**The *Picturephone*® System:**

# Crosstalk Considerations in the Transmission of Analog Signals on Paired Cable

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*Crosstalk is an important consideration in the transmission of analog baseband* Picturephone® *signals on paired cable. Crosstalk due to the worst disturbing* Picturephone *signal can cause a distinctive visible interference pattern. Crosstalk interference from other* Picturephone *and wideband systems, such as T1 and T2 digital lines, contributes to random noise. In addition, feedback via crosstalk coupling may cause spurious oscillation of cable equalizers. Methods are determined to control these impairments by placing restrictions on equalizer spacing and cable pair assignment.*

## I. INTRODUCTION

Exchange cable pairs will provide the medium for transmission of the analog baseband *Picturephone* signal over subscriber loops and most local trunks.[1] The unwanted coupling of energy, or crosstalk, between pairs within a cable places important limitations on the engineering of these equalized transmission lines.

### 1.1 *Types of Impairments*

Three types of impairments in the *Picturephone* signal which can occur in the presence of crosstalk are considered in this article. They are listed in Table I along with the source of the impairment. Near-end and far-end crosstalk coupling paths are shown in Fig. 1.[2]

#### 1.1.1 *Worst-Disturber Interference*

The first impairment is caused by interference from the single most prominent *Picturephone* interfering signal among all those present, and is called worst-disturber interference. The signal from the worst disturber, when passed through a crosstalk coupling path, will appear differentiated and superimposed on the desired signal. Since the two signals are not synchronized, the interference will drift across the screen when the desired signal is displayed. For example, if the sync pulses are the most detrimental part of an interfering *Picturephone* signal, the interference will appear as two vertical lines, one black and one white, drifting across the screen.

#### 1.1.2 *Random Noise Interference*

Even if no signal is visible as a worst disturber, each of the interfering *Picturephone* signals will contribute unwanted energy to the desired signal. This energy must be considered as a contribution to the overall system random noise, which is the second impairment whose effects are studied. The interfering signals do not necessarily have to be of the *Picturephone* type, but could be of any wideband system. In this article we consider crosstalk coupling of noise from *Picturephone,* T1 line, and T2 line signals. Interference from the digital systems is studied on trunk facilities only, since carrier systems are not usually present on subscriber loop cable.

Additional sources of interference, such as *N* carrier systems exist, but are not considered in this article. Furthermore, the effect of interference on other systems, caused by the coupling of energy from *Picturephone* signals, and subsequent limitations on this energy are not discussed here.

#### 1.1.3 *Equalizer Singing*

Finally, we investigate a third impairment due to crosstalk, referred to as singing. Singing, or spurious oscillation, can occur around the

TABLE I—IMPAIRMENTS DUE TO CROSSTALK

| Impairment | Source of Impairment |
|---|---|
| (*i*) Worst disturber | Interference from oppositely-directed *Picturephone* signals via NEXT* |
| (*ii*) Random noise | Interference from *Picturephone* signals via NEXT and FEXT*<br>Interference from T1 and T2 signals via NEXT and FEXT |
| (*iii*) Equalizer singing | Feedback around two oppositely directed equalizers via the two NEXT paths coupling them |

* NEXT and FEXT stand for near-end and far-end crosstalk, respectively.

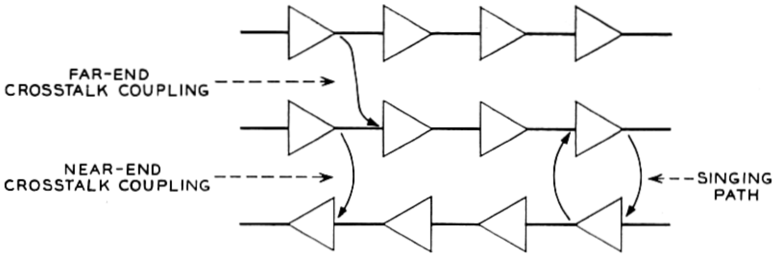


Fig. 1—Crosstalk and singing paths.

path formed by two oppositely-directed *Picturephone* equalizers and the two near-end crosstalk paths coupling the output of one equalizer to the input of the other, as shown in Fig. 1.

### 1.2 *Measures to Control the Effects of Crosstalk*

The effects of near-end crosstalk can be mitigated by reducing equalizer spacing and thus the required equalizer gain. The signal-to-interference ratio is then increased by the amount the equalizer gain is reduced. If the required spacing becomes too small to be economical, additional near-end crosstalk loss can be realized by segregating the opposite directions of transmission into separate cable units. Further reductions in interference may be achieved by limiting the number of interfering systems.

Equal-level far-end crosstalk is independent of equalizer spacing. This interference can be reduced, however, by limiting the overall line length, and by placing restrictions on the number of interfering systems, and on their placement within the cable sheath.

## II. RESULTS AND CONCLUSIONS

We present methods of analyzing the effects of crosstalk which can be used in writing engineering rules. The worst-disturber impairment is analyzed in Section IV by finding the probability that the minimum pair-to-pair crosstalk coupling loss is less than the requirement. Random noise interference is computed in Section V by passing the interfering spectra through several shaping filters and then integrating the output to find the total effective interfering power. The probability of singing, or of being close enough to singing to affect echo rating, is computed by a Monte Carlo technique in Section VI.

As an example of the use of these methods, they are applied to find the allowable equalizer spacings using the requirements and assump-

TABLE II—LIMITS ON EQUALIZER SPACING

| Type and Source of Impairment | Coupling Path | Allowable Loop Equalizer Spacing (kft) (26-gauge pulp cable) | | Allowable Trunk Equalizer Spacing (kft)* (22-gauge pulp cable) | |
|---|---|---|---|---|---|
| | | Within-Unit | Adjacent-Unit | Within-Unit | Adjacent-Unit |
| (*i*) Worst disturber | NEXT | 4.6 | >6.0 | >6.0 | >6.0 |
| (*ii*) Random noise | | | | | |
| (*a*) *Picture-phone* signals | NEXT | >6.0 | >6.0 | 3.6 | >6.0 |
| | FEXT† | >6.0 | >6.0 | 6.0 | >6.0 |
| (*b*) T1 signals | NEXT | | | 0 | 4.7 |
| | FEXT† | | | 0.3 | >6.0 |
| (*c*) T2 signals | NEXT | | | 2.7 | >6.0 |
| | FEXT† | | | 4.3 | >6.0 |
| (*iii*) Singing | NEXT | 3.6 | 5.8 | 4.4 | >6.0 |

* This applies to the trunk equalizer now being developed.
† Interference coupled through the FEXT path limits overall length but is independent of equalizer spacing. The spacings are listed for consistency and are based on allowable overall length divided into 4 and 6 equal sections for loops and trunk spans respectively.

tions stated in Section III. The results are given in Table II for the indicated source and crosstalk coupling path. The figures for trunks are for the trunk facilities being developed, not for the equalizers to be used initially.[1] The limitations differ somewhat between the two. Other design considerations restrict equalizer spacings to 6 kft on the assumed transmission media. Therefore, spacings beyond this value are not given in the table.

The controlling impairment for loops is singing or near-singing and that for trunks is random noise interference from T1 lines. Thus for both loops and trunks the desire to keep the number of equalizers to a minimum by using long equalizer spacings is offset by the necessity to limit equalizer gain to reduce the effects of near-end crosstalk. The details of the allowable spacings are given below.

Loop equalizers may be spaced 5.8 kft apart on 26-gauge cable if opposite directions of transmission are in separate cable units. When opposite directions of transmission must be placed in the same cable unit, 3.6 kft separation is possible with acceptable probability of degradation in the gain and phase characteristics due to near-singing.

To obtain 6-kft trunk equalizer spacing on 22-gauge cable, opposite directions of *Picturephone* transmission must be in separate cable units and there may be no T1 or T2 systems in any cable unit containing *Picturephone* systems. In addition, opposite directions of T1 and *Picturephone* transmission must be in non-adjacent units. Equalizer spacing of 4.7 kft may be obtained if opposite directions of T1 and *Picturephone* transmission are in adjacent units. If opposite directions of *Picturephone* transmission must be placed in the same cable unit, interference due to random noise may be controlled by maintaining an equalizer spacing no greater than 3.6 kft. In order to achieve this spacing of within-unit and oppositely directed trunk equalizers, it is important to control T1 and T2 interference by keeping *Picturephone* lines and the digital (T1 and T2) lines in separate cable units.

