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Anisotropic Scattering Due to Rain at Radio-Relay Frequencies

By DAVID E. SETZER

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L. T. Gusler and D. C. Hogg have estimated that interference coupling due to rain is a significant factor in coordinating the shared use of frequencies between satellite-communications and terrestrial microwave radio-relay systems. Their calculations are based on the assumption that rain scatters isotropically. Calculations given here, based on the exact anisotropic angular scattering functions, do not alter their conclusions. The exact scattering patterns at selected frequencies in the range 1.4 to 300 GHz for rains whose drop sizes obey the Laws-Parsons distribution are presented for the range of rain rates 1 to 150 mm/hr.

I. INTRODUCTION

Recently L. T. Gusler and D. C. Hogg[1] calculated the degree of coupling between satellite-communications and terrestrial radio-relay systems due to scattering by rain at frequencies of 4, 6, 11, 18.5 and 30 GHz. In their work they took the scattering by raindrops to be isotropic and to be based on the Laws and Parsons drop-size distribution.[2] They have pointed out correctly that it is known that raindrops do not scatter isotropically. Since the isotropic assumption is incorrect, it is of interest to examine the magnitude of the resulting error. Also, for the purposes of documentation, we present the scattering patterns at selected frequencies in the range 1.4 to 300 GHz at rain rates in the range 1 to 150 mm/hr.

It has been shown that the Mie solution to the problem of scattering from a single sphere can be used to generate the Stokes scattering matrix for atmospheric type aerosols, such as rain, and in particular for Laws and Parsons type rains.[3,4] One of the elements of this matirx is the angular scattering function $[P_1(\theta) + P_2(\theta)]$, sometimes referred to as the normalized Mie intensity function. Here, θ is the scattering angle (0° to 180°) with zero representing the forward direction. Since the aerosol is assumed to be made up of spherical particles, the scattering is symmetric about the direction of propagation; therefore, the total scattering field can be described in terms of the single variable θ. Essentially, the function $[P_1(\theta) + P_2(\theta)]$ is a prescription for the relative amount of energy scattered into a differential solid angle in the direction θ. For isotropic scattering,

$$P_1(\theta) + P_2(\theta) = 2, \tag{1}$$

and for Rayleigh scattering,

$$P_1(\theta) + P_2(\theta) = \tfrac{3}{2}(1 + \cos^2 \theta). \tag{2}$$

Generally, as the ratio of particle circumference to the wavelength of the scattered energy becomes small, the scattering function $[P_1(\theta) + P_2(\theta)]$ approaches the Rayleigh formula.

II. COMPUTER PROGRAM

We devised a computer program which was used to generate the angular scattering functions for Laws and Parsons rains of 1, 50, 100 and 150 mm/hr at 1.4, 2, 3, 6, 16, 30, 60, 100, 150 and 300 GHz.[4] The results are plotted on Figs. 1 through 4 along with plots for isotropic and Rayleigh scattering. The Laws and Parsons rains are described in Table I.

Figures 1 through 4 show that the scattering at 1.4, 2 and 3 GHz is described by the Rayleigh function. Also, for many applications it appears that the assumption of Rayleigh scattering at 6, 16 and 30 GHz will be in order; however, the scattering patterns at 60, 100, 150 and 300 GHz deviate greatly from the Rayleigh function. Note that the scattering diagrams are only mildly dependent on rain rate, the most noticeable feature being the increase in the forward peak with increase in rain rate. It is to be expected that, in general, for a Laws and Parsons rain, the angular scattering function will not be greatly influenced by rain rate. This is because the angular scattering function of an aerosol is determined by the shape of the particle size distribution and the mean particle size, rather than the density of particles. As can be seen by examining Table I, the mean particle size and the general distribution shape are not very different over the range of rain rates presented.

