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The Coupling of Degenerate Modes in Two Parallel Dielectric Waveguides

By D. MARCUSE

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Two parallel dielectric waveguides can exchange energy if the field carried by one guide reaches the other guide. We consider only the case of coupling between degenerate modes of dielectric waveguides (optical fibers, etc). Degenerate modes have equal phase velocities, but their transverse field distributions need not be identical.

The coupling theory presented in this paper applies to dielectric waveguides of arbitrary shape and arbitrary distribution of refractive index. The dielectric media of the guides as well as the surrounding medium are allowed to be lossy. The coupling coefficient is obtained by means of perturbation theory. It is shown that whereas lossless degenerate modes can exchange their power completely, lossy modes tend to equalize their power.

The theory is applied to the problem of crosstalk between cladded dielectric slab waveguides and cladded optical fibers embedded in a lossy medium.

Since a lossy surrounding medium also causes an increase in the loss of the guided modes, formulas for this additional loss are presented. It is shown that additional mode loss results even for a lossless surrounding medium if $n_3 k > \beta$. (n_3 = index of surrounding medium, k = free space propagation constant, β = propagation constant of guided mode.)

I. INTRODUCTION

Optical fibers appear attractive as waveguides for the transmission of light. Since many such fibers are likely to be bunched together to form a cable, the problem of crosstalk between the different fibers of the cable arises. There are several sources of crosstalk. Light scattered in one fiber may excite a guided mode of an adjacent fiber. This double scattering mechanism is discussed in the next paper in this issue.

The present paper is concerned with the direct coupling of power between two parallel dielectric waveguides. Even though it addresses itself to crosstalk in optical fibers the theory is more general and applies to dielectric waveguides of arbitrary distribution of the refractive index. The modes carried by the two guides need not be identical. However, we assume that the phase velocities of the modes in either guide are identical. We call modes with identical phase velocity degenerate modes. We furthermore admit the possibility that the media of the waveguides as well as the surrounding medium in which the guides are embedded may be lossy. The restriction to degenerate modes is no serious limitation for cases of practical interest. Provided that the coupling coefficients are constant, appreciable coupling of power from a mode in one guide to a mode in the other guide is possible only if the modes have identical phase velocities.

The coupling theory is based on finding the change of the propagation constant of the mode of one guide caused by the presence of the other guide. This perturbation theory is directly based on Maxwell's equations and is thus independent of the composition and geometry of the coupled waveguides.

We obtain the coupling coefficient in form of an integral over the cross section of the waveguides. The integral is taken over the scalar product of the electric field vectors of the two guides. The general coupling formula is then used to obtain the coupling between two cladded slab waveguides as well as the coupling between two cladded round optical fibers. The crosstalk between these two types of waveguides is discussed. The crosstalk can be reduced by increasing the separation between the two guides and by increasing the loss of the medium in which the guides are embedded. Coupling between dielectric fibers has previously been treated by A. L. Jones[1] and by R. Vanclooster and P. Phariseau.[2] However, these authors consider only uncladded lossless fibers.

II. COUPLED LINE EQUATIONS

The general properties of coupled waveguides can be studied with the help of coupled line equations. Neglecting the possibility of coupling to modes traveling in the opposite direction we can write the general coupled line equations of two degenerate modes as follows:[3]

$$\frac{dA}{dz} = -i\beta A + c_1 B, \tag{1a}$$

$$\frac{dB}{dz} = -i\beta B + c_2 A. \tag{1b}$$

The propagation constant β is assumed to be the same for either mode; A is the amplitude of the mode of one guide and B that of the mode of the other guide. If each guide can support more than one mode these modes would have to be included in the coupled line equations. However, it is well known that only modes with equal phase velocity exchange a significant amount of energy if they are coupled together by a length-independent coupling mechanism. The restriction to only two modes is thus an excellent approximation. The coupling coefficients c_1 and c_2 are allowed to be different since the two modes can be different even though they are degenerate. However, if the waveguides are lossless, conservation of power imposes the condition[3]

$$c_1 = -c_2^*. \tag{2}$$

The asterisk indicates complex conjugation.

In the following analysis we assume that β as well as c_1 and c_2 may be complex quantities since we want to include the case of lossy modes. Equation (2) is thus not assumed to hold exactly. Since β as well as c_1 and c_2 are constants we can immediately solve the coupled line equations and obtain solutions in the form

$$A(z) = \frac{1}{2}\left\{A_0(e^{-i\Delta\beta z} + e^{i\Delta\beta z}) + \left(\frac{c_1}{c_2}\right)^{\frac{1}{2}} B_0(e^{-i\Delta\beta z} - e^{i\Delta\beta z})\right\}e^{-i\beta z}, \tag{3a}$$

$$B(z) = \frac{1}{2}\left\{B_0(e^{-i\Delta\beta z} + e^{i\Delta\beta z}) + \left(\frac{c_2}{c_1}\right)^{\frac{1}{2}} A_0(e^{-i\Delta\beta z} - e^{i\Delta\beta z})\right\}e^{-i\beta z}, \tag{3b}$$

with

$$\Delta\beta = i\sqrt{c_1 c_2}\,. \tag{4}$$

The coefficients A_0 and B_0 are the field amplitudes A and B at $z = 0$.

It is apparent that equations (3a) and (3b) are composed of the superposition of two new normal modes with propagation constants

$$\beta_1 = \beta + \Delta\beta \tag{5}$$

and

$$\beta_2 = \beta - \Delta\beta. \tag{6}$$

For complex values of c_1 and c_2 we obtain also a complex value of the change of the propagation constant $\Delta\beta$. In that case, it is clear that the two new normal modes do not only have slightly different propagation velocities but also suffer different amounts of loss.

For the study of crosstalk we need the solution with $B_0 = 0$. That means we assume that at $z = 0$ only mode A is excited. If we restrict ourselves to the case

$$|\Delta\beta z| \ll 1 \tag{7}$$

we find that the ratio of the amplitude B to amplitude A for distances $z = L$ for which (7) is valid is given by

$$\frac{B(L)}{A(L)} = -i\left(\frac{c_2}{c_1}\right)^{\frac{1}{2}} \Delta\beta L. \tag{8}$$

The absolute square of the ratio of B/A is the power ratio at $z = L$ in the two guides. Using the letter V for this power ratio we obtain the following useful formula for the distance L at which the power ratio in the two guides is V:

$$L = \left|\frac{c_1}{c_2}\right|^{\frac{1}{2}} \frac{(V)^{\frac{1}{2}}}{|\Delta\beta|}. \tag{9}$$

For low-loss modes equation (2) must hold at least approximately so that we obtain from (9) in the low-loss case

$$L = \frac{V^{\frac{1}{2}}}{|\Delta\beta|}. \tag{10}$$

The low-loss assumption refers only to the actual loss suffered by the modes propagating inside of the guides. The loss of the surrounding medium in which the guides are embedded can be arbitrarily high.

Finally we consider two special cases. If we assume that the two guides are lossless and assume again $B_0 = 0$ we obtain from equation (3)

$$A(z) = A_0 \cos(\Delta\beta z)e^{-i\beta z}, \tag{11a}$$

$$B(z) = -i\left(\frac{c_2}{c_1}\right)^{\frac{1}{2}} A_0 \sin(\Delta\beta z)e^{-i\beta z}. \tag{11b}$$

In the lossless case assumed here $\Delta\beta$ is real and it is clear that the two guides exchange power after a distance D given by

$$D = \frac{\pi}{2\Delta\beta}. \tag{12}$$

If, on the other hand, $\Delta\beta$ is complex, as it would be in the case of lossy guides, and if we make z sufficiently large that the exponential factor $\exp(i\Delta\beta z)$ is much less than unity, we obtain again for $B_0 = 0$ from equation (3)

$$A(z) = \tfrac{1}{2}A_0 e^{-i(\beta+\Delta\beta)z} \tag{13a}$$

and

$$B(z) = \frac{1}{2}\sqrt{\frac{c_2}{c_1}}\, A_0 e^{-i(\beta+\Delta\beta)z}. \tag{13b}$$

Since the absolute value of the ratio c_2/c_1 is nearly unity we see that the power carried by the two guides equalizes in the lossy case provided that we let both modes travel far enough. This power equalization may occur after many power exchange cycles or it may occur even before one cycle is completed depending on the magnitude of the imaginary part of $\Delta\beta$. Of course both modes suffer additional loss because of the complex value of the propagation constant β. The magnitude of β is much larger than that of $\Delta\beta$ for this discussion to be meaningful.