## III. ASSUMPTIONS OF THE ANALYSIS

### 3.1 *Cable Characteristics*

Pulp-insulated cable of unit-type construction is assumed to be the medium over which analog *Picturephone* signals are transmitted. The 900-pair cable of this type is made up of eighteen 50-pair cable units, as shown in Fig. 2. Adjacent and alternate units are indicated on the figure. The loss characteristics of 26- and 22-gauge pulp-insulated cable pairs, which are assumed in this study to be the facilities for loop and trunk transmission respectively, are given in Fig. 3. Experimentally obtained crosstalk data for 22-gauge pulp-insulated cable are given in Table III.[3] The same values of coupling loss are used for 26-gauge cable. The loss, in dB, of both near-end and far-end crosstalk coupling is assumed to be normally distributed to $3.5\sigma$ with means that decrease with frequency at 4.5 and 6.0 dB/octave, respec-

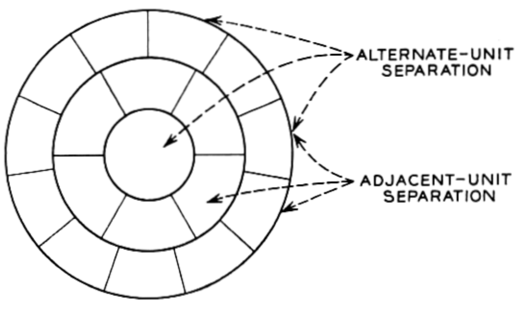


Fig. 2—Construction of unit-type cable.

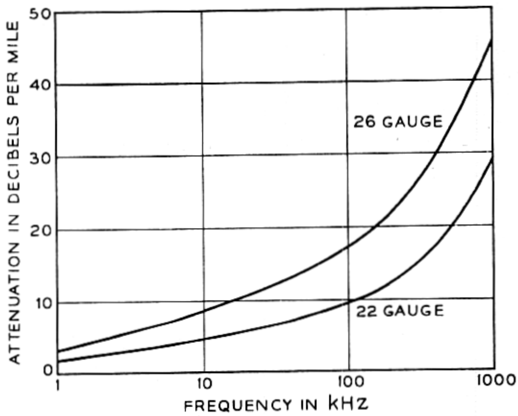


Fig. 3—Loss versus frequency of pulp-insulated cable pairs.

tively.[4] The standard deviation of the distribution is assumed to be independent of frequency.

### 3.2 *Requirements*

The requirements given by H. E. Brown[5] result from subjectively determined limits allocated among the various segments of the circuit that make up the complete connection. The equal-level crosstalk loss which allows the interference from the worst-disturbing signal to be noticeable but not objectionable is 45 dB at 150 kHz.[6] This applies to both loops and trunks as well as to all other parts of the end-to-end transmission path. The requirement for random noise on loops is given as a distribution with mean of −61.6 dBm and a standard deviation of 4.0 dB. Although Brown further allocates the loop noise requirement among the systems that may compose a loop, the loop facility here is assumed to be a direct connection between central office and subscriber and is therefore allocated the entire requirement. On trunks the mean and standard deviation per span* are specified as −64.6 dBm and 4.0 dB respectively. The minimum singing margin (see Appendix B) required to insure that an equalized line has an acceptably small probability of singing is given as 10 dB with probability of 0.99.

### 3.3 *Equivalent Number of Disturbers*

The detrimental effects of near-end crosstalk occur primarily at equalizer locations. Therefore the equivalent number of disturbers

* Defined in Fig. 2 of Ref. 5.

($nq$) that must be considered is equal to the number of interfering signals at each equalizer location, $n$, multiplied by the number of equalizer sections, $q$.

Equal-level far-end crosstalk coupling is computed for the overall line length since its effects are independent of equalizer spacing. The number of equivalent disturbers is therefore equal to $n$, the number of actual disturbers.

In the computations that follow, it is assumed that a disturbed pair in a given 50-pair cable unit may be subjected to at most $n = 25$ oppositely-directed, within-unit disturbers at any one equalizer location. It is further assumed that loops and trunk spans will be composed of at most $q = 4$ and 6 equalizer sections, respectively. If all pairs within a cable unit are transmitting in the same direction, interference on any disturbed pair, due to the effects of far-end crosstalk, is calculated assuming $n = 49$ within-unit disturbers. When disturbing and disturbed pairs are in adjacent cable units, the interference is that computed from $n = 50$ disturbers. All computations assume that the equalizers of the disturbing and disturbed circuits are located at the same point.

### 3.4 *Traffic Activity*

A traffic activity factor, defined as the percentage of pairs active of those equipped for *Picturephone* transmission, is assumed for loops and trunks. The factor for loops is chosen to be 0.1 whereas that used for trunks is conservatively taken as 1.0.

## IV. WORST-DISTURBER INTERFERENCE

The analysis of worst-disturber interference is given in Appendix A. It is shown that the equalizer spacing is dependent upon the sub-

Table III—Crosstalk Coupling Loss at 1 MHz for 22-Gauge Pulp-Insulated Cable

| Pair Separation | Near-End (>1000 ft) | | Far-End (1000 ft) | |
|---|---|---|---|---|
| | Mean (dB) | Std. Dev. (dB) | Mean (dB) | Std. Dev. (dB) |
| Within-unit | 74 | 8.8 | 78 | 12 |
| Adjacent-unit | 93 | 6.7 | 94 | 11 |
| Alternate-unit | 105 | 6.7 | 103 | 11 |

jective effect of this interference, the number of equivalent disturbers, and the traffc activity.

The equal-level loss required to make this type of interference definitely noticeable but not objectionable, denoted by $L_{REQ}$, has been subjectively determined to be 45 dB at 150 kHz (Section 3.2). As is shown in Fig. 4, a disturbing signal coupled through the near-end crosstalk path is subject to the gain of the equalizer of the disturbed circuit. Denoting this gain in dB by $G$, the required cable crosstalk coupling loss, $C_\ell$, is equal to $(L_{REQ} + G)$ dB at 150 kHz.

Knowing the cumulative probability distribution of the pair-to-pair crosstalk loss, $F_X(x)$, the following result (derived in Appendix A) can be obtained.

Assume that in a cable made up of $q$ equalizer sections, there are $n$ possible interferers and $n$ possible disturbed circuits, each of which is active with probability $p$. Then the probability that the minimum pair-to-pair loss between the $n$ interferers and a disturbed circuit chosen at random is less than the required $C_\ell$ is given by

$$F_m(C_\ell) = 1 - [1 - (1 - a)p]^n \tag{1}$$

where

$$a = [1 - F_X(C_\ell)]^q \tag{2}$$

and $F_m$ is the distribution of the minimum pair-to-pair loss.

This relation is used to obtain the curves of Fig. 5, where the probability of exceeding the requirement on loops is given as a function of equalizer spacing. The equivalent number of disturbers is treated as a parameter. If a criterion is chosen that limits the probability of exceeding the worst-disturber requirement to 5 percent, and the number of within-unit equivalent disturbers is assumed to be $25 \times 4 = 100$, the maximum equalizer spacing is found to be 4.6 kft.

Worst-disturber interference is not a problem on trunks even for the maximum trunk equalizer spacing of 6 kft. This follows since the gain of the trunk equalizer for 22-gauge cable is less than that supplied on the same length 26-gauge loop, and therefore a smaller coupling loss, $C_\ell$ is required.

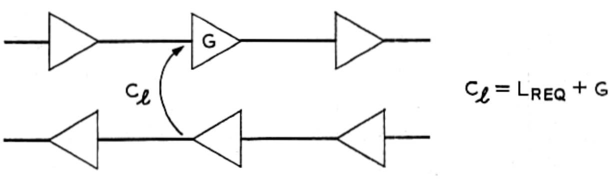


Fig. 4—Crosstalk loss, $C_\ell$, required to meet the worst-disturber requirement, $L_{REQ}$.

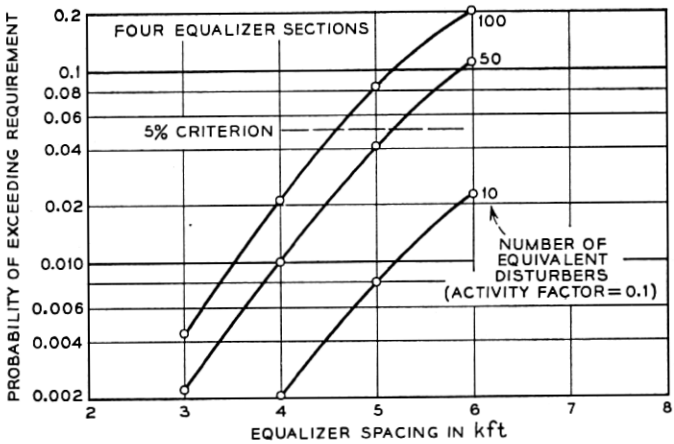


Fig. 5—Probability of exceeding the worst-disturber requirement on loops due to within-unit, near-end crosstalk.

## V. RANDOM NOISE INTERFERENCE

Even when the effects of crosstalk are not evident as a distinct pattern produced by a worst disturber, as discussed in Section IV, wide-band systems operating within the cable contribute unwanted energy through crosstalk coupling paths to a *Picturephone* circuit. This energy must be taken into account, and is treated here as random noise.

