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Statistical Circuit Design:

Large Change Sensitivities for Statistical Design

By E. M. BUTLER §

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A Monte Carlo study is an analysis in the sense that for specified tolerances, correlations, etc., empiric distributions of measures of performance are obtained. An approach is presented which addresses itself to the inverse problem, that of determining the tolerances, correlations, etc., necessary to realize acceptable performance distributions. The approach is based on the concept of large change sensitivities which are proposed as a measure of sensitivity for statistical design. The approach specifically addresses design problems such as specifying tolerances, desensitizing a nominal design, recognizing the possibilities for and specifying tuning and/or matching procedures, and verifying that a design is consistent with expected statistical correlation between parameters. We present an example illustrating several of these applications.

I. INTRODUCTION

Realistic system and circuit design must account for the fact that exact realizations of paper designs are seldom achieved. The Bell System is particularly sensitive to this problem not only because of physical and economic constraints in manufacture, but also because of the varied field environments in which the system must operate. The effects of variations in design parameters, which are usually modeled as random variables, can be investigated via a Monte Carlo study. However, a Monte Carlo study is an analysis in the sense that for specified probability density functions of design parameters (specified by "nominal" value, tolerance, correlation, etc.) an empirical distribution for various outputs or performance measures is found. The inverse problem, that of finding nominal values, tolerances, and correlation in order to obtain an acceptable performance distribution, has received relatively little attention. This paper describes an approach addressed to this problem of "closing the loop around tolerance analysis." *

There are three significant points about this approach. First, the approach does not rely on first- or second-order approximations.† Like Monte Carlo, no attempt is made to approximate measures of performance. Second, the approach can accommodate multiple-specifications. Third, the implementation of the approach is feasible with a computer and so the techniques may be thought of as computer aids to statistical design.

The approach is based on four assumptions.

(*i*) There exists a designer-specified scalar performance criterion, J, which adequately reflects the goodness of a design and which is a continuous function of the design parameters. A method for forming a single criterion from many criteria is illustrated in the examples.

(*ii*) There is a designer-specified value of this criterion beyond which designs are not acceptable.

(*iii*) There is a known nominal design which is acceptable in terms of the performance criterion.

Definition: The region of acceptability, R_A, is defined to be a connected region in parameter space such that the nominal design is in R_A and such that for all realizations in R_A the corresponding performance is acceptable.

Finally, we make a fourth assumption.

(*iv*) All realizations inside R_A are equally good; i.e., a pass/fail decision can be made for each realization.

These ideas are illustrated in the one parameter, two criterion example in Fig. 1 where J_i is the scalar value of the ith performance criterion, p^0 is the nominal design parameter value, J_i^0 is the ith performance at nominal and ϵ_i is the allowable degradation in J_i from J_i^0.‡ Since there is more than one specification which must be met, the region of acceptability, R_A, is the intersection of the individual regions of acceptability for each J_i. In this example, R_A is $[p \mid a \leqq p \leqq b]$.

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* Some of the information in this paper has appeared elsewhere.[1] It is included here in the interest of completeness.

† There have been suggestions in control theory to eschew first-order sensitivities, but the proposed concepts have been difficult to realize.[2–4]

‡ The superscript ^{0} denotes nominal value. A method for forming a single criterion from many criteria is illustrated in the examples.

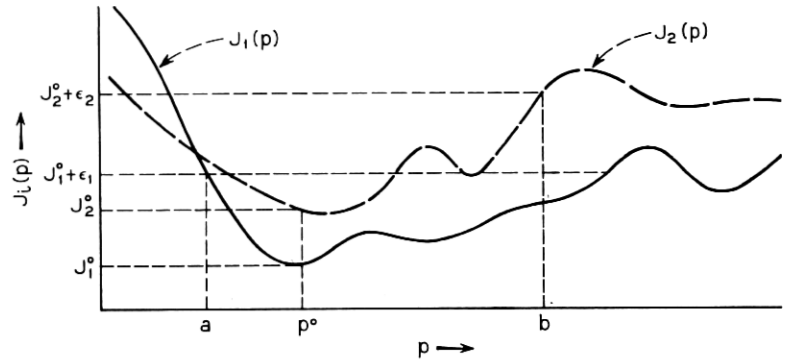

Fig. 1—A one-dimensional example.

