THE BELL SYSTEM TECHNICAL JOURNAL
Vol. 50, No. 4, April, 1971
Printed in U.S.A.

Statistical Circuit Design:

The Application of Monte Carlo Techniques to the Study of Impairments in the Waveguide Transmission System

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(Manuscript received December 3, 1970)

Monte Carlo techniques have proven to be useful in examining the effects of component variations in electronic circuits. These techniques can also be applied to the study of parameter variation in systems. As an example of this approach, this paper reports on the use of Monte Carlo techniques in the study of impairments in the waveguide transmission system, a high-capacity, long-haul communication facility now under development at Bell Laboratories. Two examples of the Monte Carlo analysis are discussed. These examples show how the Monte Carlo analysis can give the system designer insight into the effects of impairments on the system performance, and can aid the designer in setting requirements on the system components. It is expected that these techniques will find further application in other Bell System design efforts.

I. INTRODUCTION

1.1 *Monte Carlo Analysis in Circuit and System Design*

Monte Carlo tolerance analysis has proven to be a useful tool in the circuit design process. [1-3] Typically, the designer uses the statistical distributions and correlations of the circuit components in conjunction with random number generators to produce in the computer a large number of sample circuits. These circuits are then analyzed on the computer and empirical distributions of circuit performance are obtained. From these distributions, the designer can predict yield and can study the implications of altering component tolerances.

Monte Carlo techniques can also be used to analyze the effects of parameter variations in systems; in particular, these methods can be applied to the study of impairments in a transmission system. We shall first discuss some of the steps involved in the system design and specification, and then show how Monte Carlo techniques can be applied.

1.2 *System Specification and Impairment Allocation*

An important part of the design of a transmission system is the specification of the system transmission characteristics. This specification is made in two steps:

(*i*) Specification of the nominal transmission characteristics of each block of the system.
(*ii*) Specification of the allowed deviations from the nominal transmission characteristics.

The first step involves the specification of the modulation type, nominal filter shapes and bandwidths, and the nominal characteristics of other blocks such as amplifiers and equalizers, so that the nominal system meets the performance objectives.

The second step involves the specification of the allowed range of impairments (such as delay and amplitude distortions) in each block of the system so that the performance will still be satisfactory with impairments present. This step is known as the *impairment allocation.*

These specifications translate into performance requirements for the individual circuits and components comprising each block of the system. There are several difficulties: First, these requirements must be consistent with the hardware capabilities; overly stringent requirements would result in a reduced manufacturing yield. Secondly, if realistic requirements are placed on each component, a repeater assembled at random from a number of separate components may not meet the overall impairment requirements. In this situation, alternatives such as tightening component requirements, component matching, or tuning must be considered.

1.3 *Using Monte Carlo Techniques in Impairment Analysis*

The impairment allocation requires a knowledge of the effects on the system performance of impairments in each block of the system. Determining the effects of impairments one at a time (by computer simulation) is useful, but does not provide the entire answer. If there are nonlinearities in the system, the effects of many simultaneous impairments cannot be found by the superposition of their individual effects. Similarly, the effects of a particular impairment (such as quadratic amplitude distortion) may depend on the block in which the impairment occurs.

Monte Carlo analysis, on the other hand, can provide valuable insight into the effects of multiple impairments. Each individual impairment is described by a probability distribution representing its range of possible values. The system is simulated for many sets of impairments chosen from the assumed distributions, and the resulting measures of system performance are tabulated in histograms.

This procedure can be used in preliminary impairment studies and impairment allocations, using hypothetical impairment distributions, to get a feeling for the way in which impairments combine, and what maximum and minimum degradations can be expected from impairments which are constrained to lie in a certain range.

Monte Carlo analysis can also be used to get quantitative results on repeater yield and component tolerance requirements if the actual impairment distributions in each block of the system are known. Typically, this information becomes available as the hardware design progresses. For example, the designer can trade off repeater failures for looser component requirements. The tolerances required to give 100 percent acceptable repeater performance may result in overly stringent requirements on the individual hardware components. Relaxing the component requirements to give, say, 99 percent acceptable repeater performance would increase the component yield, at the expense of having a one percent repeater rejection rate. Monte Carlo analysis can thus give the designer valuable information on the various tradeoffs involved in the system specification.

1.4 *Outline of Paper*

This paper reports on the use of Monte Carlo techniques in the study of impairments in the waveguide transmission system, a high-capacity, long-haul communication facility now under development at Bell Laboratories.

The outline of this paper is as follows: Section II describes the waveguide transmission system and the sources of impairments. Section III discusses the Monte Carlo analysis program; Section IV describes the case studies which have been made; and Section V gives the results of the Monte Carlo analysis. Section VI gives a summary and conclusions.