The effects of random noise on equalizer spacing and pair placement are described in the following sections. The models used to determine the received noise power due to near-end and far-end crosstalk coupling of *Picturephone,* T1 line, and T2 line signals are shown in Figs. 6a and 6b respectively. The equalizer gain is not included in Fig. 6b because in the far-end crosstalk case, the disturbing signal is exposed to cable loss equal to the gain supplied by the equalizer of the disturbed line. As shown, the interfering source is coupled through a crosstalk path, assumed to be that described in Section III. The *Picturephone* equalizer gain (included in the near-end case only) is assumed to match the loss, at nominal temperature, of a given length of 26- or 22-gauge pulp cable. Deemphasis is used to provide increased immunity to high frequency interference[5] and noise weighting is included to account for subjective weighting of noise by the human eye. The characteristic of station set roll-off, used to enhance picture quality, attenuates the received signal at the high end of the band

and, therefore, must be included. The characteristics of noise weighting plus roll-off, and deemphasis are shown in Fig. 7.

From Fig. 6a the mean noise power at any frequency due to interference coupled through the near-end path is given by

$$N(f) = S(f)X(f)G(f)\,D(f)W(f)R(f) \tag{3}$$

where $S(f)$ is the interfering power spectrum, $X(f)$ the mean crosstalk loss function, $G(f)$ the equalizer gain, and $D(f)$, $W(f)$ and $R(f)$ the deemphasis, noise weighting, and station set roll-off characteristics respectively. The expression for noise coupled through the far-end crosstalk path is the same, except for the gain factor, $G(f)$, which is not included. Numerical integration of equation (3) yields the mean noise power on the disturbed pair due to one disturber. The amplitude distribution of $N(f)$ is based on the lognormal distribution of crosstalk loss, and is therefore also normal in dB. The probability distribution of the noise power accumulated from $nq$ equivalent disturbers is found by convolving the lognormal distribution $nq$ times.[7] The noise power exceeded with some given probability can then be determined.

The noise produced through far-end crosstalk coupling is calculated by first finding the far-end crosstalk loss distribution for the total loop or trunk span length of interest and then applying the model of Fig. 6b. The noise due to $n$ actual disturbers is determined by convolving the amplitude distribution of the noise generated by a single disturber $n$ times.

### 5.1 *Interference From* Picturephone *Signals*

Results are naturally dependent upon the source of the interference, the *Picturephone* signal. In this study the preemphasized *Picturephone*

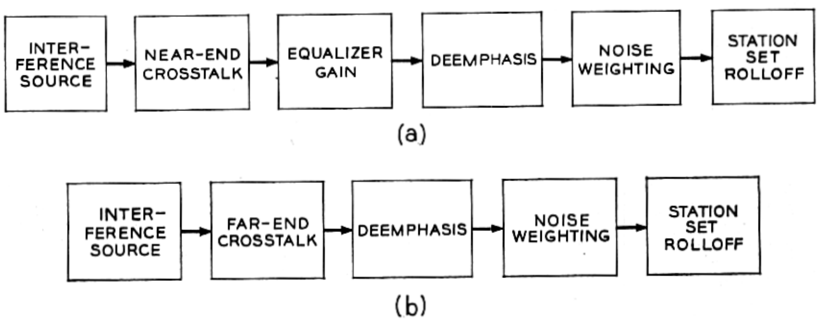


Fig. 6—Model used to compute random noise interference coupled through (a) the near-end crosstalk path and (b) the far-end crosstalk path.

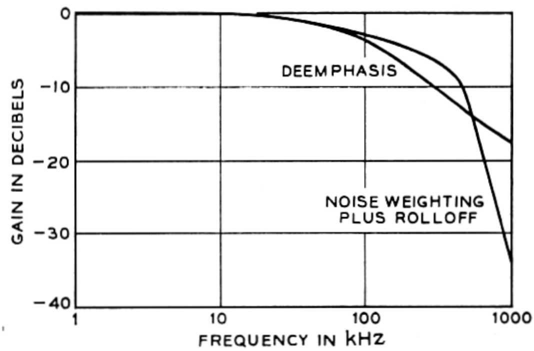


Fig. 7—Gain-frequency characteristics of noise weighting plus station set roll-off, and deemphasis network.

spectral envelope is assumed flat to a frequency of 15 kHz, and decreasing at a 6 dB/octave rate thereafter. The spectrum is scaled to produce a total signal power of 0 dBm.

Only the video information portion of the *Picturephone* signal is preemphasized while the synchronizing pulses, the dominant part of the signal, remain unaltered. The spectral envelope of the preemphasized and unpreemphasized signals are therefore very much the same.

#### 5.1.1 Picturephone *Interference on Loops*

The mean noise power due to *Picturephone* interference on loops is given as a function of equalizer spacing in Fig. 8. The average number of equivalent within-unit disturbers is shown as a parameter. The exact number of equivalent disturbers cannot be specified since each disturbed pair is assumed active with probability $p$. The average number is equal to $nqp$ where $n$ and $q$ are as previously defined and $p$ is the traffic activity factor, equal to 0.1 in the loop case.

Using the requirement (Section 3.2) that the mean noise power on loops should not exceed $-61.6$ dBm and considering the effect of 100 equivalent disturbers, it is seen that the noise produced is within that allowed even for equalizer spacing of greater than 6 kft. If it is possible to restrict each cable unit to carry only one direction of transmission, the noise produced on a given disturbed pair is primarily due to the effects of within-unit far-end crosstalk coupling. This noise is well within the requirement and results in no limitation on total loop length.

5.1.2 Picturephone *Interference on Trunks*

With the availability of the trunk facilities under development, trunks will be limited to 22-gauge pulp-insulated cable and trunk spans will be composed of a maximum of six equalizer sections of at most six kft each. The 22-gauge cable has less loss than 26-gauge and therefore with identical equalizer spacing requires less gain. Hence 22-gauge trunks have some advantage relative to 26-gauge loops in near-end crosstalk coupling. The advantage, however, is reduced by the increased activity on trunks (a traffic activity factor of 1.0 is assumed).

The allowable trunk equalizer spacings are derived using the assumption that one of the several sources of noise (*Picturephone*, T1, T2) dominates and may, therefore, be allocated the entire trunk span noise objective. If, in engineering a trunk span, it is found that the total noise from two or more sources exceeds the requirement and that the contribution from each of the sources is comparable, the noise power allocated to the span will have to be further allocated among the sources of noise, and the equalizer spacings or the number of disturbers reduced so that the total noise produced is within that permitted.

Figure 9 shows the mean within-unit near-end crosstalk noise power as a function of trunk equalizer spacing with the equivalent number of disturbers, $nq$, a parameter. As shown, with 150 equivalent dis-

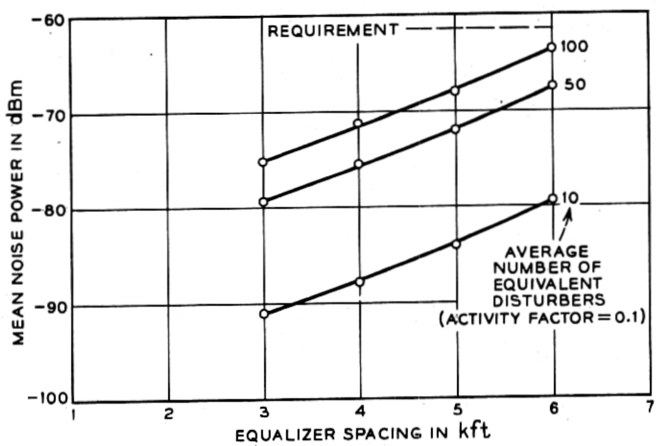


Fig. 8—Mean noise power on loops due to *Picturephone* interference coupled via within-unit, near-end crosstalk.

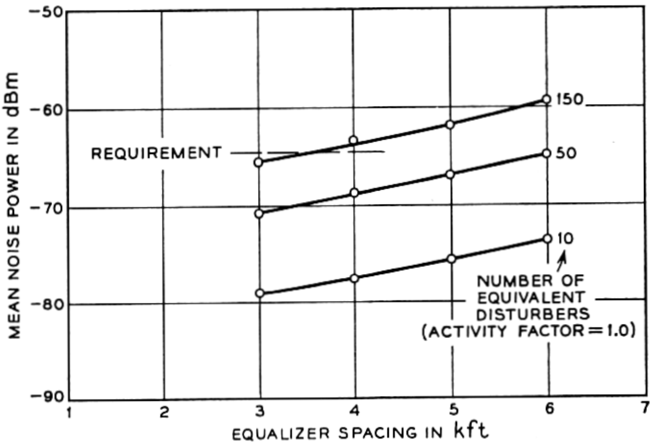


Fig. 9—Mean noise power on trunk spans due to *Picturephone* interference coupled via within-unit, near-end crosstalk.

turbers, the noise produced is within the requirement of −64.6 dBm only if the equalizer spacing is limited to 3.6 kft. The crosstalk advantage derived with adjacent-unit separation of opposite directions of transmission is sufficient to control the interference created by near-end crosstalk coupling of *Picturephone* signals and therefore allow maximum equalizer spacings. The noise on a trunk span of a given overall length, due to $n$ actual disturbers coupled through the within-unit far-end crosstalk path is shown in Fig. 10. With the maximum of 49 actual disturbers the noise produced on a 36-kft trunk span just meets the requirement.

