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A Relation for the Loss Characteristics of Circular Electric and Magnetic Modes in Dielectric Lined Waveguide

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Recent studies have shown that modes not of the circular electric type have low-loss characteristics in dielectric lined circular waveguide. It was determined that circular electric modes are low-loss for linings approximately 0, $\lambda/2$, λ, $\cdots$ wavelengths thick while circular magnetic modes are low-loss for $\lambda/4$, $3\lambda/4$, $\cdots$ thick linings. In this paper we derive a simple relationship between the loss characteristics of circular electric waves with 0, $\lambda/2$, $\cdots$ thick linings and circular magnetic waves with $\lambda/4$, $3\lambda/4$, $\cdots$ thick linings.

Specifically, we show that the minimum obtainable circular magnetic mode loss is at least four times greater than the minimum obtainable circular electric mode loss. We also show that the minimum loss for successively higher order circular electric (magnetic) modes corresponding to approximately 0, $\lambda/2$, $\cdots$ ($\lambda/4$, $3\lambda/4$, $\cdots$) thick linings is approximately the same if we neglect the dielectric losses.

I. INTRODUCTION

Recent studies[1] indicate many modes not of the circular electric type may have very low loss in dielectric lined circular waveguide. In these studies the duality principle was used to explain the low-loss characteristics of circular magnetic modes for linings having thicknesses which are an odd multiple of a quarter wavelength.

In this paper we extend this use of the duality principle and derive a simple relation between the loss characteristics of circular electric and magnetic modes in dielectric lined circular waveguide. In all cases, we find the minimum heat loss obtainable as the dielectric thickness varies is greater for circular magnetic modes in comparison with the minimum circular electric mode heat loss.

II. DISCUSSION

The waveguide under consideration is shown in Fig. 1. It consists of a highly conducting outer wall to which a thin layer of dielectric (of relative permittivity ϵ) is bonded. The liner is usually "electrically" thin and its sole function is to break the phase velocity degeneracy between the TE_{01} and TM_{11} modes in hollow metal-walled waveguide. In this paper we are concerned with the effect of thicker linings (linings which are an integral multiple of a quarter wavelength thick) on the conducting wall losses of the waveguide. We will assume that the dielectric is lossless in this study.

The metal walls of the waveguide in Fig. 1a may be modeled[2] as a low-impedance termination ($Z_l \ll \eta$) for the fields interior to the walls. Here $\eta = \sqrt{\mu_0/\epsilon_0}$ is the characteristic impedance of the interior filler (free space) in the guide. The dielectric liner is equivalent to a short section of transmission line. This transmission line has a characteristic impedance*

$$Z_{0\phi} = \eta/\sqrt{\epsilon - 1} \tag{1}$$

for the electric field polarized parallel to the wall and a characteristic impedance

$$Z_{0z} = \frac{\eta\sqrt{\epsilon - 1}}{\epsilon} \tag{2}$$

for the magnetic field polarized parallel to the wall. The appropriate transmission line propagation constant is

$$\beta_e = k_0\sqrt{\epsilon - 1}, \tag{3}$$

where k_0 is the free-space propagation constant. The equivalent impedance conditions (Z_ϕ and Z_z) at the inner face of the dielectric in Fig. 1b may be obtained from the transmission line parameters in (1), (2), and (3) in the usual manner. The wall impedance guide in Fig. 1b is equivalent to the lined guide in Fig. 1a and may be used to predict its electrical properties with a small error.[3]

R. E. Collin[4] has shown that if the fields $\bar{E}_1$, $\bar{H}_1$ are solutions of Maxwell's equations in a source-free region of free space, the dual fields $\eta\bar{H}_2$, $-(1/\eta)\bar{E}_2$ are also a solution. The same transformation is applicable to the wall impedance guide in Fig. 1b but we must also transform the wall impedances. The appropriate dual is shown in

* The expressions in (1), (2), and (3) were derived from the plane-wave scattering at grazing incidence by a grounded slab.

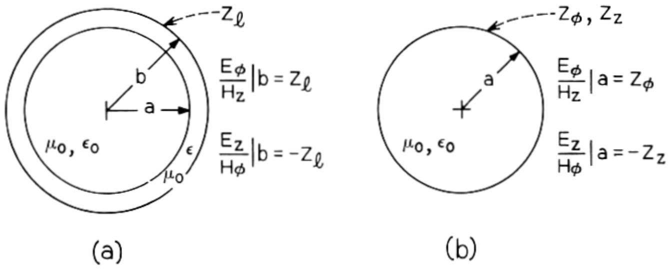

Fig. 1—(a) Dielectric lined waveguide. (b) Equivalent wall impedance waveguide.

Fig. 2. The fields are transformed as in Collin and the impedance wall is replaced by an equivalent admittance wall. We may summarize the duality principle for a guide of radius a, as in Fig. 2, as follows:

$$\bar{E}_1 \leftrightarrow \eta \bar{H}_2 \ .$$

$$\bar{H}_1 \leftrightarrow -\frac{1}{\eta} \bar{E}_2 \ .$$

$$\frac{E_{1\phi}}{H_{1z}}\bigg| a = Z_{1\phi} \leftrightarrow \eta^2 Y_{2\phi} \ , \qquad -\frac{H_{2\phi}}{E_{2z}}\bigg| a = Y_{2\phi} \ . \tag{4}$$

$$-\frac{E_{1z}}{H_{1\phi}}\bigg| a = Z_{1z} \leftrightarrow \eta^2 Y_{2z} \ , \qquad \frac{H_{2z}}{E_{2\phi}}\bigg| a = Y_{2z} \ .$$

From (4) we see that the dual of a circular electric mode in a guide with a low-impedance wall ($Z_{1\phi} \ll \eta$) is a circular magnetic mode in a guide with a high-impedance or low-admittance ($Y_{2\phi} = (1/\eta^2) Z_{1\phi} \ll 1/\eta$) wall. Since the $\bar{E} \times \bar{H}$ product is invariant under the above transformation, the loss characteristics of the two duals are identical.