II. THE WAVEGUIDE TRANSMISSION SYSTEM

2.1 *System Description*

The waveguide transmission system will utilize the vast communication potential of millimeter waves to transmit voice, data, *Picture-*

phone® service, and other signals over the telephone network. A decade from now, it is expected to play a major role in meeting the increased demand for telecommunication services.

A simplified block diagram of one link of the system for one direction of transmission is shown in Fig. 1. Present plans call for 120 broadband channels (60 in each direction) to occupy the frequency band from 40 to 110 GHz, giving the system a capacity of about a quarter million voice circuits. The baseband input signal in each channel is two-level PCM at a bit rate of 282 MHz. The modulation technique used is differentially-coherent phase-shift-keying (DCPSK), in which information is transmitted in the relative phase of the carrier.

The oscillator frequencies for the carriers are spaced at 560-MHz intervals across the 40- to 110-GHz band. Tre modulator outputs are fed into a channel-combining network, and are then transmitted over a twenty-mile* section of buried two-inch circular waveguide. The channel-separation network separates the signals from the various channels at the waveguide output.

The signal in each channel then enters a regenerative repeater. The down converter shifts the signal to an IF frequency of about 1.4 GHz; the equalizers correct for delay and amplitude distortions; and the IF filter limits the noise power entering the detector. The regenerator samples the detector output every time slot, decides if a one or a zero was transmitted, and puts out a pulse of the appropriate polarity. The regenerated pulse stream is then modulated back to the millimeter band for transmission over the next link of the system.

2.2 *Design Objective*

The design objective in each channel is a 10^{-9} error rate per repeater with a twenty-mile repeater spacing. The probability of error can be reduced by raising the received carrier power. Thus the performance measure for each channel is based on the amount of carrier power required to obtain a 10^{-9} error rate. First we define the *relative carrier power* (RCP) by

$$\text{RCP} = \frac{C}{\mu f_b}$$

where C is the undeviated carrier power at the receiver input, μ is the thermal noise per unit bandwidth and f_b is the bit frequency. The RCP is thus the carrier power relative to the noise power in a band-

* The nominal repeater spacing is twenty miles.

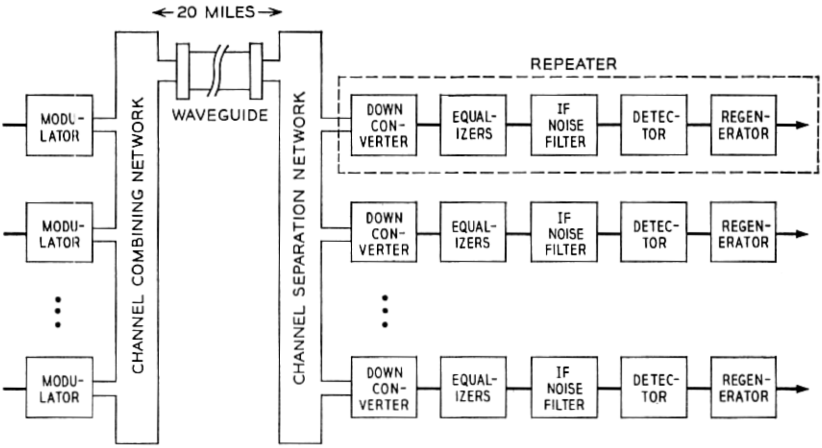

Fig. 1—Block diagram of the waveguide transmission system.

width equal to the bit frequency. The performance measure for a channel is the RCP required to obtain a 10^{-9} error rate (called the required RCP).

We now reformulate the design objective in terms of the required RCP. Taking into account the power output of millimeter wave sources, waveguide and network losses, thermal noise and amplifier noise figures, the RCP *available* for the worst channel (at 40 GHz) turns out to be 25 dB; other channels have a higher available RCP. The error rate in any channel is satisfactory if

$$\text{Required RCP} \leqq \text{Available RCP}.$$

The system is being designed to meet the worst channel requirement; thus the design objective for each channel is

$$\text{Required RCP} \leqq 25 \text{ dB}.$$

2.3 *Sources of Impairments*

There are two main causes of impairments in the system: imperfect equalization and variations in component characteristics. For example, the waveguide delay characteristic is such that in any one 560-MHz channel, the delay is almost linear with frequency. The IF delay equalizer should therefore have a linear delay characteristic with a slope which is the negative of that of the waveguide, so that their combined delay characteristic is flat.[4] However, the combined waveguide and equalizer delay characteristic typically contains some residual distortions, which are adequately modeled as having linear, quadratic, cubic, and ripple terms.

Similarly, variations from the nominal waveguide delay characteristic will produce distortion even if the nominal is perfectly equalized. Another important source of impairments is the repeater, which consists of about fifteen separate components (such as amplifiers, isolators, filters), each with an inherent variability.

The above considerations also apply to *amplitude* distortions, which must also be included in an impairment analysis of the system.