### 5.2 *Interference from Digital Carrier Systems*

Concern has been raised over the possibility of degradation of the *Picturephone* signal by noise crosstalked from the digital T1 and T2 lines. This is possible since T1 is found largely in urban areas and often on 22-gauge cable, the same environment and medium in which *Picturephone* analog trunks will initially be installed.[8] Although T2 signals will initially be transmitted on a specially constructed low capacitance cable, it is planned that future T2 designs will allow transmission over 22-gauge pulp cable.[9] The study of interference on *Picturephone* lines caused by T2, located within the same 22-gauge pulp cable sheath, is therefore included.

Compatibility with T1 carrier is of particular concern because of its widespread use and, as shown in Fig. 11, the fact that the maxi-

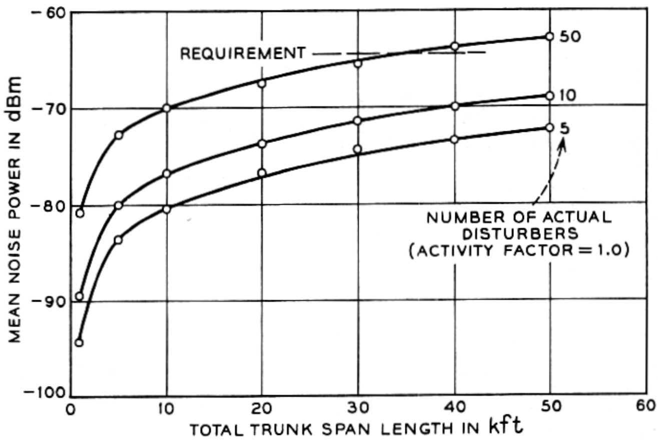


Fig. 10—Mean noise power on trunk spans due to *Picturephone* interference coupled via within-unit, far-end crosstalk.

mum energy of the typical T1 spectrum falls at 772 kHz, well within the *Picturephone* band. The 0 to 1 MHz portion of the typical T2 spectrum is also shown in Fig. 11. The noise produced by crosstalk of these digital signals on the *Picturephone* line is computed by again using the models of Fig. 6.

The mean noise power due to within-unit near-end and far-end crosstalk of T1 into *Picturephone* lines is given in Figs. 12 and 13

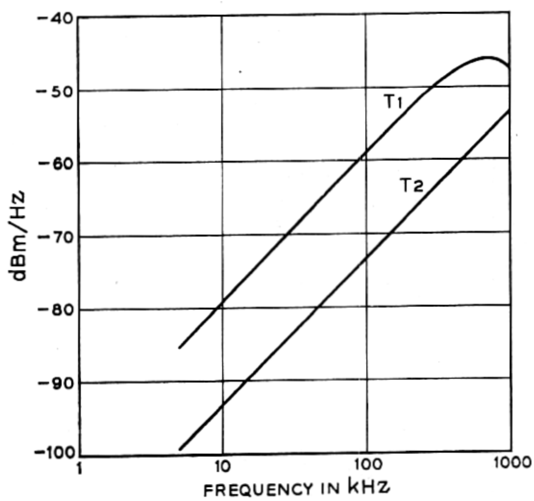


Fig. 11—Spectra of typical T1 and T2 signals.

respectively. As shown in Fig. 12, the noise generated by 150 equivalent disturbers coupled through the near-end path would not meet the requirement even if no equalizer were used and no gain employed. Similarly, Fig. 13 shows that the noise produced by 49 actual disturbers coupled through the within-unit, far-end path is within the requirement only if the total trunk span length is limited to 1.5 kft. Although adjacent-unit separation of opposite directions of T1 and *Picturephone* transmission provides some advantage, it is not sufficient to allow 6-kft equalizer spacing. As shown in Fig. 14, 300 equivalent adjacent-unit T1 disturbers (see Section 3.3) coupled through the near-end crosstalk path yield a noise power within the requirement only if the equalizer spacing is limited to 4.7 kft. If full 6-kft spacing is desired, non-adjacent-unit separation (see Fig. 2) of opposite directions of T1 and *Picturephone* transmission is required. The curves of Fig. 15, however, indicate that adjacent-unit separation of like directions of T1 and *Picturephone* transmission is sufficient to control interference from 50 disturbers coupled through the far-end path and hence allow maximum total trunk span length.

Figures 16 and 17 present the mean noise power generated by crosstalk of T2 signals. Again assuming 150 equivalent disturbers coupled through the within-unit near-end path, it is seen from Fig. 16 that the resulting noise power is within that allowed when equalizer spacing is limited to 2.7 kft. Total trunk span length must be limited to 26 kft, as indicated in Fig. 17, if the noise produced by 49 actual dis-

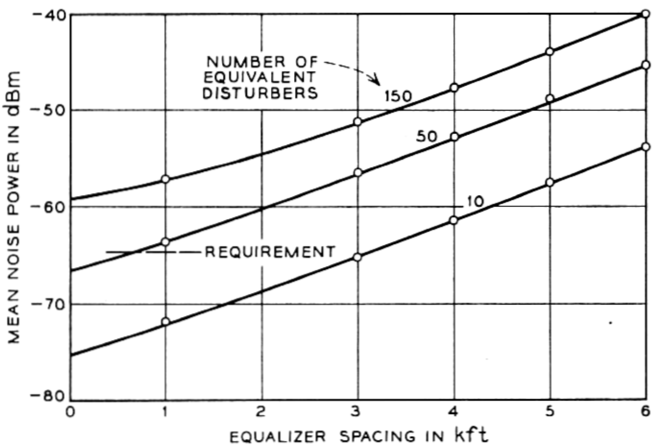


Fig. 12—Mean noise power due to T1 interference coupled via within-unit, near-end crosstalk.

turbers coupled through the within-unit far-end path is to be within that required. Interference from T2 signals can be controlled and 6-kft equalizer spacing and 36-kft trunk span length obtained if T2 and *Picturephone* signals are restricted to separate cable units.

## VI. SINGING OF BASEBAND EQUALIZERS

### 6.1 *The Feedback Mechanism*

The problem of providing an adequate singing margin for *Picturephone* equalizers is aggravated by the fact that cable loss must be equalized beyond the 1-MHz signal bandwidth. This must be done so the inband phase nonlinearity associated with the gain roll-off is small enough to meet the echo rating requirements.[1,5] The higher the frequency to which the cable is equalized, the more linear the inband phase can be made. However, this also makes the equalizer more likely to oscillate because the equalizer gain increases, while the crosstalk loss decreases with frequency. Thus equalized *Picturephone* lines may provide acceptable crosstalk loss in the 1-MHz signal bandwidth, while allowing the equalizer gain and the loss at the tail of the crosstalk loss distribution to be comparable beyond 1 MHz. This allows the possibility of singing.

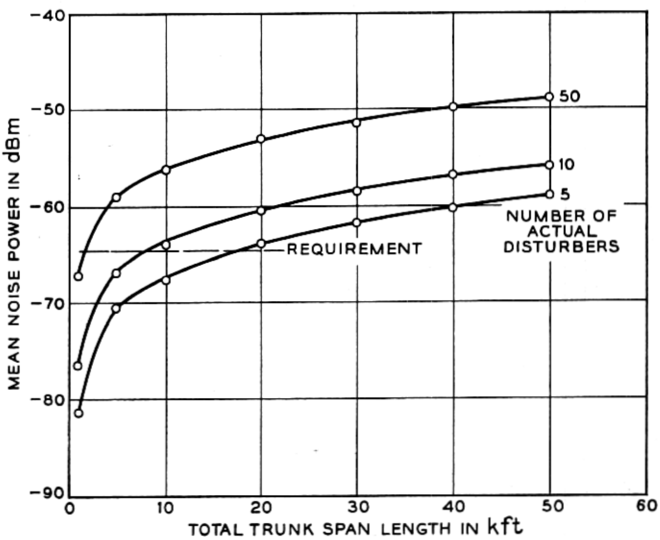


Fig. 13—Mean noise power due to T1 interference coupled via within-unit, far-end crosstalk.