Finally, note that none of the scattering patterns can be described as isotropic. However, in the range of frequency considered by Gusler

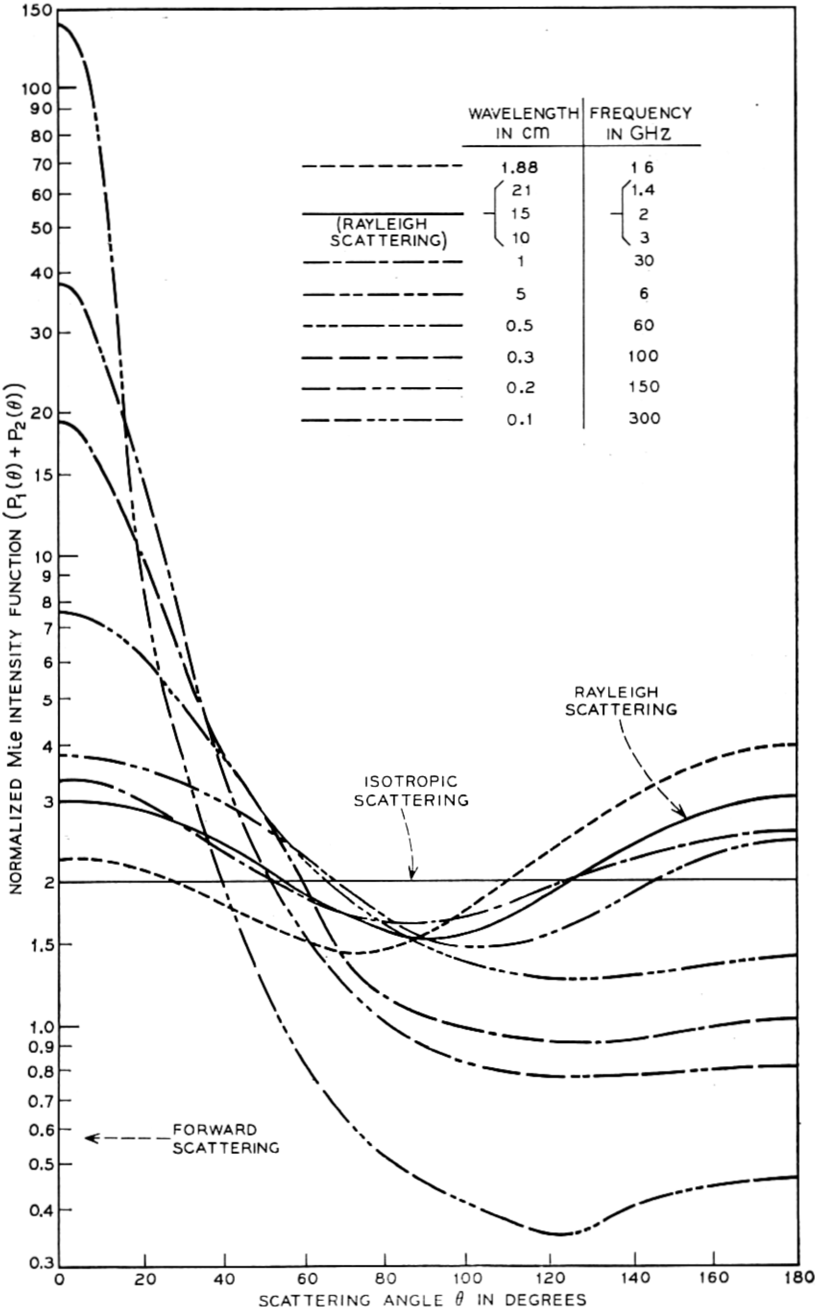

Fig. 1—Intensity versus scattering angle, rain rate = 150 mm/hr.