III. COMPUTER ANALYSIS OF THE WAVEGUIDE TRANSMISSION SYSTEM

3.1 *Computer Simulation of the System*

In order to carry out a Monte Carlo study of impairments, we need a simulation program to evaluate the system performance (i.e., compute required RCPs) given a particular set of transmission characteristics. Such a program has been written by J. H. Mullins.[5] This program has many features and options; only the ones relevant to this discussion are described below. The program simulates the transmission of two periodic streams of sixteen pulses through one channel of the system, so that intersymbol interference from all combinations of five adjacent pulses is obtained. A calculation is made of the required RCP for each time slot in the transmitted pulse stream. The effects of timing jitter are included by computing the required RCP for various regenerator timing errors, typically $-\frac{1}{8}$, 0, and $+\frac{1}{8}$ of a time slot. A record is made of the worst (i.e., highest) required RCP for each timing error, taking all time slots into account.

The effects of an interfering signal in an adjacent channel can also be included in the simulation, in which case the relative time position of the adjacent channel signal is adjusted for each time slot to produce the worst effect (highest required RCP). Thus there are several required RCPs computed for a given channel. Characterizing the performance of the channel by a single number, we define the *computed* performance measure to be the maximum of the required RCP at $-\frac{1}{8}$ or $+\frac{1}{8}$ timing error, with adjacent channel interference included in the simulation (called the required RCP with $\pm\frac{1}{8}$ timing error).

3.2 *The Monte Carlo Analysis Program*

The waveguide simulation program described in the previous section has been embedded in a Monte Carlo analysis program which selects parameter values from probability distributions, passes them to the simulation program, and forms histograms of the required RCPs computed by the simulation. The program was written for a Control Data 3500 computer. About 45 seconds of computation time is required for each run through the simulation program when adjacent channel interference is included in the simulation; and about 15 seconds when the interference is omitted. Generating histograms of 1000 samples, therefore, takes either four or 12½ hours of computer time.

IV. EXAMPLES OF MONTE CARLO IMPAIRMENT ANALYSIS

4.1 *The Monte Carlo Impairment Studies*

The Monte Carlo analysis program described in the previous section has been used to study the effects of delay and amplitude impairments on the waveguide transmission system and has aided in the setting of preliminary impairment allocations.

The nominal transmission characteristics of the system are shown in Table I; the system is assumed to be perfectly delay-and-amplitude equalized in the absence of impairments. The impairments are described in Section 4.2.

The worst-case required RCPs (with adjacent channel interference) for the nominal system are as follows:

Zero timing error: 15.20 dB,

$-\frac{1}{8}$ timing error: 16.37 dB, and

$+\frac{1}{8}$ timing error: 16.34 dB.

The computed performance measure for the nominal system is thus 16.37 dB.

TABLE I—NOMINAL TRANSMISSION CHARACTERISTICS FOR MONTE CARLO IMPAIRMENT STUDY

Modulation: DCPSK, path length modulator.
Symbol rate, polar binary PCM = 282 MBd = f_b.
Time slot = 3.5 ns = τ.
RF Channel Spacing = 560 MHz = 1.98 f_b.
RF Channel Filters:
  Two-pole, maximally flat, lossless
    3-dB bandwidth = 450 MHz = 1.6 f_b;
  In tandem with two-pole lossless rejection filter centered 560 MHz higher than passband filter.
Receiver IF Noise Filter:
  Four-pole, maximally flat
    3-dB bandwidth = 370 MHz = 1.31 f_b.
Detector: Ideal DCPSK.
Adjacent channel interference included in simulation.
System is delay and amplitude equalized.

Two Monte Carlo studies have been carried out. In the first, the distortions were assumed to originate in the waveguide section of the system (i.e., at an RF frequency in the 40- to 110-GHz range). In the second, the distortions were assumed to originate in the repeater (i.e., at the IF frequency of 1.4 GHz). The only difference between the two is that in the second case, the amplitude distortions affect the amount of noise power entering the detector, whereas in the first case the noise power is independent of the distortions. This difference is significant, as will be seen in the discussion of the results.

Adjacent channel interference was included in the simulation, and histograms of required RCPs were made for timing errors of $-\frac{1}{8}$, 0, and $+\frac{1}{8}$ of a time slot. An additional histogram was made of the required RCP with $\pm\frac{1}{8}$ timing error. One thousand samples were obtained in each histogram, and a record was made of the impairment parameter values causing the maximum and minimum entries in each histogram.

4.2 *Impairment Parameters and Distributions*

The impairments investigated in these studies are delay-and-amplitude distortions consisting of linear, quadratic, cubic and ripple components. The linear, quadratic and cubic terms are characterized by their values at the frequency $f = f_c + f_b/2$, where f_c is the carrier frequency and $f_b = 1/\tau$ is the bit frequency (282 MHz). The ripple terms are characterized by their peak-to-peak amplitude, period and phase.

