-relay system, referred to as a trendline, is assumed as a random variable. The distribution that was used for the results given in Section V is uniform between 0 and 2π. However,

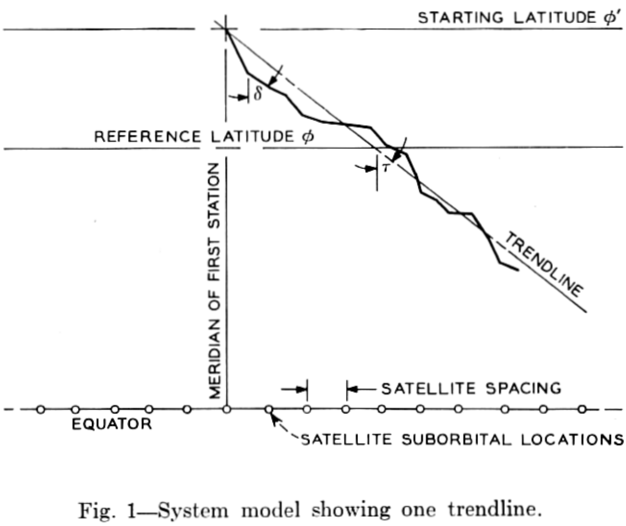

Fig. 1—System model showing one trendline.

the simulation is capable of using any distribution. For the AT&T terrestrial network, a uniform distribution is reasonable; however, for other countries the distributions will be biased (Canada's network is nearly east-west). Since the direction of each station is not the same as the trendline (intra-system interference and geography dictate a zig-zag route), the direction of each station is assumed as a random variable within some range about the direction of the trendline. Again the results in Section V were calculated for a uniform distribution within $\pm 25°$ of the trendline direction.

The radio-relay antenna pattern can be any appropriate pattern depending on the actual system. However, the angular separation between the beam of the radio-relay station and the satellite does not correspond to the off-beam angle of most radio-relay antenna patterns because the patterns are usually measured in either the horizontal or the vertical direction. The true angular separation is neither, but either pattern can be used because most of the interference results when the separation angle is small, and the pattern is nearly symmetrical in all planes for small off-beam angles.

To summarize, the model incorporates a satellite spacing, radio-relay antenna pattern, number of hops, trendline, distribution, and station pointing distribution. These parameters are the inputs for a computer program which calculates the distribution of the total radio-relay interference.

III. DESCRIPTION OF THE SIMULATION PROGRAM

The location of the radio-relay system need only be identified in latitude. Since the stationary orbit is assumed filled with a constant satellite spacing, the longitude need not be a parameter of the interference. The starting assumption is made that the first radio-relay station is on the same longitude as one of the satellites. Moreover, the latitude of the center of the radio-relay system is used as a parameter to describe the geographical location of the radio-relay system and this parameter is definitely one of the variables of the system interference. The trendline direction is a random variable that was described earlier. This variable will not appear in the final answer because a distribution will be assumed and it will be averaged out. If a symmetrical distribution is assumed about zero degrees, there are some obvious points of symmetry that should be considered in order to save computation time. Assuming that the trendline direction is measured from the south, there is no need to consider negative trendline azimuths because duplicate results will be generated. Furthermore, irrespective of the assumed distribution, only the direction of transmission having southerly pointing receivers need be considered because that direction will receive the significant interference. Consequently, the azimuth directions that should be considered are between 0 and $\pi/2$ radians from south for the north latitudes.

The computer program begins by selecting a latitude for the center of the radio-relay system. Results are provided in this memorandum for latitudes between 20° and 70° in 10° increments for 50-hop systems and at 40° for a 140-hop system. Some care is required in selecting the latitudes used in the computer program. Those that are too small may cause the radio-relay system to cross the equator requiring some of the logic of the program to be changed to account for the crossover of the equatorial plane. All the derived geometry, however, is valid. For latitudes beyond about 81.3°, the entire geostationary orbit is below the horizon. Consequently, stations beyond this latitude will receive no interference.*

After a latitude has been selected, a trendline direction, τ, is chosen from a random number generator of numbers between 0 and 1.

$$\tau = 90° \cdot RND(\zeta). \tag{1}$$

The latitude of the first station with respect to the reference latitude, ϕ_r, is

$$\phi' = \frac{T}{2} \cos(\tau) + \phi_r \tag{2}$$

where T is the great-circle angular span of the radio-relay system. The average angular span for a CCIR system (50 hops) is near 22° and for an AT&T system (140 hops) about 61°.

With the trendline and antenna direction determined, the next calculation is the total interference into the first station from all visible satellites. The limits of the visible stationary orbit must be determined and this will, in turn, determine the number of visible satellites. First, the azimuth displacement, A, to the intersection of the geostationary orbit with the horizon is calculated and then the relative longitudinal displacement, λ, from the station is determined (see Fig. 2). The azimuth displacement† (measured from the south) is

$$| A | = \cos^{-1} \left[\frac{\tan \phi}{(K^2 - 1)^{\frac{1}{2}}} \right], \tag{3}$$

* Free-space propagation is considered.
† All geometry derivations are given in Appendix A.

