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Estimated Outage in Long-Haul Radio Relay Systems with Protection Switching

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A general outage formula relating a number of important system outage factors has been derived for long-haul radio relay systems utilizing in-band or crossband automatic channel switching. This formula is used to estimate the outage of TD and TH systems with various switching strategies.

The results indicate that the crossband 2 × 18 protection-to-working channel switching system appears to be a very attractive arrangement from the standpoints of good system reliability and efficient use of the radio frequency spectrum.

I. INTRODUCTION

In order to achieve the high reliability demanded for long-haul TD and TH microwave radio relay systems, frequency diversity by automatic channel switching is used as an effective means to protect services against outages due to propagation, equipment failure, and maintenance activity. In this arrangement, each radio relay route is divided into a number of switching sections. One or two of the total number of radio channels in a given band are designated as protection channels. The automatic protection switching system uses protection channels to replace failed or otherwise unavailable regular channels by operating either IF or baseband switches at the transmitting and receiving ends of the switching section.

The first protection switching system developed for use with the long-haul TD-2 system (TDAS) is capable of protecting up to five regular working channels with one protection channel (1 × 5 switching).[1] It was first placed in service in 1953, and it accounts for about 45 percent of the long-haul radio switching systems now in the field. Another frequency diversity switching system was developed for application to the 6-GHz TH-1 system which was overbuilt on TD-2 routes or used on new routes. It was initially installed in 1960 and forms about 10 percent of the switches now existing in the field. This switching system uses two protection channels to protect up to six regular channels (2×6 switching).[2] About 1960, the TD-2 radio route capacity was expanded to a total of 12 channels through the use of channels placed interstitially between the original six assignments, and a second one-for-five switching system was initially required to provide protection for these interstitial channels. A much more reliable overall protection switching arrangement could be realized by combining the regular and the interstitial channels into a two protection and ten regular working channel system (2×10 switching), and accordingly, the 100A protection switching system was developed.[3] The 100A system was originally designed to operate with both the existing TD-2 system and the new solid-state TD-3 system. Subsequently, it was modified to provide 2×6 protection for the TH-3 system. Since 1964, the 100A system has become the primary heavy-route protection facility for the microwave network in this country.

Each switching installation is now operated exclusively either in the 4-GHz band (the TD systems) or in the 6-GHz band (the TH systems), and for reasons of reliability, with the exception of TDAS, two protection channels in each band usually are specified. Thus, a total of four channels out of 20 on a fully loaded route are reserved for system protection use. In order to utilize the frequency spectrum efficiently, the number of protection channels should be kept to a minimum consistent with the system requirements on reliability. The large-scale use of solid-state devices in TD-3 and TH-3 and higher fade margins achieved in these systems should result in less system outage* with present switching arrangements. This leads to reduced demand for protection channels and the observation that a reduction in the number of protection channels might be made possible by combining 4 and 6 GHz in one crossband switching operation because the fading correlation is relatively small between 4- and 6-GHz channels.

In this paper, outage is estimated for systems using various in-band and crossband diversity switching arrangements. A general expression for the system outage of a working channel is first derived considering the number of channels, fading, equipment failure, maintenance,

* In this paper, "outage" refers to the case in which the number of channels simultaneously unavailable because of fading, equipment failure, maintenance, or any other cause exceeds the number of protection channels. It therefore implies an interruption of service to one or more of the working loads.

and other outage factors. The derived formula is then applied to various TD and TH systems to estimate the reliability* to be achieved by automatic channel switching. Recommendations of switching strategies are made, based on comparison of reliabilities and utilization of the radio frequency spectrum.

II. OUTAGE FACTORS

2.1 *Selective Fading*

The most serious source of outage encountered in current 4- and 6-GHz microwave radio systems is considered to be selective fading caused by the multipath transmission of microwave signals. The mechanism and effects of multipath transmission have been discussed quite thoroughly in numerous articles published in the past two decades.[4–6] Selective fading is characterized by deep excursions of the received microwave signal level below its free space value. The time scale varies from a few seconds for large excursions (average duration of 40-dB fades) to a few hundred seconds for small excursions (average duration of 10-dB fades). The distribution of fading variation with time depends upon variable or temporary conditions in the transmission path in a complex manner. Enough data now exists to support an empirical formula relating the most important parameters.[4] Consideration of all pertinent and available selective fading data in general indicates that the fading distribution for a single hop is characterized by a fixed slope of 10 dB of fade per decade of probability for fade depth greater than 20 dB, i.e., the slope but not necessarily the amplitude of the Rayleigh distribution. Since this type of fading is frequency selective, the use of either frequency or space diversity would be effective in reducing its effect on outage.

2.2 *Flat Fading*

In addition to selective fading, there are occasional long periods of depressed signal in a microwave path caused by "earth bulge" or obstructive fading. This type of fading is rare and occurs only under substandard atmospheric conditions. In areas of high moisture content, stable calm air conditions at night and during the hours of early morning and ground fog are usually conducive to fading of this type. Widely separated frequencies and vertically spaced antennas are

* The term "reliability" is being used in the general sense and the more appropriate terminology might be "availability."

affected alike with regard to the average signal level (Ref. 6, page 72). Therefore, protection against flat fading can be accomplished only by providing adequate vertical path clearance and restricting the length of radio paths.

2.3 *Equipment Failure and Maintenance*

Unlike selective fading, the outages due to equipment failure and maintenance are of relatively long duration. Most microwave radio stations are unmanned, and therefore, the down time of a channel due to an equipment failure is longer than the average repair time actually used. Routine maintenance also increases the hazard of outages because the number of protection channels available in a system is reduced when routine maintenance is being done on any of the channels. It has been recognized that the equipment failures and maintenance can control the system outages in a long-haul system having only one protection channel.[7–9] In the past, inband frequency diversity with two protection channels has been used effectively for both fading and equipment protection. New solid-state systems are expected to be more reliable than vacuum tube systems. Thus, lower equipment outage rates would be expected from the newer TD-3 and TH-3 systems.

2.4 *FM Terminal Failure and Others*

Solid-state 3A FM terminal equipment together with solid-state 3A wire line entrance link equipment and the 200A protection switching system (a 1 × 12 baseband to IF system) have been introduced for maintenance and baseband circuit protection. Therefore, it is expected that the level of system outage resulting from terminal failures will be small.* Other system outage factors are power failure, human error, and switch failures.† These outages can be made small in a long-haul system.

III. ANALYSIS AND COMPUTATIONAL TECHNIQUES

Here we develop a method for the calculation of 4000-mile two-way annual outage resulting from the joint effects of selective fading, equipment failure, and maintenance for a radio system with auto-

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* Reliability analysis of the FM terminals using a 200A switch indicates that the average outage from FM terminals in a 4000-mile system is less than 0.001%.[7]

† The effect of using protection channels for restoration and occasional TV service on system outage is a separate area of interest. No attempt has been made to study this effect in detail in this paper.

matic channel switching. It is noted that, without protection switching systems, the outage from the aforementioned factors in a 4000-mile route would be several hundred (about 800 in the 2 × 10 TD-2 system) times the total outage of the system with protection switching. Thus, these three factors are among the most important in the consideration of system reliability.

3.1 *Assumptions*

The basis of any estimation of the reliability of a system is a mathematical model of that system. Assumptions are made as to the behavior of this model which we expect will give the model the attributes of the working system. The following assumptions were made in the study of radio system reliability:

3.1.1 *Fading and Fading Frequency Correlation*

Fading rates are based on the MIDAS 1966 West Unity data.[4] These data, shown for a 68-day summer period (July 22 to September 28, 1966) are given in Fig. 1. The average fading rate shown in the figure represents the percent probability of the signal being faded to a certain depth (or greater) relative to free space for the total time of 68 days. Fading versus frequency correlation is based on an empirical formula developed by W. T. Barnett.[4]

3.1.2 *Fading Activity*

Fading is assumed to occur only during a six-hour period per day. This is a reasonable assumption over the fade depth of 35-40 dB and is supported by the Bryan-Wauseon data.[10] Figure 2 shows the ratio of fading rates during the midnight to 6 A.M. period of measurement divided by that observed during the 8 P.M. to midnight period. Fading activity was negligible during the remaining times.

