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Short-Term Frequency Stability of Precision Oscillators and Frequency Generators

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We present in this paper two definitions of short-term frequency stability: (i) time domain, the expected value of the variance of the fractional frequency fluctuations from nominal frequency, $\langle \sigma_y^2(N, T, \tau) \rangle$, *in which* N *is the number of samples,* T *is the averaging time plus the dead time between samples, and* τ *is the averaging time; and (ii) frequency domain, the power spectral density of the fractional frequency departure from nominal frequency,* $S_y(f)$. *We discuss the topics of conversion from the frequency domain to the time domain and conversion among time domain measures.*

All measurements were made in the time domain, using period counting techniques. An oscillator was offset in frequency by using a specially built quartz crystal unit plated for 10 kHz below the frequency of the other sources. This oscillator was used to obtain the beat frequency required by the period counting approach.

Since the use of $\langle \sigma_y^2(2, T, \tau) \rangle$ *as a measure of short-term stability has significant advantages over* $\langle \sigma_y^2(N, T, \tau) \rangle$, *the relationship between the two quantities was investigated. For averaging times of one second or less,* $\langle \sigma_y^2(2, T, \tau) \rangle$ *and* $\langle \sigma_y^2(N, T, \tau) \rangle$ *were almost equal.*

The short-term stability of several quartz crystal oscillators and precision frequency generators was measured. The stability over the shorter averaging times was nearly equal for most of the oscillators. At longer times, the stability of each oscillator was unique. The frequency generators demonstrated similar stability over averaging times of 10 and 100 milliseconds, but were unique elsewhere. The accuracy of all measurements was limited by systematic effects from the environment and the measurement equipment itself.

I. INTRODUCTION

When the phrase "short-term frequency stability" is mentioned, one can immediately infer that frequency varies with time and that a short time interval is involved. Many questions are, at this point, unanswered: How short is the interval? How is stability defined? What system malfunctions can result from excessive instability? What definitions are appropriate for the specific application and why?

Some answers to these questions are developed here; others must be determined by the user in terms of the specific system involved. If a measurement of short-term frequency stability is to be of significant use, each of these questions must be resolved.

Although ever-increasing interest in short-term stability has existed for 25 or 30 years, no universally accepted definition of short-term stability exists. Primarily, early research in this area was extremely application-oriented. Little general work was performed with the result that many definitions were developed. Individuals and organizations defined and used stability in terms of their own applications. The resultant confusion showed that a more general definition applicable to the majority of cases was desirable in the topic of short-term frequency stability.

Today, many users talk about stable oscillators without understanding why or even if such precision is necessary. Manufacturers add to the present confusion through lack of rigor in their performance specifications. That is, they often merely quote a number without defining what, how, or why they are measuring the stability of their equipment.

Sophisticated systems exist today which require more precision than frequency standards of 10 to 15 years ago. Only recently has the precision oscillator been liberated from its position in secluded portions of carefully controlled laboratories.[1] Spacecraft applications suffice as good examples of the strides made in this area; wide temperature variations, severe mechanical vibrations, and varying oscillator voltage are often encountered. In the immediate future, these requirements will tighten even further. A proposed collision avoidance system for aircraft will require stable oscillators. With higher and higher data rates in communication systems, even the dependable telephone office will contain precision oscillators. As precision oscillators come into general use, accepted measurement theory and techniques are absolutely necessary.

In view of the above, two definitions of frequency stability are presented in this paper. One definition is in the time domain; the other is in the frequency domain. Conversion between the domains, an extremely important subject, is also treated. Every attempt has been made to be consistent with the IEEE Subcommittee on Frequency Stability.* It is anticipated that the subcommittee will publish its formal definition sometime in 1971.

II. GENERAL DISCUSSION

2.1 *General Definitions*

The general definition of instantaneous angular frequency is the time-rate of change of phase. That is

$$\omega(t) = \frac{d\Phi}{dt} = \dot{\Phi}. \tag{1}$$

An oscillator output signal may be described as

$$g(t) = [A + \epsilon(t)] \cos [\omega t + \phi(t)], \tag{2a}$$

$$g(t) = [A + \epsilon(t)] \cos [2\pi F t + \phi(t)], \tag{2b}$$

where

ω = nominal angular frequency of the oscillator,
$F = \omega/2\pi$ = nominal frequency in hertz of the oscillator,
A = nominal amplitude of the oscillator,
$\epsilon(t)$ = small, slowly time-varying amplitude fluctuations, and
$\phi(t)$ = slowly time-varying real function (phase).

Here, $g(t)$ may be considered as either a voltage or a current. If the oscillator is to qualify as a *precision* oscillator, it is required that

$$\left|\frac{\epsilon(t)}{A}\right| \ll 1, \tag{3}$$

$$\left|\frac{\dot{\phi}(t)}{2\pi F}\right| \ll 1. \tag{4}$$

2.2 *Statistical Processes*

For the present, assume that the variables of equation (2) are random processes. It is generally assumed in the study of frequency stability that amplitude deviations, $\epsilon(t)$, do not directly affect frequency or

* This subcommittee, formed in 1964 as a result of the IEEE-NASA Symposium on short-term frequency stability, is formally named the Subcommittee on Frequency Stability of the Technical Committee on Frequency and Time of the Group on Instrumentation and Measurement of the IEEE.

phase.[2] It must be stressed that $\phi(t)$ presumably contains all frequency and phase fluctuations and is the main quantity considered.

If a random process is stationary in the strict sense, it is unaffected by translations of the origin for time.[3] This implies that the probability distribution of values in the ensemble will be the same at any two instants. Hence, examination of the ensemble yields no data as to which instant of time the examination occurred.

Texts on random noise and stochastic processes assume that most noise functions may be represented or approximated as stationary gaussian random processes with zero averages.[4] The justification for assuming a gaussian distribution lies in the central limit theorem. This well-known theorem states that the distribution of the sum of a large number of independent random variables will approach a gaussian distribution. S. O. Rice showed that noise does, in general, conform to a gaussian distribution, provided that a sufficiently large sample is taken.[5] Zero averages of each random variable are assumed for convenience.

A process can be defined as ergodic when the statistics of one system over an infinite period of time are equivalent to the statistics of an infinite ensemble of systems at any given instant. (Note that stationarity is a necessary but not sufficient condition for ergodicity.) Since the processes are stationary and gaussian, the noise functions considered in this paper are assumed ergodic.

In the real world, no process can be stationary in the strict sense. To be stationary, the probability distributions across the ensemble must be the same for any selected instant of time. This stipulation includes time as it approaches infinity. If the process is terminated at some future time, the concept of stationarity in the strict sense is violated. The random process, X_t, is called stationary in the wide sense, if the first and second order moments of its random variables exist and satisfy

$$E(X_t) = \text{constant},$$

$$\sigma^2(X_t) = \text{constant},$$

$$E\{[X_{t+\tau} - E(X_{t+\tau})][X_t - E(X_t)]\} = R(\tau),$$

where

$R(\tau)$ = autocorrelation function, and
τ = some arbitrary finite time interval.

The actual verification of stationarity is not feasible. The main requirement is that the physical process be *consistent* with the concept of stationarity. That is, if measurements are to be of any use, they must reasonably describe the process at all times in the future, prior to termination of the process.

2.3 *Systematics*

When a study is made of a precision oscillator or frequency generator, great pains are often taken to isolate the circuits from electrical disturbances. This is done to minimize the systematic effects so that only the truly random noise can be observed. For evaluation of the generator itself, this is a sound practice. However, in the real world, electrical disturbances do exist. For example, in a data processing system there are card readers, magnetic tape transports, and other devices. Each of these devices has numerous electro-mechanical relays which are continually chattering. Thus, it is desirable to insure that a frequency generator has the necessary stability. This obviously means it must demonstrate adequate stability in the operating environment with the contribution of the systematic effects.

In equation (2), it was assumed that the main contribution of $\phi(t)$ came from the oscillator. Now it is necessary to loosen this assumption to include the contributions to $\phi(t)$ from the operating environment. While these added uncertainties are not caused by the oscillator, any attempt to observe $\phi(t)$ will include these effects. Therefore, $\phi(t)$ in equation (2) can be considered to include

$$\phi(t) = c(t) + s(t) + n(t), \tag{5}$$

where

$c(t)$ = phase due to aging (drift),
$s(t)$ = phase due to environmental systematic effects, and
$n(t)$ = phase due to oscillator random noise.

The contributions of $c(t)$ are strictly long term, affecting time intervals of a thousand seconds or longer. Hence, for time periods of less than a thousand seconds, $c(t)$ becomes almost constant and thus is not significant. The exact composition of $s(t)$ is unknown, although it has both long- and short-term effects. Again, the long-term component of the systematic effects becomes insignificant for short time periods. For these short intervals, $\phi(t)$ becomes

$$\phi(t) = s_n(t) + n(t), \tag{6}$$

where

$s_n(t)$ = short-term components of the systematic effects.

For the present, it will be assumed that the short-term component of the systematic effects is a random process which is stationary at least in the wide sense: gaussian, with zero averages, and ergodic. This stipulation may later fall as more is learned about the nature of systematic effects. Certainly, even if not truly random, the effects will appear as short-term instabilities.

