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Channel Spacing and Necessary Bandwidth in FDM-FM Systems

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We examine the effect of loading and filtering on adjacent channel interference noise and give the results for different cases of filtering and various amounts of bandlimiting. The results are applied to typical systems and an empirical formula relating channel spacing to loading and adjacent channel interference is derived. We give some examples computing the capacity of typical radio channels.

I. INTRODUCTION

Necessary bandwidth is defined by the International Telecommunications Union as: ". . . bandwidth sufficient to ensure the transmission of information at the rate and with the quality required for the system employed . . . ."[1] Radio channel assignments are based on necessary bandwidth, i.e., a user is authorized a band at least equal to the necessary bandwidth. If he wants two adjacent channels, he gets another band equally wide and the two are usually not allowed to overlap. For many signals this is a straightforward and accurate way to assign channels. For instance an AM broadcast signal is well contained within a specified frequency band.

In FM systems however the case is not so clear. An FM signal theoretically has infinite bandwidth and bandlimiting will cause distortion. On the other hand putting two FM signals next to each other in the frequency band and *not* bandlimiting them will cause mutual interference.

The present way of calculating the necessary bandwidth for FM signals is to use Carson's rule which states that

$$B_n = 2(f_t + f_p) \tag{1}$$

where

B_n is the necessary bandwidth,
f_t is the top modulating frequency, and
f_p is the peak frequency deviation.

This has been a very good measure to use for necessary bandwidth and channel assignments. As the radio spectrum gets more and more congested, however, some of its limitations are beginning to show up. One difficulty is that it does not relate the necessary bandwidth to the system performance. Obviously as the passband is more and more restricted, distortion will increase, and this is not reflected by the rule. Restricting the bandwidth means that channels may be put closer together but the spacing should be a function of permissible mutual interference, which is not reflected either.

Here we analyze this problem for FDM-FM systems, finding the interference as a function of channel spacing, top baseband frequency, rms frequency deviation, and RF selectivity. To trade interference against bandlimiting noise, we use some recent work by A. Anuff and M. Liou relating bandlimiting to distortion.[2] They have given an empirical formula for necessary bandwidth which we use in choosing filter bandwidths.

We find the interference by numerical convolution of RF spectra, and look at the effect of filtering and conclude that from both a theoretical and practical point of view, the receiver filter is the most important filter. We therefore concentrate on the effects of the receiver filter. Using the Anuff-Liou formula, we compute realistic filter bandwidths and find the adjacent channel interference noise as a function of loading. The end result is an empirical formula for channel spacing.

In this paper we talk about many different signal-to-noise ratios (S/N), and to lessen the confusion let us mention them right away. There are two sources of noise, independent of each other: (*i*) filter distortion, here called bandlimiting noise, which we refer to when talking about a S/N due to bandlimiting; and (*ii*) adjacent channel interference noise. There is also a baseband signal-to-interference ratio related to an RF carrier-to-interference ratio. We hope that it will be clear from the context which ratio we are talking about.

II. NECESSARY BANDWIDTH

Recently Anuff and Liou arrived at the following empirical formula relating bandlimiting distortion to rms deviation and filter bandwidth.[2]

$$B_N = 2f_t\left\{1 + 0.065 \log_{10}\left[\frac{S_{\phi_1}}{S_d}\right]\right\} + \sigma \log_{10}\left[\frac{S_{\phi_1}}{S_d}\right] \qquad (2)$$

where

B_N is the filter bandwidth,
S_{ϕ_1}/S_d is the S/N in the top baseband channel due to bandlimiting,
f_t is the top baseband frequency, and
σ is the rms deviation.

The signal was assumed to be a preemphasized bandlimited gaussian signal, and the filters were assumed to be square filters with constant delay. B_N can be interpreted as the necessary bandwidth of an FDM-FM signal because equation (2) relates performance to filter bandwidth as a function of loading which is exactly what "necessary bandwidth" is meant to do.