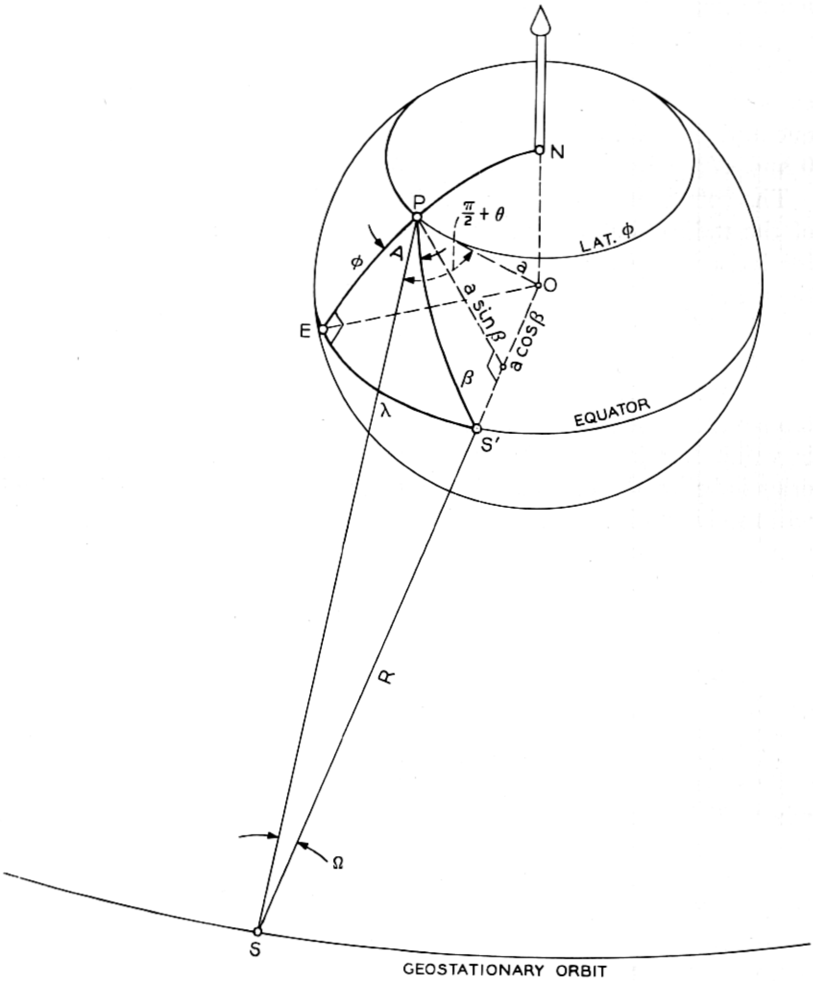

Fig. 2—Geometry of the geostationary orbit and a radio-relay station.

where

ϕ = latitude of station,
$K = R/a$,
R = geostationary orbit radius and
a = earth radius.

The above equation assumes zero degree elevation for the radio-relay antenna and zero refraction. The problem could have been carried out with a non-zero elevation and with an appropriate distribution. However, most existing systems have a mean of near zero degrees. Also, taking refraction into account would have no significance in a statistical model. Consequently, any other assumptions would not have been much more realistic.

The relative longitudinal separation of the intercept is

$$\lambda = \sin^{-1} \{\sin A\ [1 - (K^{-1})^2]^{\frac{1}{2}}\}. \tag{4}$$

Since the span of the stationary orbit is symmetrical about the zero degree azimuth line, the total longitudinal span of the orbit is 2λ. Consequently, the number of visible satellites is the total of the one on the same meridian as the first radio-relay station and all other satellites included by incrementing the given satellite separation until the critical longitude, λ, is not exceeded. Also for negative azimuths the satellite spacing is stepped in negative increments until the negative critical longitude is reached.

The problem then reduces to the calculation of the angular separation from the beam of the radio-relay station to each visible satellite, and conversion to the appropriate interference suppression.

The azimuth, A_z, to each visible satellite is

$$A_z = \cot^{-1} \{\cot \lambda_r \sin \phi\}, \tag{5}$$

where λ_r is the relative longitude to the next satellite. The elevation angle to the satellite, assuming zero-degree elevation of the radio-relay antenna, is

$$\theta = \frac{\pi}{2} - [\beta + \Omega], \tag{6}$$

where

$$\beta = \cos^{-1} [\cos \phi \cos \lambda_r] \tag{7}$$

and

$$\Omega = \tan^{-1} \left[\frac{\sin \beta}{K - \cos \beta}\right]. \tag{8}$$

Finally, the separation angle, γ, as shown in Fig. 3, is given as

$$\gamma = \cos^{-1} [\cos \theta \cos (A_z - \delta)], \tag{9}$$

where δ is the random direction of the radio-relay antenna measured

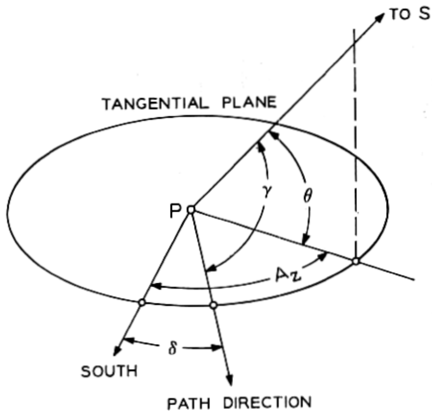

Fig. 3—Geometry determining the off-beam angle to a satellite.

from the south and is

$$\delta = (\tau + 25°) - 50° \cdot \text{RND}\,(\varsigma). \tag{10}$$

ς is a random number set as described earlier and thus allows δ to vary $\pm 25°$ about the chosen trendline.

