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Low-Loss Microstrip Filters Developed by Frequency Scaling

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Low-pass and band-pass microstrip filters up to 30 GHz have been built on silica substrates by photolithographic reduction of optimized low-frequency models. Scaling accuracies of 1.8 percent or better have been achieved. An insertion loss of 0.28 dB has been measured for a five-section low-pass filter with a cutoff frequency of 5.7 GHz. The insertion loss of a 12-percent band-pass filter at 8.18 GHz is 0.16 dB. Good agreement between measured and calculated losses has been obtained. Circuit dimensions are given to facilitate scaling of the low-pass and band-pass microstrip filters to other frequencies.

I. INTRODUCTION

Microstrip filters are used in frequency converters, frequency multipliers, and branching networks for RF systems. Microstrip filters have been built in the past with photo-etched conductor patterns on ceramic substrates such as alumina, sapphire, and beryllia.[1,2] These materials are useful in microwave integrated circuits at a few gigahertz where a desirable size reduction relative to free-space wavelength is achieved because of the high dielectric constant of the substrate. At higher frequencies, size reduction is a disadvantage because the circuit elements are approaching the limitations of photolithographic reproduction. A suitable substrate material for use at higher frequencies is clear fused silica. Its relative dielectric constant over a wide frequency range is 3.82 and its dielectric Q is 3,000 or higher at all frequencies up to 50 GHz.

The purpose of this paper is to show that optimized microstrip filters at microwave and millimeter-wave frequencies can be built by linear reduction of a low-frequency model. The flow chart of this procedure is shown in Fig. 1. The oversize microstrip filter is designed, built, and tested at a few hundred megahertz. All circuit dimensions

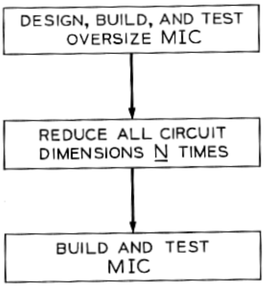

Fig. 1—MIC flow chart (general).

are then reduced N times in order to obtain the optimized microstrip filter at microwave or millimeter-wave frequencies.

The initial design and testing of the microstrip filters described in this paper are made at 285 MHz for the low-pass filter and 818 MHz for the band-pass filter. The filters are scaled by factors of 20 and 10 times, respectively, to produce 5.7-GHz and 8.18-GHz circuits. A scaling factor of 37.5 times is also used to obtain low-pass input and band-pass output filters for a 10.1-GHz to 30.3-GHz frequency multiplier which is described in another paper.[3]

II. MICROSTRIP SUBSTRATE

Fused silica, SiO_2, is selected as the microstrip substrate for its desirable properties. These properties are as follows:

(*i*) The dielectric constant and loss tangent are low.
(*ii*) The dielectric constant is independent of frequency up to 50 GHz.
(*iii*) The surface can be highly polished.
(*iv*) It is commercially available in a wide range of sizes and thicknesses.
(*v*) It has a low thermal expansion coefficient.

Table I lists the electrical and physical properties of fused silica and other common dielectric materials. Soda-lime glass, although not normally used as a microstrip substrate, is included as a low-cost material for price comparison. Detailed data on dielectric constant, loss tangent, and volume resistivity of high-purity synthetic fused silica are given in the appendix. Since silica has a low thermal conductivity of 1.4×10^{-2} watts/cm°K it is recommended for low- or medium-power applications.

III. SCALING OF MICROSTRIP CIRCUITS

The laws governing scaling of electromagnetic circuits are described by Stratton.[4] In order to obtain circuits with similar characteristics at different frequencies, each with a characteristic length l, an RF frequency ω, a conductivity σ, a permeability μ, and a permittivity ϵ, it is necessary and sufficient that:

$$\mu\epsilon(\omega l)^2 = K_1 \,, \tag{1}$$

$$\mu\epsilon\sigma\omega l^2 = K_2 \,, \tag{2}$$

where K_1 and K_2 are constants. Scaling has not been widely used for building microstrip circuits because high dielectric constant materials are expensive and difficult to obtain in large sizes and also because air gaps between these materials and metal conductors have a significant effect on the circuit performance and are difficult to scale. These problems are substantially reduced by using a low dielectric constant material such as silica.