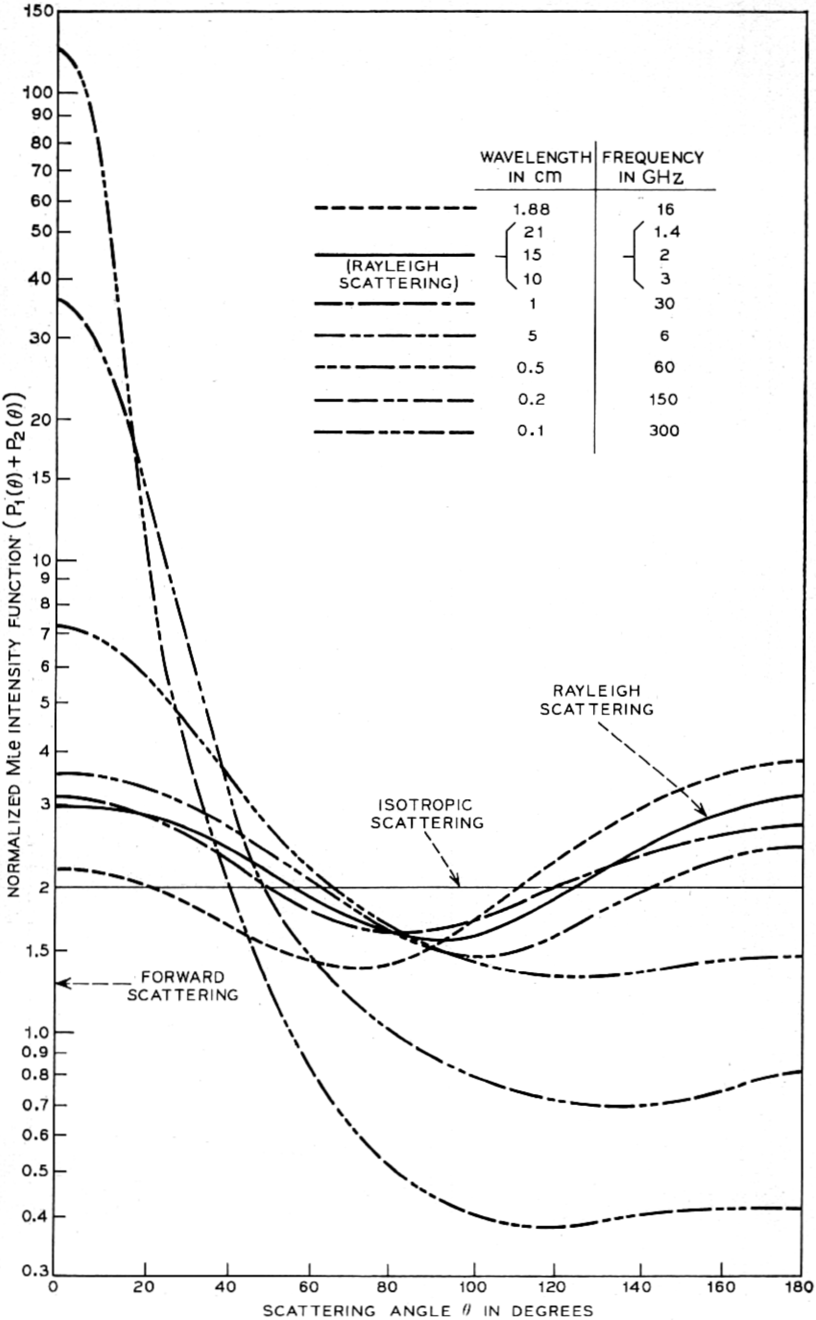

Fig. 2—Intensity versus scattering angle, rain rate = 100 mm/hr.