Any environment will contribute a term $s(t)$ to (5). Obviously, if the systematics, $s(t)$, can be minimized to the point where their contribution is several orders of magnitude less than the oscillator noise, $n(t)$ becomes the only significant measurable. On the other hand, if the environment is extremely noise contaminated, the actual oscillator instabilities may be "swamped" by the systematic effects. Conditions can exist that are so contaminated that coherent operation of many electronic systems is not possible. The systematic effects will contribute to the short-term instabilities in an additive manner on a power basis. If an estimate of the short-term stability of an oscillator or frequency generator can be obtained in a carefully controlled, low-noise environment and if a subsequent estimate can be made in an operating atmosphere, the resulting instabilities may be compared. The contribution of the noise due to the systematic effects of the environment may then be described.

III. DEFINITIONS

3.1 *General*

Many applications exist that necessitate the use of highly stable oscillators or frequency generators. Due to the accuracy requirements of these various applications, some measure of oscillator instability is imperative. Although many persons view such a quantity merely as a vehicle for oscillator selection and comparison between oscillators, these uses are only academic. Strictly speaking, a measure of instability enables prediction of oscillator performance or, if not sufficiently stable, nonperformance in a given system. More precisely, *any* deviation from the intended frequency will degrade the performance of any system. Specific cases of the effects of frequency stability on an operation are discussed by J. A. Barnes.[6, 7]

Presented below are definitions of short-term frequency stability both in the time and frequency domains. Translation between the domains is also discussed.

3.2 *Time Domain*

In this paper, short-term frequency stability in the time domain is defined as the ensemble average of the variance of fractional frequency fluctuations from nominal frequency. This definition is based on the work of D. W. Allan.[8]

It is convenient to make the following definition

$$y(t) = \frac{\dot{\phi}(t)}{2\pi F_1}, \tag{7}$$

where

$\dot{\phi}(t) = d\phi/dt = \omega(t)$ {from equations (1) and (2)}, and
F_1 = long-term average frequency, hertz, of the oscillator {from equation (2)}.

Taking the short-term average value of equation (7)

$$\bar{y}_n = \frac{1}{\tau}\int_{t_n}^{t_n+\tau} y(t)\,dt = \frac{\phi(t_n+\tau)-\phi(t_n)}{2\pi F_1\tau}, \tag{8}$$

$$\bar{y}_k = \frac{1}{\tau}\int_{t_k}^{t_k+\tau} y(t)\,dt = \frac{\phi(t_k+\tau)-\phi(t_k)}{2\pi F_1\tau}, \tag{9}$$

where $n, k = 1, 2, \cdots, m, \cdots$.

Using equations (8) and (9), an expression in terms of the sample variance is obtained

$$\sigma_y^2(N, T, \tau) = \frac{1}{N-1}\sum_{n=1}^{N}\left(\bar{y}_n - \frac{1}{N}\sum_{k=1}^{N}\bar{y}_k\right)^2, \tag{10}$$

where

N = number of samples,
T = time between the beginning of successive sample intervals, and
τ = sample time.

Short-term frequency stability in the time domain is defined as the expected value or ensemble average of equation (10), the variance of fractional frequency fluctuations from nominal frequency. That is

$$\langle\sigma_y^2(N, T, \tau)\rangle = E\left[\frac{1}{N-1}\sum_{n=1}^{N}\left(\bar{y}_n - \frac{1}{N}\sum_{k=1}^{N}\bar{y}_k\right)^2\right], \tag{11}$$

where $E[X]$ denotes the expected value or ensemble average of X. Note that this definition of frequency stability is dimensionless.

The justification of the use of the term $1/(N-1)$, located directly to the left of the first summation in equations (10) and (11), follows from the theory of estimation (see Ref. 9). Thus, in order to obtain an unbiased estimate of the population variance, the sample variance is multiplied by $N/(N-1)$, resulting in equations (10) and (11). Clearly, for large samples, the population and sample variances become very nearly equal.

A popular expression of short-term stability in the time domain is the expected value of the standard deviation (rms value) of the fractional frequency fluctuations from nominal frequency. Expressed functionally, this becomes

$$\langle \sigma_y(N, T, \tau) \rangle = E\left\{ \left[\frac{1}{N-1} \sum_{n=1}^{N} \left(\bar{y}_n - \frac{1}{N} \sum_{k=1}^{N} \bar{y}_k \right)^2 \right]^{\frac{1}{2}} \right\}. \tag{12}$$

It should be noted that the square root of equation (11), $\langle \sigma_y^2(N, T, \tau) \rangle$, is equal to equation (12), $\langle \sigma_y(N, T, \tau) \rangle$, only when the successive samples are truly stationary in the wide sense. If the samples appear only slightly nonstationary, approximate equality may be assumed as

$$(\langle \sigma_y^2(N, T, \tau) \rangle)^{\frac{1}{2}} \cong \langle \sigma_y(N, T, \tau) \rangle. \tag{13}$$

3.3 *Frequency Domain*

L. S. Cutler and C. L. Searle showed that a practical definition of short-term frequency stability can be given in terms of autocorrelation functions and power spectral densities.[10]

The autocorrelation function of phase is defined as

$$R_\phi(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_0^T \phi(t + \tau) \phi(t) \, dt. \tag{14}$$

$R_\phi(\tau)$ can also be determined using numerical methods on a digital computer if enough samples can be taken, such as

$$R_\phi(\tau) = \frac{1}{N} \sum_{k=1}^{N} \phi_k(t + \tau) \phi_k(t), \tag{15}$$

where N is some large positive integer.

Let $R_\phi(\tau)$ be the autocorrelation function of a process which is stationary in the wide sense and continuous. From the Wiener-Khintchine theorem, it is known that autocorrelation functions and power spectral densities are Fourier transform pairs. Hence, the two-sided spectral density of phase becomes[10]*

$$S_\phi(f) = \int_{-\infty}^{\infty} R_\phi(\tau) e^{-j2\pi f\tau}\, d\tau. \tag{16}$$

From this, the one-sided spectral density of phase, $S_\phi(f)$, becomes[6]

$$S_\phi(f) = 2\int_0^\infty R_\phi(\tau)\cos(2\pi f\tau)\, d\tau, \tag{17}$$

$$R_\phi(\tau) = 2\int_0^\infty S_\phi(f)\cos(2\pi f\tau)\, df. \tag{18}$$

In normal Fourier analysis,[4] differentiation in the time domain corresponds to multiplication by $j2\pi f$ in the frequency domain. In terms of power spectral densities, this becomes $(2\pi f)^2$. Thus

$$S_{\dot\phi}(f) = (2\pi f)^2 S_\phi(f). \tag{19}$$

But $S_{\dot\phi}(f)$ is the power spectral density of frequency fluctuations. To obtain the power spectral density of instantaneous fractional frequency departure from nominal frequency, equation (19) must be normalized

$$S_y(f) = \frac{S_{\dot\phi}(f)}{(2\pi F_1)^2} = \frac{f^2}{F_1^2} S_\phi(f). \tag{20}$$

Thus, $S_y(f)$, equation (20), is the definition of short-term frequency stability in the frequency domain. Note that $S_y(f)$ has the units "per hertz."

3.4 *Translations Between Domains*

3.4.1 *Frequency Domain to Time Domain*

L. S. Cutler has shown that the following equation allows calculation of the time-domain stability from the frequency domain[6]

$$\langle\sigma_y^2(N, T, \tau)\rangle = \frac{N}{(N-1)}\int_0^\infty S_y(f)\,\frac{[\sin^2(\pi f\tau)]}{(\pi f\tau)^2}\left[1 - \frac{\sin^2(\pi r f N\tau)}{N^2\sin^2(\pi r f\tau)}\right] df, \tag{21}$$

where

$$r = T/\tau.$$

* In this paper, the convention is used that the term F_1, as in equations (2) and (7), is the cycle frequency, hertz, of the oscillator. The term f is a Fourier frequency, hertz, and is not a function of time.

3.4.2 *Translations Among Time-Domain Measures*

In certain applications, translation among time-domain measures is of interest. The following method, presented by Barnes,[11] allows calculation of $\langle\sigma_y^2(N_2, T_2, \tau_2)\rangle$, given an estimate of $\langle\sigma_y^2(N, T, \tau)\rangle$, for functions which have a power law spectral density.

Since most precision oscillators and frequency generators have power law spectral densities, it is possible to define two bias functions, $B_1(N, r, \mu)$ and $B_2(r, \mu)$, as follows

$$B_1(N, r, \mu) \equiv \frac{\langle\sigma_y^2(N, T, \tau)\rangle}{\langle\sigma_y^2(2, T, \tau)\rangle}, \tag{22}$$

$$B_2(r, \mu) \equiv \frac{\langle\sigma_y^2(2, T, \tau)\rangle}{\langle\sigma_y^2(2, \tau, \tau)\rangle}, \tag{23}$$

where

μ = spectral type (related to the shape of the spectrum. In Ref. 11, Barnes shows how μ may be found, given B_1).