3.1.3 *Fading Simultaneity*

Outage resulting from simultaneous fading activity in different hops in the same switching section is calculated on a daily basis. Daily fading rate assumed as a variable is given in Fig. 3.[9]

3.1.4 *Fading Hops*

Fading is assumed to occur in only one-half of the hops.

3.1.5 *Annual Fading Distribution*

Heavy fading is assumed to occur for two months, medium fading (rates reduced by 1/2) for two months, and negligible fading is assumed for the remaining eight months of the year.

3.1.6 *Switching System Operation*

To simplify the computation, the switching system is assumed to have no hysteresis, that is, no width or threshold in the "channel good" versus "channel bad" decision maker (Initiator) and no differentiation between regular and protection channels with regard to the switch point. It is also assumed that the switch point (fade depth at which a protection switch is requested) is equal to the fade margin.

3.1.7 *Joint Fading and Maintenance*

Fading activity and maintenance activity are assumed to be mutually exclusive as a result of maintenance rules.

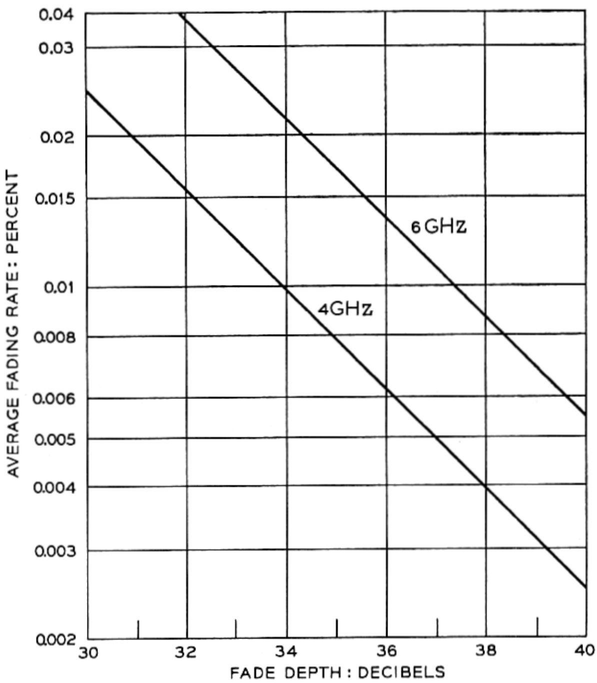

Fig. 1—Average fading rate of a 4- or 6-GHz channel based on the MIDAS West Unity data (July 22 through September 28, 1966).

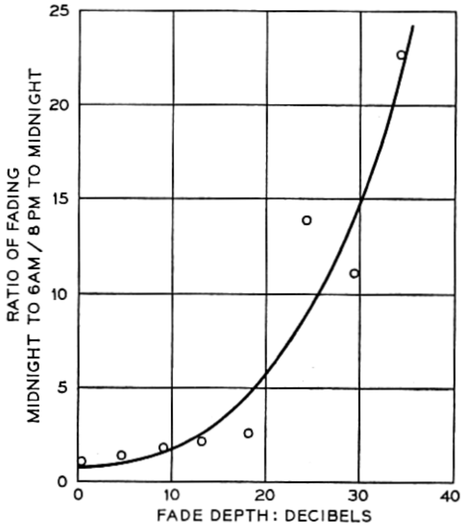

Fig. 2—Ratio of fading rates based on the Bryan-Wauseon data.

3.1.8 *Maintenance*

The maintenance rate for solid-state equipment is assumed to be the same as for TD-2, i.e., 4-1/2 hours per year per T/R bay (1-1/2 hours per T/R bay at four-month intervals). Maintenance is assumed to be concentrated during a six-hour period per day.

3.1.9 *Equipment Failure*

The failure rate for the solid-state repeater is assumed to be 0.014 failure per T/R bay per month. This represents about 3 to 1 improvement over present vacuum tube TD-2 equipment failure rates. The failure rate of TD-2 is estimated at one per T/R bay every two years, and the average time to repair an equipment failure is assumed to be 2-1/4 hours.* This corresponds to a fractional outage rate per vacuum tube T/R bay of 0.013 percent. The solid-state rate is taken to be 0.0043 percent.

3.1.10 *Two Constraints on Maintenance Operation*

(*i*) Maintenance is not to be undertaken on one channel while any other channel in the same switching section has an equipment failure.

* This is the average time required to restore a channel under equipment failure.

(*ii*) In the event of an equipment failure on any other channel while maintenance is in progress, the channel under maintenance will be restored to service within a short specified interval (say, 10 minutes).

3.1.11 *Distribution of Switching Section Lengths*

The switching section length distribution is assumed as indicated by Fig. 4. This distribution is based on the results of a 1962 TD-2 survey with the exception that 50 percent of the one-hop switching sections were deleted to compensate for the number of one-hop message sidelegs not found in a main route. The results of a 1967 TD-2 survey (with 50 percent of the one-hop sections deleted) substantially agree with the assumed distribution.

3.1.12 *Repeater Spacing*

The average repeater spacing is assumed to be 27 miles. This is based on TD-2 route data as of 6-30-65 indicating an average hop length of 26.65 miles.

3.2 *The Weighted Average Outage*

Consider a multichannel radio relay switching section having n channels of which $(m - 1)$ are protection channels. At least m simul-

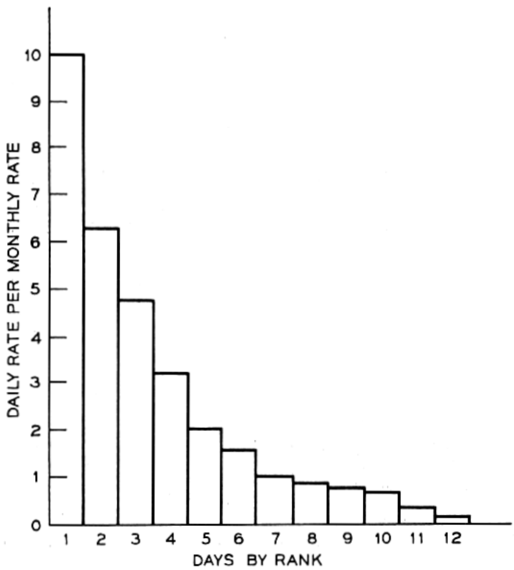

Fig. 3—Daily fading rate based on the Bryan-Wauseon data.

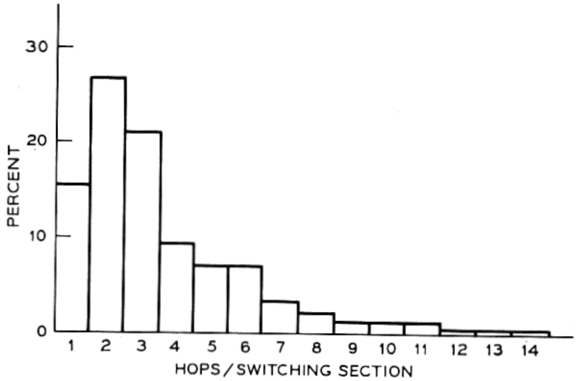

Fig. 4—Normalized distribution of switching section lengths based on the results of 1962 and 1967 TD-2 surveys with the exception that 50 percent of the one-hop sections are deleted.

taneous failures or interruptions are required before there is an actual outage or a loss of service. Those outages corresponding to more than m simultaneous channel failures should be weighted in the system outage computations since, for example, the event of two working channels failed and unprotected simultaneously is obviously twice as serious as the event of one working channel failed and unprotected at a time in a given system. Thus, we may define the weighted average outage of a working channel as follows:

ϕ_{av} = the weighted average outage of a working channel,

$$= \frac{\left[\begin{array}{c}\text{the expected number of working channels failed}\\ \text{and not protected at a time}\end{array}\right]}{[\text{the total number of working channels}]}.$$

Expressed in probability notation, we have

$$\phi_{av} = \frac{1}{n - m + 1} \sum_{j=0}^{n-m} (j + 1) \Pr (\text{exactly } (m + j)) \tag{1}$$

where Pr (exactly $(m + j)$) is the probability of exactly $(m + j)$ channels being failed simultaneously in a given system.