In a typical system a reasonable S/N might be 70 dB. In practice, however, the bandwidth of the filter has to be larger. It is difficult to achieve a well-behaved delay characteristic over the whole passband in a practical filter. Therefore, a bandwidth corresponding to 90 dB is more realistic. Even though the filter is a 90-dB filter, the noise is 20 dB greater because the delay cannot be equalized well enough.

Examination of present typical filters show this to be a reasonable assumption. As the state of the art changes, it should be possible to decrease the necessary bandwidth to that corresponding to 70 dB.

Inserting 90 dB into the formula for B_N, we get

$$B_N = 3.17f_t + 9\sigma. \tag{3}$$

III. ADJACENT CHANNEL INTERFERENCE ANALYSIS

We need to find the baseband interference from an adjacent channel as a function of spacing and loading. We assume that the two channels are identical and loaded with bandlimited gaussian signals simulating multichannel talker loads, and that the channels are far enough apart that the first-order sidebands do not overlap. The worst interference will be in the top voice channel and we need to compute the signal-to-interference ratio in that channel due to two adjacent channels, one on each side.

Many authors have shown the following formula for the baseband phase due to a small interfering signal.

$$\phi(t) = \phi_1(t) + \psi(t) \tag{4}$$

where

$$\psi(t) = k \sin (\Delta\omega t + \phi_2(t) - \phi_1(t) + \alpha) \tag{5}$$

and

k is the RF voltage ratio of the unwanted and wanted signals, $k \ll 1$,
$\Delta\omega$ is the difference frequency between carriers,
ϕ_1 and ϕ_2 are the phases due to modulation of the wanted and unwanted signals respectively, and
α is a random angle uniformly distributed from 0 to 2π.

Equation (5) is an approximation for small k, of the more general expression derived by V. K. Prabhu and L. H. Enloe.[3] ϕ and ψ are not independent but are uncorrelated because the expected value of exp (α) is equal to 0. Furthermore the time average of exp $(j\ \Delta\omega t)$ equals 0 for $\Delta\omega \neq 0$. The signal-to-interference ratio can thus be found by finding the ratio of the spectra of ϕ_1 and ψ.

ϕ_1 and ϕ_2 are assumed to be integrals of preemphasized gaussian variables. They have identical spectra and the same mean-square deviation. The spectrum of ϕ_1 is

$$
\begin{aligned}
S_{\phi_1}(f) &= \frac{\sigma^2 \cdot \mathrm{Pre}\,(f)}{2(f_t - f_b)\cdot f^2}, \qquad f_b \leqq |f| \leqq f_t\,; \\
&= 0, \qquad \text{elsewhere}; \qquad (6)
\end{aligned}
$$

where

f is the baseband frequency,
f_t is the top baseband frequency,
f_b is the bottom baseband frequency,
Pre (f) is the preemphasis function, and
σ is the rms frequency deviation.

If there is no RF filtering the spectrum of ψ, S_ψ, can be found directly by computing the spectrum of a carrier of power $k^2/2$ phase modulated by $(\phi_2 - \phi_1)$. This is similar to finding the RF spectrum of one signal. ϕ_1 and ϕ_2 are identical gaussians and $\phi_2 - \phi_1$ is also gaussian with twice the power but the same spectral shape. The spectrum for such a signal can be expressed as a series of consecutive convolutions and if we call

$$\phi_2 - \phi_1 = \theta, \qquad (7)$$

the spectrum of ψ is[4]

$$S_\psi\left(\frac{\Delta\omega}{2\pi} + f\right) = \exp\,[-R_\theta(0)]\left[\delta(f) + \sum_{n=1}^{\infty} \frac{1}{n!} S_\theta(f)^{n-1}_{*} S_\theta(f)\right]. \qquad (8)$$

When RF filtering is applied, θ is no longer gaussian and other methods have to be used to find S_ψ. The most common way is to convolve the RF spectra.[3] The original RF spectrum can be found using equation (8) substituting φ_1 for θ. The RF selectivity is then applied to the two identical spectra. Three filters may be involved; transmitter filters in each channel and a receiver filter in the wanted channel. Finally the filtered spectra are convolved to get the spectrum of ψ.