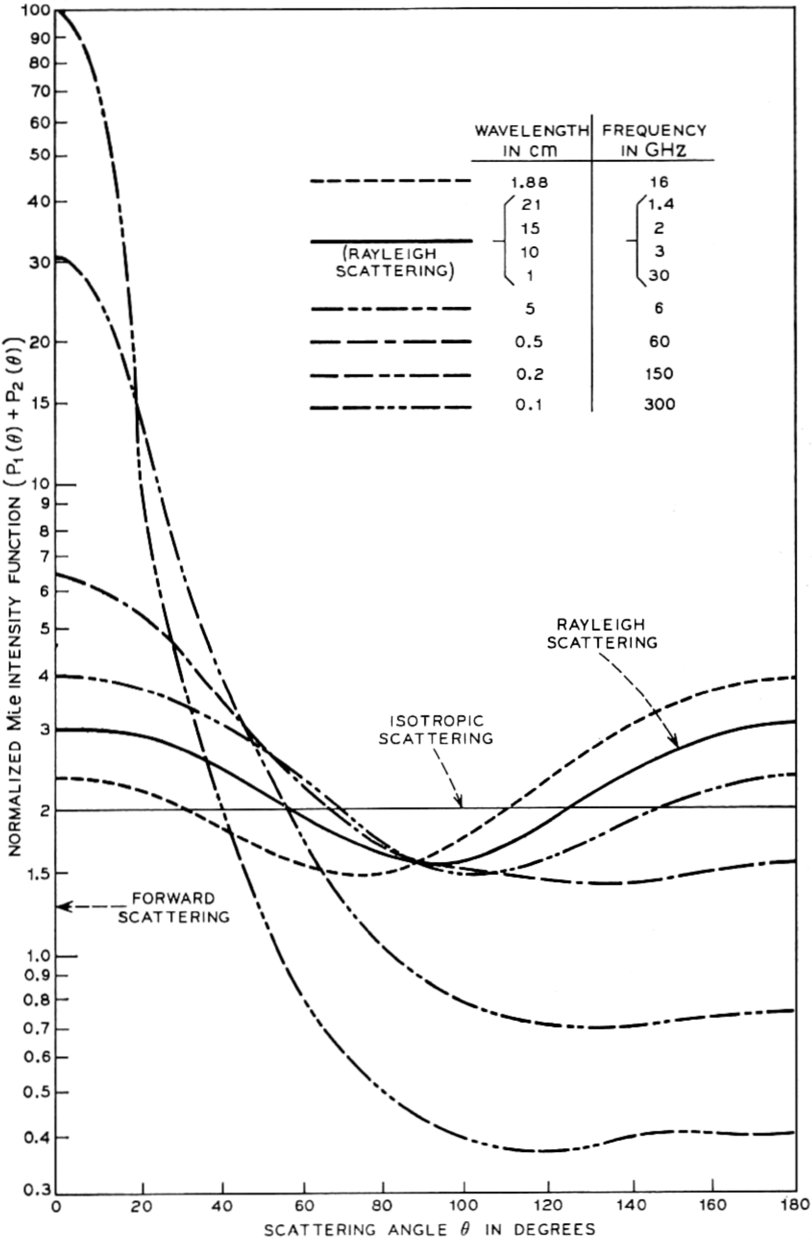

Fig. 3—Intensity versus scattering angle, rain rate = 50 mm/hr.