3.3 *Derivation of the Weighted Average Outage From a Switching Section*

Let us now compute the weighted average outage from a switching section based on equation (1). Referring to Fig. 5, the switching sec-

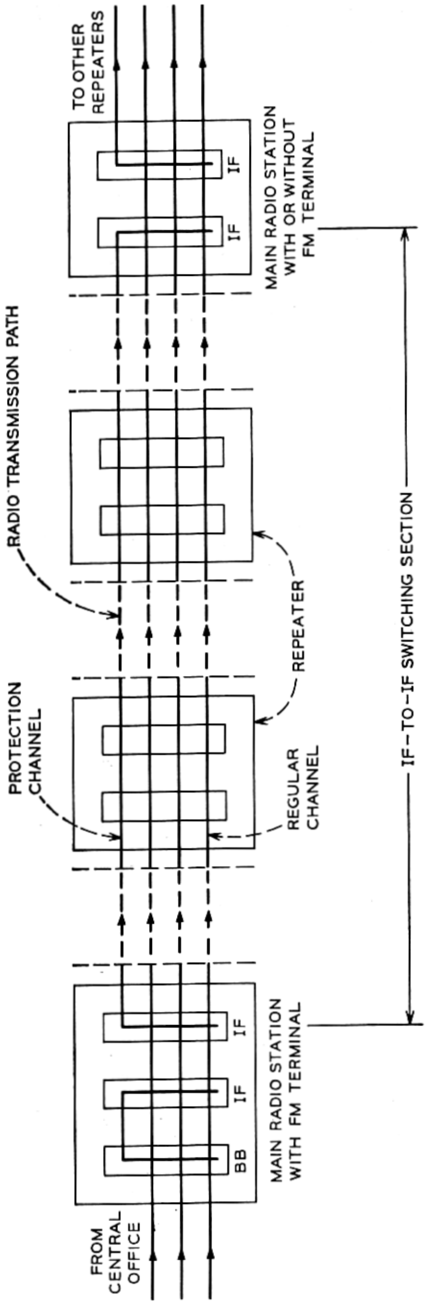

Fig. 5—Geometry of a TD or TH IF-to-IF switching system.

tion on a TD or TH route forms a link of a chain. Each switching section is completely self-contained and has no switching connection with adjacent sections. A switching section protects both directions of transmission simultaneously and independently. Thus, it is necessary only to compute one-way outage and double the result. In order to apply to MIDAS data and the empirical formulas in equation (1), it is required that we express the weighted average outage ϕ_{av} in terms of the purely simultaneous unavailability of $(m + j)$ channels, i.e., to eliminate explicit inclusion in the probability statement of channels which are not suffering a failure. It can be shown that a different representation for ϕ_{av} is

$$\phi_{av} = \frac{1}{n - m + 1} \left[\sum_{j=0}^{n-m} (-1)^j C_{m-2}^{m+j-2} \sum_{(m+j)_s} \Pr ((m + j)_s) \right] \qquad (2)$$

where C_{m-2}^{m+j-2} is the number of ways we may choose $(m - 2)$ out of $(m + j - 2)$ objects without regard to order and $(m + j)_s$ is a particular set of $(m + j)$ channels suffering simultaneous failures. The notation $\sum_{(m+j)_s} \Pr ((m + j)_s)$ means that the outage of all possible $(m + j)$ simultaneous channel failures are to be summed (there will be C_{m+j}^{n} of them). Equation (2) is a very general expression in which no restriction on channel independence has been made. Observe that equation (2) is an alternating series which converges if all channels are independent and if the failure rate of each individual channel is low. The series converges very slowly if the channel failures are highly correlated as the MIDAS data has shown. Thus, we will consider all the terms in the series related to fading for system outage computations.

The unavailability of a single channel per hop is given by

$$P_s = P_f + P_e + P_m \qquad (3)$$

where

P_f = unavailability due to multipath fading,
P_e = unavailability due to equipment failure, and
P_m = unavailability due to maintenance.

The unavailability of a single channel in a switching section is then modified as follows:

$$P_{ss} = rP_f + hP_e + hP_m , \qquad 0 \leqq r \leqq h, \qquad (4)$$

where h is the number of hops in the switching section, and r is the number of hops in the switching section which are liable to experience heavy fading.

In order to apply equation (2) for the computation of outage from a switching section, we first assume that the channels fail independently of each other, i.e., that the failure of an arbitrary set of channels does not change the availability of the other channels, the correlation among channels due to fading will be considered later in our mathematical development. When all channels are independent, we obtain from equation (2)

$$\phi_{av} = \frac{1}{n - m + 1}\left[\sum_{j=0}^{n-m} (-1)^j C_{m-2}^{m+j-2} C_{m+j}^{n} P_{ss}^{m+j}\right] \tag{5}$$

where P_{ss} is the unavailability of a channel in a switching section given by equation (4).
Let

$$\begin{aligned} P &= C_{m+j}^{n} P_{ss}^{m+j}, \\ &= C_{m+j}^{n}(rP_f + hP_e + hP_m)^{m+j}. \end{aligned} \tag{6}$$

By grouping the first and third terms in the parentheses of equation (6) and applying the binomial expansion, one gets

$$\begin{aligned} P &= C_{m+j}^{n} \sum_{k=0}^{m+j} C_k^{m+j}(rP_f + hP_m)^{m+j-k}(hP_e)^k, \\ &= \sum_{k=0}^{m+j} C_k^{n-(m+j-k)} C_{m+j-k}^{n}(rP_f + hP_m)^{m+j-k}(hP_e)^k. \end{aligned} \tag{7}$$

Further expand the common factor $(rP_f + hP_m)^{m+j-k}$ in equation (7) as follows:

$$\begin{aligned} &(rP_f + hP_m)^{m+j-k} \\ &\quad = \sum_{s=0}^{m+j-k} C_s^{m+j-k}(rP_f)^{m+j-k-s}(hP_m)^s, \qquad m + j - k \neq 0 \\ &\quad = 1, \qquad m + j - k = 0. \end{aligned} \tag{8}$$

Those cross terms in equation (8) which have factors $P^{m+j-k-s} P_m^s$ for $m + j - k - s \geqq 1$ and $s \geqq 1$ will be ignored because daytime maintenance routines have been put into effect in practice, and therefore, selective fading and maintenance are not considered to occur simultaneously. Thus, equation (8) is simply reduced to

$$\begin{aligned} (rP_f + hP_m)^{m+j-k} &= (rP_f)^{m+j-k} + (hP_m)^{m+j-k}, \qquad m + j - k \neq 0, \\ &= 1, \qquad m + j - k = 0. \end{aligned} \tag{9}$$

Substitution of equation (9) in equation (7) yields

$$P = \left\{C^n_{m+j} + \sum_{k=0}^{m+j-1} C_k^{n-(m+j-k)} C^n_{m+j-k}[(rP_f)^{m+j-k} + (hP_m)^{m+j-k}]\right\} \cdot (hP_e)^k. \tag{10}$$