Under the above assumptions, the region of acceptability is clearly important in the context of statistical design, that is, in specifying nominal values, tolerances, etc. In fact, it is the shape of this region and the placement of the nominal in it rather than the value of the performance at nominal that is important. For example, if the random deviations in the realization of p in the above example were uniformly distributed and symmetric about the nominal, then a nominal design located half way between points a and b would be better than p^0 in terms of yield and/or allowable tolerance. Furthermore, in this context of statistical design the concept of sensitivity takes on a new meaning which is introduced in Section II. Applications of this sensitivity information to closing the loop in tolerance analysis are discussed in Section III and an example is given in Section IV.

II. LARGE CHANGE SENSITIVITY

2.1 *Intercepts*

Suppose that we hold all parameters fixed at nominal except the kth. We define the upper (lower) intercept of parameter k to be the value in percent deviation from nominal of parameter k for which the performance is unacceptable for the first time as parameter k is increased (decreased) from nominal. Parameter values are expressed in percent deviation from nominal for reasonable scaling. Points b and a are the upper and lower intercepts for the one parameter example of Fig. 1. We denote these intercepts by

$I_k^+(J, \mathbf{p}^0, \epsilon) \equiv I_k^+ \equiv$ upper intercept of parameter k with respect to performance criterion J, nominal design $\mathbf{p}^0$, and allowable performance degradation ϵ.*

Similar notation holds for I_k^-.

The intercepts are simply a measure of how far a single parameter can deviate within the region of acceptability. If the kth intercepts are small, then being in R_A is very "sensitive" to the kth parameter, and vice versa. Using this observation as motivation, the following measure of sensitivity is proposed.

$L_k^+(J, \mathbf{p}^0, \epsilon) = L_k^+ =$ upper large change sensitivity of parameter k with respect to J, $\mathbf{p}^0$ and ϵ.

$$\triangleq \frac{1}{I_k^+}.$$

Similarly,

$$L_k^- \triangleq \frac{-1}{I_k^-}.$$

A single large change sensitivity for parameter k can be defined as the maximum of $[L_k^+, L_k^-]$.

2.2 *Performance Contours*

The intercepts provide information about how far a single parameter can vary while all others are fixed at nominal before the specifications are not met. This idea can be extended to two parameters. For a pair of design parameters, a line (or lines) of constant, just acceptable performance provides an indication about how the two parameters can vary simultaneously around nominal before specifications are not met. In fact, a *performance contour* for a pair of parameters is defined to be this line (or lines) which describes the edge of R_A restricted to the two-dimensional subspace defined by these parameters, while all other parameters are held fixed at nominal. Again, each parameter value is specified in terms of percent deviation from its nominal value.

The concept of a performance contour can be illustrated with a simple example. Consider the two parameter voltage divider shown in Fig. 2 where $R_1^0 = R_2^0 = 1$. The transfer function, T, is given by

$$T = 1/(R_1/R_2 + 1), \qquad T^0 = 0.5;$$

the input resistance, R, is given by $R = R_1 + R_2$, $R^0 = 2$.

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* Bold face letters denote vector quantities.

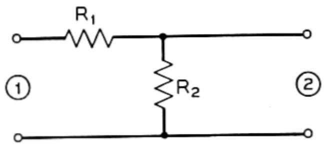

Fig. 2—A voltage divider example.

Suppose the design specifications call for $0.49 \leqq T \leqq 0.51$ and $1.8 \leqq R \leqq 2.2$. The region around nominal where the first specification is met is the shaded area in Fig. 3 and the region where the second specification is met is the cross-hatched area. The region where both specifications are met is the intersection of these regions. The edge of this acceptable region is the performance contour. The points where the contour crosses the axes are the intercepts.

Notice that the performance contour has sharp corners because of the multiple design specifications. The particular specification which determines an intercept or a section of a performance contour is said to be dominant at that point or points. For example, the upper left part of the contour in Fig. 3 is determined by the $T \leqq 0.51$ specification.