The preceding discussion shows that the crosstalk between the two coupled guides is determined by the change in propagation constant $\Delta\beta$ and not by the individual coupling constants c_1 and c_2. It is thus sufficient to determine $\Delta\beta$.

III. MODE COUPLING THEORY

We assume that two dielectric waveguides are positioned parallel to each other. Each guide is a cylindrical structure with a refractive index distribution that is independent of the z coordinate (z is the direction parallel to the axis of the structure) but which can have an arbitrary dependence on the transverse x and y coordinates. However, it is assumed that the index distribution is such that it produces a dielectric waveguide capable of supporting guided modes. We assign a refractive index distribution n_1 to guide one assuming that n_1 has the constant value of the refractive index n_3 of the background material in the region occupied by guide two. A similar assumption applies to the index distribution of guide two. It is assumed to have the constant value n_3 of the background in the region occupied by guide one. The square of the index distributions n_1 and n_2 is shown in Fig. 1. Since it is the square of the refractive index that enters Maxwell's equations we use the following expression

$$n^2 = (n_1^2 - n_3^2) + (n_2^2 - n_3^2) + n_3^2 \,. \tag{14}$$

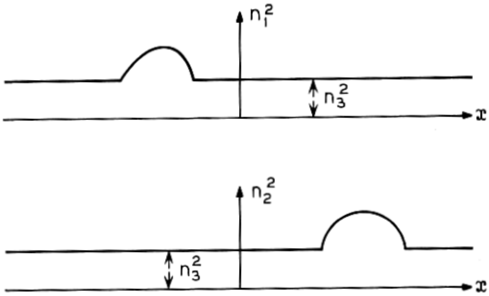

Fig. 1—Distribution of the square of the refractive index used to describe the two dielectric waveguides.

Owing to the definitions of $n_1(x, y)$ and $n_2(x, y)$ it is clear that (14) becomes $n^2 = n_1^2$ in the region of guide one, $n^2 = n_2^2$ in the region of guide two, and $n^2 = n_3^2$ outside the regions of either guide. The refractive indices n_1, n_2, and n_3 need not be real quantities but are allowed to be complex if the media are lossy.

The starting point of the perturbation theory is Maxwell's equations

$$\nabla \times \mathbf{H} = i\omega\epsilon_0 n^2 \mathbf{E} \tag{15a}$$

and

$$\nabla \times \mathbf{E} = -i\omega\mu_0 \mathbf{H}. \tag{15b}$$

The time dependence of the fields was assumed to be given as exp $(i\omega t)$. The constants ϵ_0 and μ_0 are the permittivity and permeability of the vacuum and the square of the refractive index is given by (14). Next we introduce the perturbation assumption that the electric field $\mathbf{E}$ and the magnetic field $\mathbf{H}$ are very nearly given by the superposition of the mode fields of each guide in the absence of the other. We thus write

$$\mathbf{E} = a\mathbf{E}_1 + b\mathbf{E}_2 + \mathbf{e} \tag{16}$$

and

$$\mathbf{H} = a\mathbf{H}_1 + b\mathbf{H}_2 + \mathbf{h}. \tag{17}$$

The coefficients a and b are being determined by the theory. The field $\mathbf{E}_\nu$, $\mathbf{H}_\nu (\nu = 1, 2)$ being a mode of guide ν has the z dependence exp $(-i\beta_0 z)$. It is assumed that the propagation constant is the same for the modes of either one of the guides in the absence of the other. As indicated in Section I, such modes are here called degenerate. The electric and magnetic fields of the unperturbed modes of either guide satisfy the equations

$$\nabla_t \times \mathbf{H}_\nu - i\beta_0(\hat{z} \times \mathbf{H}_\nu) - i\omega\epsilon_0 n_\nu^2 \mathbf{E}_\nu = 0 \quad \nu = 1, 2 \tag{18a}$$

and

$$\nabla_t \times \mathbf{E}_\nu - i\beta_0(\hat{z} \times \mathbf{E}_\nu) + i\omega\mu_0 \mathbf{H}_\nu = 0. \tag{18b}$$

$\hat{z}$ is a unit vector in z direction. The symbol ∇_t indicates the transverse part of the ∇ operator. The electric vector $\mathbf{e}$ and the magnetic vector $\mathbf{h}$ appearing in (16) and (17) are assumed to be small perturbations that are needed to express the change in the field shape resulting from the close proximity of the two guides. In addition to the changes in the field shape we must allow for the possibility of a change in the propagation constant which can be assumed to be small if the coupling of the two guides is sufficiently loose. We thus assume that the new fields $\mathbf{E}$ and $\mathbf{H}$ of (16) and (17) have a z dependence of the form $\exp(-i\beta z)$ with

$$\beta = \beta_0 + \Delta\beta. \tag{19}$$

We have seen in the last section that the change $\Delta\beta$ of the propagation constant determines the coupling of the two guides. It is thus the quantity we are most interested in.

We substitute (16) and (17) into (15a) and (15b) using (14). After cancellation of a number of terms with the help of (18a) and (18b) we are left with the following system of equations:

$$\begin{aligned}\nabla_t \times \mathbf{h} - i\beta_0(\hat{z} \times \mathbf{h}) - i\omega\epsilon_0[(n_1^2 - n_3^2) + (n_2^2 - n_3^2) + n_3^2]\mathbf{e} \\ = i\Delta\beta(a\hat{z} \times \mathbf{H}_1 + b\hat{z} \times \mathbf{H}_2) \\ + i\omega\epsilon_0[(n_2^2 - n_3^2)a\mathbf{E}_1 + (n_1^2 - n_3^2)b\mathbf{E}_2]\end{aligned} \tag{20}$$

and

$$\nabla_t \times \mathbf{e} - i\beta_0(\hat{z} \times \mathbf{e}) + i\omega\mu_0\mathbf{h} = i\Delta\beta(a\hat{z} \times \mathbf{E}_1 + b\hat{z} \times \mathbf{E}_2). \tag{21}$$

Products of $\Delta\beta$ with $\mathbf{e}$ and $\mathbf{h}$ have been omitted from (20) and (21) since they are small of second order.

Since we are interested in obtaining $\Delta\beta$ from our theory we must eliminate the dependence on the spatial coordinates from (20) and (21). Concentrating for the moment on the left-hand sides of these equations we obtain by scalar multiplication of (20) with $\mathbf{E}_1$ and of (21) with $\mathbf{H}_1$ and by subtracting the two equations and integration over the entire cross-sectional area

$$\int \{\mathbf{E}_1 \cdot [\nabla_t \times \mathbf{h} - i\beta_0(\hat{z} \times \mathbf{h}) - i\omega\epsilon_0((n_1^2 - n_3^2) + (n_2^2 - n_3^2) + n_3^2)\mathbf{e}] - \mathbf{H}_1 \cdot [\nabla_t \times \mathbf{e} - i\beta_0(\hat{z} \times \mathbf{e}) + i\omega\mu_0\mathbf{h}]\} \, dx \, dy. \tag{22}$$

The terms inside of the square brackets are small of first order. We now assume that the field intensity of $\mathbf{E}_1$ in the region of guide two has decayed sufficiently to render it also small of first order. Since $n_2^2 - n_3^2$ is different from zero only in a small area in the vicinity of guide two the product of $\mathbf{E}_1$ with $(n_2^2 - n_3^2)\mathbf{e}$ is a term that is small of second order. In first order of perturbation theory this term can be neglected. Performing a partial integration on the terms containing the ∇_t operator and rearranging the terms of the mixed products allows us to write

$$\int \{\mathbf{h} \cdot [\nabla_t \times \mathbf{E}_1 + i\beta_0(\hat{z} \times \mathbf{E}_1) - i\omega\mu_0\mathbf{H}_1] - \mathbf{e} \cdot [\nabla_t \times \mathbf{H}_1 + i\beta_0(\hat{z} \times \mathbf{H}_1) + i\omega\epsilon_0 n_1^2 \mathbf{E}_1]\} \, dx \, dy. \tag{23}$$