A detailed flow chart of the scaling process is shown in Fig. 2. All circuit dimensions are reduced by the same factor by which the frequency is increased, thus satisfying equation (1). Equation (2) is only partially satisfied because brass conductors have been used in the over-

TABLE I—PROPERTIES OF MIC SUBSTRATES

Property	Units	Silica SiO_2	Alumina Al_2O_3 (99.5%)	Beryllia BeO(99.5%)	Soda-Lime Glass Corning #0080
Dielectric Constant at 10 GHz, 25°C		3.82	9.8	6.8	6.71
Loss Tangent 10 GHz	$\times 10^{-4}$	1	4	3	170
Thermal Expansion	PPM/°C	0.35	6.6	7.5	9.2
Relative Cost Factor*		6	15	125	1
Surface Condition		Polished	Polished	Polished	Drawn

* Relative cost factor of substrate material compared to drawn soda-lime glass = 1 (Data from *Handbook of Thin Film Technology*, edited by L. I. Maissel and R. Glang, McGraw–Hill, 1970, Section 6–43).

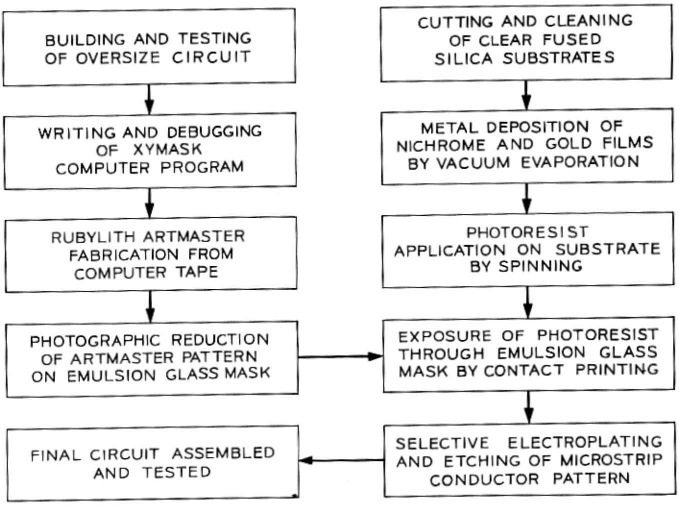

Fig. 2—MIC flow chart (detailed).

size model and gold conductors are used in the reduced size circuits. This leads to a slight increase in the scaled circuit losses because the skin effect loss increases with the square root of the frequency.

IV. LOW-PASS FILTER

The low-pass filter is a five-element network of three shunt capacitors, C_1, C_3, and C_5, and two series inductors, L_2 and L_4, designed for a 0.2-dB Chebyshev frequency response. The microstrip layout of the filter is shown in Fig. 3. The total filter length is slightly less than $\lambda_o/5$; λ_o is the free-space wavelength at the 0.2-dB cutoff frequency. The length of each element is less than one-eighth of a microstrip wavelength, which allows the initial design of the filter to be based on a lumped circuit concept. The filter parameters C_n and L_n are computed from normalized prototype parameters g_n, the input impedance $R = 50\Omega$, and the cutoff frequency $\omega_c = 2\pi f_c$:

$$C_n = \frac{1}{\omega_c R} g_n , \tag{3}$$

$$L_n = \frac{R}{\omega_c} g_n . \tag{4}$$

Table II lists the parameters of the filter. The 0.2-dB cutoff frequency is 285 MHz. The thickness of the dielectric substrate is 0.5

