

Spreading Resistance Between Constant Potential Surfaces

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We determine the spreading resistance for a sheet of homogeneous material of uniform thickness with a disk contact source on one side and with current collected (i) over the entire back plane, and (ii) at a corresponding disk on the back plane. A constant driving potential is assumed over the source resulting in mixed boundary conditions in the plane of the source. For each case, a closed form integral solution is derived and then numerically integrated for a range of geometric ratios resulting in a universal spreading resistance curve. The results are used to evaluate the spreading resistances encountered in a typical (1 Ω cm) semiconductor material.

I. INTRODUCTION

The resistance associated with the nonparallel current flow between a spacially separated source and sink is referred to as spreading resistance. Calculation of spreading resistance is often required in the analytical treatment of semiconductor devices. In particular, electrical current flow in a slice of silicon between a surface contact and a back-plane contact involves the calculation of the ohmic spreading resistance. The heat flow between an active transistor or integrated circuit and an external heat sink involves a calculation of the thermal spreading resistance in the device carrier. Also, for a given structure, the capacitance including fringing is directly related to the conductance including spreading.

Two cases are considered in this paper. The spreading resistance is determined for an infinite sheet of homogeneous material of uniform thickness with a disk contact source on one side and with current collected (i) at a completely metallized back plane, and (ii) at a corresponding disk on the back plane. D. P. Kennedy analyzed these cases for finite cylindrical volumes but with nonmixed boundary conditions.¹ He assumed a constant flux over the source region and

defined spreading resistance in terms of the maximum temperature (or potential) on the disk. In this paper, we assume a constant driving potential over the source. This assumption results in a mixed boundary condition in the source plane because the potential is specified over part of the surface, and the normal derivative is specified over the rest of the surface. In the limiting case of a cylinder of infinite radius, there appears to be some difference between Kennedy's results and those presented herein. This appears to be associated with Kennedy's definition of spreading resistance in terms of the maximum disk potential. However, the comparison is based on the extrapolation of curves given by Kennedy.

A. Gray, et al., considered certain special cases of the infinite region problem with mixed boundary conditions in the source plane.² However, they imposed one of two constraints. Either (i) there was sufficient separation between source and sink so the flux distribution at the disk could be taken equal to the limiting half-space case or (ii) the disk was small enough to be considered a point source. In essence, these constraints again reduce the problem to one with a nonmixed boundary condition in the source plane. Neither constraint is imposed in this paper.

In this paper, for each case, a closed form integral solution is derived which is numerically integrated for a range of geometric ratios resulting in a universal spreading resistance curve. For both cases, the curves approach the half-space limit for thick sheets, and the curves asymptotically approach the nonfringing limit for very thin sheets. Finally, the results are used to evaluate the spreading resistances typical of those encountered in semiconductor technology.

II. ALGEBRAIC SOLUTION

The geometry and system of coordinates are given in Fig. 1. Solving La Place's equation in cylindrical coordinates when the potential ϕ is independent of θ gives³

$$\phi(\rho, z) = \int_0^{\infty} f(k)(C_1 \cosh kz + C_2 \sinh kz)J_0(k\rho) dk, \quad (1)$$

where $f(k)$ must be determined by the boundary conditions at $z = 0$. For nonmixed boundary conditions at $z = 0$, $f(k)$ can be found by inverting the Hankel transform.¹ In the present case, however, the mixed boundary condition must be imposed at the $z = 0$ surface. Thus,

$$V = \int_0^{\infty} f(k) C_1 J_0(k\rho) dk, \quad 0 \leq \rho \leq a; \quad (2)$$

$$\left. \frac{\partial \phi}{\partial z} \right|_{z=0} = \int_0^{\infty} f(k) k (C_1 \sinh kz + C_2 \cosh kz) J_0(k\rho) dk \Big|_{z=0} = 0.$$

This results in a dual set of integral equations.