From equations (5), (6) and (10), we obtain

$$\phi_{\text{av}} = \frac{1}{n-m+1} \left\{ \sum_{j=0}^{n-m} (-1)^j C^{m+j-2}_{m-2} \cdot \left\{C^n_{m+j} + \sum_{k=0}^{m+j-1} C_k^{n-(m+j-k)} C^n_{m+j-k}[(rP_f)^{m+j-k} + (hP_m)^{m+j-k}]\right\} \cdot (hP_e)^k \right\}. \tag{11}$$

Observe that all those terms in equation (11) having factor P_m^{m+j-k} with $m + j - k \geqq 2$ vanish because maintenance is a controlled activity, and the rules of operation prohibit simultaneous maintenance of two or more channels in the same direction in a switching section (i.e., maintenance is assumed to be done on one channel at a time). It follows, therefore, from equation (11) that

$$\begin{aligned} \phi_{\text{av}} = \frac{1}{n-m+1} & \cdot \left\{ \sum_{j=0}^{n-m} \left[(-1)^j C^{m+j-2}_{m-2} \cdot \sum_{k=0}^{m+j-1} C_k^{n-(m+j-k)} C^n_{m+j-k} (rP_f)^{m+j-k} (hP_e)^k \right] \right. \\ & + \sum_{j=0}^{n-m} (-1)^j C^{m+j-2}_{m-2} C^{n-1}_{m+j-1} C^n_1 (hP_m)(hP_e)^{m+j-1} \\ & \left. + \sum_{j=0}^{n-m} (-1)^j C^{m+j-2}_{m-2} C^n_{m+j} (hP_e)^{m+j} \right\}. \end{aligned} \tag{12}$$

It may be verified that the last two series on the right hand side of equation (12) converge rapidly when the factors P_m and P_e are small and independent. Thus far, we have considered only events of outage arising from independent channels in a given system. Let us now impose multipath fading. This will modify term by term the first series on the right hand side of equation (12). We see that a common factor in the series is

$$C^n_{m+j-k}(rP_f)^{m+j-k}$$

which denotes the probability of $(m + j - k)$ uncorrelated channels fading simultaneously. Actually, simultaneous fading can occur in two ways; either all the fading may occur in one hop and be highly correlated, or it may occur in different hops in the section with less correlation. Since there are r fading hops, all fades in the same hop can occur r ways, and fades in t different hops can occur C_t^r ways (without regard to order). The probability P_{ff} of simultaneous fading of $(m + j - k)$ correlated channels becomes

$$P_{ff} = \sum_{t=0}^{r} [C_t^r \sum_{(m+j-k)_s} P_t((m + j - k)_s)] \tag{13}$$

where $\sum_{(m+j-k)_s} P_t((m + j - k)_s)$ is the unavailability of all possible $(m + j - k)$ simultaneous failures in t different hops of a switching section. For example, if three channels fade simultaneously in two hops, i.e., $m + j - k = 3$ and $t = 2$, then we have

$$\sum_{(m+j-k)_s} P_t((m + j - k)_s) = \sum_{3} P_2(3), \tag{14}$$

where $P_2(3)$ is the unavailability of three arbitrary channels in the set of n which fade simultaneously in two different hops. Therefore, if all channels are correlated due to fading, we may write equation (12) as

$$\begin{aligned} \phi_{\text{av}} = \frac{1}{n - m + 1} \Bigg\{ & \sum_{j=0}^{n-m} \sum_{k=0}^{m+j-1} \sum_{t=0}^{r} (-1)^j \, D_{m-2}^{m+j-2} C_k^{n-(m+j-k)} C_t^r \\ & \cdot [\sum_{(m+j-k)_s} P_t((m + j - k)_s)](hP_e)^k \\ & + \sum_{j=0}^{n-m} (-1)^j C_{n-2}^{m+j-2} C_{m+j-1}^{n-1} n(hP_m)(hP_e)^{m+j-1} \\ & + \sum_{j=0}^{n-m} (-1)^j C_{m-2}^{m+j-2} C_{m+j}^{n} (hP_e)^{m+j} \Bigg\}. \end{aligned} \tag{15}$$

Since the fading is assumed to occur only during a six-hour period and since we want an average outage, the terms in the first series of equation (15) having the factor

$$\sum_{(m+j-k)_s} P_t((m + j - k)_s)$$

should be multiplied by a time weighting factor τ^{t-1} where τ in this case is 4, and fading in different hops should involve a factor α_t to account for the peakedness of daily fading among t fading hops.* Let us designate the first series in equation (15) by S_1. Then the complete expression for S_1 becomes

* The expression of α_t in terms of t is given in Appendix B.

$$S_1 = \frac{1}{n-m+1}\left\{\sum_{j=0}^{n-m}\sum_{k=0}^{m+j-1}\sum_{t=0}^{r}(-1)^j C_{m-2}^{m+j-2} C_k^{n-(m+j-k)} C_t^r \tau^{t-1}\right.$$
$$\left.\cdot \alpha_t \Big[\sum_{(m+j-k)_s} P_t((m+j-k)_s)\Big](hP_e)^k\right\}. \qquad (16)$$

The formula can be modified for reduced fading rates in other months by expressing the fading rate as $b_i P_f / B_i$ where $b_i < 1$, $B_i \geqq 1$ to account for fading in other months. In nonfading months, $b_i = 2/3$, $B_i = \infty$; in medium fading months, $b_i = 1/6$, $B_i = 2$; and in heavy fading months, $b_i = 1/6$, $B_i = 1$. The joint equipment failure and maintenance outage term in equation (12) should also be modified, based on maintenance constraints. Let

$$P(m, e) = P_m P_e^{m+j-1} \qquad (17)$$

Reference to Appendix A will show that this expression may be modified as follows:

$$P(m, e) = P_m P_{EQ}^{m+j-1} \qquad (18)$$

where

P_{EQ} = the equivalent equipment failure probability;

$\quad = \frac{\tau_r}{\tau_e} P_e$;

τ_e = assumed duration of one equipment failure; and

τ_r = time required to restore a channel under maintenance to service in the event of an equipment failure on any other channel.

Summarizing then, the weighted average outage of a working channel, as defined in equation (1), in a switching section one may be expressed as:

$$\phi_{av} = \frac{1}{n-m+1}\left\{\sum_{i=1}^{l}\sum_{j=0}^{n-m}\sum_{k=0}^{m+j-1}\sum_{t=0}^{r}(-1)^j C_{m-2}^{m+j-2} C_k^{n-(m+j-k)} C_t^r \tau^{t-1}\alpha_t\right.$$
$$\cdot \frac{b_i}{B_i^t}\Big[\sum_{(m+j-k)_s} P_t((m+j-k)_s)\Big](hP_e)^k$$
$$+ \sum_{j=0}^{n-m}(-1)^j C_{m-2}^{m+j-2} C_{m+j-1}^{n-1} n h^{m+j} P_m P_{EQ}^{m+j-1}$$
$$\left.+ \sum_{j=0}^{n-m}(-1)^j C_{m-2}^{m+j-2} C_{m+j}^{n} (hP_e)^{m+j}\right\} \qquad (19)$$

where

n = total cross-sectional number of channels in the system;
$m - 1$ = number of protection channels;
h = number of hops in the switching section;
r = number of fading hops in the switching section;
τ = hourly variation parameter;
α_t = daily variation parameter;
b_i , B_i = monthly variation parameters; and
l = upper limit for the index of monthly variation parameters.

3.4 *Empirical Formulas for the Correlated Fading Channels in the Same Hop* ($t = 1$)

An empirical formula derived by W. T. Barnett[11] can be used to evaluate the simultaneous fading of an arbitrary set of $(m + j - k)$ channels at 4 and 6 GHz:

$$P_1((m + j - k)_s) = \frac{2^{(2l/m+j-k)} L^4}{\dfrac{4}{m + j - k} \sum_{ij} \Delta_{ij}} \tag{20}$$

where

$m + j - k$ = total number of channels in set;
l = number of 6-GHz channels in set; and
$\sum_{ij} \Delta_{ij}$ = sum of fractional frequency difference between all possible pairs in the set (there are C_2^{m+j-k} of them).