Now an estimate of $\langle\sigma_y^2(N_2, T_2, \tau_2)\rangle$ can be made

$$\langle\sigma_y^2(N_2, T_2, \tau_2)\rangle = \left(\frac{\tau_2}{\tau_1}\right)^\mu \frac{B_1(N_2, r_2, \mu)B_2(r_2, \mu)}{B_1(N_1, r_1, \mu)B_2(r_1, \mu)} \langle\sigma_y^2(N_1, T_1, \tau_1)\rangle. \tag{24}$$

See Ref. 11 for a listing of the bias functions for various spectral types, number of samples, and r.

IV. MEASUREMENT TECHNIQUES

4.1 *Period Counting Technique*

The basic functional description of this method is shown in Fig. 1. This arrangement is similar to that presented in Ref. 2. Two similar oscillators are offset in frequency. This offset can be produced by adjustment of the tuning control on the oscillator or use of a crystal which produces a slightly different average frequency. It is assumed that these procedures change only the long-term average frequency of the oscillators and that the short-term instabilities are not affected.

As in equation (2), the output signals from these similar oscillators are fed to a mixer or product detector. The resulting output is fed to a low pass filter. This yields an expression:

$$g_0(t) \cong \frac{A_1A_2}{2}\{\cos[2\pi(F_1 - F_2)t + (\phi_1(t) - \phi_2(t))]\}. \tag{25}$$

This signal is the nominal frequency difference between the sources plus the instabilities of each. (For a more detailed discussion, see Refs. 8, 10 or 12.) Define

$$F_0 = F_1 - F_2 ,$$

$$\Phi(t) = [\phi_1(t) - \phi_2(t)].$$

Substituting these values into equation (25)

$$g_0(t) \cong \frac{A_1 A_2}{2} \cos [2\pi F_0 t + \Phi(t)]. \tag{26}$$

This quantity is fed directly to a digital counter.

The theoretical time, τ_0, between the first and Mth positive-going zero crossings, if the signals were ideal, is

$$\tau_0 = \frac{M}{F_0}.$$

The actual elapsed time, τ, between the first and Mth zero crossing is

$$\tau = \frac{M}{F_0} - \frac{[\Phi(t_0 + \tau) - \Phi(t_0)]}{2\pi F_0}. \tag{27}$$

The uncertainties are characterized as

$$\delta\tau = \frac{\Phi(t_0 + \tau) - \Phi(t_0)}{2\pi F_0}.$$

Therefore, equation (27) becomes

$$\tau = \tau_0 - \delta\tau. \tag{28}$$

Because of equation (4), $\delta\tau$ is small. Cutler and Searle show that if

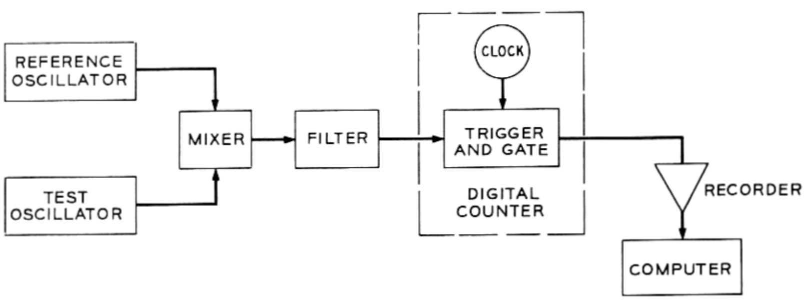

Fig. 1—Fundamental description, period counting technique.

variations of $\delta\tau$ are small, the following may be assumed

$$\Phi(t_0 + \tau) \cong \Phi(t_0 + \tau_0), \tag{29}$$

substituting

$$\delta\tau \cong \frac{\Phi(t_0 + \tau_0) - \Phi(t_0)}{2\pi F_0}. \tag{30}$$

Substituting equation (30) into equations (8) and (9) gives

$$\bar{y}_n \cong \frac{F_0}{F_1\tau_0}\left[\frac{\Phi(t_{0n} + \tau_0) - \Phi(t_{0n})}{2\pi F_0}\right] = \frac{F_0}{F_1\tau_0}\,\delta\tau_n\,, \tag{31}$$

$$\bar{y}_k \cong \frac{F_0}{F_1\tau_0}\left[\frac{\Phi(t_{0k} + \tau_0) - \Phi(t_{0k})}{2\pi F_0}\right] = \frac{F_0}{F_1\tau_0}\,\delta\tau_k\,. \tag{32}$$

For each sampling sequence, N estimates of $\bar{y}_n$ and $\bar{y}_k$ are computed. Substituting these values into equation (11)

$$\langle\sigma_y^2(N, T, \tau)\rangle = E\left[\frac{1}{N-1}\sum_{n=1}^{N}\left(\frac{F_0}{F_1\tau_0}\,\delta\tau_n - \frac{1}{N}\cdot\sum_{k=1}^{N}\frac{F_0}{F_1\tau_0}\,\delta\tau_k\right)^2\right], \tag{33}$$

$$\langle\sigma_y^2(N, T, \tau)\rangle = E\left[\left(\frac{1}{N-1}\right)\frac{F_0^2}{F_1^2\tau_0^2}\sum_{n=1}^{N}\left(\delta\tau_n - \frac{1}{N}\sum_{k=1}^{N}\delta\tau_k\right)^2\right]. \tag{34}$$

4.2 *A Special Case*

When $N = 2$, equation (11) is simplified

$$\langle\sigma_y^2(2, T, \tau)\rangle = E\left[\sum_{n=1}^{2}\left(\bar{y}_n - \frac{1}{2}\sum_{k=1}^{2}\bar{y}_k\right)^2\right]. \tag{35}$$

Likewise, as in equation (34),

$$\langle\sigma_y^2(2, T, \tau)\rangle = E\left[\frac{F_0^2}{F_1^2\tau_0^2}\sum_{n=1}^{2}\left(\delta\tau_n - \frac{1}{2}\sum_{k=1}^{2}\delta\tau_k\right)^2\right]. \tag{36}$$

Recall that $\tau = \tau_0 - \delta\tau$

$$\langle\sigma_y^2(2, T, \tau)\rangle = E\left[\frac{F_0^2}{F_1^2\tau_0^2}\sum_{n=1}^{2}\left\{(\tau_0 - \tau_n) - \frac{1}{2}\sum_{k=1}^{2}(\tau_0 - \tau_k)\right\}^2\right]. \tag{37}$$

Expanding the right side of this equation and reducing

$$\langle\sigma_y^2(2, T, \tau)\rangle = E\left[\frac{F_0^2}{F_1^2\tau_0^2}\,\frac{(\tau_2 - \tau_1)^2}{2}\right]. \tag{38}$$

Let $\tau_2 - \tau_1 = \Delta\tau_0$

$$\langle\sigma_y^2(2, T, \tau)\rangle = E\left[\frac{F_0^2}{2F_1^2}\left(\frac{\Delta\tau_0}{\tau_0}\right)^2\right]. \tag{39}$$

The following term often appears in the literature on frequency stability[13]

$$\frac{\Delta F}{F} = \frac{\Delta t}{T}, \tag{40}$$

where

ΔF = frequency difference between received and local standards (hertz),
F = nominal frequency,
Δt = accumulated time error corresponding to change in phase, and
T = averaging time.

The expression in equation (40) is frequently used in calculating the long-term stability or drift rate of an oscillator. The application of equation (40) has been well documented in many sources. The difference between two readings of equation (40) is recognized as the definition of long-term stability, yielding a peak-to-peak value of frequency drift over a specified time period.

As the time interval is compressed, the oscillator instabilities demonstrate more randomness while being less subject to drift. In the limit, equation (40) yields an instantaneous peak-to-peak value of the fractional frequency fluctuations from nominal frequency. Letting $t = \tau$ and substituting equation (40) into equation (39),

$$\langle \sigma_y^2(2, T, \tau) \rangle = E\left[\frac{F_0^2}{2F_1^2}\left(\frac{\Delta F_0}{F_0}\right)^2\right], \tag{41a}$$

$$\langle \sigma_y^2(2, T, \tau) \rangle = E\left[\frac{1}{2F_1^2}(\Delta F_0)^2\right], \tag{41b}$$

$$\langle \sigma_y^2(2, T, \tau) \rangle = E\left[\frac{1}{2F_1^2}(F_{02} - F_{01})^2\right], \tag{41c}$$

where

F_{01}, F_{02} = two successive samples of the beat frequency, F_0, over an averaging time, τ.

In a similar manner, the standard deviation becomes

$$\langle \sigma_y(2, T, \tau) \rangle = E\left\{\frac{1}{\sqrt{2}\,F_1}[(F_{02} - F_{01})^2]^{1/2}\right\}. \tag{42}$$

In Section 3.4.2, it was shown that, given the spectral type, μ, translations between time domain measures can be made. Recalling equation (22)

$$B_1(N, r, \mu) = \frac{\langle \sigma_y^2(N, T, \tau) \rangle}{\langle \sigma_y^2(2, T, \tau) \rangle}.$$

It is obvious that if

$$\langle \sigma_y^2(N, T, \tau) \rangle = \langle \sigma_y^2(2, T, \tau) \rangle, \tag{43}$$

then

$$B_1(N, r, \mu) = 1. \tag{44}$$

Barnes showed that for $r > 1$, equation (44) holds in two cases, when $\mu = -1.00$ and -2.00.[11] For $r = 1$, equation (44) holds for $\mu = -1.00$ only. Allan showed that when the spectral type, μ, is equal to minus one, white noise frequency modulation is present.[8] When μ equals minus two, flicker noise phase modulation is indicated.