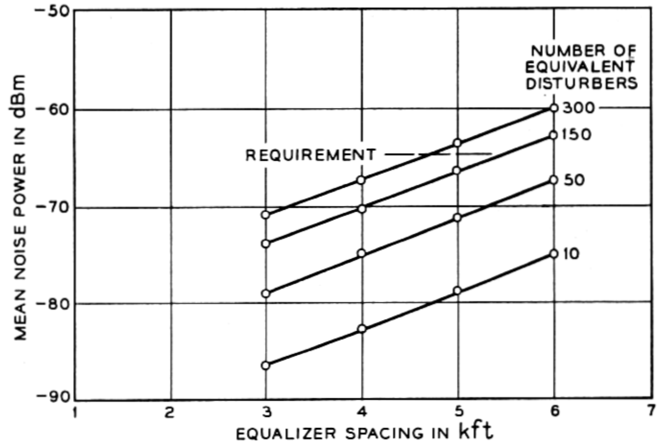


Fig. 14—Mean noise power due to T1 interference coupled via adjacent-unit, near-end crosstalk.

Figure 18 shows the mean NEXT loss and the gains of the loop and the trunk equalizers versus frequency. The significant parameter as far as singing is concerned is the difference between the crosstalk loss and the equalizer gain, denoted *CR*, the average of which is also plotted. This difference is half the average loss of the feedback loop around a pair of oppositely-directed equalizers. Since the crosstalk loss is random, the gain of the feedback loop is random. Thus, a pair of equalizers

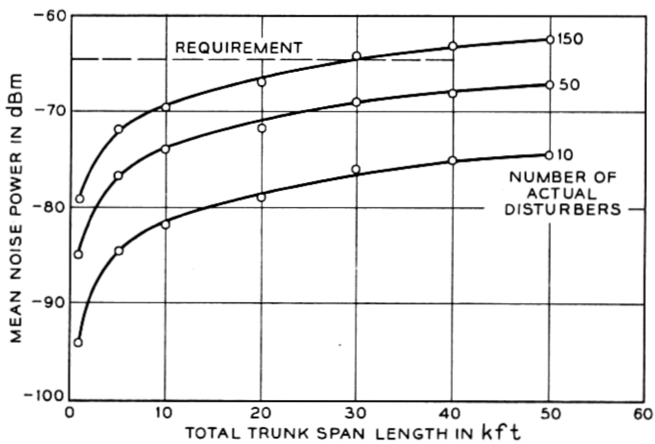


Fig. 15—Mean noise power due to T1 interference coupled via adjacent-unit, far-end crosstalk.

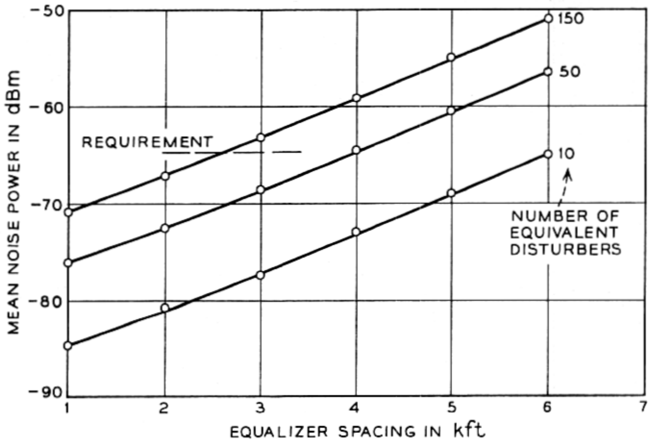


Fig. 16—Mean noise power due to T2 interference coupled via within-unit, near-end crosstalk.

could sing if the loss around the feedback path happened to be less than one when the phase was a multiple of $2\pi$. Consider, for example, the probability that a single pair of oppositely-directed, 6-kft, 26-gauge loop equalizers, when installed in the same unit of a pulp cable picked at random, will have loop gain greater than unity at 1.3 MHz. It is equal to the probability that a normal random variable of mean $2 \times CR_{1.3\,\mathrm{MHz}} = 2 \times 16 = 32$ dB and standard deviation $\sqrt{2} \times 8.8$

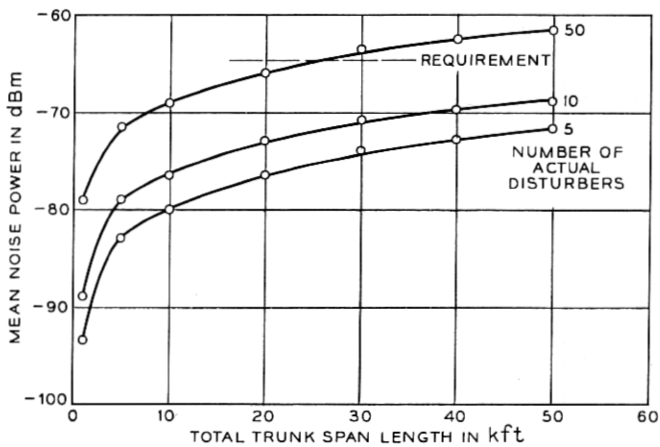


Fig. 17—Mean noise power due to T2 interference coupled via within-unit, far-end crosstalk.

(Table III) =12.4 dB is less than 0 dB, which is 0.5 percent. The possibility of singing can be decreased by reducing the equalizer spacing. Increasing the crosstalk loss has the same effect on the probability of singing and, in fact, the possibility of singing via NEXT can be eliminated by placing the opposite directions of transmission in different cables. Two-cable operation may not always be practical, however, especially on loops. In addition, singing would still be possible with two-cable operation via near-end-near-end-interaction crosstalk (NE-NE-IXT), as shown in Fig. 19. However, NEXT is the limiting singing path in single-cable operation so NE-NE-IXT will not be considered further. In two-cable operation, the equalizers are limited by their available gain rather than singing considerations.

Thus far we have considered feedback around a single 2-way equalizer. However, in a cable with many parallel equalized lines, each with several intermediate equalizers in tandem, there are a

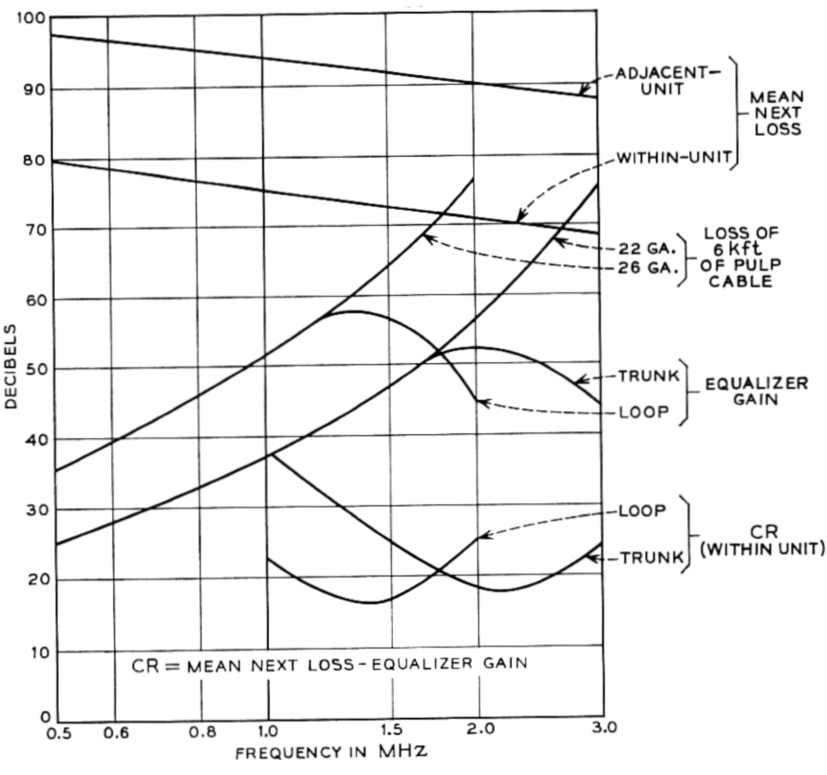


Fig. 18—Equalizer gains and crosstalk loss.

multitude of feedback paths. The feedback path for each equalizer includes all the oppositely-directed equalizers at a site, thus increasing the loop gain. Furthermore, a single feedback loop can include two separate equalizer sites. Two of the many possible paths are shown in Fig. 19.

The two directions of a *Picturephone* equalized line will be looped back on themselves through low-loss connections in several situations. For example, when a person calls his neighbor, one direction of one line may be looped through the zero-loss switching office to the opposite direction of the other line in the same cable as shown in Fig. 20. Such loop-backs act, in effect, like additional low-loss crosstalk paths and thus increase the probability of singing.