$$V = \int_0^{\infty} f(k) C_1 J_0(k\rho) dk, \quad 0 \leq \rho \leq a; \quad (3)$$

$$0 = \int_0^{\infty} f(k) C_2 J_0(k\rho) dk, \quad \rho \geq a.$$

J. D. Jackson observes that the solution of this set of equations is³

$$f(k) = \frac{2}{\pi} \frac{Va \sin ka}{C_1 ka}. \quad (4)$$

Thus

$$\phi(\rho, z) = \frac{2Va}{\pi} \int_0^{\infty} \frac{\sin ka}{ka} \left(\cosh kz + \frac{C_2}{C_1} \sinh kz \right) J_0(k\rho) dk. \quad (5)$$

2.1 Case 1—Back Plane Grounded

In the first case considered, the second electrode is a completely metallized or grounded back plane. The boundary conditions are given in Fig. 2. This situation is representative of heat flow from an integrated circuit through an insulated header to a can acting as

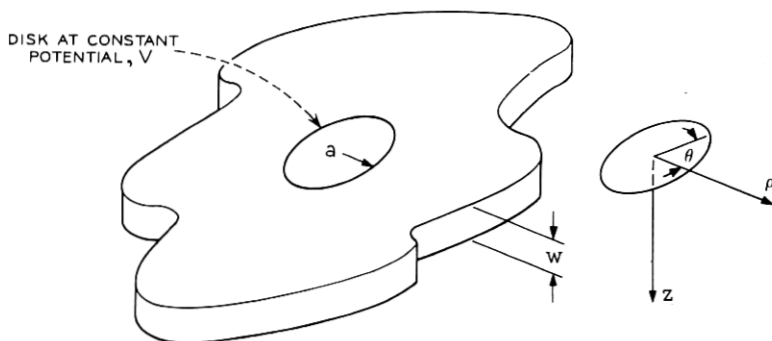


Fig. 1—Physical geometry and coordinate system for calculating spreading resistance.

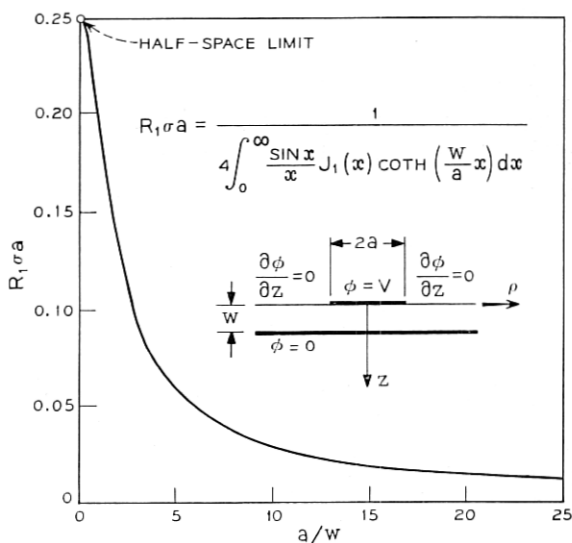


Fig. 2—Spreading resistance R_1 between disk and grounded back plane.

a thermal radiator.

$$\text{At } z = w, \phi(\rho, z)|_{z=w} = 0,$$

and thus

$$\int_0^\infty \frac{\sin ka}{ka} J_0(k\rho) \left(\cosh kw + \frac{C_2}{C_1} \sinh kw \right) dk = 0. \quad (6)$$

Since this must hold for all w ,

$$\frac{C_2}{C_1} \sinh kw = -\cosh kw. \quad (7)$$

Thus,

$$\phi(\rho, z) = \frac{2Va}{\pi} \int_0^\infty \frac{\sin ka}{ka} J_0(k\rho) (\cosh kz - \coth kw \sinh kz) dk. \quad (8)$$