The value of (Δ_{ij}) can be calculated when the pair is from the same common carrier band. For the crossband case when one channel is from the 6-GHz band, a value of 0.05 has been used.[12]

3.5 *Annual Outage From a 4000-mile Two-Way Circuit*

Results given in this work are based on annual outage* for a 4000-mile two-way system. In order to evaluate the equations derived so far, it is necessary to assume values for h, the number of hops in a switching section. In actual computation, the distribution of hop length and switching section in a 4000-mile route are used. The computation procedure involves computing switching section performance for various combinations of h and r, multiplying the (h, r) reliability estimate by the appropriate weighting factor determined from the data, summing the results and extrapolating the answer to give an estimate of 4000-mile two-way performance.

* Including outage due to flat fading.

IV. RELIABILITY OBJECTIVES

The usual measure of reliability is outage, i.e., the total time that the circuit does not deliver the signal information within the limits of acceptable performance. This is generally considered to be at a noise level of 55 dBrnc0 or worse, or no service. The outage that can be tolerated is generally based on the worst fading month. However, there is an approximate relation between the outages in the worst month and yearly outage, so the radio relay system requirement is specified on a yearly basis. The present reliability objective for a long-haul microwave system (4000 miles two-way) is that outage time of a channel should not exceed 0.02 percent or 1-3/4 hours per year. This objective has historically been allocated among contributory causes as follows:

(*i*)	Outage due to flat fading	$\leqq$ 0.01%
(*ii*)	Outage due to selective fading equipment failure and maintenance	$\leqq$ 0.008%
(*iii*)	Outage due to FM terminal failure and others	$\leqq$ 0.002%
(*iv*)	Total	$\leqq$ 0.02%

V. RESULTS

The general expression developed in Section III was used to compute the outages for various TD and TH systems. Figure 6* shows the TD-2 1 $\times$ 5 switching performance at 4 GHz for the six original channels 1, 2, 3, 4, 5, 6 or the six interstitial channels 7, 8, 9, 10, 11, 12. A value of 0.013 percent is assumed for the unavailability due to equipment failure (see Section 3.1.9). It is seen that, for these systems, the 0.02 percent outage objective cannot be met even with very high fade margin because of the high equipment failure rate for vacuum tube systems. The effect of equipment failure can be seen by comparing the bottom two curves in the figure. As a measure of the sensitivity of these results to changes in equipment failure rate, the outage was recomputed after doubling and then after redoubling the equipment failure rate. For high fade margins, the equipment failure rate controls the system outage.

* In this figure, the annual average outage in percent is expressed in terms of fade margin which is a fixed number for a given system design. For example, the fade margin of a given system may be 35 dB. The reason for assuming the fade margin to be a variable is to show the controlling factors and how they change with different switching strategies.

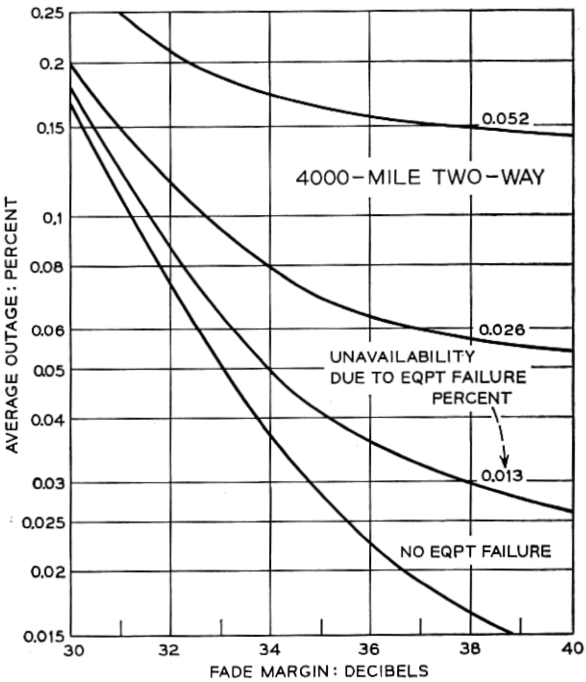

Fig. 6—Expected average outage of a vacuum tube 4-GHz system with 1 × 5 switching.

In Fig. 7, 1 × 5 switching is contrasted with 2 × 10 switching. It is noted that a reduction in the outage floor is obtained by virtue of the second protection channel. In contrast to the 1 × 5 system, the outage from a 2 × 10 system is a strong function of fade margin, and equipment failure no longer controls the system outage. The 1 × 11 curve illustrates the result of full time use of one protection channel of a 2 × 10 system; the "1 × 11 no maintenance" condition is obtained during part time use of one of the two protection channels in a 2 × 10 system when maintenance is deferred as for example when restoration is in progress. It is seen that the objective can be met at fade margin of 35.9 dB.

Figure 8 shows a set of comparable curves for 6-GHz systems.* The 2 × 6 system meets the reliability objective at 39.6-dB fade margin. Since the outage from selective fading at 6 GHz is higher than at 4

* In Fig. 8, the value of 0.0043 percent is the unavailability per T/R bay per channel due to solid state equipment failure as indicated in Section 3.1.9.

GHz, the outage is a strong function of fading margin for both the 2 × 6 and 1 × 7 arrangements.

Figure 9 shows the limits for a vacuum tube system operating in crossband diversity up to 2 × 18 capacity and, of course, 2 × 12, 2 × 14, etc., would fall between these curves.

The curves in Fig. 10 show the insensitivity of a 2 × 18 system to reasonable equipment failure rates. Again, an unavailability of 0.013 percent is assumed for vacuum tube equipment failures. Note the effect of doubling this rate once, twice, three times. The conclusion to be made is that 2 × 18 reliability is relatively insensitive to the equipment failure rate.

The effect of using one protection channel for other service, full or part time, in a 2 × 18 system is indicated in Fig. 11.

The outage from a 3 × 17 system was also computed as shown in Fig. 12 in contrast with the 2 × 10 and 2 × 18 systems. The improvement using three protection channels is not large because, in this sys-

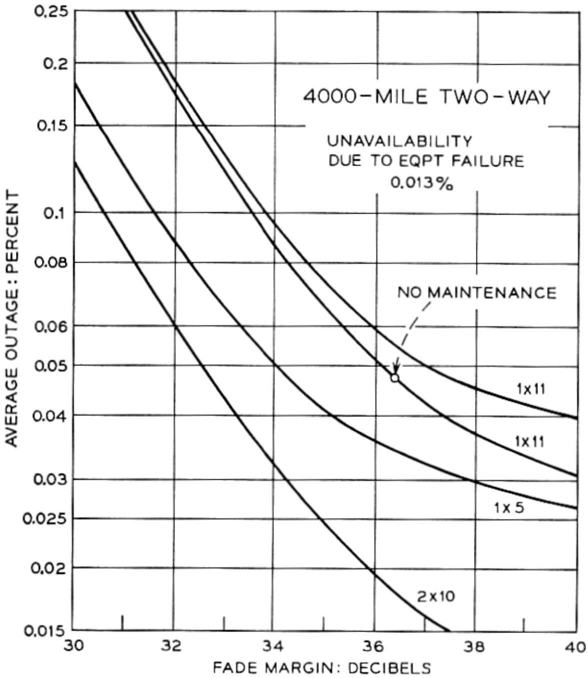

Fig. 7—Expected average outage of a vacuum tube 4-GHz system with various switching strategies.

tem, outage is controlled by the fading, and it is indicated from the MIDAS data that fading is a highly correlated phenomenon.