Comparison of (23) with (18) shows that this expression would vanish if instead of $\mathbf{E}_1$ and $\mathbf{H}_1$ we had used the complementary fields $\mathbf{E}_1^-$ and $\mathbf{H}_1^-$ that result from the original field if β_0 and ω are replaced by $-\beta_0$ and $-\omega$. We have thus shown that by scalar multiplication with $\mathbf{E}_1^-$ and $\mathbf{H}_1^-$, subtraction, and integration over the entire cross-sectional area the field terms $\mathbf{e}$ and $\mathbf{h}$ can be made to vanish from (20) and (21). A similar procedure can, of course, be carried through using $\mathbf{E}_2^-$ and $\mathbf{H}_2^-$. We thus obtain from (20) and (21) the following set of equations:

$$a\left[\Delta\beta\hat{z} \cdot \int (\mathbf{H}_1^- \times \mathbf{E}_1 + \mathbf{H}_1 \times \mathbf{E}_1^-) \, dx \, dy + \omega\epsilon_0 \int (n_2^2 - n_3^2)\mathbf{E}_1^- \cdot \mathbf{E}_1 \, dx \, dy\right] + b\left[\Delta\beta\hat{z} \cdot \int (\mathbf{H}_1^- \times \mathbf{E}_2 + \mathbf{H}_2 \times \mathbf{E}_1^-) \, dx \, dy + \omega\epsilon_0 \int (n_1^2 - n_3^2)\mathbf{E}_1^- \cdot \mathbf{E}_2 \, dx \, dy\right] = 0 \tag{24a}$$

and

$$a\left[\Delta\beta\hat{z} \cdot \int (\mathbf{H}_2^- \times \mathbf{E}_1 + \mathbf{H}_1 \times \mathbf{E}_2^-) \, dx \, dy\right.$$

$$+ \omega\epsilon_0 \int (n_2^2 - n_3^2)\mathbf{E}_2^- \cdot \mathbf{E}_1 \, dx \, dy\Bigg]$$

$$+ b\Bigg[\Delta\beta\hat{z}\cdot\int (\mathbf{H}_2^- \times \mathbf{E}_2 + \mathbf{H}_2 \times \mathbf{E}_2^-) \, dx \, dy$$

$$+ \omega\epsilon_0 \int (n_1^2 - n_3^2)\mathbf{E}_2^- \cdot \mathbf{E}_2 \, dx \, dy\Bigg] = 0. \tag{24b}$$

Equations (24a) and (24b) still contain a number of second-order terms. Products of field terms with different indices are small of first order since the modes of the two guides are supposed to overlap only slightly. Products of field terms with different indices that are multiplied by the first-order quantity $\Delta\beta$ can thus be neglected as being of second order. The term $(n_2^2 - n_3^2)\mathbf{E}_1^- \cdot \mathbf{E}_1$ is also of second order since it involves the square of the amplitude of $\mathbf{E}_1$ at the location of the opposite guide. Neglecting it and a similar term results in a consistent equation system containing only first-order terms.

$$a\Delta\beta\hat{z}\cdot\int (\mathbf{H}_1^- \times \mathbf{E}_1 + \mathbf{H}_1 \times \mathbf{E}_1^-) \, dx \, dy$$

$$+ b\omega\epsilon_0 \int (n_1^2 - n_3^2)\mathbf{E}_1^- \cdot \mathbf{E}_2 \, dx \, dy = 0 \tag{25a}$$

and

$$a\omega\epsilon_0 \int (n_2^2 - n_3^2)\mathbf{E}_2^- \cdot \mathbf{E}_1 \, dx \, dy$$

$$+ b\Delta\beta\hat{z}\cdot\int (\mathbf{H}_2^- \times \mathbf{E}_2 + \mathbf{H}_2 \times \mathbf{E}_2^-) \, dx \, dy = 0. \tag{25b}$$

The equation system (25) allows us to determine the coefficients a and b. Since we have a homogeneous system of equations we must require that the system determinant vanishes. This latter condition leads to a determination of $\Delta\beta$.

$$\Delta\beta = \pm\omega\epsilon_0 \cdot \left\{\frac{\int (n_1^2 - n_3^2)\mathbf{E}_1^- \cdot \mathbf{E}_2 \, dx \, dy \int (n_2^2 - n_3^2)\mathbf{E}_2^- \cdot \mathbf{E}_1 \, dx \, dy}{\int \hat{z}\cdot(\mathbf{E}_1 \times \mathbf{H}_1^- + \mathbf{E}_1^- \times \mathbf{H}_1) \, dx \, dy \int \hat{z}\cdot(\mathbf{E}_2 \times \mathbf{H}_2^- + \mathbf{E}_2^- \times \mathbf{H}_2) \, dx \, dy}\right\}^{\frac{1}{2}}. \tag{26}$$

The ratio of a/b is of no interest to us so that we do not need to write down the solution of (25).

Equation (26) is the final result of our coupling theory. In the general case of lossy dielectric media $\Delta\beta$ is a complex quantity. Equation (26) is very general and holds for any two dielectric waveguides carrying degenerate modes. The field expressions $\mathbf{E}_i^-$ and $\mathbf{H}_i^-$ are obtained by changing the sign of β and ω in the regular expressions for $\mathbf{E}_i$ and $\mathbf{H}_i$. In the special case of lossless media $\mathbf{E}_i^-$ and $\mathbf{H}_i^-$ become simply the complex conjugates of the original fields.

$$\left.\begin{aligned}\mathbf{E}_i^- &= \mathbf{E}_i^* \\ \mathbf{H}_i^- &= \mathbf{H}_i^*\end{aligned}\right\} \text{ for real } n_i . \tag{27}$$

Equation (26) can be simplified in another special case. If both waveguides are identical, carrying the same mode, the two types of integrals appearing in the numerator as well as in the denominator of (26) become identical for reasons of symmetry. The expression for identical modes of two coupled identical waveguides can thus be written more simply

$$\Delta\beta = \pm\omega\epsilon_0 \frac{\int (n_2^2 - n_3^2)\mathbf{E}_2^- \cdot \mathbf{E}_1 \, dx \, dy}{\int \hat{z}\cdot(\mathbf{E}_1 \times \mathbf{H}_1^- + \mathbf{E}_1^- \times \mathbf{H}_1) \, dx \, dy}. \tag{28}$$

For real refractive indices the denominator of (28) is equal to $4P$, P being the power carried by the mode of one of the guides. It is necessary to normalize the mode fields of each guide so that field 1 as well as field 2 carry equal power P. This normalization was already implied in converting (26) to (28). Even if the medium surrounding the two waveguides is lossy it is possible to use the relations (27) and the identification of the denominator of (28) as $4P$ provided that the surrounding medium has a very small influence on the mode fields carried by each guide. It is thus permissible to use (28) in the form

$$\Delta\beta = \pm\frac{\omega\epsilon_0}{4P} \int (n_2^2 - n_3^2)\mathbf{E}_2^* \cdot \mathbf{E}_1 \, dx \, dy \tag{29}$$

in the special case of lossless dielectric media or, at least approximately, even in the case that the surrounding medium is lossy if only the fields are so well guided that only a very small amount of power reaches the lossy surrounding medium.

IV. APPLICATION TO CLADDED SLAB WAVEGUIDE

The dielectric slab is the simplest type of dielectric waveguide. Because of its simplicity much insight into the performance of dielectric waveguides can be gained by studying the slab waveguide. We consider two identical cladded slab waveguides as shown in Fig. 2a. Instead of solving the mode problem of the cladded slab waveguide exactly, we assume that the cladding is thick enough so that only a small amount of power reaches the surrounding medium. This assumption allows us to obtain approximate field solutions that are much simpler than the exact solutions. This case is, moreover, of the most practical interest since the cladding is meant to protect the guided mode from the disturbing influence of the surroundings. Our approximation thus coincides with the case of real practical interest. Without going into the details of the calculation we state only the results. The field of one of the two guides can be expressed as follows: (For simplicity the field of one guide is expressed in the coordinate system shown in Fig. 2b.)

$$E_y = \begin{cases} A \cos \kappa x & 0 \leqq x \leqq d \\ Be^{\gamma x} + Ce^{-\gamma x} & d \leqq x \leqq D \\ Fe^{-\rho x} & D \leqq x < \infty . \end{cases} \tag{30}$$

The field is continued symmetrically for $x < 0$. Equation (30) gives the E component of the symmetric TE modes of the slab waveguide.

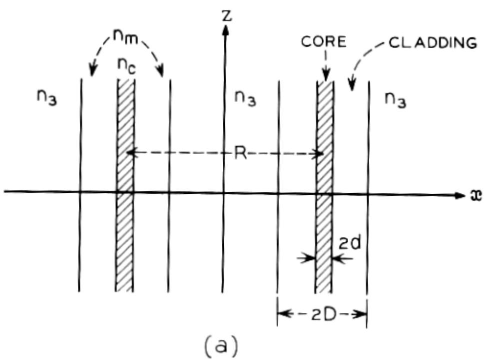

Fig. 2a—Sketch of two coupled cladded slab waveguides.