A performance contour can be interpreted as providing "second-

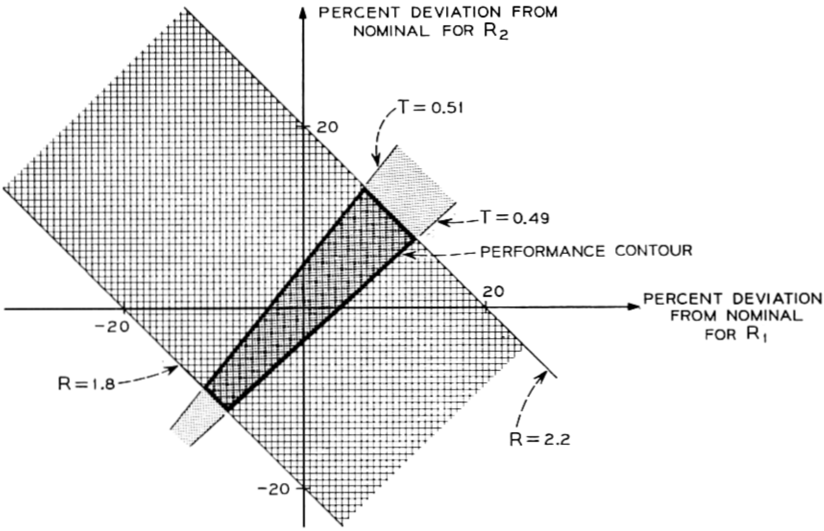

Fig. 3—Performance contour for the voltage divider example.

order" large change sensitivity information since it indicates how L^+ and L^- for one parameter will change for a change in the nominal value of the second parameter.

2.3 *Comparison with Classical Sensitivities*

One can argue that large change sensitivities are similar to first-order sensitivities in the sense that only one parameter is varied while others are held fixed. There is, however, a fundamental difference in approach. The latter sensitivities are proportional to the change in performance due to similar changes in the individual parameters. The large change sensitivities, on the other hand, are based on the change necessary in each parameter to bring about a particular (but similar) change in performance. Finally, it should be noted that if J is a linear function of the parameters, the two sensitivities are similar. This lends credence to the definition of large change sensitivity as proportional to the inverse of the intercepts.

It is interesting to note that large change and classical sensitivities each provide a characterization. The intercepts and performance contours (first- and second-order large change sensitivities) provide a characterization of R_A in one and two dimensions in parameter space, while the first two terms in a Taylor series of performance about nominal (first- and second-order classical sensitivities) provide a characterization of the performance near nominal. In statistical design, attention is (or should be) focused on R_A rather than on the performance at nominal. The large change sensitivities provide a characterization of R_A, and hence, a measure of parameter sensitivity for statistical design.

III. APPLICATIONS

3.1 *Preliminary Remarks*

Information provided by large change sensitivities can enhance a designer's insight into a problem. This can be especially important when complicated specifications exist and intuition becomes hard pressed. One might question the amount of useful information derivable from performance contours since they represent R_A for only pairs of parameters. It should be pointed out, however, that electrical properties tend to depend on parameters in pairs such as RC products and resistor ratios.

In this section, several specific applications of large change sensitivity information to design problems are discussed. The problems which are addressed include desensitizing a nominal design, specifying tolerances, recognizing the need for and specifying tuning and/or matching, and verifying that a design is consistent with known statistical correlation or tracking.

3.2 *Desensitizing a Design*

3.2.1 *The Problem*

We have seen in the example of Fig. 1 that the design could tolerate larger parameter variations from nominal if the nominal were centered in R_A. That is, we could desensitize (in a large change sense) the design by placing the nominal half-way between the intercepts rather than at p^0. We extend this notion to N dimensions and base our measure of being centered in R_A on the intercepts, or equivalently, on the large change sensitivities.

We have investigated an algorithm to automatically desensitize an initial design which satisfies the performance specifications but is not necessarily centered. Two pertinent observations which influenced the formulation of our algorithm are:

(*i*) If we change the nominal values of more than one parameter simultaneously in an attempt to center based only on intercept information, it is possible to move outside of R_A inadvertently. Consider the hypothetical two-parameter example described by its performance contour in Fig. 4. If we center both p_1 and p_2

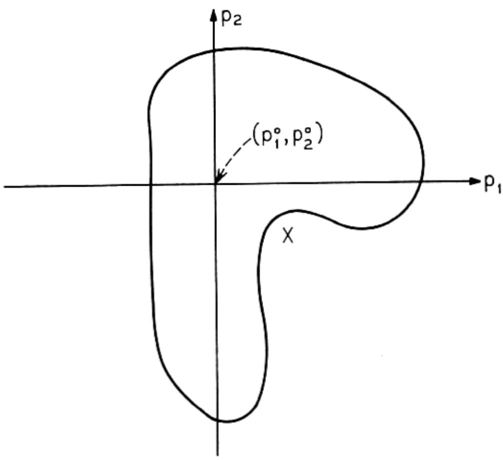

Fig. 4—Performance contour for a hypothetical two-parameter example.

simultaneously based on the intercepts at point (p_1^0, p_2^0), we move to point X which is outside R_A.