The angle, γ, is one of the functions of the interference. However, before computing the interference, it is necessary to determine the location for the next station because it is at a different latitude, and it is improbable that it is on the same meridian as one of the satellites. The procedure is to calculate the new latitude and longitude of each succeeding station and start calculating the interference from the satellite that was used at the first radio-relay station. Naturally new azimuthal limits of the orbit, $|A|$, and longitudinal limits, λ, will be calculated. Furthermore, since the starting satellite is the same as that of the first station, the number of visible satellites on each side of the starting satellite will not be symmetrical.

The latitude shift of the next station is

$$\Delta\phi = \phi - \sin^{-1}\{\sin\phi\cos\rho - \cos\phi\sin\rho\cos\delta\}, \tag{11}$$

where ρ is the great-circle angular span to the next station. Since the total angular span was previously given as 22° for a 50-hop system, ρ is approximately 0.45°.

The longitude shift of the next station is

$$\Delta\lambda = \sin^{-1}\left\{\frac{\sin\rho\sin\delta}{[1-(\sin\phi\cos\rho-\cos\phi\sin\rho\cos\delta)^2]^{\frac{1}{2}}}\right\}. \qquad (12)$$

The geometry of the latitude and longitude shifts is shown in Fig. 4. From these values the azimuthal limit of the orbit and the relative longitude to each satellite can be calculated. Consequently, the beam separation angle from each radio-relay station to each visible satellite can be calculated.

To summarize briefly, the program starts at a given latitude and chooses a trendline direction. Then it shifts the latitude of the first station in order to make the center of the trendline lie near the middle of the chosen latitude, and it picks the beam direction of the first radio-relay station. It calculates the longitudinal limits of the visible stationary orbit and calculates the interference from each visible satellite taking into account the beam separation angle. The coordinates of the next station are calculated and the new set of visible satellites determined. The interference is once again calculated starting at the same satellite as the one used for the first station and proceeding in positive and negative satellite spacing increments until all visible satellites are exhausted. The interference to all radio-relay stations of a system is similarly calculated. The program proceeds with other sample trendline directions and chooses a sufficient number of direc-

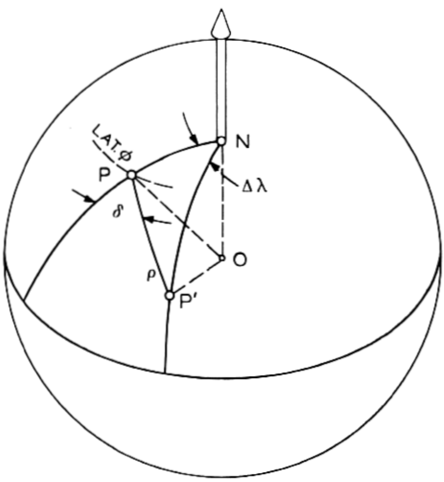

Fig. 4—Geometry determining the latitude and longitude shift.

tions to accurately arrive at an interference distribution. The results in Section V were derived for a sample of 50 trendlines at each latitude. Furthermore, the procedure is repeated at each chosen latitude.

The size of the sample appeared adequate for calculation of the mean; however, it was too small for calculation of the tails. Consequently, some smoothing was applied to the results, and some cases were rerun with larger samples to assure that the suggested trend of the 50-sample results was accurate. Increasing the sample size to greater than 50 was not too meaningful because the uncertainty of the radio-relay and satellite characteristics was large enough so that more accurate computation with higher samples would not have been more enlightening.

IV. COMPUTATION OF INTERFERENCE

The exact calculation of the interference between the satellite and radio-relay system involves the convolution of the two received spectral densities, with the suppression of the interfering spectral density being a function of the beam separation angle, γ. However, an approximate calculation is possible because the satellite signal is one of relative high index. Consequently, the simplifying assumption can be made that the interfering spectrum is substantially flat over the receiver bandwidth. It is possible that this assumption may not be valid when the index is not sufficiently high, or when the top baseband frequency of the satellite system is much smaller than that of the radio-relay system. However, even for these cases an upper bound calculation can be made by assuming a flat spectrum with the same magnitude as the peak of the actual spectrum. Also the assumption is made that carrier dispersal is used during periods of light loading.

The actual convolution could have been easily carried out, but the reason the computations were simplified is that the peak of the baseband interference is a function of the spectral densities, frequency separation of the two signals, and the de-emphasis of the radio-relay system. Hence, it would have introduced additional dimensions in the number of variables. Moreover, the effect in the results would have been insignificant. It is also true that a satellite signal without carrier dispersal and low-index of modulation may generate more interference. In any event, the flat spectrum approach will be used in order to make the results more tractable.

The baseband interference can be related to the thermal noise of the system by

$$\frac{i_4}{n_4} = \frac{i_c}{n_c}, \tag{13}$$

where i_4 and n_4 are the interference and thermal noise power respectively in a 4-kHz bandwidth at the input of the receiver, and i_c and n_c are the interference and noise power respectively in a voice channel.

The baseband interference can be calculated as

$$i_c = \frac{i_4}{n_4} n_c . \tag{14}$$

The thermal noise input to the receiver is a function of the radio-relay system noise temperature

$$n_4 = kT_s b, \tag{15}$$

where

k is Boltzmann's constant,
T_s is the system noise temperature and
b is the voice channel bandwidth.

The interference power input to the receiver is a function of the satellite power flux density, f, the elevation angle, θ, from the interfered radio-relay station to the interfering satellite, and the gain of the radio-relay antenna in the direction of the satellite. Hence

$$i_4 = f(\theta)g(\gamma)\cdot\frac{\Lambda^2}{4\pi\ell}. \tag{16}$$

Equation (16) contains the factor $\Lambda^2/4\pi$ because $f(\theta)$ is in units of $W/m^2\cdot 4$ kHz (Λ is the wavelength of the carrier). Hence, what is required is the radio-relay station effective antenna aperture in the direction of the satellite. The factor ℓ is feeder losses. For the results of Section V, 3dB was assumed.