In these preliminary studies, the impairments are assumed to originate in a single block (transfer function) of the system. The overall delay of this block is the sum of the four delay components, and is given by:

$$d(f) = d_1[2(f - f_c)/f_b] + d_2[2(f - f_c)/f_b]^2 + d_3[2(f - f_c)/f_b]^3 + (1/2)d_R \sin [2\pi(f - f_c)/P_D + \theta_D].$$

The distributions of the delay distortion parameters, d_1, d_2, d_3, d_R, P_D and θ_D assumed in this study are given in Table II.

The amplitude response of the impaired transfer function is unity plus the four amplitude distortions:

$$A(f) = 1 + A_1[2(f - f_c)/f_b] + A_2[2(f - f_c)/f_b]^2 + A_3[2(f - f_c)/f_b]^3 + (1/2)A_R \sin [2\pi(f - f_c)/P_A + \theta_A].$$

The distributions assumed in this study for the amplitude impairment

TABLE II—ASSUMED DELAY IMPAIRMENT DISTRIBUTIONS*

Impairment Type	Parameter	Distribution Limits
Linear Delay	d_1 = value at $f_c + f_b/2$	-0.2τ to 0.2τ
Quadratic Delay	d_2 = value at $f_c + f_b/2$	-0.2τ to 0.2τ
Cubic Delay	d_3 = value at $f_c + f_b/2$	-0.2τ to 0.2τ
Ripple Delay	d_R = peak-to-peak value	-0.3τ to 0.3τ
	P_D = period	$0.125 f_b$ to $1.0 f_b$
	θ_D = phase	0° to 90°

* All distributions are uniform and independent.

parameters A_1, A_2, A_3, A_R, P_A, and θ_A are given in Table III. All delay and amplitude distributions are assumed to be independent.

The impairment parameters are assumed to be uniformly distributed, since the actual distributions are not known. However, this is adequate for our purposes, since a primary goal of these preliminary studies is to determine the effects of multiple impairments which are constrained to lie within certain limits.

A few words are in order concerning the distribution limits specified in Tables II and III. These limits were arrived at after a study had been made of the effects of individual impairments on the system performance. From this study, a preliminary set of impairment limits was obtained for which it was felt that the overall required RCP with $\pm 1/8$ timing error would not exceed 22 dB or so, leaving a margin of about 3 dB for other types of impairments. It is these *preliminary* impairment limits which are tabulated in Tables II and III. These should not be construed as final requirements on the system, or as anything other than first estimates of the impairment levels which the system can tolerate. In fact, the Monte Carlo results in Section V indicate that these limits are probably too tight, and satisfactory performance can be achieved with somewhat larger impairment limits.

TABLE III—ASSUMED AMPLITUDE IMPAIRMENT DISTRIBUTIONS*

Impairment Type	Parameter	Distribution Limits
Linear Amplitude	A_1 = value at $f_c + f_b/2$	-0.15 to 0.15
Quadratic Amplitude	A_2 = value at $f_c + f_b/2$	-0.2 to 0.2
Cubic Amplitude	A_3 = value at $f_c + f_b/2$	-0.1 to 0.1
Ripple Amplitude	A_R = peak-to-peak value	-0.06 to 0.06
	P_A = period	$0.125 f_b$ to $1.0 f_b$
	θ_A = phase	0° to 90°

* All distributions are uniform and independent.

V. RESULTS OF THE MONTE CARLO ANALYSIS

5.1 *Case Study #1: Impairments at RF*

The histograms of required RCP resulting from the Monte Carlo analysis with the distortions occurring at RF are shown in Figs. 2 through 5. Also shown are the mean required RCP, the nominal (no distortion) value, the value which was exceeded in only one percent of the cases, and the percentage of cases in which the required RCP was lower than nominal.

Figure 2 shows the histogram of the required RCP for zero timing error. The mean required RCP is 1.1 dB higher than the nominal value. The highest value which occurred was 19.0 dB, but only one percent of the cases exceeded 17.55 dB, or 1.45 dB lower. There were three cases (0.3 percent) in which the required RCP was less than the nominal value, indicating that there are some combinations of dis-