(*ii*) The intercepts for a particular parameter are simple functions of that parameter's nominal value, but they can be complicated, even noncontinuous functions of the nominal values of other parameters. For example, in Fig. 4 the upper intercept of parameter 1 (for $p_1 = p_1^0$) is not a continuous function of p_2.

3.2.2 *An Algorithm for Desensitizing*

In view of the above comments, it was decided to iterate towards a desensitized design by changing only one nominal at a time using a simple algorithm. We have taken as our measure of being centered,

$$E(\mathbf{p}) \equiv \max_i |I_i^+(J, \mathbf{p}, \epsilon) + I_i^-(J, \mathbf{p}, \epsilon)| \equiv \max_i e_i ,$$

$$\equiv | I_M^+ + I_M^- | .*$$

Suppose we start at $\mathbf{p}$ and that the error, $E(\mathbf{p}) = E$, is attributable to parameter k, i.e., $M = k$. Let us call p_k the "worst offender." First, we center parameter k and compute the new error $E(\mathbf{p}') = E'$. Note that centering p_k insures $e_k = 0$, but the intercepts for the other parameters can, and probably will, change. If E' is less than E, $\mathbf{p}'$ becomes our new starting point with error E', and we then center that parameter which is the current worst offender. (It cannot be p_k since p_k is centered.) If E' is not less than E, it means that centering p_k has altered the other intercepts enough to cause a larger error. We thus step p_k back half-way between its present value and its original value, and again compute the error. This process continues until a lower error has been found or until p_k has been stepped back seven times at which point the error is accepted and the algorithm starts over. Note that if this happens the worst offender will not be parameter k.

This algorithm has been implemented in a computer program called XCENTRIC (Experimental Centering Program) and some results will be given in Section IV. No claims are made about convergence of the algorithm; rather, its strength lies in its simplicity.

3.3 *Specifying Tolerances*

Typical approaches to specifying tolerances in the past have been either to set tolerances roughly inversely proportional to first-order

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* We have assumed symmetry in the probability density function for p_i. This is not necessary since one can weight the intercepts accordingly. For example, if the density function for p_i were rectangular with twice as much probability above nominal as below, one could set e_i proportional to $(I_i^+ + 2I_i^-)$.

sensitivities or to set them to the tightest available. The former approach requires linear approximations while the latter can be unnecessarily expensive. The intercepts and performance contours provide a designer with information about how far parameters can deviate and stay within R_A. This is what a designer really needs to better specify tolerances. Furthermore, if 100 percent yield is desired, the intercepts and contours provide upper bounds and pairwise constraints respectively on feasible parameter tolerances. This fact has been utilized to help solve a version of the minimum cost tolerance specification problem; the method and results are presented in another paper in this issue.[5]

3.4 *Parameter Correlation with Respect to the Performance Specifications*

Consider again the performance contour shown in Fig. 3 for the voltage divider example. The shape is an indication that the two resistors are "correlated" with respect to the design specifications. Two parameters are qualitatively defined to be correlated with respect to the performance specification if the region of acceptability for one parameter (specified by its intercepts) depends strongly on the value of the second parameter. If the contour were rectangular and parallel to the axes, the two parameters would be uncorrelated. This correlation information can be useful in two ways.

First, a design may be evaluated by determining whether parameter correlation with respect to the performance specifications is consistent with statistical parameter correlations. A design should be insensitive to, and in fact, it should take advantage of known statistical correlation. For example, it would be desirable if R_1 and R_2 in the voltage divider example tracked each other (or could be chosen to track) because of manufacture and/or environment. In addition, it is suggested that if one is investigating a design with statistical correlation between many parameters, it might be advantageous to find intercepts and contours for the independent and correlation determining random variables since they are really the design parameters.