The baseband interference can now be written as

$$i_c = \frac{f(\theta)g(\gamma)\Lambda^2/4\pi}{kT_s b\ell}\cdot n_c . \tag{17}$$

It can be seen that interference can be expressed as the product of three factors:

(*i*) The gain of the radio-relay antenna in the direction of the interfering satellite.
(*ii*) The power flux density of the satellite, $f(\theta)(W/m^2\cdot 4\text{ kHz})$:

where θ is the elevation angle from the interfered station to the interfering satellite.

(*iii*) A constant, μ, that depends on the parameters of the radio-relay system.

$$i_c = \mu f(\theta) g(\gamma). \tag{18}$$

μ can be easily derived from equations (13), (14), (15), (16) and (17) as

$$\mu = \frac{n_c \cdot (\lambda^2/4\pi)}{kT_s b\ell}. \tag{19}$$

V. INTERFERENCE RESULTS

At the 1969 CCIR Interim Meeting in Geneva, Switzerland, a suggestion was generated to study the effect of a new power flux-density limitation. The proposed limitation is

$$F^*(\theta) = \begin{cases} -152 & 0^\circ \leqq \theta \leqq 5^\circ \\ -152 + 0.5(\theta - 5) & 5^\circ \leqq \theta \leqq 25^\circ \text{ dB Rel. } 1W/m^2 \cdot 4 \text{ kHz.} \\ -142 & \theta \geqq 25^\circ \end{cases} \tag{20}$$

This limit was used in the simulation program for both the CCIR and AT&T systems.

The parameters used for the CCIR reference system are

Antenna gain =

$$\left.\begin{matrix} 0^\circ \leqq \gamma \leqq 0.48^\circ \\ 0.48^\circ \leqq \gamma \leqq 57.5^\circ \\ \gamma \geqq 57.5^\circ \end{matrix}\right\} G(\gamma) = \begin{cases} 42 \text{ dB} \\ 34 - 25 \log_{10} \gamma \quad \text{dB}, \\ -10 \text{ dB} \end{cases} \tag{21}$$

$$\begin{aligned} \text{Hops} &= 50, \\ T_s &= 1750^\circ\text{K}, \\ \mu &= 5.8 \times 10^{13} \text{ at 4 GHz}, \\ M &= 10 \log_{10} \mu = 137.6, \\ I &= F(\theta) + G(\gamma) + 137.6. \end{aligned} \tag{22}$$

The parameters used for the AT&T system approximate a TD-3

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* Capitals are used for logarithmic quantities.

system and are

Antenna gain =

$$\left.\begin{array}{r} 0^\circ \leqq \gamma \leqq 0.575^\circ \\ 0.575^\circ \leqq \gamma \leqq 57.5^\circ \\ \gamma \geqq 57.5^\circ \end{array}\right\} G(\gamma)^* = \begin{cases} 40 \text{ dB} \\ 34 - 25 \log_{10} \gamma \quad \text{dB}, \\ -10 \text{ dB} \end{cases} \qquad (23)$$

$$\text{Hops} = 140,$$

$$T_s = 1160^\circ\text{K},$$

$$\mu = 7 \times 10^{13} \text{ at 4 GHz},$$

$$M = 10 \log_{10} \mu = 138.5,$$

$$I = F(\theta) + G(\gamma) + 138.5. \qquad (24)$$

Calculations were performed for the above systems using satellite spacings of 3° and 6°. Calculations were also carried out with the additional constraint that the radio-relay antenna not point within two degrees of the stationary orbit. This restriction was accounted for in the program by setting the angular separation between the radio-relay station and satellite to two degrees whenever it calculated to be less than two degrees. This extra set of calculations were performed because CCIR already has a recommendation at 6 GHz for new stations not to point within 2° of the stationary orbit whenever possible. Since Bell System 4-GHz routes are, in many cases, also 6-GHz routes, it is informative to evaluate the results of this restriction.

The interference distributions were calculated and are shown in Figs. 5–11. Figs. 5–10 include distributions for CCIR systems at latitudes of 20° through 70° for satellite spacings of 3° and 6°, and with and without the stationary orbit pointing restriction. The AT&T system is calculated at a latitude of 40° only; this result is shown in Fig. 11. Since it is a 4000-mile system, inclusion of other latitudes that span the U.S.A. would not have been more enlightening.

The attached results are not included as a check of the validity of the assumed power flux density, but as example calculations for the assumed statistical models. The program is general and could incorporate other radio-relay azimuthal distributions, satellite spacings, antenna patterns, and radio-relay system characteristics. It is also possible to use the program to calculate an appropriate power flux

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* This pattern is pessimistic and not representative of the horn reflector antenna which has better off-beam discrimination.