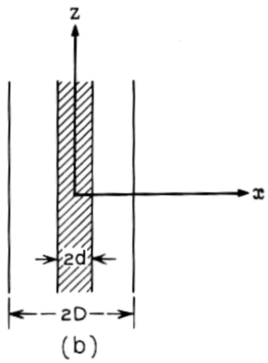

Fig. 2b—Sketch of a single slab waveguide used for equation (30).

The other field components are obtained from E_y by the relations

$$H_x = -\frac{i}{\omega\mu_0}\frac{\partial E_y}{\partial z} \tag{31}$$

and

$$H_z = \frac{i}{\omega\mu_0}\frac{\partial E_y}{\partial x}. \tag{32}$$

The dependence of the field on time t and on the z coordinate is, as always, given by the factor

$$e^{i(\omega t - \beta z)} \tag{33}$$

which is being omitted from the equations. The parameters appearing in (30) are defined as follows: ($k = \omega\sqrt{\epsilon_0\mu_0}$)

$$\kappa = (n_c^2 k^2 - \beta^2)^{\frac{1}{2}}, \tag{34}$$

$$\gamma = (\beta^2 - n_m^2 k^2)^{\frac{1}{2}}, \tag{35}$$

$$\rho = (\beta^2 - n_3^2 k^2)^{\frac{1}{2}}. \tag{36}$$

n_c, n_m, and n_3 are the refractive indices of core, cladding, and outer medium. The boundary conditions lead to relations between the constants A, B, C, and F:

$$B = \frac{A}{2}\frac{\kappa}{(\kappa^2 + \gamma^2)^{\frac{1}{2}}}\frac{\gamma - \rho}{\gamma + \rho}e^{-\gamma d}e^{-2\gamma(D-d)}, \tag{37}$$

$$C = Ae^{\gamma d}\cos \kappa d, \tag{38}$$

$$F = \frac{2\kappa\gamma A}{(\gamma + \rho)(\kappa^2 + \gamma^2)^{\frac{1}{2}}} e^{\rho D} e^{-\gamma(D-d)}. \tag{39}$$

The coefficient A can be related to the power P carried by the mode,

$$A = \left\{\frac{2\omega\mu P}{\beta d + \frac{\beta}{\gamma}}\right\}^{\frac{1}{2}}. \tag{40}$$

The coefficient F depends exponentially on the factor $-\gamma(D - d)$. For increasing cladding thickness this factor becomes very small. However, the factor B vanishes even more rapidly since it depends on the square of the same exponential decay factor. This shows that the field inside of core and cladding is altered only very slightly if the cladding is sufficiently thick. The eigenvalue equation differs from the eigenvalue equation for infinite cladding only by a term that also vanishes as the square of the aforementioned exponential decay factor. To a good approximation we are thus justified to use the eigenvalue equation for the infinite cladding guide.

$$\tan \kappa d = \frac{\gamma}{\kappa}. \tag{41}$$

Once the mode fields are known it is an easy matter to evaluate the coupling coefficient (29). We obtain

$$\Delta\beta = \frac{4\kappa^2\gamma^3\rho}{\beta(1 + \gamma d)(\gamma + \rho)^2(\kappa^2 + \gamma^2)} e^{-2\gamma(D-d)} e^{-\rho(R-2D)}. \tag{42}$$

(R is the distance between the core centers of the two slab waveguides.) In case that the cladding material is identical to the surrounding medium we have $\rho = \gamma$ and (42) reduces to the expression for coupled slab waveguides to be found in N. S. Kapany's book.[4]

The same mechanism that is responsible for the coupling between the two slab waveguides also causes loss to the modes in each of the guides. Coupling occurs because some of the field in one guide reaches the region of the other guide. However, fields reaching out into the lossy medium between the guides also cause power loss to the mode traveling inside of the guide. Only if the surrounding medium is truly lossless can there be no additional loss to the guided mode. I calculated the mode loss contributed by the surrounding medium assuming that it is infinitely extended and also ignoring the presence of the other guide. The power loss can be obtained to a good approximation by computing the power flow in transverse direction to the guide axis at the boundary between the cladding and the surrounding medium. The validity of this approach hinges again on the fact that most of the power is carried inside of core and cladding. The approximate calculation leads to the following expression for the power loss 2α of the mode (α is the amplitude loss coefficient).

$$2\alpha = \frac{8\kappa^2\gamma^3 \operatorname{Im}(\rho)e^{-2\gamma(D-d)}}{\beta(1+\gamma d)(\kappa^2+\gamma^2)\,|\,\gamma+\rho\,|^2}. \tag{43}$$

It is assumed that n_c and n_m are both real; n_3 is allowed to assume complex values. This loss expression has two interesting features. It shows that the loss decreases exponentially with increasing cladding thickness. It also shows that the loss is dependent only on the imaginary part $\operatorname{Im}(\rho)$ of ρ. For ρ to be real, two conditions must be satisfied. First of all, the refractive index of the surrounding medium, n_3, must be purely real. This means that the mode suffers loss whenever the surrounding medium is lossy. However, even if the surrounding medium is lossless, it is still possible for ρ to be purely imaginary. This happens when $n_3k > \beta$. In this case there is no total internal reflection between the cladding and the surrounding medium and the evanescent field tail reaching into the surrounding medium does no longer decay exponentially but begins to radiate. Power is thus lost from the guided mode even though all the materials of the guide and the surrounding medium are lossless.

Finally, we state the result of the coupling calculation and mode loss for the cladded slab waveguide carrying the symmetric TM mode. For the details of the field distribution of this mode in an infinite cladding guide see Ref. 5. The result of the evaluation of (29) in this case is

$$\Delta\beta = \frac{n_c^2 n_m^2 \kappa^2 \gamma^2 [2e^{(\gamma-\rho)(D-d)} - 1]e^{-2\gamma(D-d)}e^{-\rho(R-2D)}}{\beta[(n_m^4\kappa^2 + n_c^4\gamma^2)\gamma d + n_c^2 n_m^2(\kappa^2+\gamma^2)]}. \tag{44a}$$

The power loss caused by the lossy surrounding medium is

$$2\alpha = \frac{8n_c^2 n_m^4\,|\,n_3\,|^4 \kappa^2\gamma^3 \operatorname{Im}\left(\frac{\rho}{n_3^2}\right)e^{-2\gamma(D-d)}}{\beta[\gamma d(n_m^4\kappa^2 + n_c^4\gamma^2) + n_c^2 n_m^2(\kappa^2+\gamma^2)]\,|\,n_m^2\rho + n_3^2\gamma\,|^2}. \tag{44b}$$

It is apparent that the coupling expression (42) for the TE mode differs from that for the TM mode. The coupling as well as the loss, thus turn out to be polarization dependent.

V. COUPLING OF THE HE_{11} MODES OF ROUND CLADDED FIBERS

In complete analogy to the cladded slab waveguide we now proceed to a discussion of coupling between HE_{11} modes in two round cladded

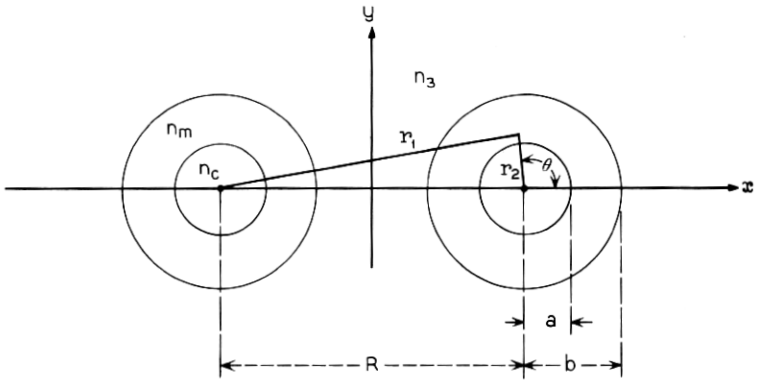

Fig. 3—Cross section of two coupled cladded fibers.

fibers, Fig. 3. Again, we assume that the cladding is sufficiently thick to allow us to treat the field inside of core and cladding as being affected only very little by the presence of the surrounding medium. With this approximation in mind, we state the relevant equations for the mode field of a cladded fiber.