This is the approach taken in this paper. We found the RF spectra by using equation (8) using the Fast Fourier Transform algorithm in performing the convolutions. The spectra were filtered by square filters all with the same RF bandwidth chosen to cause a specified amount of bandlimiting noise. Equation (2) was used to determine the necessary bandwidth and the filters were assumed to have infinite loss outside this bandwidth. Since the FFT deals with samples of spectra the square filters removed the samples outside the passband. The final convolution was performed numerically on the remaining samples to get the interference noise in the top channel, and the signal sample at that frequency was divided by the interference to get the signal-to-interference ratio.

We use 64 samples in the baseband and 15 terms in the series in equation (8). All results were normalized to f_t and we found the signal-to-interference ratio as a function of channel spacing for various σ and for various amounts of bandlimiting noise and various combinations of filters. f_b/f_t was assumed to be 0.1 which is a reasonable approximation of a multiplex load. The preemphasis was that recommended by the CCIR.[5]

$$\text{Pre}\,(f) = \cfrac{3.15}{1 + \cfrac{6.9}{1 + \cfrac{5.25}{\left[\cfrac{1.25\,f_t}{f} - \cfrac{f}{1.25\,f_t}\right]^2}}}. \tag{9}$$

3.1 *Results*

We have computed interference noise from two adjacent channels, one on each side of the wanted channels, as a function of spacing, deviation, and bandlimiting distortion. We studied four cases of channel selectivity; no filters, transmitter filters only, receiver filters only, and both transmitter and receiver filters. All the results are not included in this paper because a very large amount of data was generated. We will show some of the important results and apply the results to typical radio relay systems.

The computer programs were written to compute the top channel interference noise as a function of increasing channel spacing and for various σ/f_t and (S/N) due to bandlimiting. The programs are special versions of a general program which computes the RF spectra of two FDM-FM systems, applies filtering, performs the convolution, and shifts the spectra to the frequency difference. This yields the complete baseband interference spectrum, and, to save computer time, the programs were trimmed to compute only top channel noise.

The output of the programs was the signal-to-distortion ratio at the top baseband frequency due to interference from two adjacent channels normalized to an RF ratio of 0 dB. Figure 1 shows this for the case of no bandlimiting. A family of curves, each for different σ/f_t, is shown. For each case of filtering and a specified amount of bandlimiting distortion similar curves have been generated. From those curves other curves showing the channel spacing as a function of deviation for constant baseband signal-to-interference ratio can be generated, and Fig. 2 shows this for the case of no filtering.

3.2 *The Effect of Filters*

The maximum amount of filtering possible is when both transmitter and receiver filters are used and this naturally will generate the least

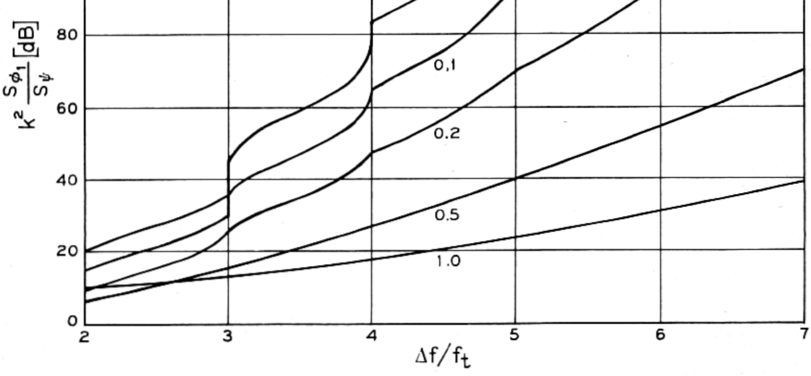

Fig. 1—Signal-to-Noise Ratio as a function of channel spacing. No filtering.