### 6.2 *Method of Calculation of Singing Margin*

A random variable called the singing margin, defined in Appendix B, allows one to estimate how close an equalized line is to singing. In order to calculate the distribution of the singing margin when there are several equalized lines in a single cable, a matrix formulation of the effects of near-end crosstalk is also presented in Appendix B. This model, which allows one to calculate the singing margin taking into account all near-end singing paths, has been implemented in a computer program. The program is used as part of a Monte Carlo procedure to calculate the distribution of the singing margin. The crosstalk is simulated by pseudo-random variables with lognormal ampli-

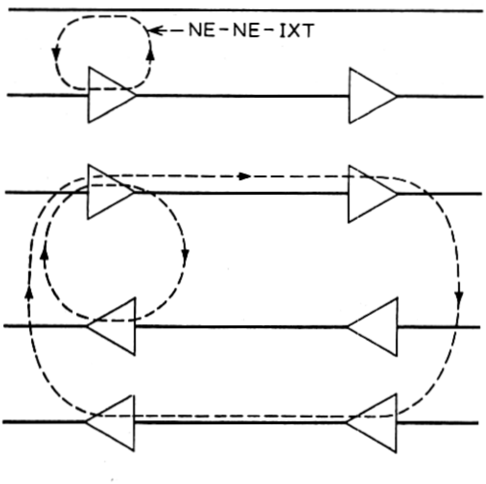


Fig. 19—Some possible singing paths.

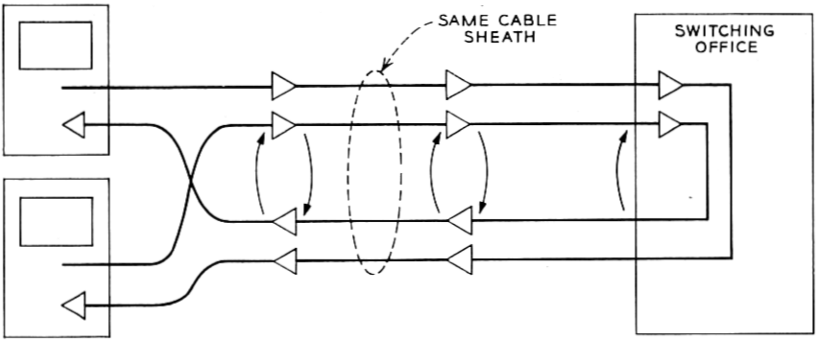


Fig. 20—Loopback of a line through a switching machine.

tude density and uniform phase density. The effect of this crosstalk on the through transmission of an equalized line with specified parameters is calculated on a computer many times, each with new crosstalk samples. This Monte Carlo procedure allows one to estimate the probability that a line is close to singing and to determine the effects of various parameters of the line on this probability.

### 6.3 *Limitations Necessary to Avoid Singing*

#### 6.3.1 *Required Singing Margin*

The criterion used to insure that an equalized line has an acceptably small probability of singing is to require its singing margin to be at least 10 dB with probability 0.99. The small probability of exceeding the limit is used because singing is a severe degradation. The 10-dB value for the limit is adopted for computational convenience, as described in Appendix B. The method of analysis and the criteria of acceptability described here are somewhat different from those mentioned in Ref. 1. The results of these two approaches are quite similar however. Through the use of the computer program this 10-dB requirement can be translated into a requirement on the minimum *CR*, i.e. the minimum difference between the mean crosstalk loss and the repeater gain. As shown in Fig. 21, the required *CR* in dB is proportional to the logarithms of the number of intermediate equalizers in tandem and the number of systems in the cable. Once the required *CR* has been determined, it can be translated into a requirement on equalizer spacing for the type crosstalk being encountered. Since singing paths can traverse a switching office, one must consider the possibility of singing for an entire connection.

The difference between crosstalk loss and equalizer gain must also be limited inband, in order to avoid degradation of a line's echo rating due to gain and phase deviations caused by feedback via crosstalk. As a guideline, the inband difference should, at all frequencies, exceed the minimum difference required for singing protection by 10 dB for loops and 15 dB for trunk spans. These limits are roughly those at which a line's echo rating would begin to be degraded. Either the inband difference or the singing requirement can be the limiting one, depending mostly on the gain roll-off characteristic of the equalizer in question.

#### 6.3.2 *Trunk Equalizer Spacing*

For 25 within-unit trunk spans, each with five intermediate equalizers in tandem, the required *CR*, found from Fig. 21, is about 31.5 dB. From Fig. 18, the trunk equalizer *CR* at the worst frequency is about 19 dB for 6-kft spacing. By scaling the equalizer gain, we find that the spacing would have to be reduced to 4.4 kft to raise the *CR* to the required level. The same method applied to 50 adjacent-unit trunk spans indicates that there is adequate singing margin with 6-kft equalizer spacing because of the increased crosstalk loss between adjacent units.

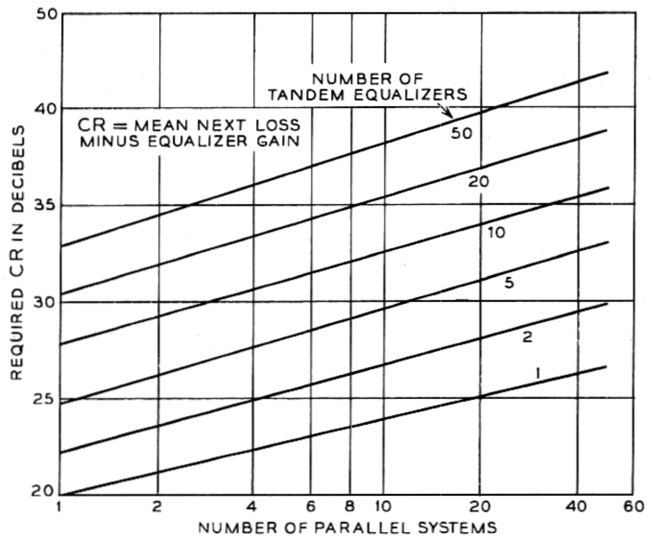


Fig. 21—System parameters required to give adequate singing protection.

Another possible singing situation is with the maximum of six analog trunk spans connected in tandem. This could result in 35 intermediate equalizers. Assuming that only one system would be in parallel over the entire length of such a connection, it can be seen from Fig. 21 that the required *CR* is again about 31.5 dB. Hence, the equalizer spacings derived above, 4.4 kft for within-unit operation and 6 kft for adjacent-unit operation, will provide an adequate singing margin for this connection also.

Because the gain roll-off occurs so far out of band, the singing requirements are dominant over the inband requirements.

### 6.3.3 *Loop Equalizer Spacing*

The controlling limitation on loop equalizer spacing is due to the possibility of connecting two lines in the same cable through an office switch. Such loopbacks change the functional dependence of *CR* on the numbers of systems and equalizers from that shown in Fig. 21. Thus, new computations including the effects of a loopback were made to determine the required *CR* on loops.

For adjacent unit operation on the maximum length line composed of three intermediate equalizers with 6-kft spacing, the singing margin is adequate even with two lines looped back. To provide singing protection for within-unit operation, the equalizer spacing must be reduced to 4.2 kft. Since the singing margin with a loopback is very insensitive to the number of systems, this suffices for 25 systems also, the maximum number possible. However, this reduction to 4.2-kft spacing only provides adequate singing protection; it does not give adequate protection against echo rating degradation. To do this, the equalizer spacing must further be reduced to about 3.6 kft. This increases the one percent point of the distribution of the singing margin at 1 MHz to 20 dB for two systems looped back through a switching office. Out-of-band loss in the office equalizers could increase the minimum spacing required by singing limitations from 4.2 kft to perhaps 4.4 kft, the limit with no loopback. Such loss cannot be used inband, however, so it cannot affect the 3.6-kft limit due to echo rating protection. This limit can be increased only by improving the crosstalk separation.

## VII. ACKNOWLEDGMENTS

We wish to acknowledge valuable discussions with many of our colleagues at Bell Laboratories. In particular, those with T. V. Crater on worst-disturber interference, M. W. Baldwin, Jr., on random noise weighting, and L. Hochgraf on carrier system interference have been especially helpful.

## APPENDIX A

## *Worst-Disturber Interference*

### A.1 *Characteristics of Worst-Disturber Interference*

When a *Picturephone* signal is coupled via a crosstalk path into another cable pair, it is approximately differentiated. Sharp transitions in the interfering signal appear as spikes in the disturbed pair. In particular, the sync pulses at the end of each scan line of the video signal represent step functions displaced by the horizontal blanking time. Differentiating both step functions results in two delta functions, one positive and one negative, in the disturbed video signal. Hence each scan line in the disturbed signal will have a positive and negative delta function superimposed upon it. When displayed, the interference will manifest itself as a white vertical line and a black vertical line. Since the oscillators generating the interfering and disturbed signals will not be at precisely the same frequency, the interference pattern will drift across the raster of the displayed disturbed signal. The loss required to render this interference just perceptible is denoted by $L_{\text{REQ}}$ and is equal to 45 dB at 150 kHz. (See Section 3.2).