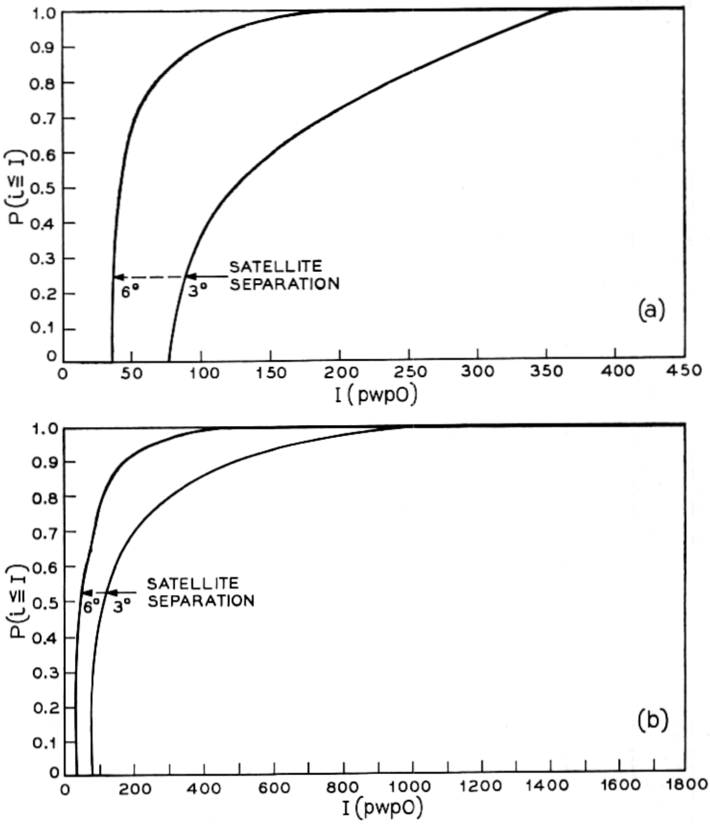


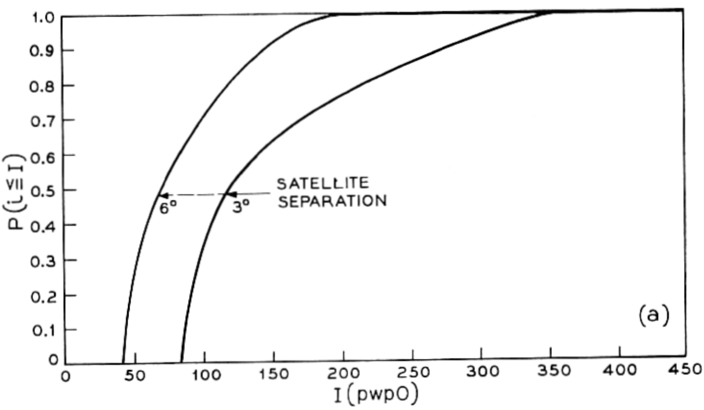
Fig. 5—Distribution of interference into CCIR reference systems centered at 20° latitude with (a) 2° pointing restriction, and (b) no restriction.

density limitation to meet an interference objective. The answer is derived by iteration and it is not unique. However, it converges very quickly to a desired answer. Some parameters of the desired density may also be specified prior to computation and the program can generate the remaining parameters. For example, the power flux density at the horizon may be specified ($\theta = 0$), and the simulator can determine the slope and maximum density. Furthermore, joint distributions of parameters other than latitude can be calculated. Joint distributions with elevation angle as a parameter may be just as useful.

It may well be that simpler models can be formulated, but it may be difficult to convince the radio-relay community of their validity. The program that has been derived can be made quite realistic, and perhaps the insight that can be gained from this model could lead to simpler models.

VI. CONCLUSIONS

The suggested analytical approach for the evaluation of satellite interference has practical value due to its generality and realism. The calculation techniques have been reported widely in the literature and have been substantiated by experiment. The contribution of this paper is a method for calculation of the probability of exposure to satellite



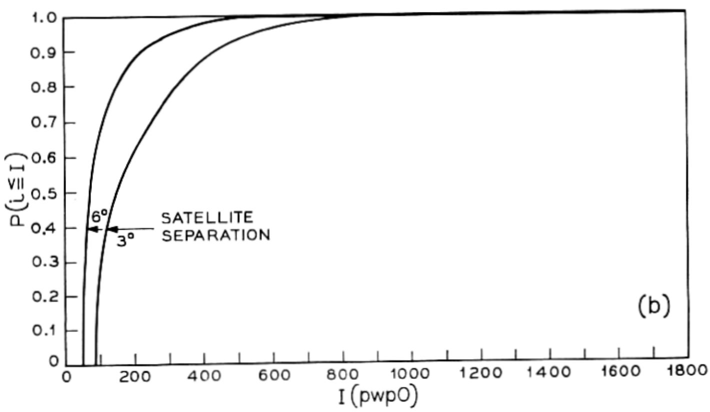


Fig. 6—Distribution of interference into CCIR reference system centered at 30° latitude with (a) 2° pointing restriction, and (b) no restriction.