As shown by E. A. Gerber and R. A. Sykes,[14] and Cutler and Searle,[10] there are three main sources of noise in oscillators. These are:

(*i*) additive noise (added to signal),
(*ii*) thermal and shot noise (perturbs oscillation), and
(*iii*) flicker $(1/f)$ noise frequency modulation. (See Ref. 15 for a discussion of flicker noise.)

For very short time intervals, the additive (white) noise predominates. For longer intervals, flicker noise frequency modulation prevails.[16] Thermal and shot noise are overpowered by the other two types.[10]

For oscillators, μ normally assumes two values

$$\mu \cong 0,$$

$$\mu \cong -1.$$

When the latter is true, $B_1(N, r, \mu) \cong 1$ and $\langle \sigma_y^2(N, T, \tau) \rangle \cong \langle \sigma_y^2(2, T, \tau) \rangle$. When $\mu \cong 0$, however, this does not hold.

There are many indications that the most basic parameter of frequency stability in the time domain is $\langle \sigma_y^2(2, \tau, \tau) \rangle$ (no dead time between samples). Initially, there is no guarantee that $\langle \sigma_y^2(\infty, T, \tau) \rangle$ will converge. Secondly, even if it were possible to assume convergence, it is not practical to take enough samples at the longer intervals to assure meaningful results. Next, it can be shown[6] that $\langle \sigma_y^2(2, \tau, \tau,) \rangle$ converges even for divergent $\langle \sigma_y^2(N, T, \tau) \rangle$. Therefore, the greatest significance of $\langle\sigma_v^2(2, T, \tau)\rangle$ as a measure of short-term frequency stability is that it eliminates the embarrassing divergence of flicker noise contributions as $N \to \infty$ in $\langle\sigma_v^2(N, T, \tau)\rangle$.

In addition, some noise functions have extremely long periods, even beyond one cycle per year. These low frequency components affect estimates of $\langle\sigma_v^2(N, T, \tau)\rangle$ even though their periods are too long to have any influence on an actual system. As a result, $\langle\sigma_v^2(2, \tau, \tau)\rangle$ is more constant in time than $\langle\sigma_v^2(N, T, \tau)\rangle$.

In physical applications, $\langle\sigma_v^2(2, \tau, \tau)\rangle$ is often more directly applicable than other expressions. For example, in radar it can be shown[6] that the expression for calculating doppler range errors is directly proportional to $\langle\sigma_v^2(2, \tau, \tau)\rangle$. In timing, the mean-square second difference of phase is often useful.[7,15] It can be shown[6] that $\langle\sigma_v^2(2, \tau, \tau)\rangle$ is directly proportional to the mean-square second difference of phase.

In a practical sense, measurement of $\langle\sigma_v^2(2, T, \tau)\rangle$ is easier to achieve than $\langle\sigma_v^2(N, T, \tau)\rangle$. Storage requirements for the latter become excessive as N becomes large. Usually, the most practical method of making estimates of $\langle\sigma_v^2(N, T, \tau)\rangle$ is the use of a magnetic tape unit for storage of data. The data may then be processed off-line on a special purpose or commercial computer.

For $\langle\sigma_v^2(2, T, \tau)\rangle$, storage requirements are small. Neglecting processing requirements, only one summation register is necessary to obtain the ensemble average. Resulting equipment is small and portable. Computations can be made on-line, in real time. Using this approach, reliable measurements of short-term stability in the time domain are practical at field locations.

V. MEASUREMENTS

5.1 *General*

Of prime interest to this paper is the measurement of the short-term frequency stability of a frequency and time of year generator to be used in a large, special purpose computer. This generator consists of a rack of equipment which produces a total of 17 different square wave signals ranging from 9.5367 Hz to 20 MHz. These signals are used as clock frequencies for the computer.

The primary frequency source consists of three 5-MHz quartz crystal oscillators.[1] Two of these oscillators, designated as slave and standby, are phase locked to the other oscillator, called the master. The outputs of the master-slave pair are combined in three digital mixers and fed to three counter chains. Each counter chain output is compared to the corresponding outputs of the other two counters. The two signals that very nearly coincide are combined and fed to external users via cables. The output from the standby oscillator is not used. The standby oscillator is present only because of the five day initial warm-up period. It is phase locked to the master oscillator to speed switching into the system should a failure of one of the other two oscillators occur.

The time of year generator simply uses a 1-MHz combined output and generates a 42-bit BCD parallel time of year code.

A VLF receiver is used to check the 5-MHz oscillators against precision VLF stations. Daily frequency checks reduce the actual frequency error. The specified long-term stability of each oscillator is 1 pp 10^{10} per day. [This is a measure of $\Delta F/F$ as shown in equation (40). The expression 1 pp 10^{10} is a commonly used abbreviation for 1×10^{-10}.] Thus, in theory at least, long-term error (drift) never exceeds that which can be accumulated in a single day.

Provisions have been made to indicate to operating personnel if errors have occurred. These include sensors to detect missing pulses, the loss of an output, counter synchronization errors, or a phase lock circuit approaching its limits. These techniques are present merely to add additional reliability to the system.

The phase-lock circuit permits two oscillators to be phase locked over a frequency change of ± 1 pp 10^8 with a maximum phase error between the two oscillators of less than ± 60 milliradians. The circuit can "capture" over a range of ± 2.5 pp 10^8 with a phase error of ± 160 milliradians.

Whether instabilities are caused directly by an oscillator or by associated circuitry, various problems can result from excessive instabilities. These include (*i*) loss of phase lock, (*ii*) loss of counter chain synchronization resulting in one or more false outputs, (*iii*) degradation of stability delivered to users, and (*iv*) failure of the frequency generator. Loss of phase lock will occur when linear drift or phase errors caused by excessive instability of an oscillator exceed the above-given limits. When phase lock is lost, short-term stability is degraded even further. When a counter chain loses synchronization, the resulting false outputs could have serious consequences to the data processing system.

Once the stability is degraded to some critical level, the data processing system will begin to lose accuracy and resolution; time-of-year errors will accumulate. Pulses propagating through delay lines will not coincide with intended clock pulses. Loss of data and parity errors will result. When the parity count is sufficiently large, a software program will attempt to regain coherence of the data processing system, which will cause destruction of data. Since the generator is the system clock, failure of the generator will cause failure of the system. Possibly the most serious problems occur when the stability is marginal, causing the system to respond with false data, without any indication to operating personnel.

Since the generator is located in the operating environment, all electrical disturbances caused by peripheral equipment add to the instabilities (see Section 2.3). The signals are distributed to users via cables. Each cable, up to 300 feet in length, acts as an "antenna" to the systematic noise, resulting in greater degradation of stability.

It was desirable to determine the short-term frequency stability in several configurations:

(*i*) Oscillator output in a "quiet" environment (sinusoid).
(*ii*) Output at terminals of generator in a "quiet" environment (square wave).
(*iii*) Oscillator output in an operating environment (sinusoid).
(*iv*) Output at terminals of generator in an operating environment (square wave).
(*v*) Generator output at end of a 200-foot cable in an operating environment (square wave).
(*vi*) Generator output at end of a 200-foot cable in a "quiet" environment (square wave).

If a comparison between oscillator stability in a "quiet" and an operating environment is made, an estimate of the noise contributed by the environment is possible. Comparison between the results for the oscillator and the generator yields an estimate of the instabilities contributed by the generator itself.

Comparison of the stabilities of the generator outputs over short and long cables yields an estimate of the degradation due to the cable in the environment. Finally, comparison between the stability at the end of a long cable in both operating and "quiet" environments yields an estimate of the absolute contribution of the cable and another estimate of environmental noise.

Almost all subsystems using outputs from this frequency generator are defined and operated in terms of real time. Therefore, short-term frequency stability is more appropriately defined and measured in the time domain.

5.2 *Measurement Systems*

In Fig. 2, a measurement system similar to that of Cutler and Searle is shown. The optional multipliers are used for two reasons. First, it is often desirable to use similar oscillators. If the oscillators are truly of high precision, they are tunable only over a small range (one hertz or less). One oscillator is tuned to nominal frequency. The other is offset by some predetermined amount. Using frequency multipliers, both oscillator outputs are multiplied up until the frequency difference between the two is 10 kHz. This permits measurement down to intervals of 100 microseconds. Secondly, in addition to the basic frequencies, all instabilities are multiplied as well. The use of multipliers enables the use of a counter that is less accurate than otherwise required. For example, assume that it is necessary to measure the stability of an oscillator to 1 pp 10^7 in one millisecond, using a beat frequency of 10 kHz. To be able to resolve this quantity, the counter must be accurate to one millihertz in a millisecond. If multipliers are used, however, the instabilities are increased by the multiplication factor. If the factor used is 100, the counter need resolve only 0.1 Hz. For higher multiplication factors, counter requirements are proportionately reduced.