Second, problems of specifying tuning or matching are inherently linked to the tolerance specification problem; parameter correlation with respect to performance specifications is an indication that tuning or matching might be desirable. In addition, the contour information can indicate how to match parameters and/or the specification to which to tune. In the case of tuning, the constraint which is dominant along the "long" side of the contour is the criterion to which to tune. In the voltage divider example this would be the transfer function specification. For the case of matching, correlation can indicate that the performance specifications depend strongly on, or are most sensitive to, a particular combination of the two parameters such as their ratio. Properties of performance contours which relate certain correlated contour shapes to particular combinations of the two parameters are given elsewhere.[1] However, two of the more important properties are stated in the Appendix. Design parameters presumably would be matched according to the particular combination. For example, the contour for the voltage divider lies along and contains the 45° line and so one should consider matching the ratio of the two resistors to their nominal ratio (see Section A.2). Finally, it should be pointed out that the contour information can be used to suggest sequential tuning or matching procedures since contours for a tuned or matched design might suggest further tuning or matching.

3.5 *Computation of Performance Contours*

A program, CONTOUR, has been written to compute intercepts and performance contours for user supplied subroutines which compute design performance. The program has been written for an interactive CDC 3500 facility. On-line scope displays of the contours as well as Calcomp plots are options which complement printer output. A Monte Carlo program to estimate yield is also part of the entire package. Thus, a user can verify immediately whether any changes made based on contours or intercepts did, in fact, increase the yield. The choice of parameter pairs for which performance contours are to be computed is up to the user.

The intercepts are found via a search and the contours are found via a performance contour following algorithm. Since many performance evaluations are necessary, the speed of computation depends strongly on the time required for a circuit analysis. For the example to be presented, a general purpose analysis routine for ladder networks was used to analyze the circuit and the computation of a performance contour typically took on the order of a few seconds. No advantage was taken of the fact that during the search for intercepts or computation of a contour, only one or two circuit parameter values are changed between each analysis.

IV. AN EXAMPLE

4.1 *Preliminary Remarks*

In the last section we indicated that large change sensitivity information could be used to desensitize an initial design, to help specify tolerances, and to indicate parameter correlation with respect to the specifications. In this section we present an example which illustrates the insight provided by intercepts and contours as well as their utility in suggesting matching and in desensitizing an initial design. CONTOUR was used to compute the intercepts and contours, and XCENTRIC was used to desensitize the initial design. The yields were estimated using TAP[6] with components assumed to be independent, uniformly distributed random variables. This example is not contrived; it was a recent B.T.L. design.

4.2 *The Problem*

The circuit is a low frequency bandpass filter with insertion loss specifications shown in Fig. 5. This is an example of the multicriteria case. We obtain a single criterion by defining

$$J_i \triangleq \frac{IL_i - IL_i^0}{IL_i^{\text{limit}} - IL_i^0},$$

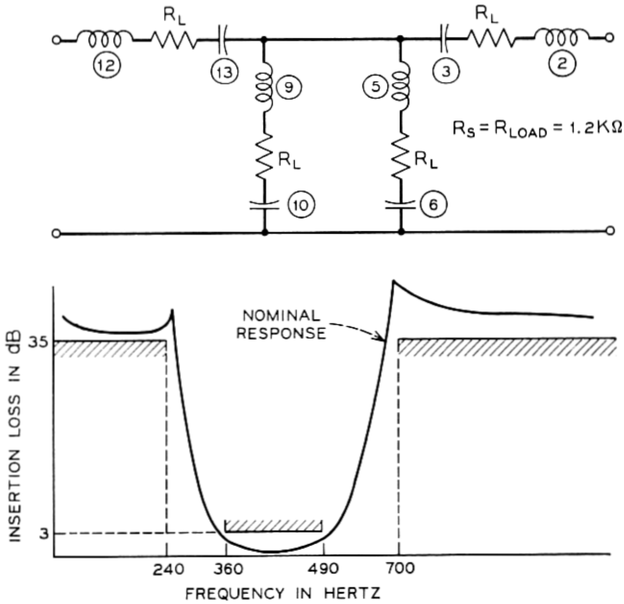

Fig. 5—The circuit and specifications for the example. (Parameters 5 and 6 correspond to the loss peak at 700 Hz. Parameters 9 and 10 correspond to the loss peak at 240 Hz. Parameters 2, 3, 12, and 13 correspond to the bandpass loss minimum at 420 Hz.)

where IL_i is the insertion loss at the ith frequency (for this example there were 30 different frequencies of interest), and

$$J \triangleq \max_{i=1,30} \{J_i\}.$$

Thus, $J^0 = 0$ and $\epsilon = 1$.

4.3 *Results*

Two of the performance contours for this example are shown in Fig. 6. The numbers which are written alongside the contours indicate the frequency at which the insertion loss specifications failed first, i.e., which criterion determined or was dominant along that edge of the region of acceptability.