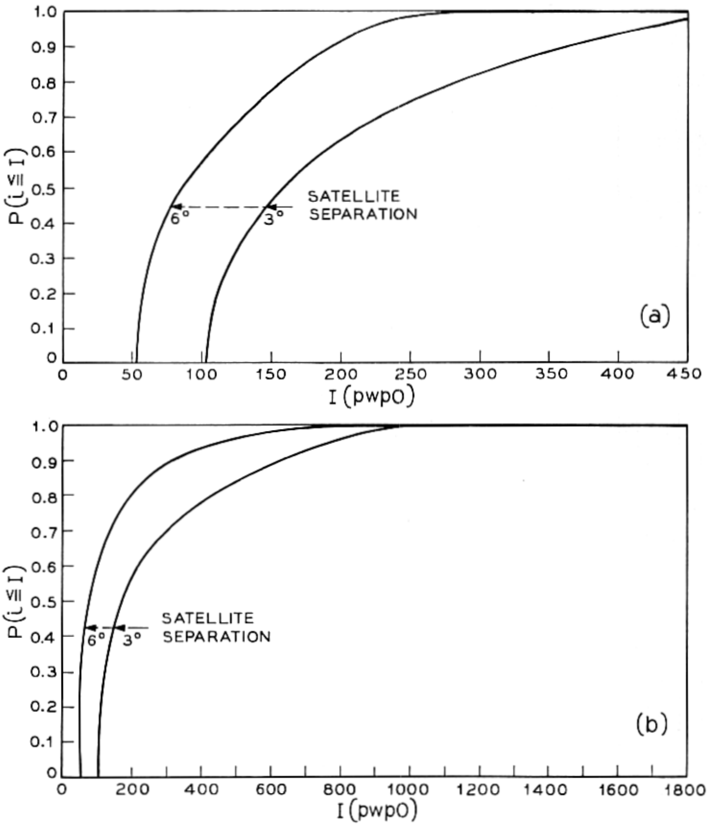


Fig. 7—Distribution of interference into CCIR reference systems centered at 40° latitude with (a) 2° pointing restriction, and (b) no restriction.

interference. The interference is a random variable and is a function of several variables.

The results were presented as a function of latitude because this appeared as a useful presentation for radio-relay operators. However, in attemping to reduce the number of variables, some very important parameters were averaged out.

One of these parameters is the elevation angle to the satellite. The simulation was also used to compute distributions as a function of the elevation angle, and the results showed the dominating effect of a direct exposure. Even at latitudes as low as 20°, more than 50 percent of the interference was due to elevation angles less than 5°, and for latitudes near 70° the contribution of exposures of less than 5° elevation accounted for nearly 90 percent of the interference (See Fig. 12). The results for the high latitudes are not surprising. However, the lower-latitude results are surprising because the probability of intercepting the geostationary orbit within 5° is not high. Even though the number of off-beam contributions is very high compared to the contributions within 5° elevation angles, the radio-relay antenna directivity suppresses these contributions to account for less than 50 percent of the total interference. Nevertheless, their contribution is not negligible as assumed in previous interference models.

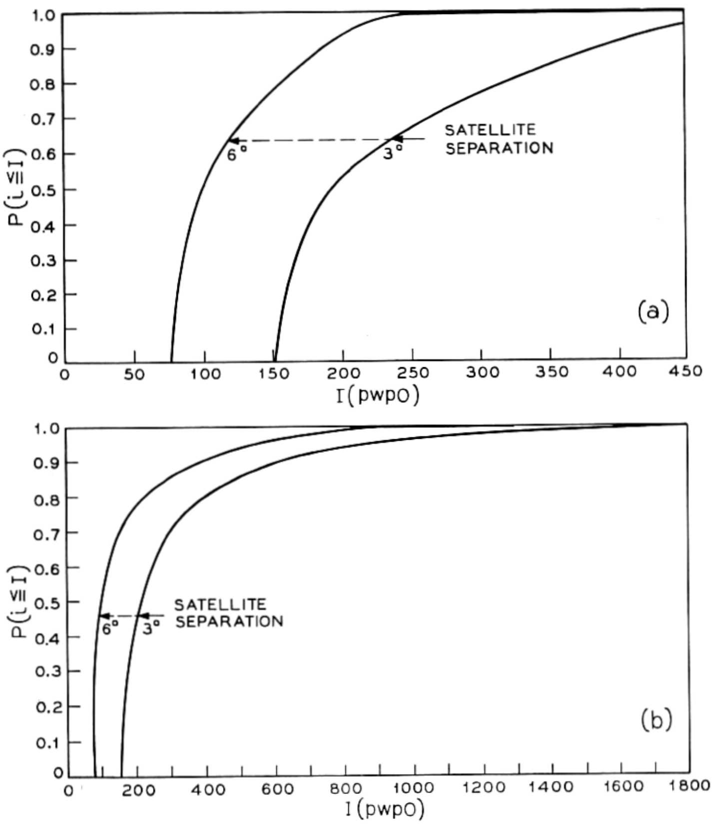


Fig. 8—Distribution of interference into CCIR reference systems centered at 50° latitude with (a) 2° pointing restriction, and (b) no restriction.

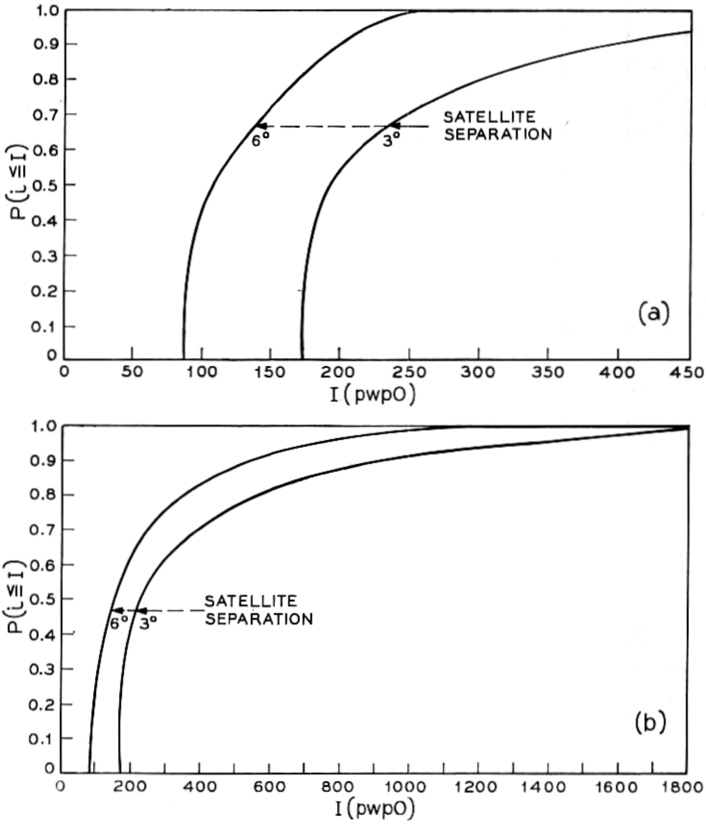


Fig. 9—Distribution of interference into CCIR reference systems centered at 60° latitude with (a) 2° pointing restriction, and (b) no restriction.