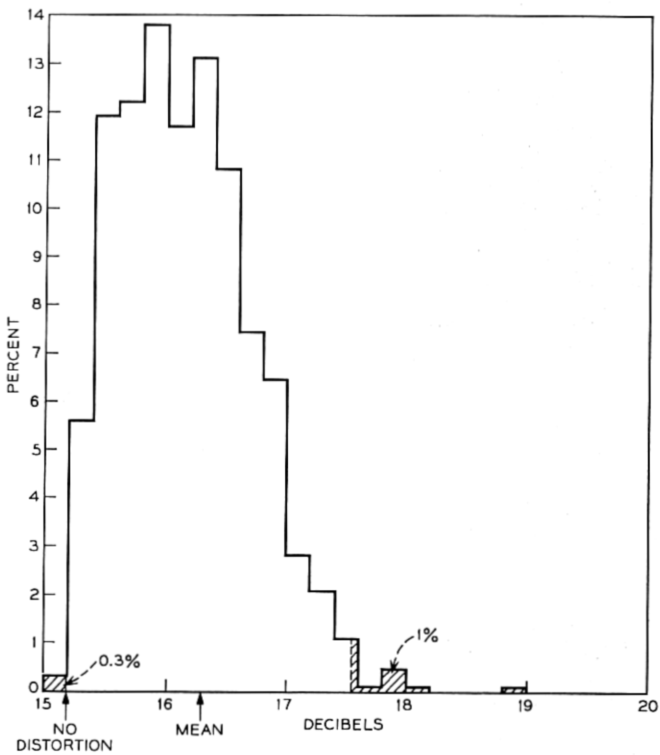

Fig. 2—Histogram of required RCP for zero timing error with impairments at RF.

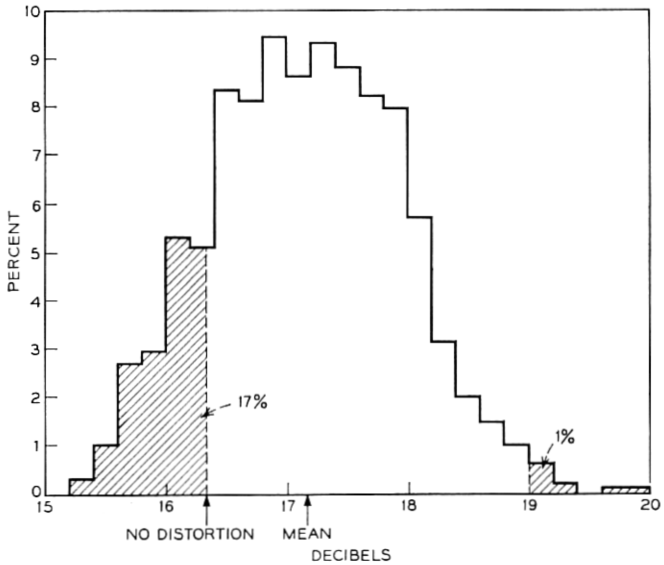

Fig. 3—Histogram of required RCP for −1/8 timing error with impairments at RF.

tortions which produce better results than no distortions at all. This is not too surprising, since the nominal transmission characteristics were not chosen to minimize the required RCP at zero timing error, but rather to minimize the required RCP with ±1/8 timing error.

Figure 3 shows the histogram of the required RCP for −1/8 timing error. The mean value is 0.75 dB higher than nominal. However, 17 percent of the distortion combinations reduced the required RCP below its no distortion value. This is primarily due to the effects of delay distortion. Large positive quadratic delay distortion shifts the pulse stream sufficiently so that better results are obtained by sampling with −1/8 timing error rather than with zero timing error. At the lowest value in the histogram in Fig. 3, the quadratic delay distortion is $d_2 = 0.16\ \tau$. However this distortion is not desirable, since it seriously degrades the required RCP at +1/8 timing error. The *highest* value in the histogram in Fig. 4 (+1/8 timing error) was obtained for $d_2 = +0.19\ \tau$. Similarly, large *negative* quadratic delay distortion improves the required RCP at +1/8 timing error, but degrades it at −1/8 timing error.

Figure 5 shows the histogram of the required RCP with ±1/8 timing error when the impairments occur at RF. The mean required

RCP in the impaired system is about 1.1 dB higher than nominal. The maximum value that occurred was 20.6 dB, or 4.2 dB higher than nominal. The impairment parameter values which resulted in this maximum degradation are given in Table IV. The linear and cubic delays are both negative and have about the same value (and hence are reinforcing) and the quadratic delay is almost at its maximum positive value. The delay ripple has a large period and a large peak-to-peak amplitude. Looking at the amplitude impairments in Table IV, we see that the quadratic amplitude has a large negative value, as does the peak-to-peak amplitude ripple.

The most interesting feature of the histogram in Fig. 5 is that four percent of the distortion combinations produce a lower required RCP than nominal, the lowest being 15.8 dB, 0.55 dB below nominal. The impairment values producing this minimum required RCP are also given in Table IV. All the distortions are quite small, except for the quadratic amplitude distortion which is almost at its maximum positive value. These results indicate that positive quadratic amplitude distortion occurring at RF improves the performance of the system.