VI. OUTAGE FROM A SYSTEM IN THE ABSENCE OF SELECTIVE FADING

A question arises as to whether, by use of space diversity,[12] one protection channel might be enough to achieve the operational objectives in a long-haul system. To answer this question, we compute the outage by suppressing the selective fading terms in the total system outage expression, i.e., assume that there is no selective fading on the propagation paths. Table I shows the results of this computation. Note that the circled items exceed the basic 0.02 percent objective. All the circled items correspond to vacuum tube systems having only one protection channel. We may thus say that one protection channel is not enough for vacuum tube equipment protection in a long-haul system.

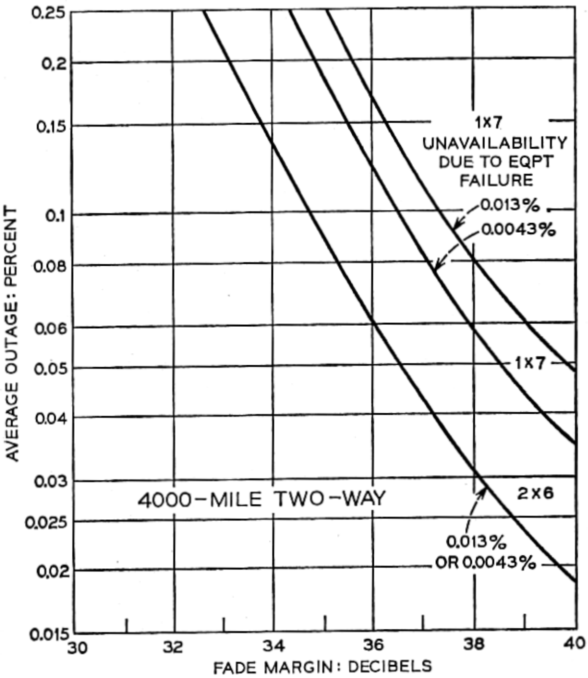

Fig. 8—Expected average outage of a vacuum tube or solid-state 6-GHz system with 2 × 6 and 1 × 7 switching.

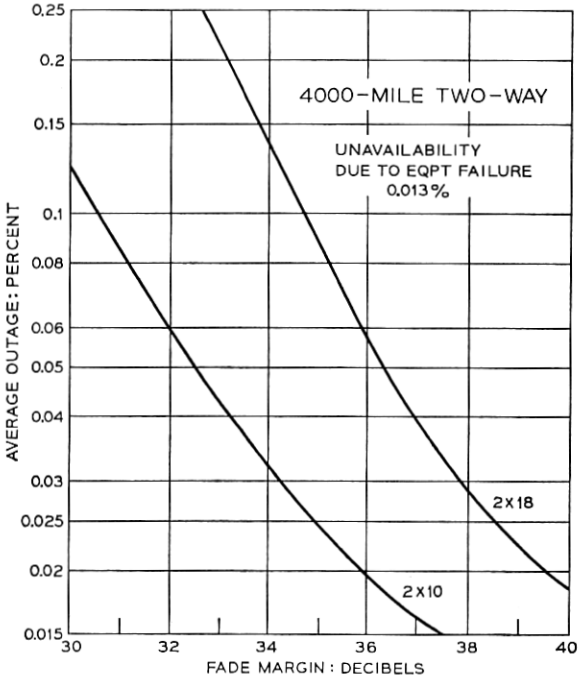

Fig. 9—Expected average outage of a 4- and 6-GHz crossband 2 × 18 system versus the 2 × 10 system.

VII. CONCLUSIONS AND RECOMMENDATIONS

The reliability of long-haul radio relay systems utilizing automatic channel switching is estimated under a few realistic assumptions. The results are illustrated in figures for various arrangements for comparison. The conclusions and recommendations drawn from these reliability curves are as follows:

(*i*) System outage is a strong function of fade margin when the fade margin is low (~ 30 dB) for all cases.

(*ii*) In systems having one protection channel, the variation of outage is strongly influenced by the equipment failure rate. With two protection channels, the outage rate is not very sensitive to equipment failure.

(*iii*) A reduction of the total number of protection channels can be achieved by crossband operation without seriously affecting the system reliability.

(*iv*) For vacuum tube systems, one protection channel does not offer adequate reliability and should not be used for long-haul applications.

(*v*) Two protection channels are sufficient to protect up to at least 18 crossband working channels (vacuum tube or solid state) assuming a fade margin of 40 dB. Multipath fading will be the primary outage variable beyond the 0.012 percent flat fading and terminal allowances.

VIII. ACKNOWLEDGMENT

The author would like to thank Messrs. H. D. Griffiths, F. H. Lanigan, and R. O. Davidson for helpful discussions on the switching systems, U. A. Matson for discussions on FM terminal equipment and switching, Mrs. B. A. Fink and Mrs. J. M. Littlefield for computer programming, and Messrs. A. C. Longton and R. P. Slade for suggestions and guidance during the course of this work.

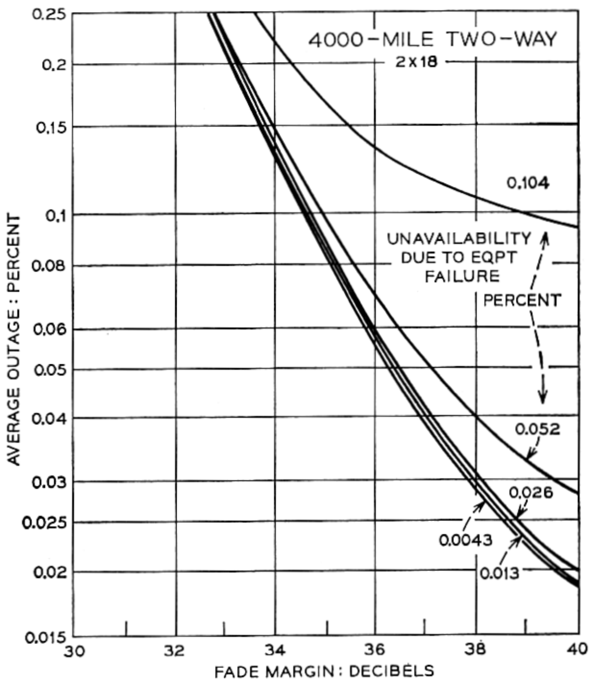

Fig. 10—Sensitivity of a 2 × 18 system to equipment failures.

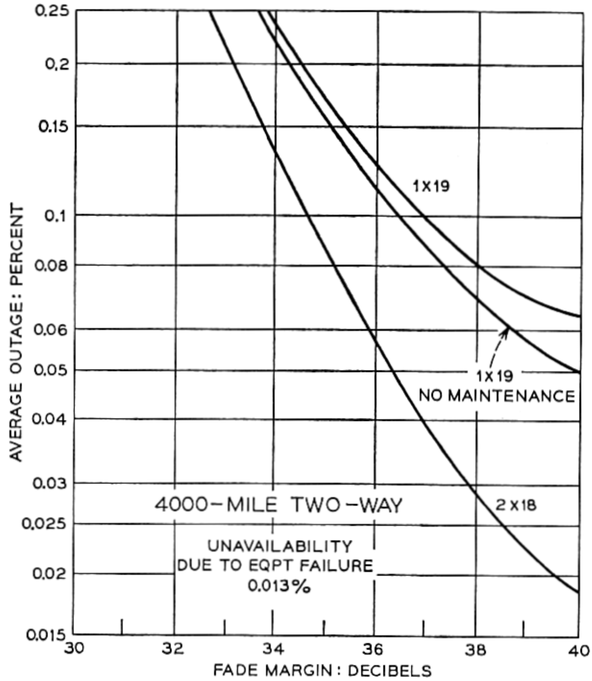

Fig. 11—Expected average outage of crossband 1 × 19 and 2 × 18 vacuum tube systems.

APPENDIX A

Outage Due to Equipment Failure and Maintenance with Constraints

This appendix develops a method to compute the outage in a multichannel radio system due to the joint effects of equipment failures and maintenance when two maintenance constraints are applied.