Up to this point, it has been assumed that the multipliers are perfect. Unfortunately, this is not the case. Highly accurate multipliers are extremely difficult to construct. Multipliers are susceptible to temperature variations and, if not properly designed, are sources of phase instability. When using multipliers, there is no guarantee that the measured instabilities are due to the oscillators. They may be due to instabilities within the multipliers.

The problem of obtaining a large enough frequency offset is easily solved. Assuming two similar precision oscillators are present, it is possible to replace the quartz crystal in one oscillator with a similar type plated for either 10 kHz above or below 5 MHz.

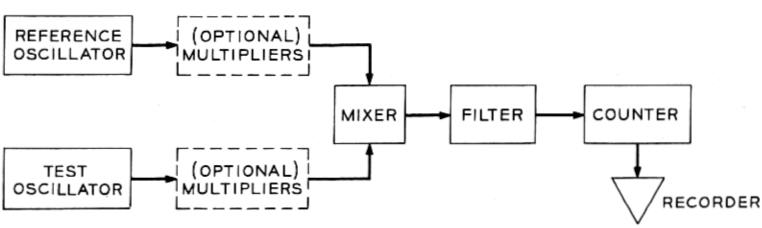

Fig. 2—Measurement system of L. S. Cutler and C. L. Searle.[10]

The problem of obtaining the necessary resolution can be solved by using commercially available counters. There are at least two types available that have sufficient accuracy to enable precise measurement at short averaging times, without multipliers. The first, a Hewlett-Packard 5248L counter, has a 100-MHz clock, allowing measurements down to one millisecond with an offset of 1000 hertz. The second, a Hewlett-Packard 5360A Computing Counter, allows measurements down to 100 microseconds with an offset of 10 kHz.

In an effort to verify the assumptions made in Section 4.2, the arrangement shown in Fig. 3 was used. The HP5248L was modified to have a one-millsecond recycle delay time. It was not possible to use the HP5360A in this configuration due to slower data transfer capabilities to the computer and interface incompatibility. A large number of samples were taken at various averaging times and the data recorded on magnetic tape. The quantities $\langle\sigma_y(N, T, \tau)\rangle$ and $\langle\sigma_y(2, T, \tau)\rangle$ were computed from the same data and compared.

Using the configuration shown in Fig. 4, measurements were made with the HP5360A Computing Counter and associated keyboard. This unit can be programmed to calculate $\langle\sigma_y(2, T, \tau)\rangle$ from its measured data. It is portable and was transported by automobile to test oscillators and frequency generators in use at various locations.

5.3 *Precision Oscillators*

The measurement systems used by the author are shown in Figs. 3 and 4. Note that a 5-MHz square wave signal from the frequency generator was used as an external time base for the counter in Fig. 4. A slight improvement was noted due to the superior stability of the frequency generator over the internal time base of the counter. It was originally planned to use the 5-MHz oscillator being tested to drive the external time base. Unfortunately, the additional load degraded measurable stability.

In the system shown in Fig 3, the oscillators, synthesizer, mixer, and video amplifier were located inside a shielded room. The counter, computer, and magnetic tape unit were located immediately outside. No frequency standard was available at the counter for use as an external time base.

In Section 2.2., the assumption was made that instabilities in oscillators are independent random processes. It can be shown[9] that the variance of the sum of two independent or uncorrelated random variables is equal to the sum of the variances, provided that the variances exist.

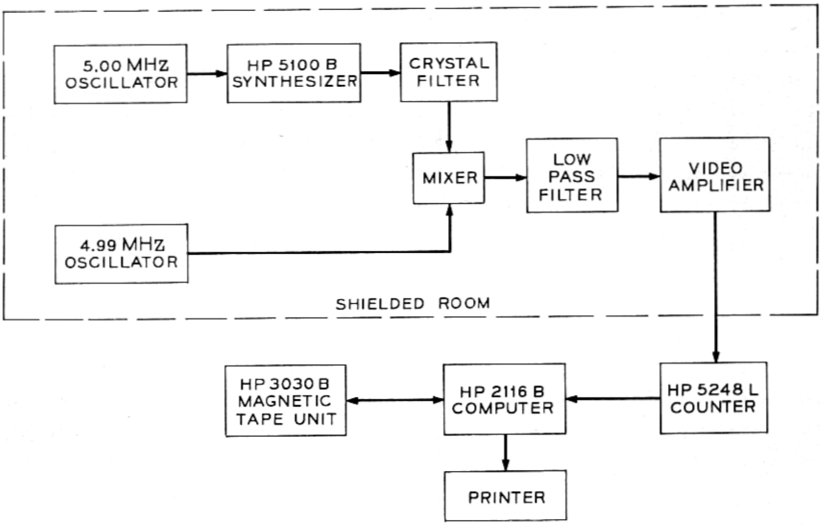

Fig. 3—Measurement system used for comparison of $\langle\sigma_y(N, T, \tau)\rangle$ and $\langle\sigma_y(2, T, \tau)\rangle$.

It was previously stated that it is often desirable to use similar oscillators. The process of selecting two oscillators at random from a large population of oscillators can be approximated by procuring two oscillators from a large supplier. Since the manufacturing processes are identical, it is assumed that the probability density and distribution functions of the oscillators are the same. The total measurable instabilities of the pair are then equal to twice the instabilities of either oscillator. Therefore, the variance measured is divided by two. Likewise, the standard deviation can be divided by $\sqrt{2}$. In the case of the

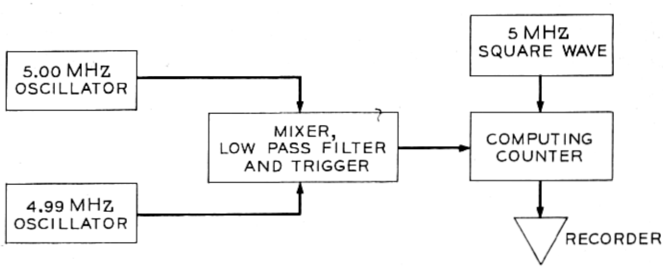

Fig. 4—Measurement system used by the author.

offset oscillator, it is assumed that modification of the crystal did not change the statistics by a significant amount.

Using the arrangement shown in Fig. 3, a comparison between $\langle\sigma_y^2(N, T, \tau)\rangle$ and $\langle\sigma_y^2(2, T, \tau)\rangle$ can be made. These quantities represent the sum of the variance contributed by the oscillator plus the variance contributed by the synthesizer, since the processes are independent.

When the arrangement shown in Fig. 4 is used, a very good estimate of the short-term stability in the time domain will result.

5.4 *Frequency Generators*

It was desirable to investigate the short-term stability of the 5-MHz square wave output of the frequency generator, described briefly in Section 5.1. The measurement system used was the same as that shown in Fig. 4 except that the 5-MHz square wave was substituted for the 5-MHz oscillator.

It seems logical that the stability of the generator would be worse than that of an oscillator, since the original sine wave produced by an oscillator has gone through shapers, counters, and other circuits used to generate the square wave signal. The exact magnitude of the generator instabilities can be easily determined. First, the total instabilities of the oscillator-generator combination are determined. The contribution of the oscillator, discussed in Section 5.3, may be subtracted out. The remainder is a good estimate of the variance of fractional frequency fluctuations from nominal frequency of the 5-MHz square wave.

5.5 *Synthesizers*

Short-term stability must often be measured when no auxiliary precision oscillators are available. By using a synthesizer, it is possible to obtain the necessary offset without the use of multipliers. Subsequently, it is possibile to arrive at an estimate of the short-term stability, provided a sufficiently accurate counter is available (see Figs. 3 and 5). Several problems are introduced, however. In the preceding examples, the stability of each oscillator or frequency generator could be determined. Such is not possible using a synthesizer since the contributions of the oscillator and synthesizer to equation (37) are unknown.

If two similar synthesizers were present, these could be driven by an external precision oscillator and the stability determined as in the case of two similar oscillators. With only one synthesizer, an estimate of the stability of the oscillator-synthesizer combination can be deter-

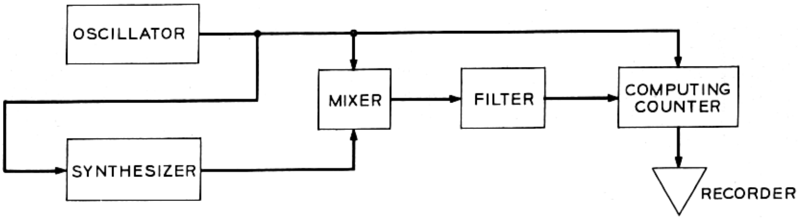

Fig. 5—Measurement system using a frequency synthesizer.

mined. Then, it can be stated that the actual stability of the oscillator is at least as good as that measured for the combination. If the synthesizer was much better than the oscillator, the majority of the instabilities measured would be contributed by the oscillator. Unfortunately, the reverse is usually true; the instabilities of the oscillator are small compared with those of the synthesizer. In some cases, however, the measured stability using such a configuration may be useful in evaluating system performance.