Now,

$$\bar{J} = \sigma \bar{E} = -\sigma \nabla \phi \quad \text{at } z = 0, \quad (9)$$

$$I = \int_0^{2\pi} \int_0^a J_\rho d\rho d\theta, \quad (10)$$

and

$$\nabla\phi \Big|_{z=0} = \frac{\partial\phi}{\partial z} \Big|_{z=0} = -\frac{2V}{\pi} \int_0^\infty \sin ka J_0(k\rho) \coth kw \, dk. \quad (11)$$

Combining the last three equations and carrying out the indicated integrations, yields

$$I = 4V\sigma a \int_0^\infty \frac{\coth kw \sin ka J_1(ka)}{k} \, dk. \quad (12)$$

Thus

$$\frac{1}{R_1} \equiv \frac{I}{V} = 4\sigma a \int_0^\infty \frac{\coth kw \sin ka J_1(ka)}{k} \, dk \quad (13)$$

and finally,

$$R_1\sigma a = \frac{1}{4 \int_0^\infty \frac{\sin x}{x} J_1(x) \coth\left(\frac{wx}{a}\right) dx}. \quad (14)$$

2.2 Case 2—Disk on Back Plane Grounded

The case where the sink consists of a disk of radius a coaxial with the source can be derived by image theory from Case 1. A drawing of this configuration is shown in Fig. 3. The values of spreading resistance in Case 2 can be derived from Case 1 by setting

$$R_1\sigma a = \frac{R_2\sigma a}{2} \quad (15)$$

and

$$\frac{a}{w} \Big|_{\text{case 1}} = \frac{2a}{w} \Big|_{\text{case 2}}. \quad (16)$$

The resulting spreading resistance equation is

$$R_2\sigma a = \frac{1}{2 \int_0^\infty \frac{\sin x}{x} J_1(x) \coth\left(\frac{wx}{2a}\right) dx}. \quad (17)$$

III. NUMERICAL EVALUATION

3.1 Numerical Integration—Universal Curves

The integrals in equations (14) and (17) can be evaluated numerically for various a/w ratios if the upper limit is finite and the

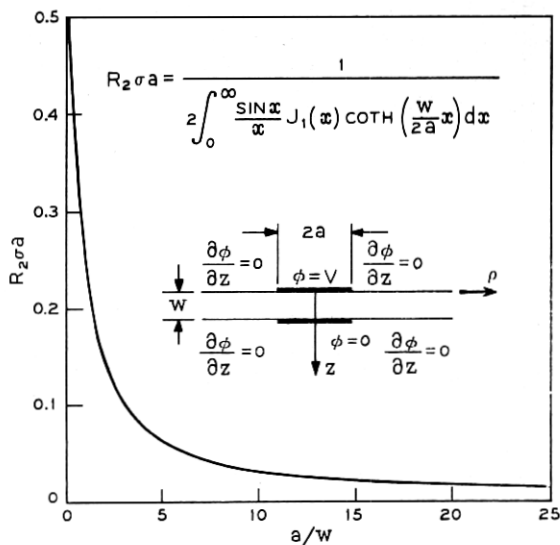


Fig. 3—Spreading resistance R_2 between disk and grounded disk on back plane.

integrands behave well at the lower limit. Investigation of the behavior of the integrands over the entire range of integration, indicated the range could be split into three ranges: 0 to 0.1, 0.1 to 300, and 300 to ∞ . For the 300 to ∞ range, and $a/w \geq 1$, the integrals for both cases reduce to

$$\int_{300}^{\infty} \frac{\sin x}{x} J_1(x) dx \equiv I_3. \quad (18)$$

But

$$I_3 = \int_0^{\infty} \frac{\sin x}{x} J_1(x) dx - \int_0^{300} \frac{\sin x}{x} J_1(x) dx \quad (19)$$

or⁴

$$I_3 = 1 - \int_0^{300} \frac{\sin x}{x} J_1(x) dx \equiv 1 - I_4. \quad (20)$$

I_4 was evaluated numerically on a digital computer and found to be 0.96439. Thus $I_3 = 0.03561$ and is independent of a/w for $a/w \geq 1$.