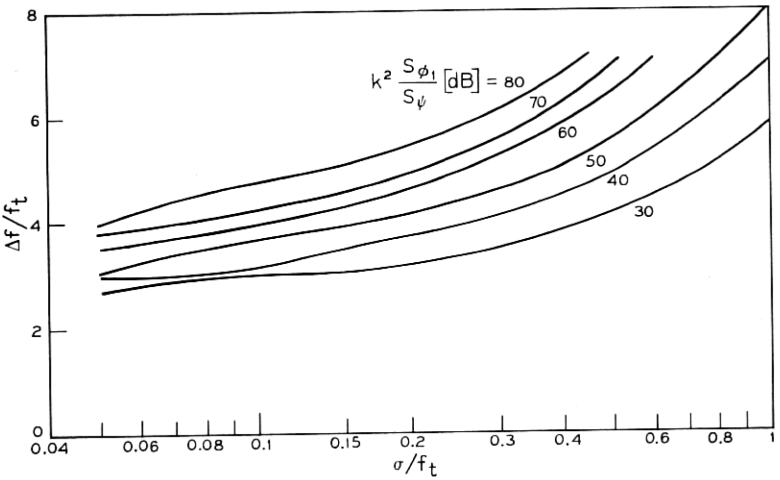

Fig. 2—Channel Spacing as a function of deviation. No filters.

amount of interference. When the channel spacing exceeds the filter bandwidth there will be no interference noise because the transmitter filter will remove the tails of the unwanted spectrum and the receiver filter the rest.

When only transmitter filters are used there will be no interference when the channel spacing exceeds the filter bandwidth *plus* the top baseband frequency. For less channel spacing there will be beat products falling into the baseband.

The receiver filter will remove most of the adjacent channel energy, and in particular the carrier. For low-index systems, the most important part of the convolution is the beating of one carrier with the sidebands of the other. If the unwanted carrier is removed, the interference power will be reduced by a factor of 2. For high-index systems, the situation is more complicated because the sidebands extend quite far from the carrier frequency and the carrier itself is very small.

Figure 3 shows the channel spacing as a function of deviation for the cases of no filtering, transmitter filters only, receiver filters only, and both transmitter and receiver filters. The filters are chosen such that the S/N due to bandlimiting is 70 dB. The signal-to-interference ratios are 40 and 60 dB which with an RF ratio of −30 dB corresponds to baseband signal-to-interference ratios of 70 and 90 dB.

As we can see, there is some effect of bandlimiting. The transmitter filter by itself has the least effect and the receiver filter does most of the

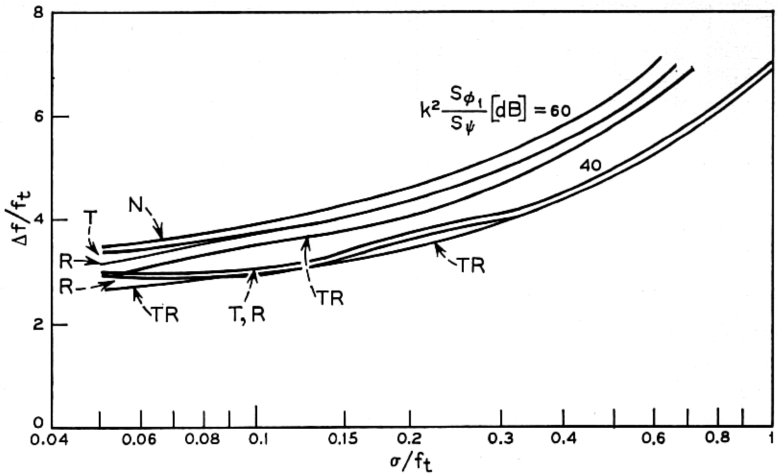

Fig. 3—Effect of Filtering. (N = no filtering, T = transmitter filter only, R = receiver filter only, and TR = both transmitter and receiver filters.)

interference rejection and the combination of both of course provides the best protection.