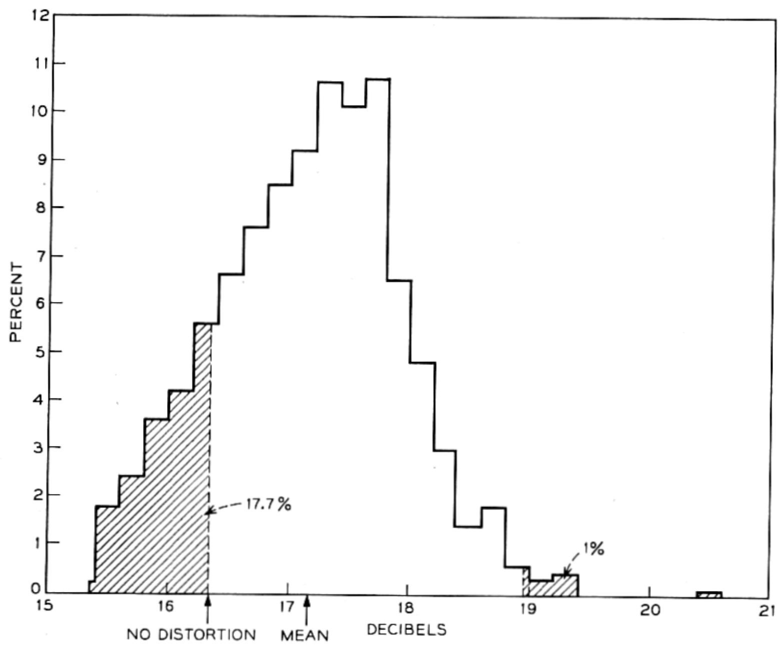

Fig. 4—Histogram of required RCP for +1/8 timing error with impairments at RF.

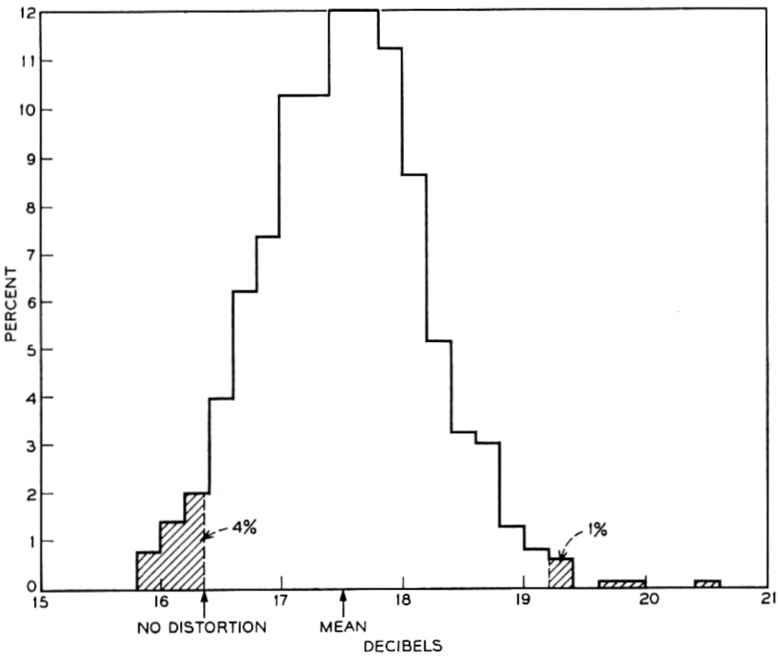

Fig. 5—Histogram of required RCP with ±1/8 timing error with impairments at RF.

This indication is quite true, as is shown by the graph of the required RCP with ±1/8 timing error versus quadratic amplitude at RF in Fig. 6. (All other impairments are set to zero.) A quadratic amplitude shaping at RF with parameter $A_2 = 0.3$ reduces the required RCP to 15.45 dB, 0.9 dB below the nominal.

It would seem desirable to build this amplitude shaping into the nominal system to take advantage of the improved performance. Yet any intentional amplitude shaping would have to be done in the IF strip, since it is extremely difficult to build filters and provide gain at RF. Unfortunately, the beneficial effect of quadratic amplitude shaping is lost when we apply it at the IF frequency. The shaping filter would follow the primary noise source in the system (the IF input amplifier), and would increase the amount of noise power entering the detector. The shaping filter would require gain,

$$A(f) = 1 + 0.3[2(f - f_c)/f_b]^2,$$

and hence increases the noise bandwidth of the system. The resulting degradation in the system performance from the increased noise power is greater than the improvement gained by amplitude shaping, as is shown in Fig. 6. The system performance is degraded by *any* quadratic amplitude distortion introduced at IF, but the effects of a negative distortion are relatively small. If quadratic shaping were introduced by a passive filter, there would be a decrease in signal power which would similarly raise the required RCP. Amplitude shaping introduced at IF degrades the system performance, and we are left in the position of having some "impairments" at RF which improve performance, but being unable to build the improvements into the system.