For the 0 to 0.1 range, the integrands were replaced by their small argument approximations. For $x \ll 1$,

$$J_1(x) \rightarrow \frac{x/2}{\Gamma(2)} = \frac{x}{2}, \quad (21)$$

$$\frac{\sin x}{x} \rightarrow 1, \quad (22)$$

$$\coth \frac{wx}{a} \rightarrow \frac{a}{wx}, \quad (23)$$

and,

$$\coth \frac{wx}{2a} \rightarrow \frac{2a}{wx}. \quad (24)$$

It follows that the small argument approximations for the integrands are $a/2w$, and a/w for Cases 1 and 2, respectively, and the values of the integrals over the range 0 to 0.1 are $a/20w$ and $a/10w$, respectively. This is a good approximation because the integrands are very flat functions of x near $x = 0$.

Thus for numerical purposes,

$$R_1 \sigma a = \frac{1}{4 \left[\frac{a}{20w} + \int_{0.1}^{300} \frac{\sin x}{x} J_1(x) \coth \left(\frac{wx}{a} \right) dx + 0.03561 \right]} \quad (25)$$

and

$$R_2 \sigma a = \frac{1}{2 \left[\frac{a}{10w} + \int_{0.1}^{300} \frac{\sin x}{x} J_1(x) \coth \left(\frac{wx}{2a} \right) dx + 0.03561 \right]}. \quad (26)$$

These results hold for $a/w \geq 1$. For $a/w = 0$, the half-space limiting case for finite a and infinite w can be used to find values for R_1 and R_2 . The resultant values are $\frac{1}{4}$ and $\frac{1}{2}$ for cases 1 and 2, respectively. This result follows directly from Jackson's work.³

Using the half-space limits and carrying out the numerical integrations for various values of a/w results in the universal spreading resistance curves given in Figs. 2 and 3.

3.2 Asymptotic Limits for Cases 1 and 2

As the ratio a/w becomes larger, the components of R_1 and R_2 due to fringing become progressively smaller. Neglecting fringing, the resistance for either case is

$$\bar{R} = \frac{w}{\sigma \pi a^2} \quad (27)$$

or

$$\bar{R}\sigma a = \frac{w}{\pi a}. \quad (28)$$

The results given in this section can be presented more clearly in terms of conductances because the fringing and nonfringing components are in parallel and $\bar{R}^{-1} \equiv \bar{G}$ is a linear function of a/w . Let the conductances corresponding to R_1 and R_2 be designated G_1 and G_2 , respectively. For a given a and w , there should be more fringing in Case 1 than in Case 2, and no fringing for \bar{R} . Thus for any a/w ,

$$G_1 > G_2 > \bar{G}. \quad (29)$$

Also G_1 and G_2 should asymptotically approach \bar{G} as a function of a/w . These observations are supported by the curves of G_1 , G_2 , and \bar{G} versus a/w given in Fig. 4. The differences of fringing components for Cases 1 and 2 are given in Fig. 5. G_1 , G_2 and \bar{G} are within one percent for $a/w \geq 10$.

3.3 Typical Example

The results given above have been used to evaluate spreading resistances typical of those encountered in semiconductor technology. A

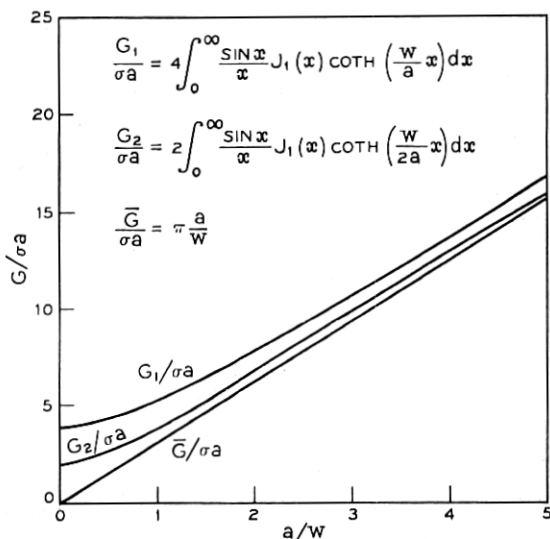


Fig. 4—Spreading conductances with and without fringing between disk and grounded back plane and between disk and grounded disk on back plane.