The field inside of the fiber core is described by the set of equations[6]

$$\left.\begin{aligned} E_z &= AJ_1(\kappa r)\cos\phi \\ H_z &= BJ_1(\kappa r)\sin\phi \end{aligned}\right\} \qquad 0 \leqq r \leqq a, \tag{45}$$

the field inside of the cladding is given by

$$\left.\begin{aligned} E_z &= [CH_1^{(1)}(i\gamma r) + DH_1^{(2)}(i\gamma r)]\cos\phi \\ H_z &= [FH_1^{(1)}(i\gamma r) + GH_1^{(2)}(i\gamma r)]\sin\phi \end{aligned}\right\} \qquad a \leqq r \leqq b, \tag{46}$$

and the field in the surrounding medium is

$$\left.\begin{aligned} E_z &= MH_1^{(1)}(i\rho r)\cos\phi \\ H_z &= NH_1^{(1)}(i\rho r)\sin\phi \end{aligned}\right\} \qquad b \leqq r < \infty. \tag{47}$$

The parameters κ, γ and ρ are given by the equations (34), (35), and (36). The other field components are obtained from the longitudinal components by differentiation

$$E_r = -\frac{i}{\Gamma^2}\left(\beta\frac{\partial E_z}{\partial r} + \omega\mu_0\frac{1}{r}\frac{\partial H_z}{\partial\phi}\right), \tag{48}$$

$$E_\phi = -\frac{i}{\Gamma^2}\left(\beta\frac{1}{r}\frac{\partial E_z}{\partial\phi} - \omega\mu_0\frac{\partial H_z}{\partial r}\right), \tag{49}$$

$$H_r = -\frac{i}{\Gamma^2}\left(\beta \frac{\partial H_z}{\partial r} - \omega\epsilon_0 \frac{n^2}{r}\frac{\partial E_z}{\partial \phi}\right), \tag{50}$$

$$H_\phi = -\frac{i}{\Gamma^2}\left(\beta \frac{1}{r}\frac{\partial H_z}{\partial \phi} + \omega\epsilon_0 n^2 \frac{\partial E_z}{\partial r}\right). \tag{51}$$

The symbol Γ stands either for κ, $i\gamma$, or $i\rho$ depending on the medium in which the field is evaluated. The same is true for n, it represents either n_c, n_m, or n_3. The boundary conditions establish the following relations between the amplitude coefficients:

$$\frac{B}{A} = \left(\frac{\epsilon_0}{\mu_0}\right)^{\frac{1}{2}} \frac{ka\kappa^2\gamma^2}{\beta(\kappa^2+\gamma^2)}\left\{\frac{n_c^2}{\kappa}\left[\frac{1}{\kappa a} - \frac{J_0(\kappa a)}{J_1(\kappa a)}\right] + \frac{n_m^2}{\gamma a}\left[\frac{1}{\gamma a} - \frac{iH_0^{(1)}(i\gamma a)}{H_1^{(1)}(i\gamma a)}\right]\right\}, \tag{52}$$

$$\frac{C}{A} = \frac{F}{B} = \frac{J_1(\kappa a)}{H_1^{(1)}(i\gamma a)}, \tag{53}$$

$$D \approx G \approx 0, \tag{54}$$

$$M = \left(\frac{\rho}{\gamma}\right)^{\frac{3}{2}} e^{(\rho-\gamma)b} \cdot\left\{C\left[\left(\frac{\beta}{b}\right)^2\left(1-\frac{\gamma^2}{\rho^2}\right)^2\left(\frac{\gamma}{\rho}-1\right) - \gamma^2k^2\left(1+\frac{\gamma}{\rho}\right)\left(n_m^2 + n_3^2\frac{\gamma^2}{\rho^2} + \frac{\gamma}{\rho}(n_m^2 - n_3^2)\right)\right] + 2\omega\mu_0 F\gamma\frac{\beta}{b}\left(1-\frac{\gamma^2}{\rho^2}\right)\right\} \cdot\left\{\left[\frac{\beta}{b}\left(1-\frac{\gamma^2}{\rho^2}\right)\right]^2 - k^2\gamma^2\left(1+\frac{\gamma}{\rho}\right)\left(n_m^2 + n_3^2\frac{\gamma}{\rho}\right)\right\}^{-1}, \tag{55}$$

$$N = -\left(\frac{\rho}{\gamma}\right)^{\frac{1}{2}} e^{(\rho-\gamma)b} \frac{2Fk^2\gamma^2\left(n_m^2 + n_3^2\frac{\gamma}{\rho}\right) + 2\omega n_m^2\epsilon_0\gamma\frac{\beta}{b}\left(\frac{\gamma^2}{\rho^2}-1\right)C}{\left[\frac{\beta}{b}\left(1-\frac{\gamma^2}{\rho^2}\right)\right]^2 - k^2\gamma^2\left(1+\frac{\gamma}{\rho}\right)\left(n_m^2+n_3^2\frac{\gamma}{\rho}\right)}. \tag{56}$$

The symbols J_0 and J_1 indicate Bessel functions of zero and first order; $H_0{}^{(1)}(i\gamma a)$ and $H_1{}^{(1)}(i\gamma a)$ are Hankel functions of zero and first order and of the first kind. We use here the notation of a Hankel function with imaginary argument used by Jahnke and Emde.[7] The coefficients M and N given by (55) and (56) do not contain any Hankel functions because the approximation for large arguments was used. The relation between the power P carried by the HE_{11} mode in the waveguide and the amplitude coefficient A is given by the equation

$$P = \frac{\pi}{4}\left\{\frac{k\beta_0}{\kappa^4}\left[(a\kappa)^2(J_0^2(\kappa a) + J_1^2(\kappa a)) - 2J_1^2(\kappa a)\right]\left(n_c^2 + \frac{\mu_0}{\epsilon_0}\frac{B^2}{A^2}\right)\right.$$
$$+ \frac{k\beta}{\gamma^4}\left[(a\gamma)^2\left\{1 - \left(\frac{iH_0^{(1)}(i\gamma a)}{H_1^{(1)}(i\gamma a)}\right)^2\right\} + 2\right]J_1^2(\kappa a)\left(n_m^2 + \frac{\mu_0}{\epsilon_0}\frac{B^2}{A^2}\right)$$
$$\left. + 2\left(\frac{\mu_0}{\epsilon_0}\right)^{\frac{1}{2}}\frac{B}{A}\left[\frac{\beta_0^2 + n_c^2k^2}{\kappa^4} - \frac{\beta_0^2 + n_m^2k^2}{\gamma^4}\right]J_1^2(\kappa a)\right\}\left(\frac{\epsilon_0}{\mu_0}\right)^{\frac{1}{2}}A^2. \tag{57}$$

For sufficiently large cladding radius b the propagation constant of the mode can still be obtained from the eigenvalue equation for a rod embedded in a medium with refractive index n_m.

$$\left\{n_c^2\frac{a\gamma^2}{\kappa}\left(\frac{J_0(\kappa a)}{J_1(\kappa a)} - \frac{1}{\kappa a}\right) + n_m^2\left(\gamma a\frac{iH_0^{(1)}(i\gamma a)}{H_1^{(1)}(i\gamma a)} - 1\right)\right\}\left\{n_m^2\frac{a\gamma^2}{\kappa}\left(\frac{J_0(\kappa a)}{J_1(\kappa a)} - \frac{1}{\kappa a}\right)\right.$$
$$\left. + n_m^2\left(\gamma a\frac{iH_0^{(1)}(i\gamma a)}{H_1^{(1)}(i\gamma a)} - 1\right)\right\} = \left[(n_c^2 - n_m^2)\frac{\beta n_m k}{\kappa^2}\right]^2. \tag{58}$$

This collection of formulas is sufficient to calculate the coupling term (29) for the coupling of the HE_{11} modes in two parallel cladded optical fibers. We distinguish two cases. We assume that the modes are either both polarized perpendicular to the plane connecting the two fiber core centers or that they are polarized parallel to this plane. The coupling is different in the two cases, corresponding to the difference in coupling between the TE modes and the TM modes of the slab waveguide treated in the preceding section. It must be pointed out that a perpendicularly polarized mode does not couple with a horizontally polarized mode. The two-mode assumption is thus still valid.