Using the duality relations in (4), we can now obtain a simple relation

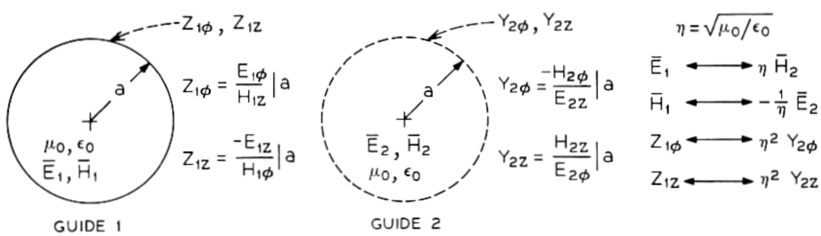

Fig. 2—Dual waveguides.

between the losses of circular electric and magnetic modes in dielectric lined guide as shown in Fig. 3. We first consider the loss for a TE_{01} mode in Fig. 3a. Since there is no lining at $r = a$, we have $Z_{1\phi} = Z_l$ and the loss $\alpha_0^{TE_{01}}$ for a TE_{01} mode in hollow guide is thus[5]

$$\alpha_0^{TE_{01}} = C \operatorname{Re}(Z_l). \tag{5}$$

We now consider the loss for a TM_{02} mode in Fig. 3b. The input admittance $Y_{2\phi}$ at $r = a$ is easily determined for a quarter-wave lining from the transmission line impedance in (2) as

$$Y_{2\phi} = \frac{H_{2\phi}}{E_{2z}} = \frac{\epsilon^2 Z_l}{\eta^2(\epsilon - 1)}. \tag{6}$$

But this circular magnetic mode will have the same loss [see equation (4)] as a circular electric mode in a guide with wall impedance

$$Z_{1\phi} = \frac{\epsilon^2}{(\epsilon - 1)} Z_l .$$

We see that the equivalent wall impedance for the circular electric mode dual is higher than the wall impedance of the unlined waveguide. Hence, the loss for the TM_{02} mode in the guide shown in Fig. 3b is greater than that for the TE_{01} mode in Fig. 3a and is given by

$$\alpha_{\lambda/4}^{TM_{02}} = C \frac{\epsilon^2}{\epsilon - 1} \operatorname{Re} Z_l . \tag{7}$$

If we now consider the TE_{02} mode in Fig. 3c we see it has the same fields for $r < a$ as the TE_{01} mode in Fig. 3a and, hence, its loss will be the same. We, thus, have

$$\alpha_{\lambda/2}^{TE_{02}} = C \operatorname{Re}(Z_l). \tag{8}$$

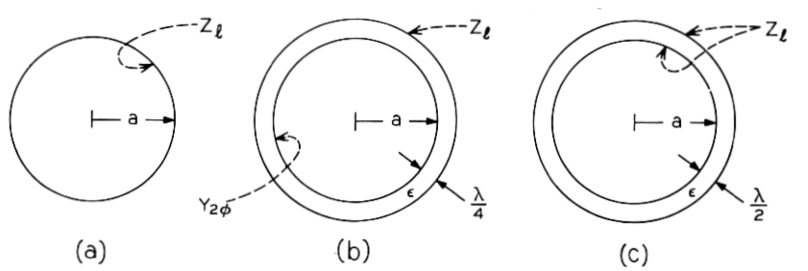

Fig. 3—Low-loss dielectric waveguides: (a) TE_{01} mode; (b) TM_{02} mode; (c) TE_{02} mode.

In the above analysis we have neglected the power carried in the dielectric regions of the guide. This is a reasonable approximation if the lining thickness is much less than the guide radius. We have also neglected the dielectric losses. Recent computer-generated results[1] indicate that this is a reasonable assumption since a quarter-wave polyethyline liner ($\epsilon = 2.34$, $\tan \delta = 83 \times 10^{-6}$) in 50mm-diameter circular guide at 100 GHz has a metal-wall loss for Cu walls of 1.79 dB/km, while the dielectric loss is 0.43 dB/km for the TM_{02} mode.

The present study shows that the minimum circular magnetic mode heat loss $\alpha_{\min}^{TM_0}$ and the minimum circular electric mode heat loss $\alpha_{\min}^{TE_0}$ are related by

$$\frac{\alpha_{\min}^{TM_0}}{\alpha_{\min}^{TE0}} = \epsilon^2/\epsilon - 1 \tag{9}$$

in dielectric lined guide. This ratio is a minimum at $\epsilon = 2$ and has a value of 4. Relation (9) was found to agree well with computer-generated results for the modal[1] heat loss in dielectric lined guide.

III. CONCLUSION

We have seen that circular magnetic modes have a higher minimum heat loss than circular electric modes in dielectric lined waveguide. The ratio of the two losses was shown to be a simple function of the linings relative permittivity. The ratio was shown to have a minimum value of 4 for a lossless dielectric of permittivity 2.

The results indicate a TM_0 mode has possibilities as a long-haul carrier, in dielectric lined guide. The heat loss, however, is always at least four times greater than that for a comparable TE_0 mode. The mode conversion loss for the two systems also can be shown to be comparable by use of the duality principle.[6]

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