In terms of channel spacing or increased capacity, the effect of filters is only moderate. On the 40-dB curve at $\sigma/f_t = 0.1$, the relative change of $\Delta f/f_t$ is $0.2/3 \simeq 7$ percent. The curves are somewhat deceptive, however, because if we look at Fig. 2, an increase of $0.2\ f_t$ in Δf without changing f_t means a substantial reduction of noise, even if we ignore the local variation in the curves and assume that they are evenly spaced.

3.3 *Bandlimiting Distortion Versus Adjacent Channel Interference*

In most radio relay systems the bulk of the filtering is at the receiver and we shall therefore concentrate on this case. Figures 4–7 show the baseband signal-to-interference ratio for signal-to-noise due to bandlimiting equal to 70, 80, 90, and 100 dB respectively. We see that going from 70 dB to 100 dB does not change the curves by very much. In a practical system the 70 dB curve (Fig. 4) might be of greatest interest because this is a reasonable limit on noise. Unfortunately, as we mentioned in Section II, filters cannot be designed which have the bandwidth corresponding to 70 dB without generating additional noise due to delay distortion. Instead one has to go to a bandwidth corresponding to 90 dB. In designing a system, therefore, the curves in Fig. 6 would be of greatest interest at this time. As the

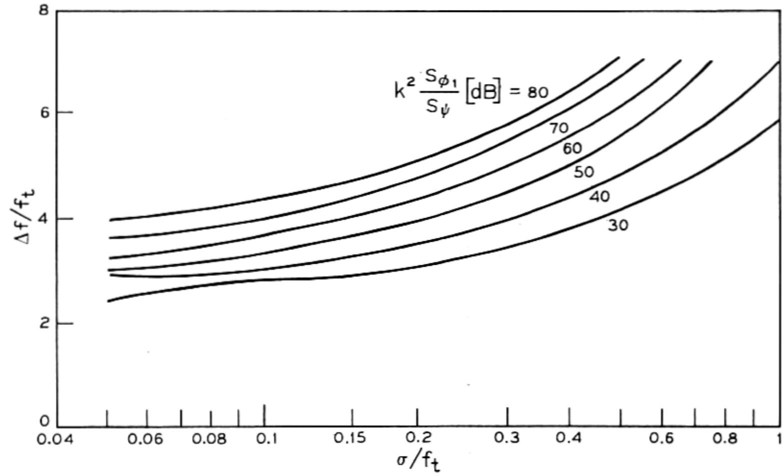

Fig. 4—Receiver filter only. Signal-to-noise from bandlimiting = 70 dB.

state of the art changes, however, we might approach the curves in Fig. 4.

3.4 *Channel Spacing Formula*

From the curves of Fig. 6, we have derived the following empirical formula

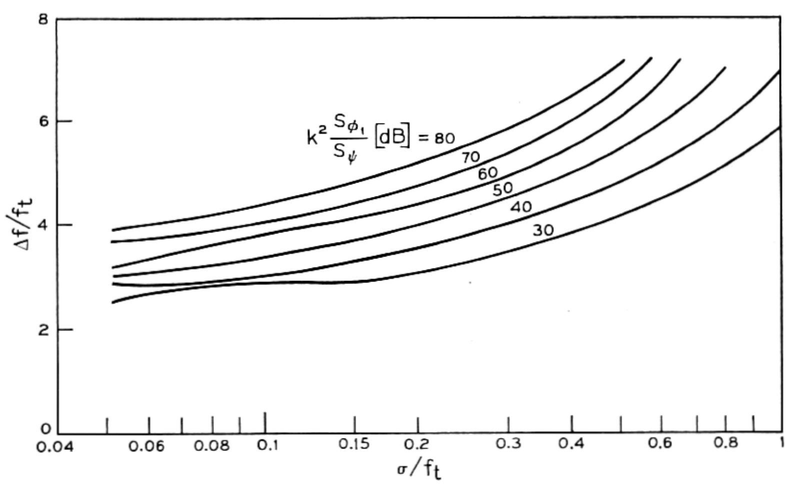

Fig. 5—Receiver filter only. Signal-to-noise from bandlimiting = 80 dB.