The integral occurring in (29) cannot be solved generally for two coupled HE_{11} modes. However, if we assume that the distance R separating the two cores is large compared to the core radius "a," we are justified in expanding the expression for the radial coordinate r (reaching from the center of guide one to the vicinity of guide two) in the following way (Fig. 3),

$$r_1 = \sqrt{R^2 + r_2^2 + 2r_2R\cos\theta} \approx R + r_2\cos\theta. \tag{59}$$

The angle θ describes the departure of the direction of r_2 from the line connecting the core centers, r_2 is the distance from the center of guide two to the endpoint of r_1. When the expansion (59) is used, the angular integration can be performed leading to Bessel functions of the imaginary argument $i\rho r_2$. The remaining integration over r_2 now involves only products of cylinder functions and can be carried out. We obtain the following result for the case of horizontal polarization:

$$\Delta\beta_h = \left(\frac{\epsilon_0}{\mu_0}\right)^{\frac{1}{2}} \frac{A^2}{2P} \frac{M}{A} \frac{1}{k} \left(\frac{2\pi}{\rho R}\right)^{\frac{1}{2}} e^{-\rho R} \left\{ \frac{e^{(\rho-\gamma)b}}{\pi(\rho\gamma)^{\frac{1}{2}}} \frac{J_1(\kappa a)}{H_1^{(1)}(i\gamma a)} \left[\left(1 + \frac{\beta^2}{\gamma\rho}\right)(\gamma + \rho) + \frac{\beta^2}{b\gamma^2\rho^2}(\rho^2 - \gamma^2)\left(1 - \frac{k}{\beta}\sqrt{\frac{\mu_0}{\epsilon_0}}\frac{B}{A}\right)\right] + iJ_1(i\rho a)\left[a\left(\frac{\beta^2}{\kappa} - \kappa\right)J_0(\kappa a) + a\left(\frac{\beta^2}{\gamma} + \gamma\right)\frac{iH_0^{(1)}(i\gamma a)}{H_1^{(1)}(i\gamma a)}J_1(\kappa a) - \frac{\beta^2}{\kappa^2\gamma^2}(\kappa^2 + \gamma^2)\left(1 - \frac{k}{\beta}\sqrt{\frac{\mu_0}{\epsilon_0}}\frac{B}{A}\right)J_1(\kappa a)\right]\right\}. \tag{60}$$

For the case of vertically polarized modes we get

$$\Delta\beta_v = \frac{A^2}{2P}\frac{N}{A}\frac{1}{\rho}\left(\frac{2\pi}{\rho R}\right)^{\frac{1}{2}} e^{-\rho R}\left\{\frac{e^{(\rho-\gamma)b}}{\pi\gamma b(\rho\gamma)^{\frac{1}{2}}}\left[k\left(\frac{\mu_0}{\epsilon_0}\right)^{\frac{1}{2}}\frac{B}{A}\left((\rho+\gamma)b + \frac{\rho^2 - \gamma^2}{\gamma\rho}\right) - \frac{\beta}{\gamma\rho}(\rho^2 - \gamma^2)\right]\frac{J_1(\kappa a)}{H_1^{(1)}(i\gamma a)} + i\rho J_1(i\rho a)\left[ka\left(\frac{\mu_0}{\epsilon_0}\right)^{\frac{1}{2}}\frac{B}{A}\left(\frac{1}{\kappa}J_0(\kappa a) + \frac{1}{\gamma}\frac{iH_0^{(1)}(i\gamma a)}{H_1^{(1)}(i\gamma a)}J_1(\kappa a)\right) + \frac{\beta}{\kappa^2\gamma^2}(\kappa^2 + \gamma^2)\left(1 - \frac{k}{\beta}\sqrt{\frac{\mu_0}{\epsilon_0}}\frac{B}{A}\right)J_1(\kappa a)\right]\right\}. \tag{61}$$

Finally, we present also the expression for the power loss 2α of the HE_{11} mode caused by the lossy surrounding medium. The calculation follows the idea described in connection with the slab waveguide.

$$2\alpha = \left(\frac{\epsilon_0}{\mu_0}\right)^{\frac{1}{2}} \frac{e^{-2\rho_r b}}{|\rho| P}\left\{\frac{\beta}{b}\frac{1}{|\rho|^4}\,\mathrm{Im}\left[\left(\frac{\mu_0}{\epsilon_0}\right)^{\frac{1}{2}} MN^*(\rho^{*2} - \rho^2)\right] - k\,|M|^2\,\mathrm{Im}\left(\frac{n_3^2}{\rho}\right) - \frac{\mu_0}{\epsilon_0}k\,|N|^2\,\mathrm{Im}\left(\frac{1}{\rho}\right)\right\}. \tag{62}$$

The coefficients M and N are given by (55) and (56), ρ_r is the real part of ρ, and the symbol Im( ) indicates the imaginary part of the expression in parenthesis (or brackets).

VI. NUMERICAL RESULTS

To illustrate the consequences of the theory presented in this paper a few sample cases have been computed. For comparison purposes I have checked the results of my theory against the curves for the coupling of uncladded fibers published by Jones.[1] The agreement between his theory and similar curves obtained from (60) and (61) is excellent.

The amount of coupling between two optical fibers depends on the type of mode they are carrying, on the separation between the fiber cores, and on the loss of the surrounding medium in which the fibers are embedded. Figure 4 shows a set of curves describing the coupling parameter $a\,|\Delta\beta|$ as a function of the loss in the surrounding medium. The abscissa labeled "loss" measured in db indicates the plane wave loss in going through the surrounding medium for a distance $R - 2b$; that means it is the loss that is encountered in going perpendicular to the fiber axis from the cladding boundary of one fiber to the other. There are two curve parameters in Fig. 4. The two sets of curves differ by the cladding thickness normalized with respect to the core radius,

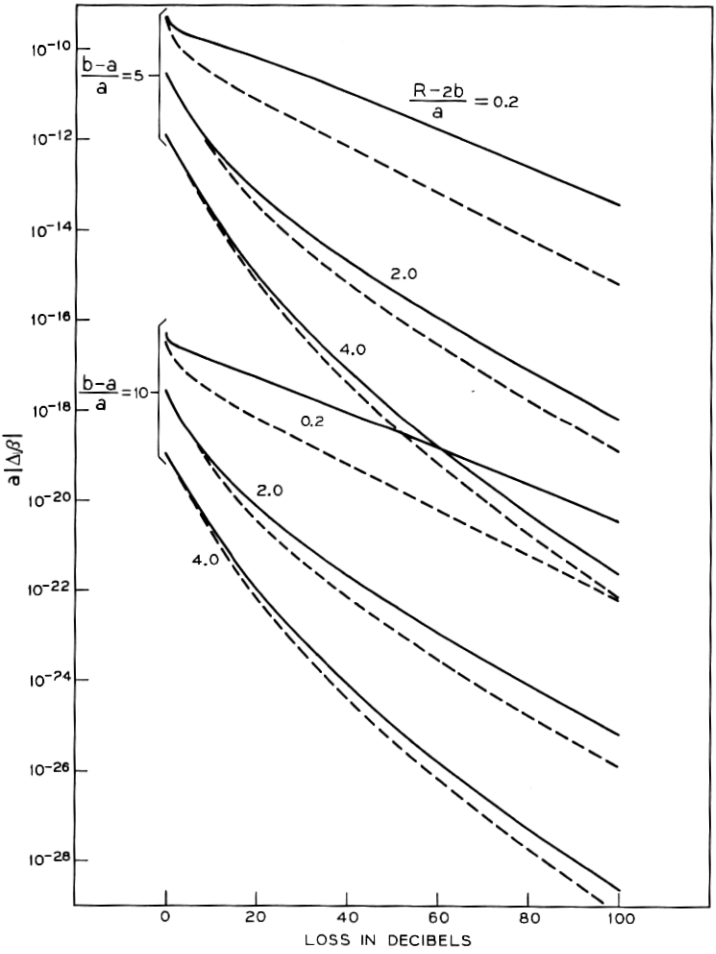

Fig. 4—Coupling of round fibers. $a|\Delta\beta|$ is the coupling parameter of equations (60) and (61), the abscissa indicates the loss (in db) that a plane wave would experience in going from the boundary of one fiber to the boundary of the other fiber. The solid lines hold for horizontal polarization, equation (60), while the dotted lines apply to the case of vertical polarization, equation (61). $n_m ka = 16$, $\gamma a = 1.594$, $n_c/n_m = 1.01$, $\mathrm{Re}(n_0) = n_m$.