5.2 *Case Study #2: Impairments At IF*

Another Monte Carlo analysis was made with the impairments in the IF strip instead of at RF. This was the only difference between the two studies; the same sets of impairments were generated in both. The histogram of the required RCP with ±1/8 timing error for the impairments at IF is shown in Fig. 7. There were *no* combinations of distortions for which the system performance was better than nominal.

The impairment parameter values producing the maximum and

TABLE IV—IMPAIRMENT VALUES—MAXIMUM AND MINIMUM OBTAINED REQUIRED RCP WITH ±1/8 TIMING ERROR (IMPAIRMENTS AT RF)

	Maximum Obtained Required RCP = 20.6 dB	Minimum Obtained Required RCP = 15.8 dB
Impairment Parameter	Value at Maximum	Value at Minimum
Linear Delay d_1	-0.13τ	-0.06τ
Quadratic Delay d_2	0.19τ	0.01τ
Cubic Delay d_3	-0.12τ	0.03τ
Ripple Delay		
Peak-to-peak value d_R	-0.29τ	-0.18τ
Period P_D	$0.99 f_b$	$0.44 f_b$
Phase θ_D	29.4°	23.4°
Linear Amplitude A_1	−0.06	0.004
Quadratic Amplitude A_2	−0.18	0.19
Cubic Amplitude A_3	−0.02	0.06
Ripple Amplitude		
Peak-to-peak value A_R	−0.06	−0.01
Period P_A	$0.16 f_b$	$0.83 f_b$
Phase θ_A	58.6°	63.4°

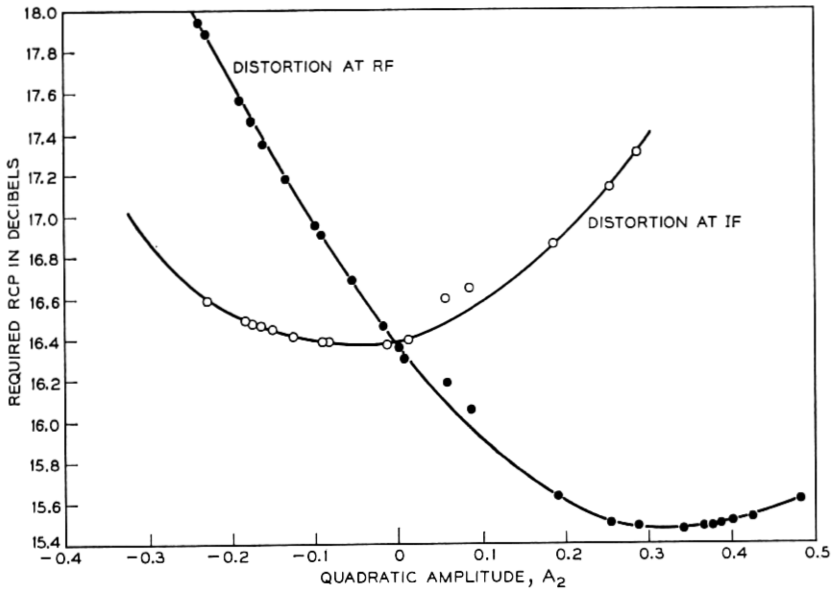

Fig. 6—Required RCP with ±1/8 timing error as a function of quadratic amplitude distortion.

minimum required RCP with ±1/8 timing error are given in Table V. The maximum required RCP obtained in the Monte Carlo analysis was caused by large negative linear, quadratic, and cubic delay components and a large slow delay ripple. The amplitude impairments, however, were quite small.

The impairments producing the minimum obtained required RCP of 16.4 dB are also tabulated in Table V. The quadratic and ripple delays are quite small, and the linear and cubic delays practically cancel each other out. The cubic and ripple amplitude distortions are very small, but the linear amplitude distortion is quite large and the quadratic amplitude is of moderate size.

Comparing the histograms in Figs. 5 and 7, we see that impairments at IF have, in general, a slightly worse effect on the system performance than those at RF: The mean of the distribution at IF is 0.1 dB higher than at RF, and the one percent point is 0.35 dB higher. Also, there were no distortions at IF for which the system performance was better than nominal, whereas four percent of the distortions at RF did improve the system performance.

The range of the distribution at RF was greater than that at IF

(15.8 dB—20.6 dB at RF, 16.4 dB—20.1 dB at IF). This is primarily due to the effects of quadratic amplitude distortion (Fig. 6). At RF, a quadratic distortion A_2 in the range -0.2 to 0.2 produces a required RCP ranging from about 15.6 dB to 17.7 dB; whereas at IF the range is from 16.4 dB to 16.9 dB.

In summary, the histograms in Figs. 5 and 7 show that the maximum required RCP observed in the two case studies was 20.6 dB, which is 1.4 dB below the 22-dB objective. Hence the eventual system impairment requirements could conceivably be less stringent than the distribution limits assumed in these Monte Carlo studies (Tables II and III).