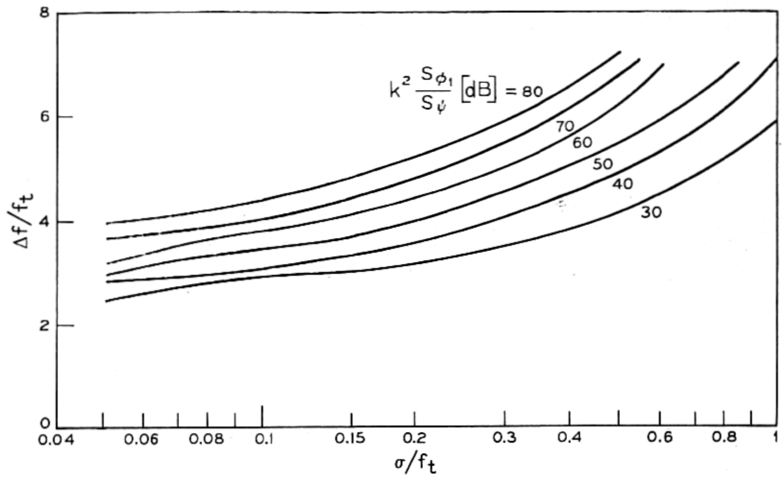

Fig. 6—Receiver filter only. Signal-to-noise from bandlimiting = 90 dB.

$$B_S = 2f_t\left(1 + 0.0711 \log_{10}\left(k^2 \frac{S_{\phi_1}}{S_\psi}\right)\right) + 1.156\sigma \log_{10}\left(k^2 \frac{S_{\phi_1}}{S_\psi}\right). \quad (10)$$

This formula is not well fitted to the curves in all the details because the curves are not well behaved. For the purpose of design however we believe that the formula is adequate. For very detailed study, the best

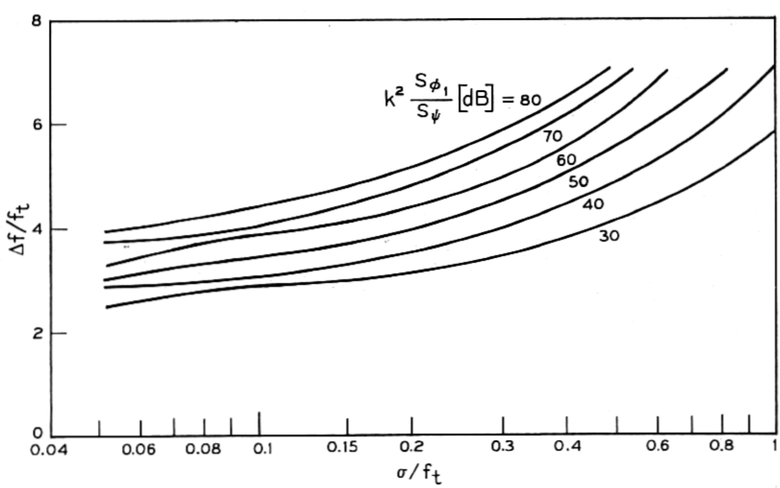

Fig. 7—Receiver filter only. Signal-to-noise from bandlimiting = 100 dB.

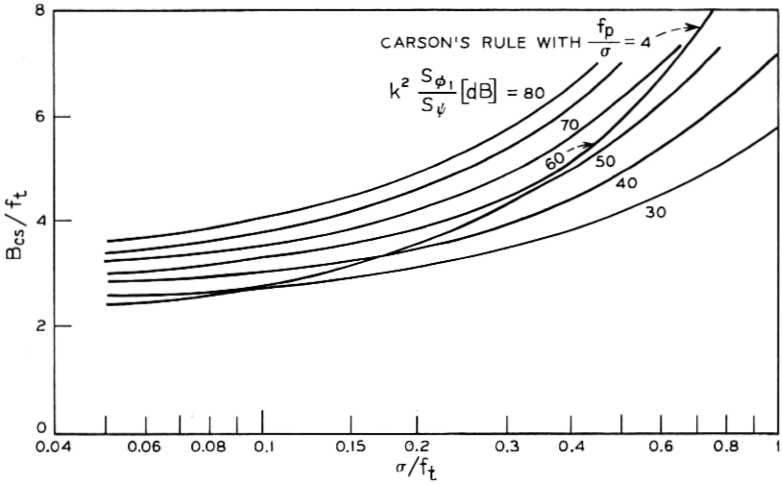

Fig. 8—Minimum channel spacing formula.