$(b - a)/a$. The coupling is, of course, reduced for increasing cladding thickness since the fiber cores are farther removed from each other. The second parameter is the separation between the fiber claddings normalized with respect to the fiber core radius, $(R - 2b)/a$. Increasing values of this parameter again cause the fiber cores to be separated farther from each other. The loss values that form the abscissa hold regardless of fiber separation, always indicating the total loss that exists between the fiber claddings on their closest approach. The curves show clearly the decrease in coupling with increasing loss in the surrounding medium. The solid curves apply to the horizontal polarization of the two modes and are obtained from (60). The dotted curves apply to vertical polarization and were computed from (61). Figure 4 holds, of course, only for one particular pair of round optical fibers. It was assumed that $n_m ka = 16$, and that the ratio of core index to cladding index is $n_c/n_m = 1.01$. For simplicity, it was furthermore assumed that the real part of the index n_3 is identical to the index n_m of the cladding material. The imaginary part of n_3 is varied in order to obtain the variable loss values of the abscissa. The abscissa can be expressed by 8.68 Im $(n_3)k(R - 2b)$.

With the help of (10), Fig. 4 can be used to evaluate the crosstalk between the two optical fibers to which the conditions of Fig. 4 apply. Let us assume that we can tolerate a power ratio $V = 10^{-6}$ at the end, $z = L$, of the two fibers. If we use, for example, $a = 1$ μm and allow $L = 1$ km we find that we must not exceed

$$a \mid \Delta\beta \mid \leqq 10^{-12}. \tag{63}$$

Any value appearing in Fig. 4 below this line thus leads to acceptable crosstalk levels. It is apparent that all curves with $(b - a)/a = 10$ are acceptable. For $(b - a)/a = 5$ a certain amount of loss is required to reduce the coupling and consequently the crosstalk below the desired level.

Figure 5 presents the same data as Fig. 4 applied to two identical slab waveguides. It was assumed that the refractive indices as well as the guide dimensions of the slab waveguides correspond exactly to the round optical fibers of Fig. 4. ($a = d$, $b = D$). However, the two slab waveguides are less tightly coupled than the corresponding round fibers. The reason for this behaviour is the difference in the decay parameter γa (or γd). For the round fibers, we obtained from the eigenvalue equation (58) the value $\gamma a = 1.594$ while (41) results in $\gamma d = 1.997$ for the slab waveguides. A larger value of the decay parameter indicates that exponential decrease of the field amplitude outside of the fiber core is more rapid. The more rapid decrease of the field of the slab waveguide leads to less coupling. The solid and dotted lines have the same meaning as in Fig. 4. The solid lines apply to the TM mode which is polarized horizontally while the dotted lines apply to the vertically polarized TE mode. For simplicity we have assumed that the values of γd are identical for the two modes which, strictly speaking, is not quite true. However, we see from Fig. 4, as well as

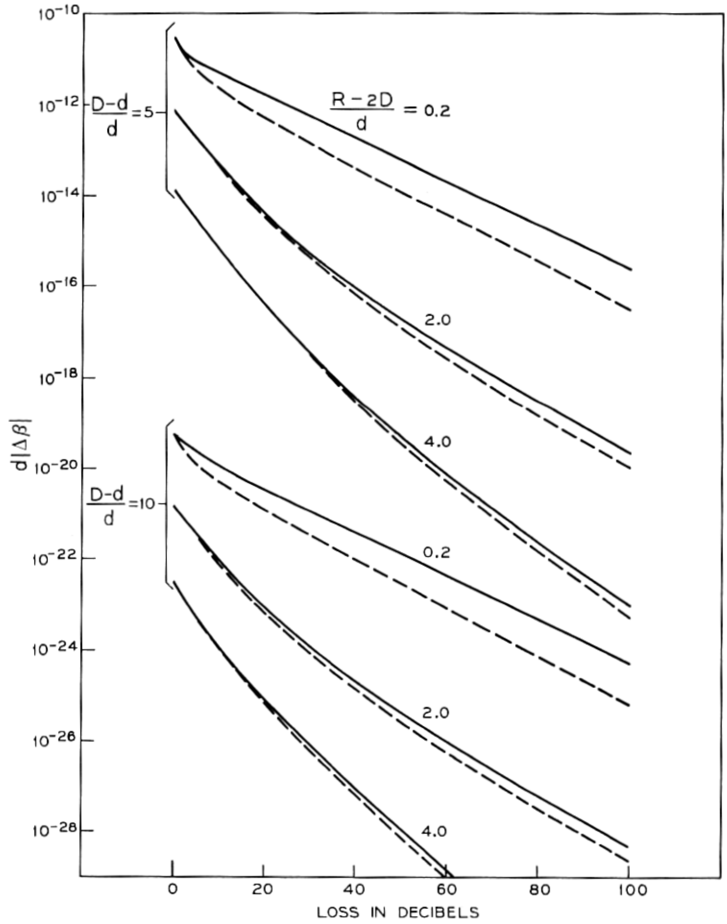

Fig. 5—Coupling of slab waveguides. Coupling parameter and loss have the same meaning as in Fig. 4. The solid lines indicate the TM mode (horizontal polarization) while the dotted lines apply to the TE mode (vertical polarization). $n_m ka = 16$, $\gamma a = 1.997$, $n_c/n_m = 1.01$.

from Fig. 5, that the horizontally polarized modes are coupled more tightly than the vertically polarized modes.

Figure 6 presents a comparison between the coupling strength of round fibers and slab waveguides. Since equal geometry does not lead to identical values of the decay parameter we have arbitrarily used equal values of $\gamma a = \gamma d = 1.594$ for both types of waveguide. The solid lines indicate the results for the round fiber while the dotted lines apply to the slab waveguides. It is apparent that the slab waveguides are more tightly coupled when the conditions are such that both guides have equal decay parameters. This appears reasonable if we consider that the slab waveguides have constant distance from each other over an infinite length while the round fibers have a point of closest approach so that the total amount of field overlap is less in this case.

The lossy medium does not only serve to reduce the coupling between the waveguides but it also has the adverse effect of increasing the loss of the modes traveling in the guides. Figure 7 is a plot of

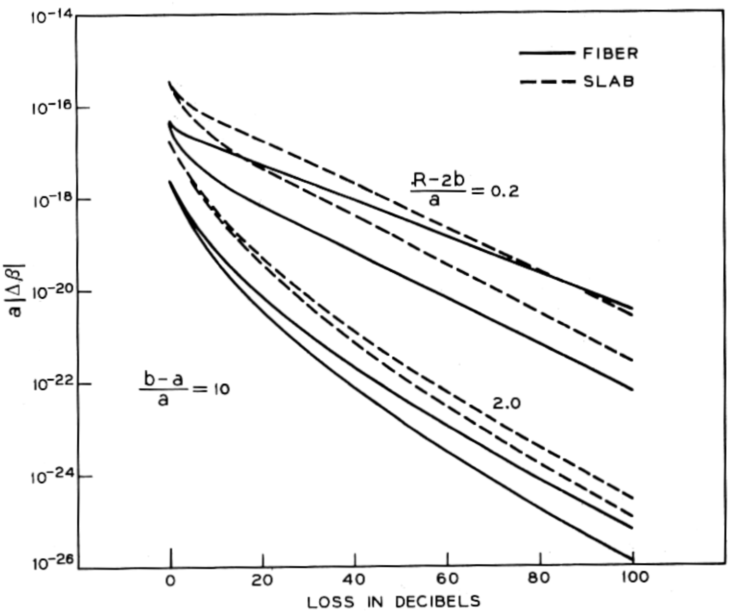

Fig. 6—Comparison between the coupling of round fibers and slab waveguides ($a = d$, $b = D$). The parameter value $\gamma a = 1.594$ is correct for the round fibers. It has also been used for the slab waveguides to enable a realistic comparison. $n_m ka = 16$, $n_c/n_m = 1.01$.

mode power loss versus plane wave power loss $2k[\text{Im}(n_3)a]$ in the surrounding medium obtained from (43), (44b), and (62). Both types of waveguides are represented in this figure. The loss values of abscissa and ordinate are both expressed in db. The decay parameters γa and γd (we use $a = d$ and $b = D$ in Fig. 7) are solutions of the respective eigenvalue equations and are not adjusted arbitrarily. We have seen that the coupling of the slab waveguides was weaker than the coupling of the fibers under these conditions. It is thus not surprising to see from Fig. 7 that the losses of the fiber are larger than the slab waveguide losses. It is interesting to see that the symmetric TM mode of the slab waveguide is slightly lossier than the symmetric TE mode.