A.1 *Outage Due to Joint Equipment Failure and Maintenance Without Constraints*

Let us consider the simultaneous unavailability of $(l + 1)$ radio channels in which l channels experience equipment failure and one channel is being maintained. Over an interval of time τ, the per-bay unavailabilities due to equipment failure and maintenance are given by

$$P_e = \frac{\rho_e \tau_e}{\tau} \tag{21}$$

$$P_m = \frac{\rho_m \tau_m}{\tau} \tag{22}$$

where

τ_e = duration of one equipment failure;
τ_m = duration of one maintenance routine on a radio bay;
ρ_e = equipment failure rate over the interval τ; and
ρ_m = maintenance rate over the interval τ.

When P_e and P_m are independent, the outage rate due to equipment failure and maintenance without constraints is given by

$$P(m, e) = P_m P_e^l = \frac{\rho_m \rho_e^l}{\tau^{l+1}} \tau_m \tau_e^l . \tag{23}$$

A.2 *Evaluation of Outages with Constraints*

Let us consider first the case of one protection channel ($l = 1$). When $l = 1$, equation (23) becomes

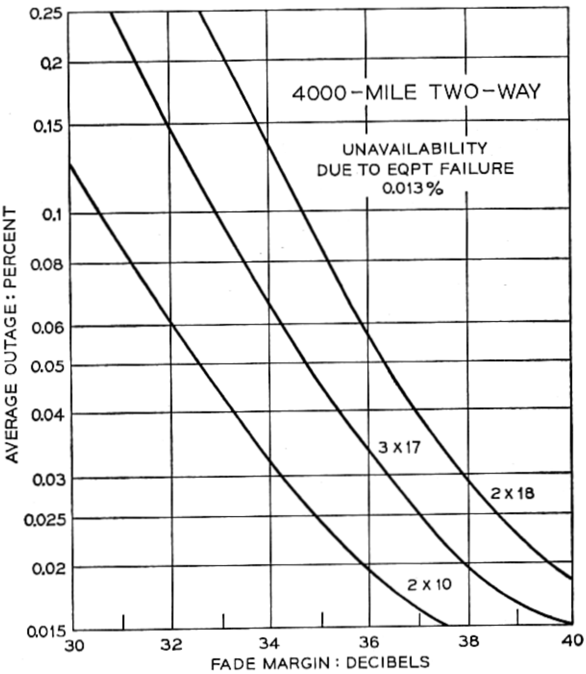

Fig. 12—Expected average outage of a 3 × 17 crossband system versus the 2 × 10 and 2 × 18 systems.

TABLE I—4000-MILE TWO-WAY OUTAGE DUE TO FLAT FADING, EQUIPMENT FAILURE, MAINTENANCE, AND FM TERMINALS, NO SELECTIVE FADING INCLUDED

Switching System	Band	Outage in Percent		
		$P_e = 0.0043\%$ (Solid State)	$P_e = 0.013\%$ (Vacuum Tube)	$P_e = 0.026\%$ (Vacuum Tube)
1×5	4	0.0143	0.0241	0.0515
2×10	4	0.0120	0.0121	0.0124
1×11	4	0.0166	0.0362	0.0910
2×6	6	0.0120	0.0120	0.0122
1×7	6	0.0151	0.0281	0.0647
2×18	$4 + 6$	0.0120	0.0122	0.0132
1×19	$4 + 6$	0.0197	0.0523	0.1440

$$P(m, e) = \frac{\rho_m \rho_e}{\tau^2} \tau_m \tau_e \,. \tag{24}$$

Let S be the starting time of an equipment failure with respect to the starting time of maintenance. It can be shown that equation (24) can be expressed in a different form:

$$P(m, e) = \frac{\rho_m \rho_e}{\tau^2} \int_{-\infty}^{\infty} F(S)\, dS \tag{25}$$

where $F(S)$ is the outage distribution shown in Fig. 13a. From equation (25), we see that $P(m,e)$ is proportional to the area under the curve $F(S)$.

Now impose the first constraint: scheduled maintenance will not be started during an equipment failure on another channel. This constraint implies that an outage will result only for equipment failure starting after maintenance is started, i.e., $S \geqq 0$. The outage distributions with the first constraint are reduced to those shown in Fig. 13b. The outage $P(m, e)$ can thus be obtained by integrating over the appropriate regions. The results are given in column 4 of Table II. It is seen that, with the first constraint, the expression of outage is multiplied by a factor which is less than 1.

In addition to the first constraint, apply the second constraint: if there is an equipment failure during maintenance, the channel under maintenance is to be restored to service in a period of time τ_r where $\tau_r < \tau_m$, τ_e. This constraint implies that rapid restoration of the maintained channel limits the outage interval to τ_r or less. The outage dis-

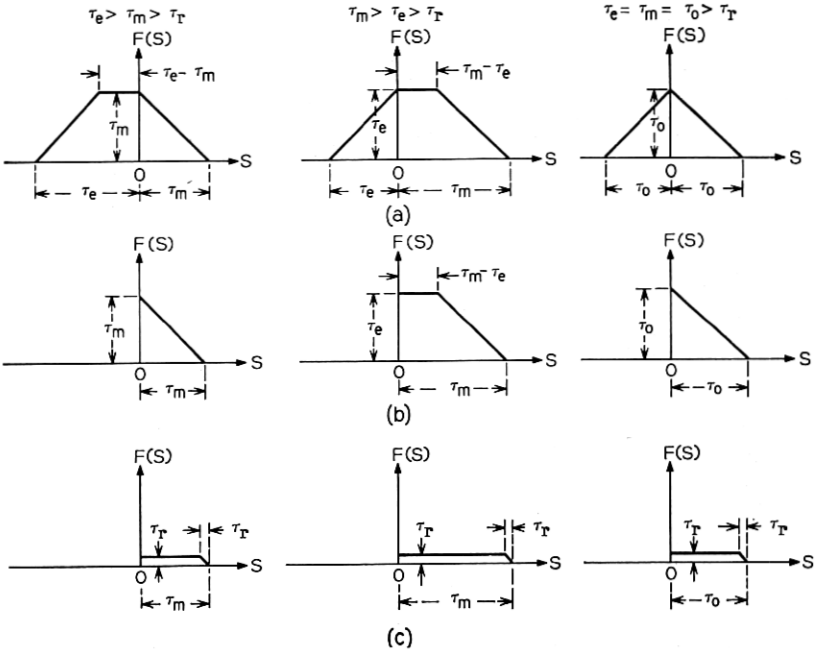

Fig. 13—Outage distributions of one variable. (a) Without maintenance constraints. (b) With the first constraint. (c) With both the first and second constraints.

tributions $F(S)$ when both constraints are applied are shown in Fig. 13c. The solutions are listed in column 5 of Table II. It is noted that the solution with both constraints is in the same form for all cases, although this is not true for the solutions obtained with the first constraint alone. Therefore, it is concluded that the first and second constraints are interrelated. The effect of the first constraint on outage is partially wiped out when the second constraint is imposed. When $\tau_r \ll \tau_m$ which is the usual case, the solutions in column 5 of Table II can be approximated by those corresponding ones in the last column of the same table. These approximations indicate that the effect of the constraints is equivalent to limiting each equipment outage to τ_r minutes.