way to find the noise is to use computer programs and find the band-limiting distortion and interference.

Figure 8 shows B_S as a comparison with the actual data shown on Fig. 6. The fit is good for $S_{\phi_1}/S_\psi = 30 - 60$ dB which is the range of interest to most system designers. We therefore have some confidence in the formula as a tool for system design.

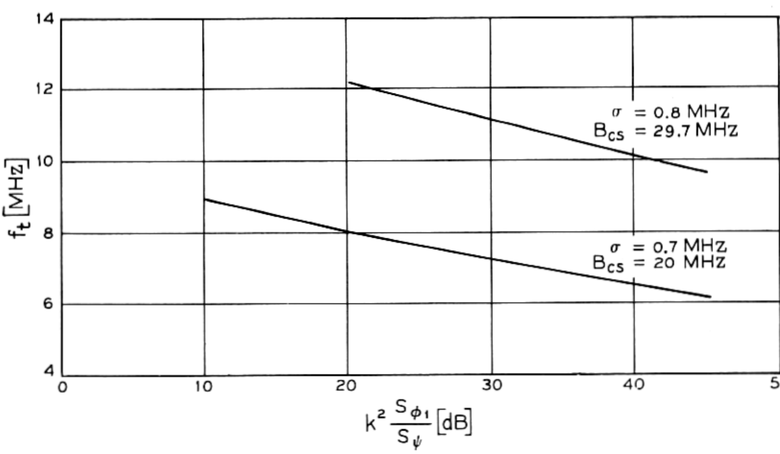

Fig. 9—Maximum top baseband frequency as a function of normalized adjacent channel interference noise.

We also show Carson's Rule on Fig. 8. The peak-to-rms ratio is 4 which is commonly used. We see that in determining channel spacing using Carson's Rule, the adjacent channel interference will vary a great deal depending on the deviation. Using Carson's Rule will give a channel spacing for low deviations that is too small and for large deviations too large.

We can rewrite equation (10) as

$$f_t = \frac{1}{2}\left[\frac{B_S - 1.156\sigma \log_{10}\left(k^2 \dfrac{S_{\phi_1}}{S_\psi}\right)}{1 + 0.0711 \log_{10}\left(k^2 \dfrac{S_{\phi_1}}{S_\psi}\right)}\right]. \tag{11}$$

Figure 9 shows this relationship for some typical systems using 29.7- and 20-MHz spacing with 0.8- and 0.7-MHz rms deviation respectively. Assuming that the RF discrimination between channels, $1/k^2$, is 30 dB and that the tolerable signal-to-interference ratio is 70 dB, i.e. $S_{\phi_1}/S_\psi =$ 40 dB, we get the top baseband frequencies to be about 10.1 and 6.5 MHz. With Bell System multiplex, this corresponds to approximately 2100 and 1320 voice channels. A 10-dB further reduction in the signal-to-interference ratio corresponds to about 2340 and 1500 voice channels.

IV. CONCLUSION

We have shown results of adjacent channel interference computations in FDM-FM systems. We looked at the effect of filters and showed intereference as a function of loading, channel spacing, and bandlimiting noise. We studied in particular the case of having only a receiver filter which has the greatest practical merit. By means of curve fitting we arrived at an empirical formula for channel spacing as a function of loading and interference. This formula should be very useful to microwave system designers in determining the noise from adjacent channels. It should also be useful in determining the maximum capacity of a channel and we gave some examples for typical channels.

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