Assuming again that $a = d = 1$ μm and limiting the allowable loss caused by the lossy surrounding medium to 1 db/km, we find that we can accept those curves with losses of less than $2\alpha a = 10^{-9}$ db. Since the loss curves are nearly independent of the actual value of the loss in the surrounding medium (after an initial rise from zero mode loss in a lossless surrounding medium) we can exclude all curves above the 10^{-9} line as unacceptable in case of a lossy surrounding medium. We thus see that the slab waveguide works well under all conditions shown. For the more important round fiber we must exclude the case of a cladding with $(b - a)/a = 5$. This means that the curves of Fig. 4 that lead to excessive coupling in the absence of sufficient loss in the surrounding medium are also unacceptable from the point of view of additional mode loss. We see that additional mode loss and mode coupling have similar trends. When the coupling between the fibers is too large we also have to contend with additional mode loss in excess of what we want to tolerate. A guide with low mode loss on the other hand is also sufficiently protected by its cladding so that coupling between adjacent fibers is not critical.

The two remaining curves, Figs. 8 and 9, show the coupling parameter and the additional mode loss for the case of a guide with lower core-to-cladding ratio. Only the case of the higher coupling for horizontally polarized modes has been plotted in Fig. 8. The remarks about coupling strength and additional mode loss hold true also for these conditions as can be seen from the figures.

VII. CONCLUSIONS

We have presented a perturbation theory for the coupling between two arbitrary dielectric waveguides made of lossy materials and embedded in a lossy environment. Only the case of two degenerate

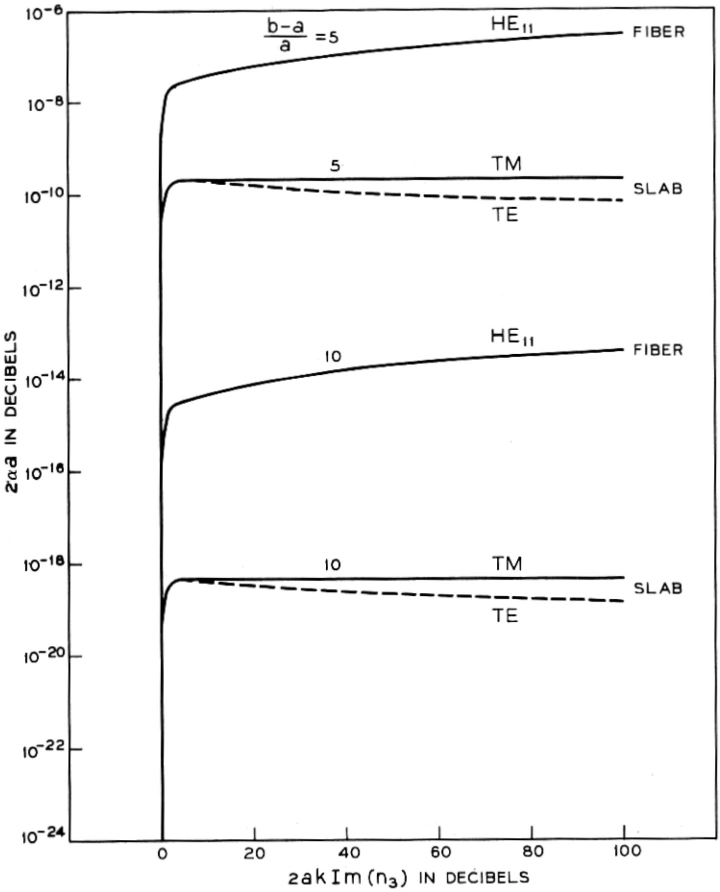

Fig. 7—Additional mode loss caused by the lossy surrounding medium. The abscissa indicates the loss (in db) that a plane wave would suffer by traveling a distance equal to the core radius in the surrounding medium. $n_m ka = 16$, $n_c/n_m = 1.01$.

modes has been treated. The general theory has been applied to cladded slab waveguides and cladded round optical waveguides (fibers) with lossless cladding material but in the presence of lossy surroundings. We have found that the coupling between fibers depends very strongly on the separation between the two fiber cores. Far separated cores result in loose coupling. The coupling between the fibers can be reduced by increasing the losses of the surrounding medium. However, a lossy surrounding medium introduces losses to the waveguide modes to such an extent that the mode losses are unacceptably high if the loss in the surrounding medium is necessary to uncouple the fibers. From this point of view it appears as though losses in the surrounding medium are unsuitable to achieve uncoupling or reduction of crosstalk between optical fibers. However, the coupling mechanism discussed in this paper is not the only contributor to crosstalk. Imperfections in the fibers cause light scattering that can also be a source of crosstalk.[8] This mechanism is less strongly dependent on the separation of the fiber cores so that it may not be possible to uncouple fibers sufficiently simply by protecting them with a cladding of sufficient thickness.

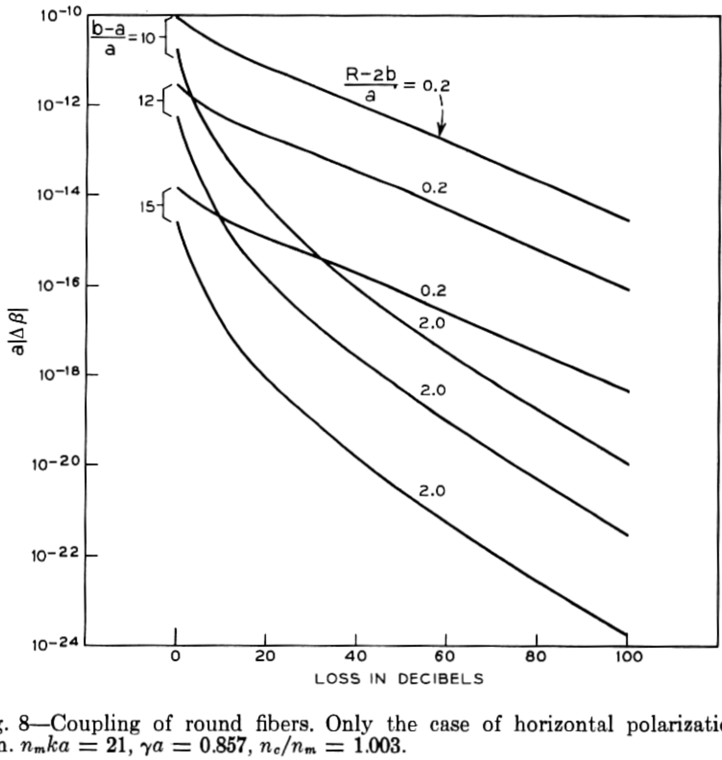

Fig. 8—Coupling of round fibers. Only the case of horizontal polarization is shown. $n_m ka = 21$, $\gamma a = 0.857$, $n_c/n_m = 1.003$.

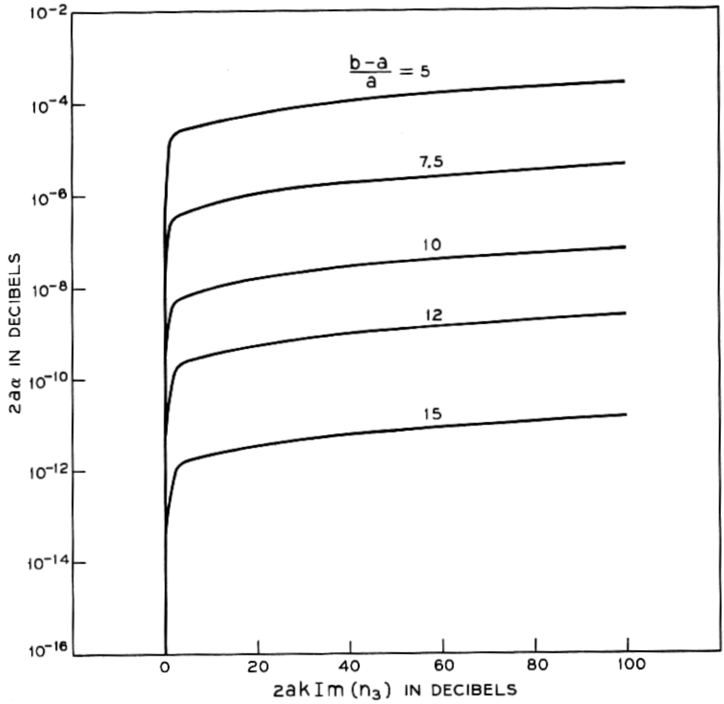

Fig. 9—Additional mode loss caused by the lossy surrounding medium. $n_m ka = 21$, $n_c/n_m = 1.003$.

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