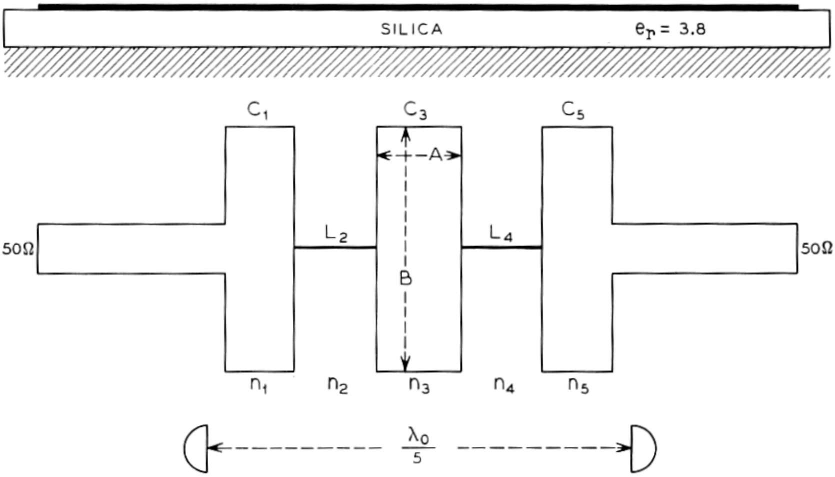

Fig. 3—Low-pass filter (circuit).

inch and its relative dielectric constant is 3.82. The computed filter area is based on the choice of a specific cutoff frequency and does not take into account distributed line effects at higher frequencies. The experimental filter has been modified in order to optimize the performance up to $4\omega_c$.

The computed dimensions of the capacitive plates shown in Fig. 3 are found as follows:

TABLE II—LOW-PASS FILTER PARAMETERS

Design Parameters				Computed Filter Dimensions		Experimental Filter Dimensions		
n	g_n	C_n pF	L_n nH	Aspect Ratio a/b	Capacitor Area A in.² or Inductance Length l_n in.	a in.	b in.	Area in.²
1	1.339	15		3.52	A_1 = 5.67	1.42	5.0	7.1
2	1.337		37.4		l_2 = 1.82	0.70	1.62	
3	2.166	24.2		2.91	A_3 = 10.23	1.72	5.0	8.6
4	1.337		37.4		l_4 = 1.82	0.70	1.62	
5	1.339	15		3.52	A_5 = 5.67	1.42	5.0	7.1

(*i*) An effective area A_{eff} is defined as the area of a capacitor plate which has no fringing fields. This area is given by

$$A_{eff} = \frac{C_n t}{\epsilon_o \epsilon_r}, \tag{5}$$

where t = dielectric substrate thickness = 0.5 inch,
ϵ_r = relative dielectric constant, and
ϵ_o = 8.85 pF/m.

(*ii*) The actual area A of the capacitor plate is smaller because of fringe field effects. If a and b are the width and length of the capacitor plate shown in Fig. 3, then A_{eff} is given by

$$A_{eff} = (a + \alpha t)(b + \alpha t), \tag{6}$$

where α is a normalized factor which describes the effect of the fringe field.

(*iii*) The aspect ratio of the capacitor plate shown in Fig. 3 is defined as $r = a/b$ and the factor β by

$$\beta = \sqrt{r} + \frac{1}{\sqrt{r}}. \tag{7}$$

(*iv*) The area A is obtained from equations (5), (6), and (7):

$$A = \left\{ \left[\frac{C_n t}{\epsilon_o \epsilon_r} + \frac{(\beta^2 - 4)\alpha^2 t^2}{4} \right]^{\frac{1}{2}} - \frac{\alpha \beta t}{2} \right\}^2. \tag{8}$$

One finds by empirical methods that the effective increase in each plate dimension due to the fringe field is approximately equal to the substrate thickness t; thus, $\alpha = 1$ and equation (8) is simplified to

$$A = \frac{t^2}{4} \left\{ \left[\frac{4C_n}{\epsilon_o \epsilon_r t} + \beta^2 - 4 \right]^{\frac{1}{2}} - \beta \right\}^2. \tag{9}$$

V. MICROSTRIP INDUCTORS

The inductance L per unit length for a narrow microstrip conductor with a width w and a substrate thickness h is

$$L = \frac{60 \ln \left(\frac{8h}{w} + \frac{w}{4h} \right)}{v_o}, \tag{10}$$

where v_o is the velocity of light in free space, $v_o = 3 \times 10^{10}$ cm/s = 1.18×10^{10} ips. The length l_n of the inductor element is given by the ratio L_n/L where L_n is the required inductance given in Table II,

$$l_n = \frac{L_n}{L} = \frac{v_o R}{60\omega_c \ln\left(\dfrac{8h}{w} + \dfrac{w}{4h}\right)} \cdot g_n \; . \tag{11}$$

The computed length for the inductive elements is 1.82 inches. The actual length of the final inductive elements in the filter is 1.61 inches because the current constrictions in the capacitor plates increase the effective length of each element.