VI. RESULTS

6.1 *Precision Oscillators*

In order to compare the actual relationship between $\langle \sigma_v^2(N, T, \tau) \rangle$ and $\langle \sigma_v^2(2, T, \tau) \rangle$, the arrangement shown in Fig. 3 was used. The two oscillators shown are similar except for the modified crystal in the 4.99-MHz oscillator. It was assumed that the instabilities of the oscillators were similar. Therefore, the channel containing the synthesizer was inherently more noisy due to the mere presence of the synthesizer.

In an attempt to minimize the measurable stability, a narrowband crystal filter was inserted between the synthesizer output and the mixer input. At averaging times shorter than the reciprocal of the bandwidth of the filter, significant improvement in measurable stability of the system was noted. At times longer than the reciprocal of the bandwidth, the presence of the filter caused a slight degradation of measurable stability. As a result, for these longer averaging times, the filter was removed from the synthesizer output.

The total recycle delay time of the counter is about one millisecond. Therefore, $T = \tau + 1$ millisecond. For longer averaging times, this delay time is not significant. It was assumed for the longer intervals that $T \cong \tau$.

The quantities observed are listed in Table I. Note the close corre-

TABLE I—COMPARISON BETWEEN $\langle \sigma_y(N, T, \tau) \rangle$ AND $\langle \sigma_y(2, T, \tau) \rangle$

Time Interval	No. of Groups	Samples per Group (N)	$\langle \sigma_y(N, T, \tau) \rangle$	$\langle \sigma_y(2, T, \tau) \rangle$	$B_1(N, r, \mu)$	Remarks
1 millisecond	10	250	8.27 pp 10^9	8.26 pp 10^9	1.0016	See Fig. 3 for Equipment Arrangement →
2 milliseconds	10	250	4.92 pp 10^9	4.88 pp 10^9	1.0164	
4 milliseconds	10	250	3.08 pp 10^9	3.03 pp 10^9	1.0336	
10 milliseconds	10	250	1.31 pp 10^9	1.28 pp 10^9	1.046	
20 milliseconds	10	250	6.34 pp 10^{10}	6.52 pp 10^{10}	0.9434	
40 milliseconds	10	250	3.00 pp 10^{10}	3.03 pp 10^{10}	0.98	
100 milliseconds	10	250	1.34 pp 10^{10}	1.31 pp 10^{10}	1.0429	Crystal Filter Removed From Synthesizer Output →
200 milliseconds	10	250	6.88 pp 10^{11}	7.14 pp 10^{11}	0.9287	
400 milliseconds	10	250	3.73 pp 10^{11}	3.78 pp 10^{11}	0.9713	
1 second	10	250	1.72 pp 10^{11}	1.73 pp 10^{11}	0.9843	
10 seconds	8	100	1.57 pp 10^{12}	6.47 pp 10^{13}	5.9	Synthesizer Removed From Circuit, 4999.95 kHz Oscillator Substituted for 4990.0 kHz Oscillator
108 seconds	10	54	1.61 pp 10^{12}	6.13 pp 10^{13}	6.8765	
200 seconds	20	11	8.19 pp 10^{12}	5.56 pp 10^{12}	2.1682	Standard Oscillator Versus an Experimental, Oscillator
1080 seconds	31	11	1.85 pp 10^{12}	9.94 pp 10^{13}	3.4482	Same Configuration as 10 and 108 seconds

spondence of $\langle \sigma_y(N, T, \tau) \rangle$ and $\langle \sigma_y(2, T, \tau) \rangle$ for averaging times of one second and less. Since neither quantity is consistently larger than the other at all sampling times, it appears that correspondence would become even better if more samples were taken.

At averaging times of ten or more seconds, $\langle \sigma_y(2, T, \tau) \rangle$ becomes smaller than $\langle \sigma_y(N, T, \tau) \rangle$. It can be seen, in these longer intervals, that $B_1(N, r, \mu)$ is in the range predicted by Barnes[11] for flicker noise frequency modulation and superpositions of white and flicker noise frequency modulation.

More precise measurements of $\langle \sigma_y(2, T, \tau) \rangle$ over all time intervals were possible by using the configuration shown in Fig. 4. The improved precision was achieved by the elimination of the synthesizer and the use of a more accurate counter. The recycle delay time of the HP5360A is on the order of 1.5 milliseconds. As before, for the longer averaging times, it was assumed that $T \cong \tau$.

First, measurements were made using several standard oscillators (averaging times from 0.1 millisecond to 10 seconds). Although the instabilities contributed by similar oscillators are theoretically the same, differences in stability were observed. Among the six standard oscillators tested, using the system shown in Fig. 4, one oscillator demonstrated generally better characteristics than the other five at the short averaging times. At longer time intervals, the measured stability of each oscillator was unique. For example, at a one second averaging time, the best oscillator demonstrated a stability of better than 6.3 pp 10^{12}. [Here, the quantity measured was $\langle \sigma_y(2, T, \tau) \rangle$.] The worst was about 1.4 pp 10^{11}. The only differences in the oscillators were that oscillators one, two and three used solder-mounted crystals. The offset oscillator and oscillators four, five and six all used thermally bonded crystal units. It appears that the thermally bonded units exhibit more similarity between crystals. It is interesting to note, however, that the best stability at an averaging time of 10 seconds was observed using an oscillator containing a solder-mounted crystal unit. Likewise, the oscillator which demonstrated the worst stability also used a solder-mounted unit. Although the differences in the measurable stability of the oscillators using the thermally bonded crystal units were significant, these differences were still somewhat small. The stabilities measured are summarized in Table II and Fig. 6.

In an attempt to determine the contribution of the environment to oscillator instabilities, all equipment in the system was de-energized. The only equipment which remained under power was the frequency

	Averaging Time						
Configuration	0.1 ms	0.4 ms	1 ms	10 ms	100 ms	1 s	10 s
Oscillator No. 1 Operating Environment	3.3 pp 10^8	1.3 pp 10^8	6.1 pp 10^9	6.6 pp 10^{10}	6.8 pp 10^{11}	1.41 pp 10^{11}	1.5 pp 10^{11}
Oscillator No. 1 Quiet Environment	3.3 pp 10^8	1.3 pp 10^8	6.0 pp 10^9	6.46 pp 10^{10}	6.8 pp 10^{11}	1.1 pp 10^{11}	1.2 pp 10^{11}
Oscillator No. 2 Operating Environment	3.2 pp 10^8	1.3 pp 10^8	6.0 pp 10^9	7.4 pp 10^{10}	6.7 pp 10^{11}	7.0 pp 10^{12}	3.0 pp 10^{12}
Oscillator No. 2 Quiet Environment	3.2 pp 10^8	1.3 pp 10^8	6.0 pp 10^9	7.5 pp 10^{10}	6.7 pp 10^{11}	7.0 pp 10^{12}	3.0 pp 10^{12}
Oscillator No. 3. Operating Environment	3.1 pp 10^8	1.2 pp 10^8	5.7 pp 10^9	6.3 pp 10^{10}	6.1 pp 10^{11}	6.4 pp 10^{12}	2.2 pp 10^{12}
Oscillator No. 3 Quiet Environment	3.1 pp 10^8	1.2 pp 10^8	5.7 pp 10^9	6.2 pp 10^{10}	5.8 pp 10^{11}	6.4 pp 10^{12}	2.2 pp 10^{12}
Oscillator No. 4 Operating Environment	2.9 pp 10^8	1.14 pp 10^8	5.2 pp 10^9	5.8 pp 10^{10}	5.86 pp 10^{11}	7.0 pp 10^{12}	3.93 pp 10^{12}
Oscillator No. 5 Operating Environment	3.05 pp 10^8	1.23 pp 10^8	5.7 pp 10^9	6.17 pp 10^{10}	6.2 pp 10^{11}	6.25 pp 10^{12}	4.0 pp 10^{12}
Oscillator No. 6 Operating Environment	3.26 pp 10^8	1.25 pp 10^8	5.95 pp 10^9	6.33 pp 10^{10}	6.17 pp 10^{11}	6.53 pp 10^{12}	3.44 pp 10^{12}