### A.2 *Minimum Pair-to-Pair Loss Distribution for n Interfering Signals*

Let $F_X(x)$ be the pair-to-pair coupling loss distribution between one interferer and one disturbed signal. We are interested in obtaining the distribution of the minimum loss when there are $n$ interferers, $F_m(x)$. Let the probability density corresponding to $F_X(x)$ be $f_X(x)$. Then the probability that a pair-to-pair loss is in the interval $(x, x + dx)$ is $f_X(x)\,dx$. For a given disturbed pair, the probability that a given coupling loss is in this interval and all other losses are greater is given by

$$p(x) = [f_X(x)\,dx][1 - F_X(x)]^{n-1}. \tag{4}$$

Since there are $n$ possible disturbing pairs, the probability that the coupling loss between the given disturbed pair and any disturbing pair is in the interval $(x, x + dx)$ and all other coupling losses are greater, is the sum of $n$ terms of the form (4), or

$$np(x) = n[f(x)\,dx][1 - F_X(x)]^{n-1}. \tag{5}$$

The associated distribution function, $F_m(x)$, is the integral of equation (5) or

$$F_m(x) = 1 - [1 - F_X(x)]^n. \tag{6}$$

Since the given disturbed pair is equally likely to be any of the $n$ possible disturbed pairs, this is the required result.

### A.3 *Effect of Repeater Spacing*

We have denoted the required loss as $L_{\text{REQ}}$ at a frequency of $f_L$ Hertz. Suppose at this frequency the repeater gain is $G$ dB. The gain $G$ depends on the length of cable that the repeater must equalize. To meet the requirement, the cable crosstalk loss must be, as shown in Fig. 4, equal to $L_{\text{REQ}} + G$. Equation (6) can be used to calculate the probability that the coupling loss between a disturbed pair and the worst single interferer is less than $L_{\text{REQ}} + G$.

### A.4 *Cable Activity, and the Effect of Several Disturbers*

With $n$ two-way systems operating in a cable, there are $n$ interferers and $n$ disturbed circuits. Assume that during the busy hour each system initiates $p$ erlangs of traffic. If there is a negligible probability of blocking, this can be interpreted by stating that the probability that a given system is in operation is $p$. The probability that $j$ systems are in operation is then

$$P(j) = C_j^n p^j (1 - p)^{n-j} \tag{7}$$

where

$$C_j^n = n!/(n - j)!j! \tag{8}$$

is the binomial coefficient.

If the cable is made up of $q$ equalizer sections, then when one interferer is active, the disturbed circuit is subjected to $q$ interferences. Hence with $j$ interferers active over the cable of $q$ sections, the minimum pair-to-pair loss distribution becomes, from (6),

$$F_m(x) = 1 - [1 - F_X(x)]^{jq}. \tag{9}$$

Combining equations (4) and (6) according to the law of conditional probabilities gives

$$\text{Prob}\,(L < C_t) = \sum_{j=1}^{n} P(j)\{1 - [1 - F_X(C_t)]^{jq}\} \tag{10}$$

where Prob $(L < C_t)$ is the probability that the crosstalk loss $L$ is less than the required $C_t$.

This can be written in closed form by using the binomial expansion to give

$$\text{Prob}\ (L < C_t) = F_m(C_t) = 1 - [1 - (1 - a)p]^n \tag{11}$$

where

$$a = [1 - F_X(C_t)]^q. \tag{12}$$

APPENDIX B

*Calculation of Singing Margin*

B.1 *Criterion for Near-Singing Condition*

There are many near-end crosstalk paths between equalized lines in a cable. Since the closed paths thus formed around a single equalizer may be considered as many positive feedback loops, a single open-loop gain, including crosstalk, is not defined. Hence such quantities as gain or phase margin are not appropriate. The criterion adopted as an indication of closeness to singing is the deviation from unity of the normalized closed loop gain in the presence of crosstalk, denoted $G_n \exp (j\theta)$. The normalizing gain is that in the absence of crosstalk. The larger the magnitude of the gain change due to crosstalk, the closer the line is to singing. We shall define the log of this quantity to be the singing margin, i.e.,

$$SM = -20 \log | G_n \exp (j\theta) - 1 |. \tag{13}$$

A plot of the singing margin versus the loop gain is shown for three types of feedback configurations in Fig. 22. When there is no feedback, $SM = +\infty$; singing corresponds to $SM = -\infty$. Near the point where singing occurs, the singing margin changes very rapidly. Note that $SM = 0$, contrary to intuition, does not indicate singing in this context.

Note, also, that the singing margin of an equalized line is a random variable, since it depends on crosstalk losses which are themselves random variables. Thus, we cannot guarantee that a line will not sing; we can only insure that its chances of singing are below some value. Another important fact about the singing margin is that it is a function of frequency. In order that a line not sing, its margin must be greater than $-\infty$ for all frequencies.

Computing such an overall probability by combining the probabili-

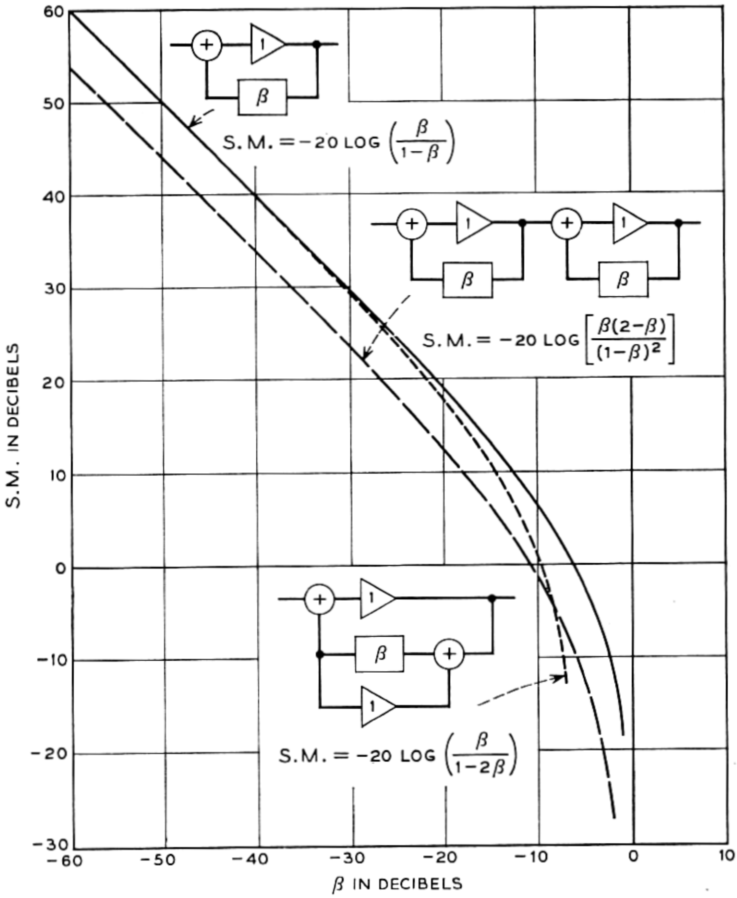


Fig. 22—Singing margin versus loop gain for several feedback configurations.

ties computed at many frequencies using a Monte Carlo technique is time-consuming and inaccurate. Instead, it can be shown that it is reasonable to assume that the overall probability of singing is less than one percent if the one percent point of the distribution of singing margin is at least 10 dB at the worst frequency.

## B.2 *Calculation of the Singing Margin*

### B.2.1 *Only One Intermediate Equalizer*

We are interested in the singing margin of an equalized line in a cable containing $n$ identical, 2-way equalized lines. Let $c_{jk}(b, c)$, a complex number, denote the near-end crosstalk voltage transfer function from point $b$ of system $k$ to point $c$ of system $j$. Let $v_i(b)$, also complex, represent the Fourier transform of the voltage at point $b$ of system $i$. Then the voltage due to crosstalk from point $b$ to point $c$ in a particular system, say $r$, is the sum of all the voltages at point $b$ times the crosstalk gain between each of them and point $b$ of system $r$, i.e.,

$$v_r(c) = \sum_{k=1}^{n} c_{rk}(b, c) v_k(b). \tag{14}$$

We can simultaneously write the equations for all the voltages at point $b$ due to crosstalk from point $a$ by utilizing matrix notation. Let $\mathbf{V}(b)$ be a column matrix of $n$ elements, the $k$th of which is $v_k(b)$. Let $\mathbf{C}(b, c)$ be an $n \times n$ matrix, the element of its $j$th row and $k$th column being $c_{jk}(b, c)$. Then

$$\mathbf{V}(c) = \mathbf{C}(b, c)\mathbf{V}(b). \tag{15}$$

Referring to Fig. 23, what we are interested in is a relationship of the form

$$\mathbf{V}(b) = \mathbf{T}(a, b)\mathbf{V}(a), \tag{16}$$

where $\mathbf{T}(a, b)$ is an $n \times n$ matrix which relates the input and output voltages, including all the effects of feedback via near-end crosstalk