Calculated and measured transmission loss for a low-pass filter with a cutoff frequency of 285 MHz is shown in Fig. 4. The transmission is plotted as a function of normalized frequency ω/ω_c. Figure 5 shows the frequency response after scaling the filter by a factor of 20X. The new cutoff frequency is 5.6 GHz and the scaling accuracy is 1.8 percent.

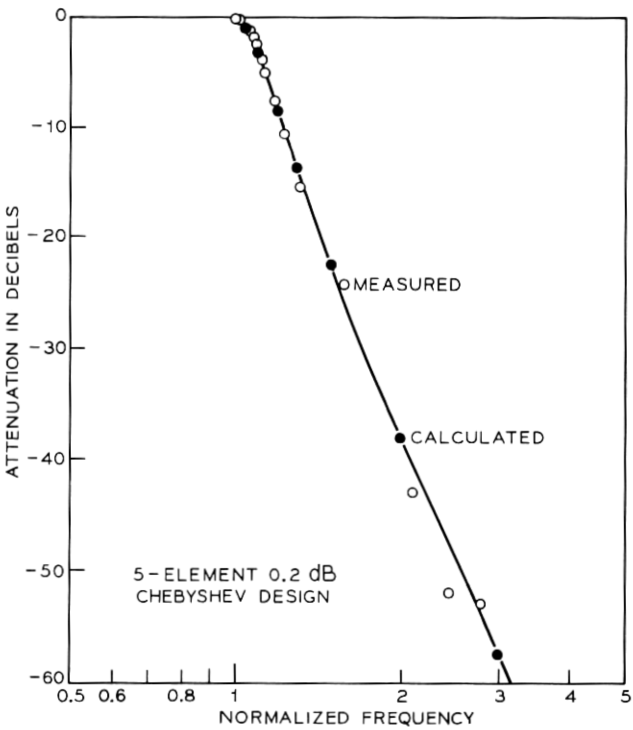

Fig. 4—Low-pass filter (comparison of calculated and measured data points).

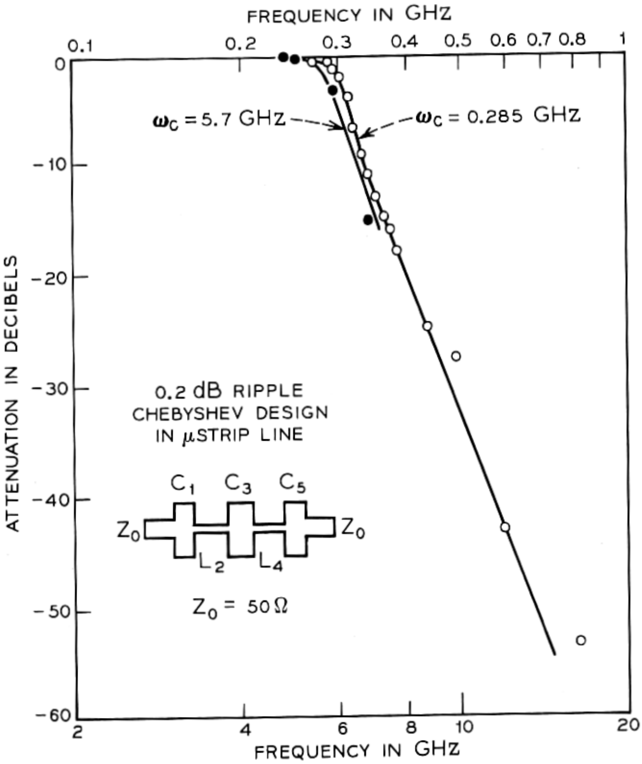

Fig. 5—5-element low-pass filter (results of scaling from 285 MHz to 5.7 GHz).