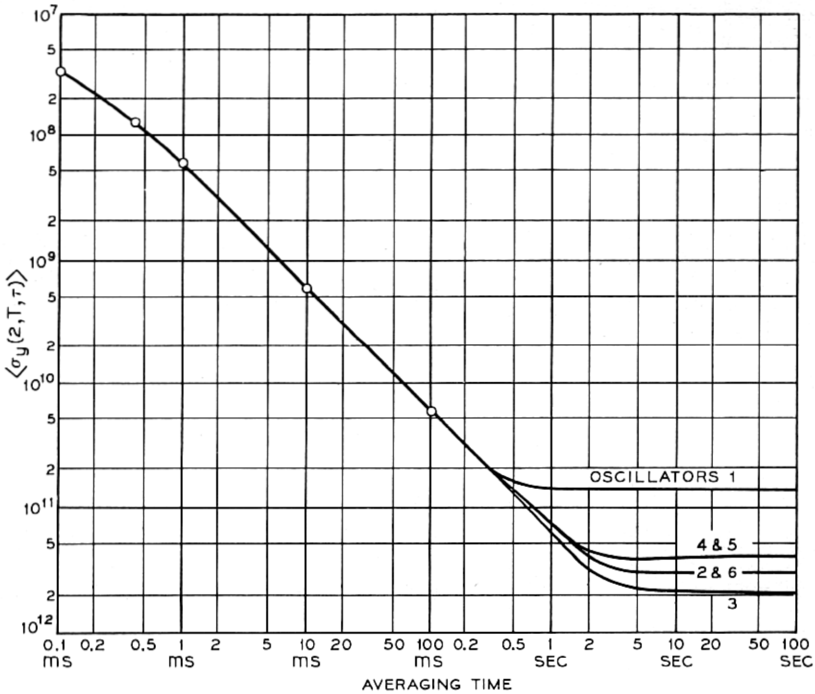

Fig. 6—Short-term stability of six precision oscillators.

generator, the offset oscillator, measurement equipment, and the air conditioning equipment required for forced air cooling of the frequency generator. Measurements were repeated for all time intervals. At the shorter intervals, no differences were apparent. In some instances, stability was slightly improved at intervals of 100 milliseconds or longer but only by an amount of questionable significance.

Since these results were not as anticipated, the causes for the disparity were investigated. The only obvious reason was inherent in the design of the mixer circuit. The mixer contains four small ferrite cores. About twenty feet from the frequency generator is a large power transformer which is used in the power distribution network for the entire data processing laboratory. Even with the remainder of the data processing system idle, this transformer must be energized, as it supplies power to the generator. Ferrites have been found to adversely affect short-term stability when used in the presence of electromagnetic fields.[17] The effect of the field created by the power transformer on mixer performance is unknown.

Other than the ferrites, no reason for the discrepancy was apparent. The most logical reason, however, would be that the measurable stability is limited by the noise characteristics of the measurement system itself. If the noise characteristics of the system were much worse than that of the oscillators, no differences in measurable stability would occur. Since differences between oscillators can be observed, the oscillators must contribute a measurable amount of the noise. As a result, if the noise characteristics of the measurement system were improved, measurable stability may improve, but probably by less than an order of magnitude.

6.2 *Frequency Generators*

6.2.1 *General*

Each frequency generator contains three precision oscillators. Any of the three oscillators may assume any of the three functions. The functions are switched by means of control panel and associated relays. The normally closed positions of the relays constitute the usual arrangement and is referred to as the "normal" mode. These are

Oscillator #1 = MASTER,
Oscillator #2 = SLAVE, and
Oscillator #3 = STANDBY.

The outputs from the phase-locked master-slave pair are digitally mixed and sent through counter chains and frequency multipliers to arrive at the various output frequencies. As mentioned above, it follows that the stability of the generator should, in general, follow the stability of the oscillators used as the frequency sources.

The observed stability of the six oscillators differed for time intervals of one second or longer (see Section 6.1). Ironically, the oscillator installed in the number one position in frequency generator #1 demonstrated the worst stability at the longer intervals. Oscillator number three demonstrated the best.

The oscillator functions were redesignated

Oscillator #3 = MASTER,
Oscillator #2 = SLAVE, and
Oscillator #1 = STANDBY.

As anticipated, the observed stability improved. Most measurements were taken using the "normal" mode, as this is the usual configuration employed during operation of the data processing system.

6.2.2 *5-MHz Square Wave, Ten-Foot Cable*

On frequency generator #1, measurements were taken in both operating and idle environments. As was the case of precision oscillators, little difference was apparent. On frequency generator #2, measurements were taken in an operating environment only.

To make estimates of the short-term stability of the generator, it was necessary to obtain the variance of the fractional frequency fluctuations of the offset oscillator-frequency generator pair. When these were determined, the variance contributed by the offset oscillator was subtracted out of the data.

The question at this point was, which oscillator-data should be subtracted out to arrive at a reasonable estimate of the square wave stability? The data from the best oscillator pair from each frequency generator was used. If the data from the worst pair was used, the stability of the generator may be too optimistic. Since the offset oscillator used a thermally bonded crystal unit, the actual stability is probably similar to the other oscillators using thermally bonded units.

In extremely critical applications, the measured stability between the oscillator-generator pair can be considered to be the stability of the generator itself. It may be safely assumed that the actual stability of the generator is no worse than the measured stability of the oscillator-generator pair.

See Table III for a listing of the quantities observed. Figure 7 shows the same data in graphical form.

6.2.3 *5-MHz Square Wave, 200-Foot Cable*

On frequency generator #2, measurements were made only in an operating environment. Measurements were taken on frequency generator #1 in both operating and idle environments. Little, if any, difference was noted between the two environments except as noted below.

At intervals of 100 and 400 microseconds, little difference between the short and long cables was apparent. At averaging times of 1 millisecond, the measured instabilities began to increase. At 10 milliseconds, the additional instabilities reached a peak about forty percent above the short cable. For 100 milliseconds, the figure had declined, although not to the level present using the short cable. At intervals

TABLE III—SHORT-TERM STABILITY OF TWO FREQUENCY GENERATORS

	Averaging Time						
Configuration	0.1 ms	0.4 ms	1 ms	10 ms	100 ms	1 s	10 s
Generator No. 1 10-Foot Cable Operating	5.6 pp 10^8	2.3 pp 10^8	1.1 pp 10^8	1.4 pp 10^9	1.4 pp 10^{10}	2.2 pp 10^{11}	2.54 pp 10^{11}
Generator No. 1 10-Foot Cable Quiet	5.6 pp 10^8	2.26 pp 10^8	1.1 pp 10^8	1.3 pp 10^9	1.3 pp 10^{10}	2.2 pp 10^{11}	2.0 pp 10^{11}
Generator No. 1 200-Foot Cable Operating	5.5 pp 10^8	2.3 pp 10^8	1.13 pp 10^8	1.91 pp 10^9	1.51 pp 10^{10}	2.2 pp 10^{11}	2.4 pp 10^{11}
Generator No. 1 200-Foot Cable Quiet	5.5 pp 10^8	2.25 pp 10^8	1.12 pp 10^8	1.81 pp 10^9	1.4 pp 10^{10}	2.1 pp 10^{11}	2.3 pp 10^{11}
Generator No. 2 10-Foot Cable Operating	7.0 pp 10^8	2.6 pp 10^8	1.24 pp 10^8	1.46 pp 10^9	1.36 pp 10^{10}	1.65 pp 10^{11}	7.7 pp 10^{12}
Generator No. 2 200-Foot Cable Operating	6.8 pp 10^8	2.57 pp 10^8	1.25 pp 10^8	1.84 pp 10^9	1.65 pp 10^{10}	2.2 pp 10^{11}	6.64 pp 10^{12}

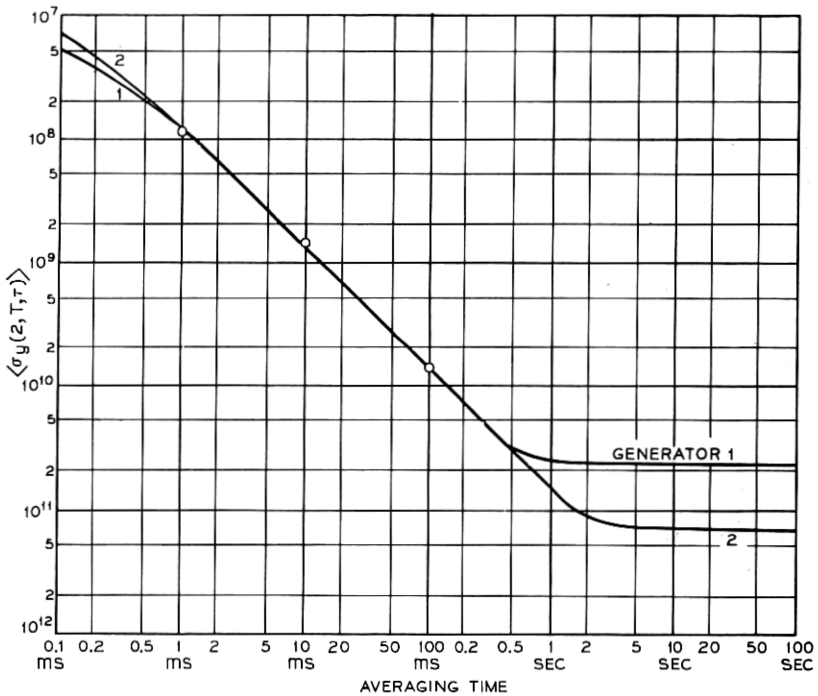

Fig. 7—Stability of two frequency generators, ten-foot cable.

of one and ten seconds, instabilities returned to the level observed using the short cable.