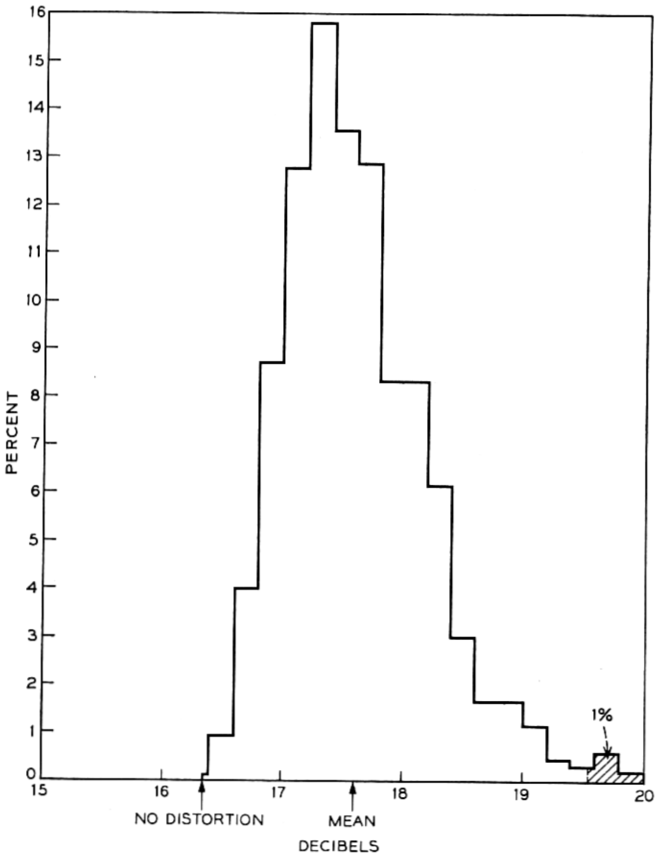

Fig. 7—Histogram of required RCP with ±1/8 timing error with impairments at IF.

TABLE V—IMPAIRMENT VALUES—MAXIMUM AND MINIMUM OBTAINED REQUIRED RCP WITH ±1/8 TIMING ERROR (IMPAIRMENTS AT IF)

	Maximum Obtained Required RCP = 20.6 dB	Minimum Obtained Required RCP = 16.4 dB
Impairment Parameter	Value at Maximum	Value at Minimum
Linear Delay d_1	-0.14τ	-0.08τ
Quadratic Delay d_2	-0.18τ	-0.002τ
Cubic Delay d_3	-0.15τ	0.12τ
Ripple Delay		
Peak-to-peak value d_R	0.28τ	0.003τ
Period P_D	$0.94 f_b$	$0.40 f_b$
Phase θ_D	82.8°	70.8°
Linear Amplitude A_1	0.008	−0.13
Quadratic Amplitude A_2	−0.003	−0.09
Cubic Amplitude A_3	0.07	0.006
Ripple Amplitude		
Peak-to-peak value A_R	0.03	−0.02
Period P_A	$0.41 f_b$	$0.82 f_b$
Phase θ_A	5.1°	84.0°

VI. SUMMARY AND CONCLUSIONS

Monte Carlo techniques can be used to study the effects of parameter variability in systems as well as circuits: Statistical distributions are used to model variations in system characteristics rather than tolerances on circuit components. In particular, this paper has shown how Monte Carlo methods can be applied to the analysis of impairments in the waveguide transmission system. Two examples of the Monte Carlo analysis have been presented. These examples show how the Monte Carlo technique provides insight into the effects of impairments on the system performance, and can aid in determining the system impairment allocations.

Monte Carlo analysis can be used in the solution of future waveguide system problems, such as the problems of component variability and tolerance allocation. The effects of a particular choice of component tolerance requirements and the percentage of randomly assembled repeaters which meet the RCP objectives can be determined by Monte Carlo analysis. The effects of tighter tolerances and the need for component matching or tuning can also be determined. As more information becomes available on component variability in manufacturing and aging, it can be incorporated into the Monte Carlo analysis and its effects can be readily evaluated. This capability for obtaining quick results from new information is expected to be of significant value to the waveguide system design effort.

Employing Monte Carlo methods at an early stage in the waveguide system design and development is expected to shorten the design interval and reduce the design effort. It is expected that Monte Carlo techniques will find further application in the study of parameter variability in other Bell System projects.

VII. ACKNOWLEDGMENTS

I have worked closely with Mark Bonomi, Joe Mullins and Peter Tu on this project and want to thank them for their many helpful discussions on the waveguide system and the waveguide system simulation program. I also want to thank Charlie Semmelman, John Logan, and Barry Karafin for their comments on and criticisms of earlier versions of this paper.

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