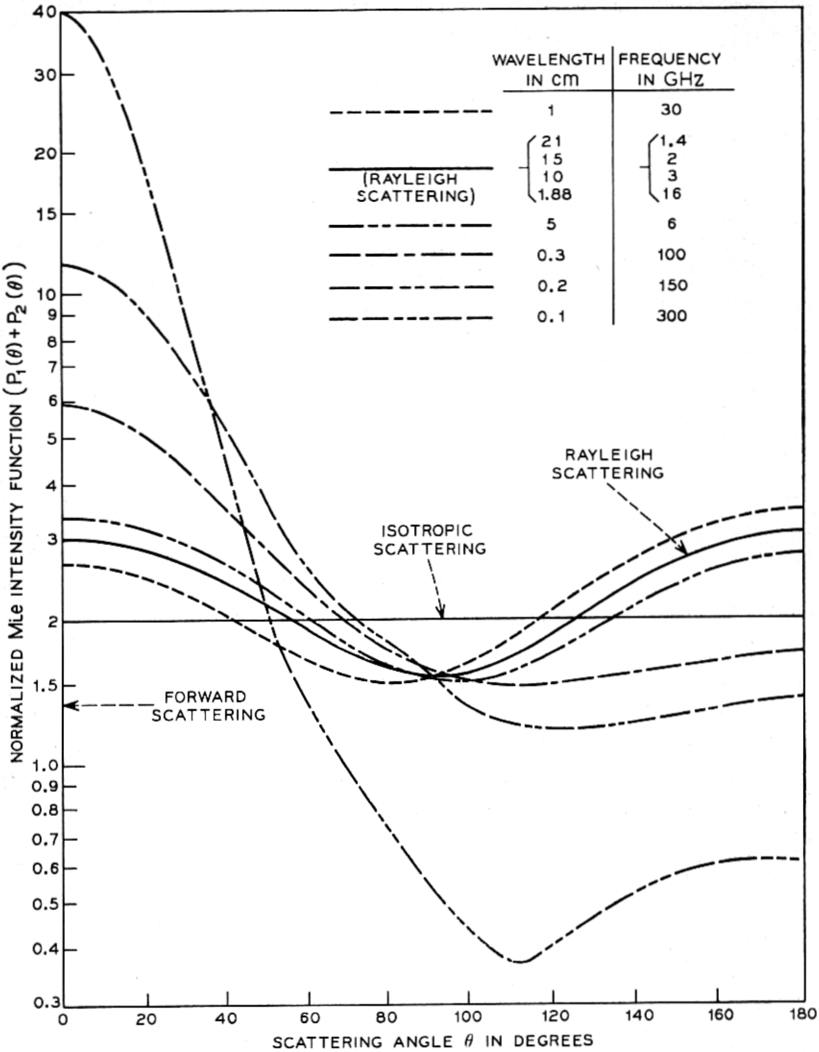

Fig. 4—Intensity versus scattering angle, rain rate = 1 mm/hr.

and Hogg, 4 to 30 GHz, the actual scattering functions are everywhere within a factor of two (3 dB) of the isotropic function. Since the Gusler-Hogg scattering model predicts an interference level between radio relay and satellite systems proportional to the scattering function, the solutions can be in error by at most 3 dB. It is important

Table I—Laws and Parsons Drop-Size Distributions for Various Precipitation Rates

Drop Diameter (cm)	Rain Rate (mm/hour)								
	0.25	1.25	2.5	5	12.5	25	50	100	150
	Percent of Total Volume								
0.05	28.0	10.9	7.3	4.7	2.6	1.7	1.2	1.0	1.0
0.1	50.1	37.1	27.8	20.3	11.5	7.6	5.4	4.6	4.1
0.15	18.2	31.3	32.8	31.0	24.5	18.4	12.5	8.8	7.6
0.2	3.0	13.5	19.0	22.2	25.4	23.9	19.9	13.9	11.7
0.25	0.7	4.9	7.9	11.8	17.3	19.9	20.9	17.1	13.9
0.3		1.5	3.3	5.7	10.1	12.8	15.6	18.4	17.7
0.35		0.6	1.1	2.5	4.3	8.2	10.9	15.0	16.1
0.4		0.2	0.6	1.0	2.3	3.5	6.7	9.0	11.9
0.45			0.2	0.5	1.2	2.1	3.3	5.8	7.7
0.5				0.3	0.6	1.1	1.8	3.0	3.6
0.55					0.2	0.5	1.1	1.7	2.2
0.6						0.3	0.5	1.0	1.2
0.65							0.2	0.7	1.0
0.7									0.3

to remember that here we are speaking only of the error due to invoking the isotropic scattering assumption. Nothing is intended to be said about any other aspect of the model and, most importantly, this work does not imply anything about the magnitude of the scattering cross section other than that the Rayleigh cross section is probably a reasonable assumption at those wavelengths where the angular scattering function is Rayleigh. For a detailed look at the Mie cross sections for Laws and Parsons rains see Ref. 4.

III. SUMMARY

Gusler and Hogg have estimated maximum coupling between satellite communications and terrestrial radio-relay systems due to scattering by rain to be of the order of −150 dB. A 3-dB uncertainty in those results is not really significant, considering all of the other uncertain aspects of modeling such a complicated physical problem. In summary, the results of Gusler and Hogg are not significantly altered if the exact angular scattering functions rather than isotropic scattering functions are used in their calculations.

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