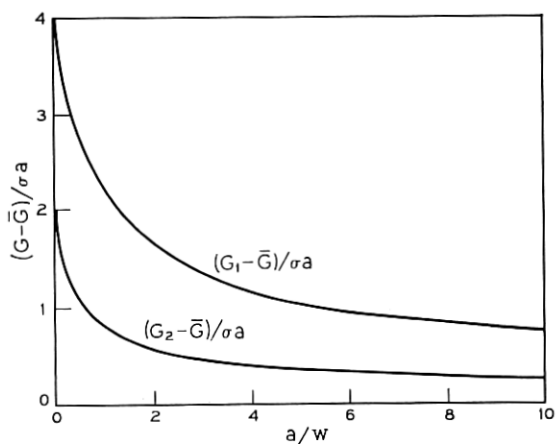
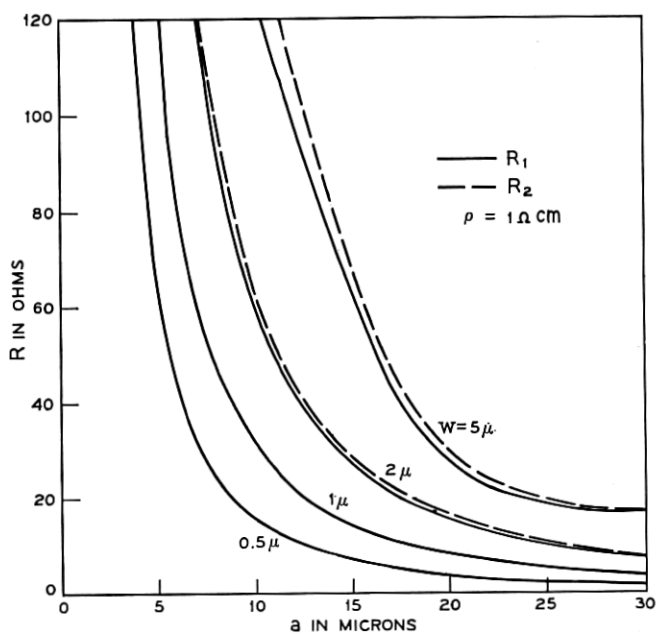


Fig. 5—Spreading conductance fringing components.

Fig. 6—Spreading resistances R_1 and R_2 for $1 \Omega \text{ cm}$ material.

1 Ω cm slice of silicon with circular contacts was considered. Figure 6 gives the calculated R_1 and R_2 for such a configuration.

IV. SUMMARY AND CONCLUSIONS

An investigation was made to determine the spreading resistance in a sheet of homogeneous material of uniform thickness with a disk contact source on one side and with current collected (i) at the entire back plane, and (ii) at a corresponding disk on the back plane. These cases were analyzed and evaluated exactly by solution of a dual set of integral equations. The method represented the boundary conditions as they physically exist. In contrast to previous work, a single, universal, spreading resistance curve was presented for each case. These curves should be useful in designing devices and analyzing materials. In particular, the curves could be used to determine conductivity of a sheet of semiconductor material if its thickness is known. Also, the curves could be used directly in the calculation of total capacitance and fringing capacitance by relabeling the ordinates with $\epsilon a/C$ instead of $R\sigma a$ and $C/\epsilon a$ instead of $G/\sigma a$.

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