VII. ACKNOWLEDGEMENT

The authors wish to thank Mrs. S. L. Fennick for programming assistance.

APPENDIX A

Derivation of Equations

Figure 2 illustrates the geometry of the problem. The azimuth displacement from the meridian through station P to a satellite suborbital location S' is identical to angle A of spherical triangle PES'. The elevation angle θ(angle OPS $- \pi/2$) to a satellite at S is dependent upon the relative longitude λ between E and S' and the latitude ϕ.

A.1 *Azimuth to Intercept*

From the laws for right spherical triangles

$$\cos A = \tan \phi / \tan \beta, \tag{25}$$

where ϕ is the station latitude and β is the arc equivalent of angle O of plane triangle OPS. For the case of orbit intercept, and when θ is

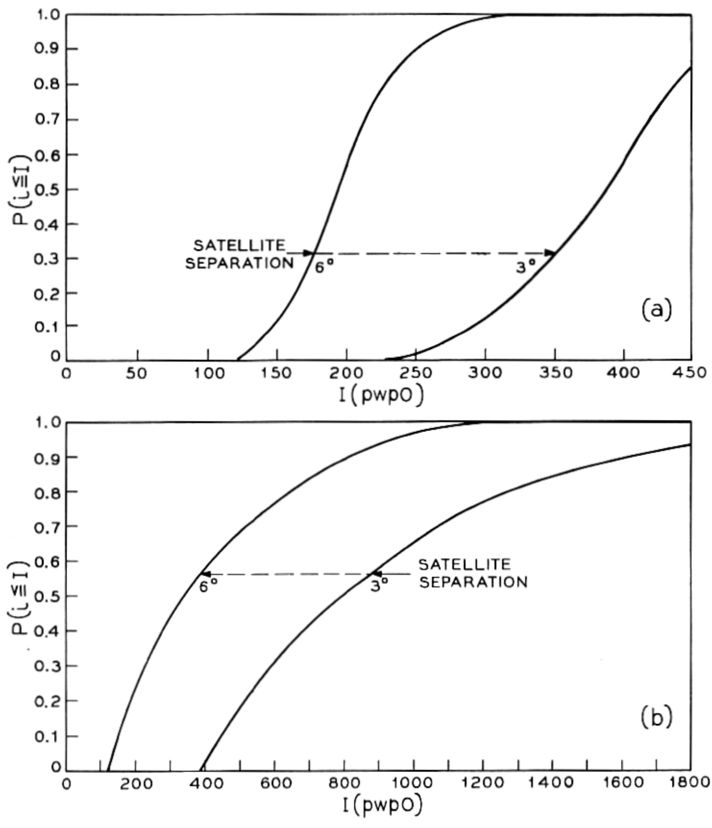


Fig. 10—Distribution of interference into CCIR reference systems centered at 70° latitude with (a) 2° pointing restriction, and (b) no restriction.

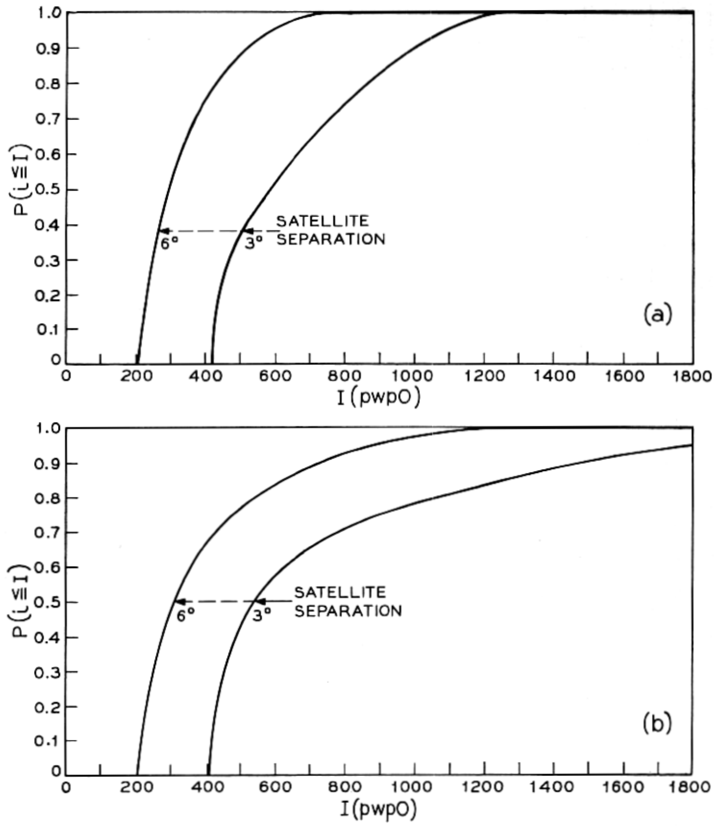


Fig. 11—Distribution of interference into 140-hop systems centered at 40° latitude with (a) 2° pointing restriction, and (b) no restriction.