Some of the causes of the additional instabilities centered at ten milliseconds are identifiable in this instance. In many data processing systems, pulse rates of 100 bits per second are used. Since this rate has a duty cycle of ten milliseconds, it is not surprising that averaging times of ten milliseconds would be affected. Comparing the data between quiet and operating environments, it became apparent that averaging times of ten milliseconds were indeed affected by the operation of the data processing system. In addition, the period of the second harmonic of the power line frequency occurs near this rate. Much longer time intervals are not affected by either of these noise sources.

The stabilities observed for both frequency generators are plotted in Fig. 8. The same data is presented in tabular form in Table III.

6.3 *Synthesizers*

A Hewlett-Packard model 5105A Frequency Synthesizer/5110B Synthesizer Driver was arranged as shown in Fig. 5. A model 5100A Frequency Synthesizer/5110B Synthesizer Driver was used in Fig. 3. A 5-MHz precision oscillator was used as an external frequency standard input to the driver in both applications. In the neighborhood of 5 MHz, the short-term stability of the 5100A is much better at all time intervals than that of the 5105A. As a result, the 5100A was more useful in evaluating performance of oscillators and frequency generators. The 5105A has a higher frequency capability than the 5100A, and produces the best short-term stability at these frequencies.

Using the 5105A, the synthesizer output frequency was adjusted for 4.99 MHz. The output was mixed with the 5-MHz oscillator which was being used as the external reference. Here, the stability of the pair was about 2 pp 10^{10} over one second. A 5-MHz square wave was then

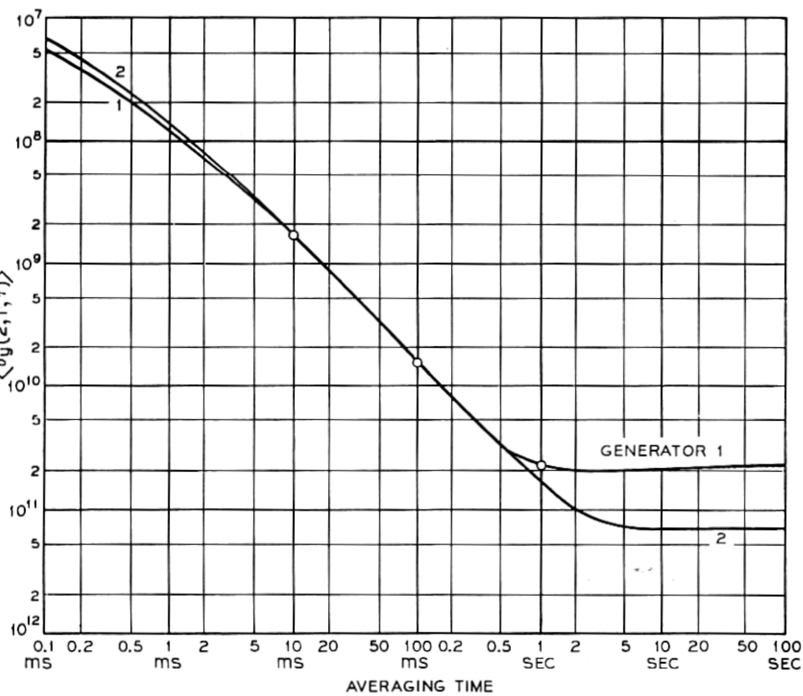

Fig. 8—Stability of two frequency generators, 200-foot cable.

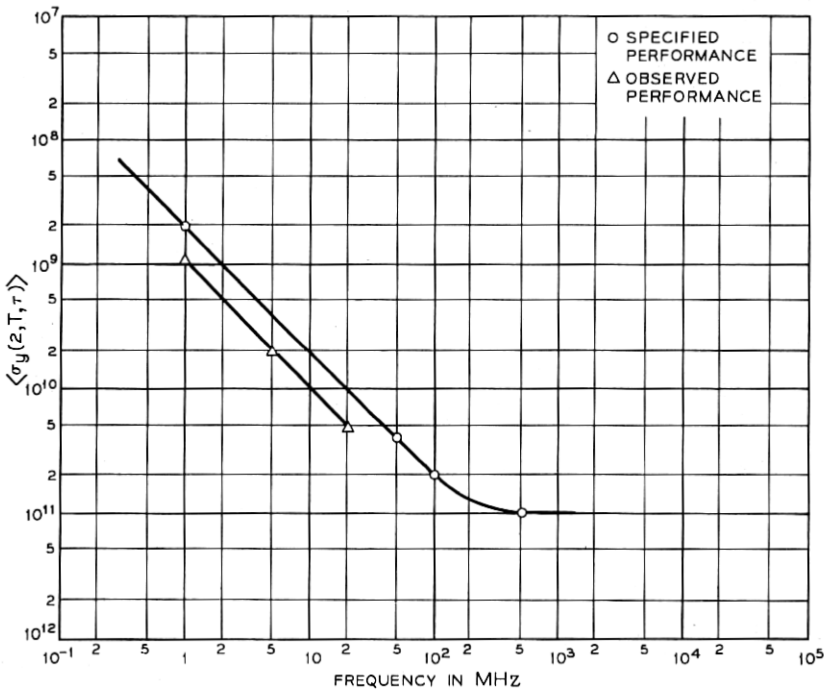

Fig. 9—Performance of frequency synthesizer, one-second averaging time.

mixed with the synthesizer output with no significant change in results.

The frequency of the synthesizer was then readjusted for 990 kHz and the output was mixed with a 1-MHz square wave signal from the generator. The stability was now reduced to 1.5 pp 10^9 over one second.

The frequency was then increased to 19.99 MHz and the output was mixed with a 20-MHz square wave signal from the generator. In this configuration, measurable stability improved to 5 pp 10^{11} over one second.

Figure 9 shows a plot of the frequency versus the standard deviation of fractional frequency fluctuations for a one-second averaging time. The specifications from the manufacturer's catalog are plotted on the same graph. Since the slopes appear the same, the synthesizer is the most likely source of the majority of noise. It may be assumed that the stability of all the frequencies measured are of the same order as the stability observed for the 20-MHz square wave. In reality, all may be somewhat better. Similar data for the 5100A Synthesizer was not taken.

6.4 *Systematic Effects*

From the experimental data, it seems that systematics from the data processing system do not contribute as large a portion of the instabilities as previously suspected. It must be noted, however, that the data processing environments used for testing purposes were only small, developmental varieties of the larger system (to be constructed at a later date). It is anticipated that the larger system will contribute more systematic noise to all frequency sources within the system.

Several discussions took place with individuals involved in short- and long-term stability measurements. Some of these individuals had made such measurements in areas where machine tools or other heavy electrical equipment were operated. There seemed to be a general consensus that stability is degraded at 8:00 a.m. and 4:30 p.m. or whenever the equipment is being started or stopped. Since no heavy machinery was located in the near vicinity of the data processing systems, these effects were not observed.

Each day, somewhere between 4:00 and 4:15 p.m., it was noted that observable stability became two to three times worse than that normally observed. About 4:30 p.m., the instabilities returned to their former level. Occasionally, the same effects were observed at other times during system operation.

Investigation yielded two causes for these observations. First, many individuals are dumping data on the various input-output devices to prepare for the change in shifts. Second, a concept of "rotation" is present to make the system available to individuals wishing to run software tests. Time intervals of five minutes are scheduled in advance. Individuals are generally prepared with magnetic and paper tapes and subject the system to heavy use during their five-minute allotment. It appeared that the increased instabilities observed during mid-day operation occurred during rotation periods.

Accuracy of measurements was probably reduced due to systematic noise introduced by the measurement system itself. For a good characterization of the noise due strictly to the oscillator or frequency generator plus induced noise from the environment, the system should have a noise level capability of at least two orders of magnitude better than the anticipated stability of the unit being tested.

VII. RECOMMENDATIONS AND CONCLUSIONS

The aspects of short-term frequency stability applicable to most situations have been discussed. Although definitions were made in both the frequency and time domains, the time domain definitions were used for measurement.

With the vast increase in the use of precision oscillators, measurement techniques must be fast, accurate, and easy to perform. Frequency domain measurements are extremely difficult to make accurately. In addition, such measurements take a long time to gather sufficient data.

The actual observable relationship between $\langle \sigma_y(N, T, \tau) \rangle$ and $\langle \sigma_y(2, T, \tau) \rangle$ was discussed. It was shown that the two quantities are very nearly equal for averaging times of one second or less. For longer times, the results are in agreement with those predicted by Barnes.[11]

Time domain measurements, using a device such as the HP5360A computing counter, are fast and consistent. The accuracy limitations of such measurements depend mainly on mixing and filtering equipment. A good method for determination of the short-term stability of precision oscillators and frequency generators is the use of an offset oscillator to obtain a frequency difference.

Further investigation of systematic effects is in order. Initially, systematic noise due to the measurement system must be minimized. Then, estimates of the stability of the oscillators in a quiet environment should be performed. Next, estimates should be made in an operating environment under controlled activity levels. Using this data, it may be possible to generate a mathematical model of the contributions of the systematic effects of the data processing system to the short-term frequency stability.

Investigation of the distribution of the instabilities would be extremely useful in evaluating performance of the measurement system. Such computations can be made by appropriate programming of the HP2116B computer shown in Fig. 3.

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