The one-dimensional model can be extended to a two-dimensional one for the case of two protection channels ($l = 2$). The outage rate due to maintenance and double equipment failure will be of the form $P_m P_e^2$. Let S_1 and S_2 denote the starting time of equipment failures #1 and #2, respectively, with respect to the starting time of maintenance; the

TABLE II—SOLUTIONS OF OUTAGES WITH CONSTRAINTS

Number of Protection Channels	Relative Lengths of Intervals	Outage Due to Equipment Failure and Maintenance			
		No Constraints	First Constraint	First and Second Constraints	
		Exact	Exact	Exact	Approx.
1	$\tau_m < \tau_e$	$P_m P_e$	$\frac{1}{2}\frac{\tau_m}{\tau_e} P_m P_e$	$\left(1 - \frac{\tau_r}{2\tau_m}\right) P_m P_{EQ}$	$P_m P_{EQ}$
	$\tau_m = \tau_e$	$P_m P_e$	$\frac{1}{2} P_m P_e$	$\left(1 - \frac{\tau_r}{2\tau_m}\right) P_m P_{EQ}$	$P_m P_{EQ}$
	$\tau_m > \tau_e$	$P_m P_e$	$\left(1 - \frac{1}{2}\frac{\tau_e}{\tau_m}\right) P_m P_e$	$\left(1 - \frac{\tau_r}{2\tau_m}\right) P_m P_{EQ}$	$P_m P_{EQ}$
2	$\tau_m < \tau_e$	$P_m P_e^2$	$\frac{1}{3}\left(\frac{\tau_m}{\tau_e}\right)^2 P_m P_e^2$	$\left(1 - \frac{2}{3}\frac{\tau_r}{\tau_m}\right) P_m P_{EQ}^2$	$P_m P_{EQ}^2$
	$\tau_m = \tau_e$	$P_m P_e^2$	$\frac{1}{3} P_m P_e^2$	$\left(1 - \frac{2}{3}\frac{\tau_r}{\tau_m}\right) P_m P_{EQ}^2$	$P_m P_{EQ}^2$
	$\tau_m > \tau_e$	$P_m P_e^2$	$\left(1 - \frac{2}{3}\left(\frac{\tau_e}{\tau_m}\right)^2\right) P_m P_e^2$	$\left(1 - \frac{2}{3}\frac{\tau_r}{\tau_m}\right) P_m P_{EQ}^2$	$P_m P_{EQ}^2$

outage is then a function of S_1 and S_2. Assume that the durations of the two equipment failures are equal. The outage distribution for various situations is shown pictorially in Fig. 14. The outage rate for maintenance and double equipment failure can be derived by using the following integral which is an extension of Equation (25).

$$P(m, e) = \frac{\rho_m \rho_e^2}{\tau^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(S_1, S_2)\, dS_1\, dS_2 \tag{26}$$

where $F(S_1, S_2)$ is the outage distribution of two variables given in Fig. 14. Observe that $P(m, e)$ is proportional to the volume under the surface $F(S_1, S_2)$. The effect of the first and second constraints on $F(S_1, S_2)$ is also shown in Fig. 14. The solutions for various cases are presented in the lower half of Table II.

It is seen that, when $\tau_r < \tau_m, \tau_e$, an approximate solution for $P(m, e)$ with both maintenance constraints for arbitrary l can be expressed by

$$P(m, e) = P_m P_{EQ}^l \tag{27}$$

where

$$P_{EQ} = \frac{\tau_r}{\tau_e} P_e . \tag{28}$$

Equations (27) and (28) indicate that the effect of the two constraints is once again equivalent to limiting the durations of equipment failures to a short period equal to τ_r. The equivalent equipment unavailability is then equal to $(\tau_r/\tau)\rho_e$ which is equivalent to equation (28). Comparing with the outage given by equation (23), we see that the outage with constraints can be treated by replacing P_e by P_{EQ} in the expression without constraints.

APPENDIX B

Derivation of the Daily Variation Parameter, α_t

The purpose of this appendix is to explain the use of the parameter α_t in the outage expressions for fading in different hops.[10]

Consider t channel failures due to fading which occur simultaneously in t different hops of a switching section. Assuming independence of fading among hops, the probability of t simultaneous failures is P_f^t, where P_f is the probability of a fade on a single channel and fades in different hops on different channels are assumed here to have the same probability, since P_f is normally given in terms of a monthly average. If data on which the P_f is based is examined in greater detail, it will be

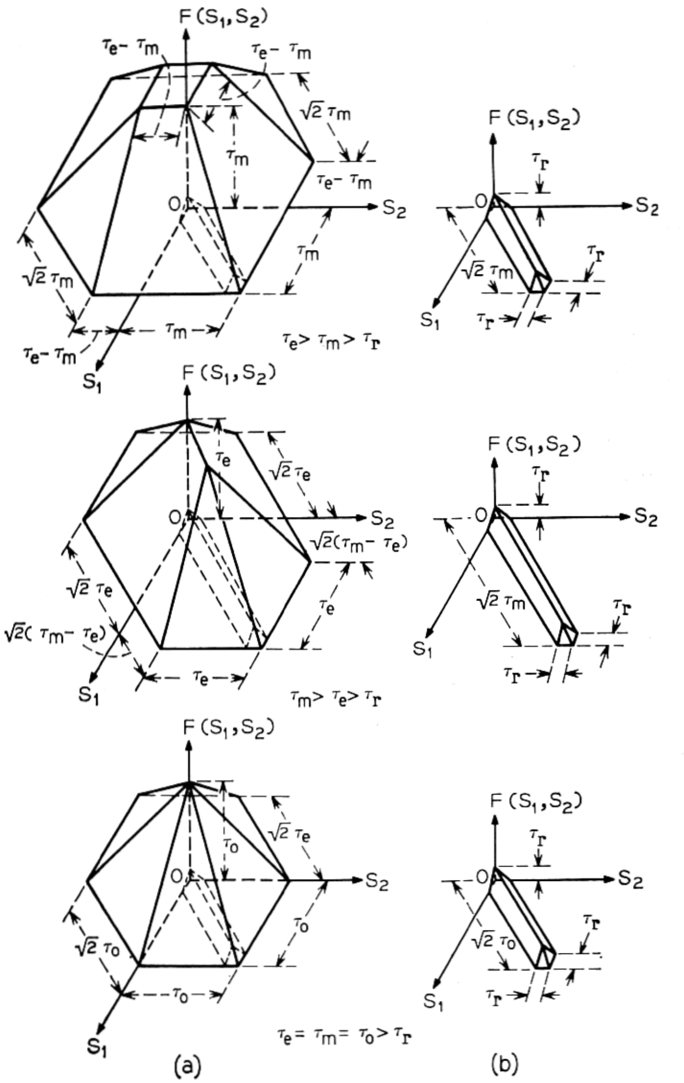

Fig. 14—Outage distributions of two variables. (a) Without constraints. (b) With constraints.

noticed that this fading is not equally distributed among the days of the month. Some days have considerably more fading than others. If P_{fi} represents fading on the ith day, then P_f may be expressed as

$$P_f = \frac{1}{T} \sum_{i=1}^{T} P_{fi} . \tag{29}$$

The joint fading on the ith day is

$$P_{fi}^t, \tag{30}$$

and the average daily fading rate for the period would then be

$$\frac{1}{T} \sum_{i=1}^{T} P_{fi}^t . \tag{31}$$

Each day's fading can be related to the monthly fading rate by the expression

$$P_{fi} = \overline{\alpha_i} P_f \tag{32}$$

so that equation (31) could be written as

$$\frac{1}{T} \sum_{i=1}^{T} (\overline{\alpha_i} P_f)^t . \tag{33}$$

Since P_f is independent of i, the actual joint fading (33) becomes

$$\left(\frac{1}{T} \sum_{i=1}^{T} \overline{\alpha_i}^t\right) P_f^t = \alpha_t P_f^t \tag{34}$$

where

$$\alpha_t = \frac{1}{T} \sum_{i=1}^{T} \overline{\alpha_i}^t . \tag{35}$$

Thus, in computing simultaneous fading activity on t channels in t different hops where fading probabilities are given in terms of monthly averages, a factor α_t should be included to account for the daily fading variation. The model for daily fading used in this paper is given in Fig. 3. Based on this model, we have

$$\alpha_2 \cong 6$$

and

$$\alpha_3 \cong 46.$$

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