The insight gained from the contours agrees with one's intuitive feel for the circuit. Consider the transmission zeroes which we know depend on LC products and in particular consider parameters 5 and 6, which determine the loss peak at 700 Hz. If both 5 and 6 increase (first quadrant in 5, 6 contour), the loss peak moves down in frequency and a specification is in trouble at 490 Hz, the upper edge of the passband. If both 5 and 6 decrease, the loss peak moves up in frequency and the 35-dB insertion loss at 700-Hz criterion is the first to be violated. Similar observations can be made by looking at the 9, 10 contour.

Let us now consider possible component matching to increase yield or to permit loosening tolerances. It turns out that the shape of the contour for parameters 5 and 6 is an indication that the specifications are sensitive to the product of these parameters. With tolerances of

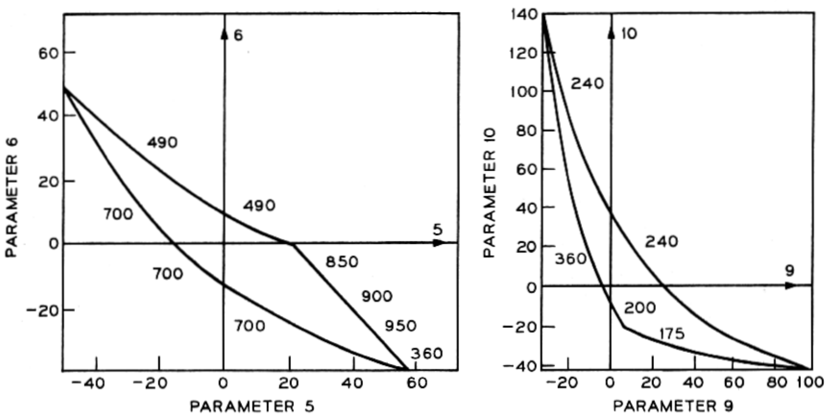

Fig. 6—Two performance contours for the example.

30 percent on the Rs, 5 percent on parameters 2, 3, 12, and 13, 1 percent on 9, 2 percent on 10, and 10 percent on 5 and 6, *the yield went from 85 percent with no matching to 99 percent with matching* 5 and 6 to their nominal product. The contour for parameters 9 and 10 also indicates that their product is important. With tolerances of 30 percent on the Rs, 10 percent on parameters 5, 6, 9, and 10, and 5 percent on parameters 2, 3, 12, and 13, *the yield went from 67 percent (no matching) to 94 percent with matching* 5, 6 and 9, 10 to their respective nominal products. It is true that in this sample example, the desirability of matching might be intuitively obvious to a designer. The purpose, however, was to illustrate how matching based on contour information can dramatically affect the yield.*

Finally, let us consider desensitizing this design since it is not centered as is obvious from the contour for parameters 9 and 10. The centering would hopefully effect an increase in yield for a particular set of tolerances or an increase in tolerances for a particular yield. XCENTRIC was run for two cases: centering only the inductors, and centering both inductors and capacitors. Yields for various tolerances were estimated for the original and both of the centered designs. The results are shown in Table I. (30 percent tolerances were used for the Rs.)

The following comments are pertinent:

(*i*) The tolerances for the original design were 1 percent Ls and 2 percent Cs which gave the desired 100 percent yield.

(*ii*) The error function for XCENTRIC as defined in Section 3.2.2 went from 21 to 0.6 for centering only the inductors and from 25 to 2.5 for centering all Ls and Cs. The changes from original nominal parameter values to centered values were typically on the order of a few percent.

(*iii*) For any of the tolerance combinations shown, the yield increased significantly as a result of "centering."

(*iv*) The original tolerances on the inductors could have been loosened to virtually 5 percent if the centered nominals had been used. This would have been physically feasible since the inductors were wound to desired values for this particular circuit.

(*v*) Ignoring the problem of preferred capacitor values, the tolerances for both Ls and Cs could have been loosened to 5 percent as a result of applying XCENTRIC.

* For other examples of this, see Ref. 1.

TABLE I—YIELDS FOR VARIOUS TOLERANCES FOR ORIGINAL AND CENTERED NOMINAL VALUES.