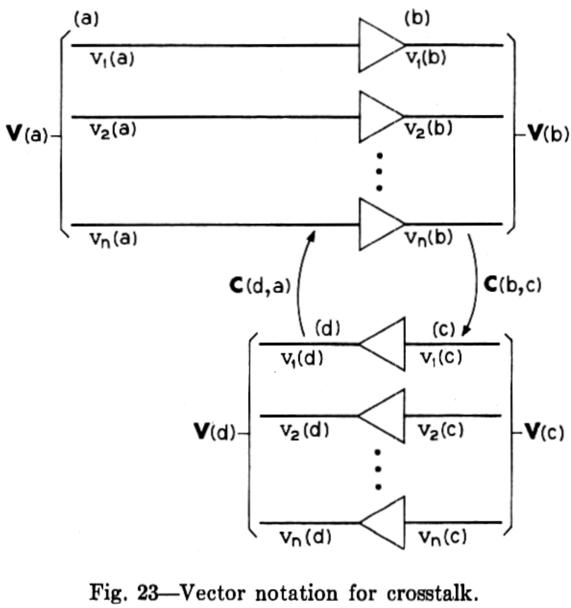


Fig. 23—Vector notation for crosstalk.

paths and the oppositely directed equalizers. Without crosstalk,

$$\mathbf{T}(a, b) = ga\mathbf{I} = q\mathbf{I}, \tag{17}$$

where $g$ is the equalizer gain, $a$ is the cable gain, $q$ is the overall gain of an equalizer section, and $\mathbf{I}$ is the identity matrix. (Throughout this appendix it should be understood that the quantities involved are functions of frequency, although this functional dependence is not made explicit.)

To find $\mathbf{T}$ in the presence of crosstalk, referring to Fig. 23, note that

$$\mathbf{V}(b) = q\mathbf{I}\mathbf{V}(a) + g\mathbf{I}\mathbf{C}(d, a)g\mathbf{I}\mathbf{C}(b, c)\mathbf{V}(b). \tag{18}$$

Rearranging and factoring out $\mathbf{V}(b)$ gives

$$\mathbf{V}(b) = q[\mathbf{I} - g^2\mathbf{C}(d, a)\mathbf{C}(b, c)]^{-1}\mathbf{V}(a), \tag{19}$$

where the exponent indicates the inverse of the matrix. Thus, in the presence of feedback,

$$\mathbf{T}(a, b) = q[\mathbf{I} - g^2\mathbf{C}(d, a)\mathbf{C}(b, c)]^{-1}. \tag{20}$$

If we are interested in the output of the $k$th system, $v_k(b)$, due only to an input to that system, $v_k(a)$, it is given by

$$v_k(b) = t_{kk}(a, b)\, v_k(a) \tag{21}$$

where $t_{kk}(a, b)$ is the $k$th diagonal element of $\mathbf{T}(a, b)$. In other words, the diagonal elements of the $n \times n$ matrix $\mathbf{T}$ are the transfer functions of the individual equalized lines, including the effects of feedback via near-end crosstalk paths and oppositely directed equalizers.

### B.2.2 *Several Intermediate Equalizers in Tandem*

When one considers a cable containing $n$ two-way equalized lines, each having $m$ intermediate equalizers in tandem, the problem of calculating the singing margin becomes even more complicated because feedback paths are possible between the equalizers in tandem. There are $2mn^2$ possible near-end crosstalk paths in such a configuration. These form a total of $\frac{1}{2}n^2m(m + 1)$ partially-overlapping, closed loops.

Let us find an expression for $\mathbf{T}(0, m) \equiv \mathbf{T}(m)$, the $n \times n$ matrix relating the inputs and outputs for one direction of the equalized lines in a cable containing $n$ two-way systems, each with $m$ intermediate equalizers in tandem. The crosstalk matrices $\mathbf{C}_u(k)$ and $\mathbf{C}_d(k)$ apply to the locations and in the directions shown in Fig. 24. We must solve the following set of equations which apply to that figure:

$$\mathbf{V}(k) = q\mathbf{V}(k-1) + g^2\mathbf{C}_u(k) \sum_{i=k}^{m} q^{i-k}\mathbf{C}_d(i)\mathbf{V}(i), \quad k = 1, 2, \cdots m. \tag{22}$$

Define

$$\mathbf{P}(k) = [\mathbf{I} - g\mathbf{Q}(k)\mathbf{C}_d(k)]^{-1} ; \tag{23}$$

$$\mathbf{Q}(k) = q^2\mathbf{P}(k-1)\mathbf{Q}(k-1) + g\mathbf{C}_u(k), \qquad k = 2, 3, \cdots m; \tag{24}$$

$$\mathbf{Q}(1) = g\mathbf{C}_u(1). \tag{25}$$

Then, beginning with the equation for $V(1)$, we find

$$[\mathbf{I} - g\mathbf{Q}(1)\mathbf{C}_d(1)]\mathbf{V}(1) = q\mathbf{V}(0) + g\mathbf{Q}(1) \sum_{i=2}^{m} q^{i-1}\mathbf{C}_d(i)\mathbf{V}(i) \tag{26}$$

whence

$$\mathbf{V}(1) = q\mathbf{P}(1)\mathbf{V}(0) + qg\mathbf{P}(1)\mathbf{Q}(1) \sum_{i=2}^{m} q^{i-2}\mathbf{C}_d(i)\mathbf{V}(i). \tag{27}$$

Now, substituting this expression for $\mathbf{V}(1)$ into the equation for $\mathbf{V}(2)$, we get

$$\mathbf{V}(2) = q^2\mathbf{P}(1)\mathbf{V}(0) + q^2g\mathbf{P}(1)\mathbf{Q}(1) \sum_{i=2}^{m} q^{i-2}\mathbf{C}_d(i)\mathbf{V}(i) + g^2\mathbf{C}_u(2) \sum_{i=2}^{m} q^{i-2}\mathbf{C}_d(i)\mathbf{V}(i); \tag{28}$$

$$[\mathbf{I} - g\mathbf{Q}(2)\mathbf{C}_d(2)]\mathbf{V}(2) = q^2\mathbf{P}(1)\mathbf{V}(0) + g\mathbf{Q}(2) \sum_{i=3}^{m} q^{i-2}\mathbf{C}_d(i)\mathbf{V}(i);$$

$$\mathbf{V}(2) = q^2\mathbf{P}(2)\mathbf{P}(1)\mathbf{V}(0) + gq\mathbf{P}(2)\mathbf{Q}(2) \sum_{i=3}^{m} q^{i-3}\mathbf{C}_d(i)\mathbf{V}(i). \tag{29}$$

This expression for $\mathbf{V}(2)$ is now substituted into the equation for $\mathbf{V}(3)$. The process can be continued until we get

$$\mathbf{V}(m) = q^m\mathbf{P}(m-1) \cdots \mathbf{P}(1)\mathbf{V}(0) + g[q^2\mathbf{P}(m-1)\mathbf{Q}(m-1) + g\mathbf{C}_u(m)]\mathbf{C}_d(m)\mathbf{V}(m); \tag{30}$$

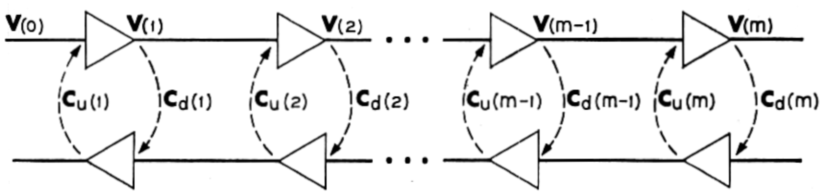


Fig. 24—Vector line of $n$ parallel systems each with $m$ intermediate equalizers in tandem.

$$\mathbf{V}(m) = q^m \mathbf{P}(m)\mathbf{P}(m-1) \cdots \mathbf{P}(1)\mathbf{V}(0). \tag{31}$$

Hence

$$\mathbf{T}(m) = q^m \mathbf{P}(m)\mathbf{P}(m-1) \cdots \mathbf{P}(1), \tag{32}$$

where $q^m$ is the gain of the overall equalized line. This gives a formula for calculating the transfer functions, including the effects of feedback via near-end crosstalk through equalizers transmitting in the opposite direction, of each of $n$ systems with $m$ intermediate equalizers in tandem. This allows calculation of the normalized gain, from which the singing margin is calculated by equation (13).

#### B.2.3 *Loopbacks*

Occasionally a two-way equalized line is looped around at a terminal, i.e., the output of an incoming line is connected to the input of the outgoing line in the same cable, as shown in Fig. 20. This can be handled by simply setting the crosstalk matrices at these locations equal to identity matrices times the gain of the loopback.

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