zero, triangle *OPS* is a right triangle and β may be expressed as

$$\beta = \cos^{-1} (K^{-1}) \tag{26}$$

where $K = R/a$ (orbit radius/earth radius). From equations (25) and (26) the azimuth to intercept is given by

$$A = \cos^{-1} [\tan \phi/(K^2 - 1)^{\frac{1}{2}}]. \tag{27}$$

A.2 *Relative Longitude to Intercept*

The relative longitude to a suborbital intercept corresponding to azimuth A may also be obtained from spherical triangle *PES'*. From geometry

$$\sin \lambda = \sin A \sin \beta. \tag{28}$$

Combining equations (26) and (28) gives

$$\lambda = \sin^{-1} \left[\sin A \left(1 - \left(\frac{1}{K} \right)^2 \right)^{\frac{1}{2}} \right]. \tag{29}$$

The total longitudinal span is 2λ.

A.3 *Azimuth to a Chosen Orbit Position*

For a chosen orbit position the relative longitude λ_r is known. Then, from triangle PES',

$$A_z = \cot^{-1} [\cot \lambda_r \sin \phi]. \tag{30}$$

A.4 *Elevation Angle to a Chosen Orbit Position*

From Fig. 2 the following relationships are established.

$$\theta = \frac{\pi}{2} - (\beta + \Omega), \tag{31}$$

$$\beta = \cos^{-1} (\cos \phi \cos \lambda_r), \tag{32}$$

$$\Omega = \tan^{-1} [\sin \beta/(K - \cos \beta)]. \tag{33}$$

Manipulating equations (31), (32) and (33) yields:

$$\theta = \frac{\pi}{2} - \cos^{-1} (\cos \phi \cos \lambda_r) - \tan^{-1} \left[\frac{(1 - \cos^2 \lambda_r \cos^2 \phi)^{\frac{1}{2}}}{K - \cos \lambda_r \cos \phi} \right]. \tag{34}$$

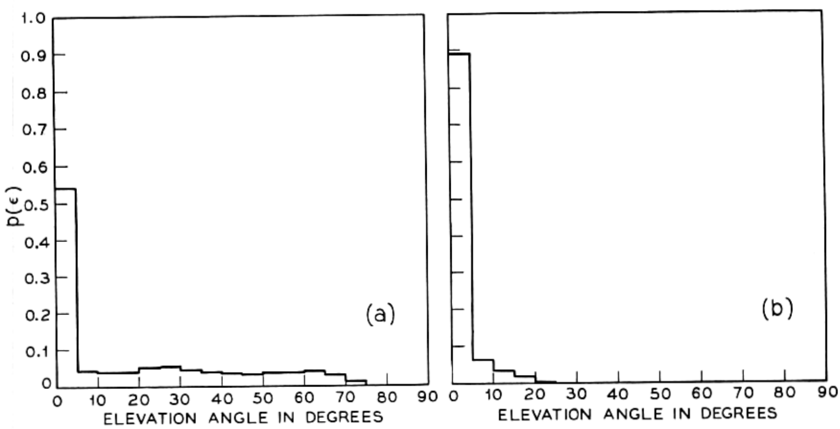


Fig. 12—Interference density vs. elevation angle into CCIR reference system centered at (a) 20° latitude and (b) 70° latitude. Satellite spacing = 6°.

A.5 *Angle Between Beams*

The off-beam angle between the radio-relay direction and a satellite is shown as angle γ in Fig. 3. Since the elevation angle θ has been determined in a plane perpendicular to a tangential plane at P, γ is given by

$$\gamma = \cos^{-1} [\cos \theta \cos (A_z - \delta)], \tag{35}$$

where δ is the direction of the radio-relay beam.

A.6 *Determination of the Latitude and Longitude Shift Between Adjacent Radio-Relay Stations*

In Fig. 4, P and P' represent locations of adjacent stations of a radio-relay system and ρ is the great circle angular span between them. If the average angular span of the entire system is T, then ρ is T/n where n is the number of hops. The value of ρ for a line of sight radio-relay system is about 0.45°.

It is seen that the latitude shift between P and P' is the angular difference between $\widehat{NP}$ and $\widehat{NP'}$ and that the longitude shift is equal to angle N of the spherical triangle NPP'. Since $\widehat{NP} = \pi/2 - \phi$, $\widehat{NP'} = \pi/2 - (\phi - \Delta\phi)$, and angle $NPP' = \pi - \delta$, the law of cosines gives

$$\Delta\phi = \phi - \sin^{-1} (\sin \phi \cos \rho - \cos \phi \sin \rho \cos \delta) \tag{36}$$

and the law of sines

$$\Delta\lambda = \sin^{-1} [\sin \rho \sin \delta / \cos (\phi - \Delta\phi)]. \tag{37}$$

Hence, from equations (36) and (37),

$$\Delta\lambda = \sin^{-1} \left\{ \frac{\sin \rho \sin \delta}{[1 - (\sin \phi \cos \rho - \cos \phi \sin \rho \cos \delta)^2]^{\frac{1}{2}}} \right\}. \tag{38}$$

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3. Lundgren, C. W., and May, A. S., "Radio-Relay Antenna Pointing for Controlled Interference with Geostationary Satellites," B.S.T.J., *48*, No. 10 (December, 1969), pp. 3387–3422.