Tolerances	Original Nominal Values	Centered L Original C Nominal Values	Centered L and C Nominal Values
1% L 2% C	100	—	—
5% L 2% C	95	99.9	—
10% L 2% C	75	89.0	—
5% L 5% C	92	97.5	100

V. CONCLUSION

In this paper we have discussed an approach to "closing the loop" in tolerance analysis. The approach specifically addresses design problems such as specifying tolerances, desensitizing a nominal design, specifying adjustment procedures, and verifying that a design is consistent with manufacturing and environmental component tracking. The approach is particularly applicable to Bell System designs not only because in such designs deviations from nominal must be anticipated, but also because the designs many times have complicated, multiple specifications. The approach does not rely on first- or second-order approximations. In addition, since they are feasible via computer implementation, the techniques may be thought of as computer aids to (statistical) design.

Under the reasonable and realistic assumptions stated in Section I and in the context of statistical design, it was seen that the shape of the region of acceptability and the placement of the nominal design in it are more important than the performance of the nominal design. Furthermore, in view of this the concept of sensitivity has a meaning different from the classical first-order one, and so large change sensitivity was introduced. Intercepts and performance contours were seen to provide "first- and second-order" large change sensitivity information. In fact, they provide a characterization of the region of acceptability in somewhat the same way that classical first- and second-order sensitivities provide a characterization of performance near nominal. Thus, because of the attention focused on the region of acceptability in statistical design, the large change sensitivities provide a measure of sensitivity for statistical design.

Two computer programs, CONTOUR and XCENTRIC, have been written to compute and utilize large change sensitivity information. An example was presented to illustrate the utility of the approach in providing insight, in suggesting possible adjustment procedures, and in desensitizing a nominal design. In the two latter applications the yield increased significantly when adjustments and changes were made based on large change sensitivity information.

In short, for the realistic design problem, attention is (or should be) focused on the region of acceptability. Large change sensitivities provide a measure of parameter sensitivity for this region and design techniques based on them are addressed to the problem of "closing the loop" in tolerance analysis.

VI. ACKNOWLEDGMENT

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APPENDIX

Properties of Performance Contours

The notation used is p_i for the value of parameter i, p_i^0 for nominal value, v_i for percent deviation from p_i^0, and $\Gamma(v_i, v_j)$ or Γ_{ij} for the contour of the i, j parameters.

A.1 *Product Property*

If J depends only on the product of two parameters p_i, p_j, then

(*i*) $\Gamma(v_i, v_j)$ contains the curve shown in Fig. 7 and is not bounded at either end;

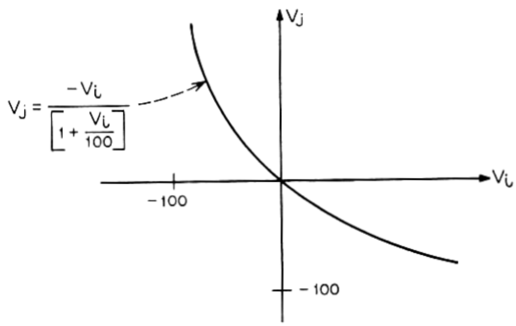

Fig. 7—The constant parameter product curve in percent deviation space.

(*ii*) $\Gamma(v_i, v_k) = \Gamma(v_j, v_k)$, for all v_k.

One could qualitatively relax this exact property into the following corollary.

A.2 *Product Property Corollary*

If J as a function of p_i and p_j is most sensitive (in a large change sense) to the product of p_i and p_j, then Γ_{ij} will roughly follow the curve in Fig. 7 and $\Gamma_{ik} \approx \Gamma_{jk}$, for all k.

A.3 *Ratio Property*

If J depends only on the ratio of two parameters p_i, p_j, then

(*i*) $\Gamma(v_i, v_j)$ contains the 45° line in the v_i, v_j plane and is unbounded;

(*ii*) for v_i and $v_j \ll 100$, $\Gamma(v_i, v_k)$ looks like $\Gamma(-v_j, v_k)$, for all v_k.

A.4 *Ratio Property Corollary*

If J is most sensitive to the ratio of two parameters p_i, p_j, then Γ_{ij} will roughly follow the 45° line, and $\Gamma(v_i, v_k) \approx \Gamma(-v_j, v_k)$ for all k, for v_i, $v_j \ll 100$.

The motivation behind parts (*i*) of these two properties is simply that the hyperbola in Fig. 7 and the 45° line are loci in percent deviation space of constant parameter product and ratio, respectively.

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