VI. BAND-PASS MICROSTRIP FILTER

Band-pass filters utilizing two parallel coupled $\lambda/2$ resonators have been designed, built, and optimized at 820 MHz and scaled to 8.2 GHz and 30 GHz. Figure 6 shows the microstrip lay-out of the conductor pattern on the silica substrate. The filter has a 0.2-dB Chebyshev ripple and 20 dB attenuation at the $\pm$33-percent bandwidth points. The calculated and measured filter responses are shown in Fig. 7 and the filter characteristics, after scaling by a factor of 10X, are plotted in Fig. 8. The scaling accuracy for the midband frequency is better than 1 percent. The 7-percent decrease in bandwidth is due to undercutting of the conductor elements during processing of the microstrip pattern. Undercutting is etching of the conductor pattern beneath the edge of the protective photoresist layer. It decreases

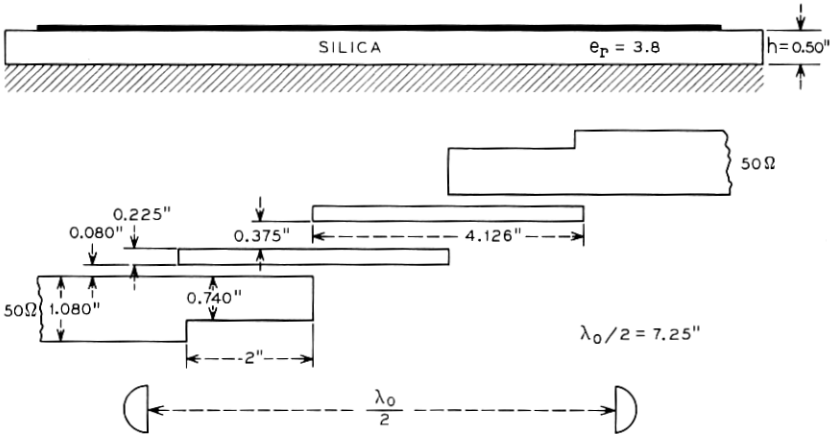

Fig. 6—Band-pass filter (circuit).

all conductor dimensions by a small distance. The effect of undercutting is most noticed on small critical circuit dimensions such as the 0.008-inch-wide coupling gap of the $\lambda/2$ resonator. Undercutting can be controlled by establishing the undercut dimension and adding this to all circuit elements on the original artmaster.

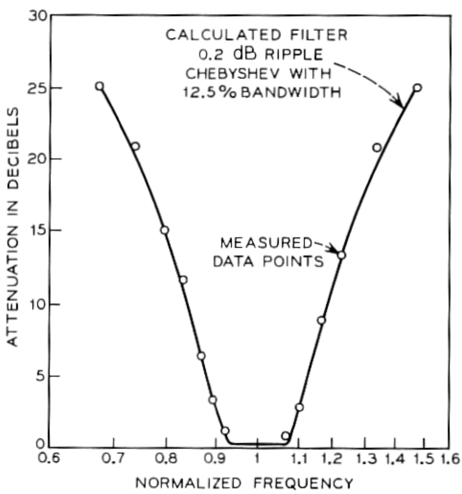

Fig. 7—Experimental and theoretical attenuation of parallel coupled band-pass filter.

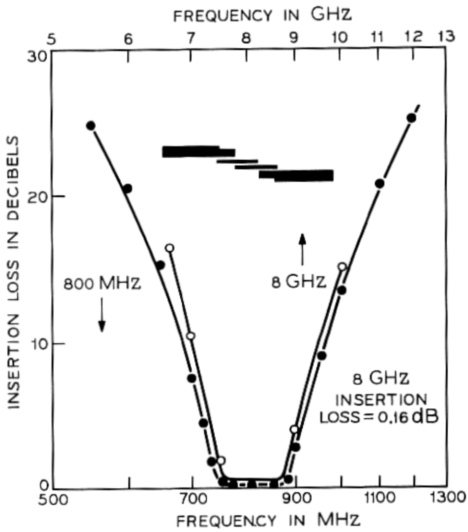

Fig. 8—Microstrip band-pass filter (results of scaling from 800 MHz to 8 GHz).

VII. ATTENUATION OF MICROSTRIP FILTERS

The dissipative loss in the pass band of a multiple-resonator filter is[5]

$$L_o = \frac{10}{w' \ln 10} \sum_{i=1}^{n} \frac{g_i}{Q_i} \text{ dB}, \tag{12}$$

where w' is the fractional bandwidth of the filter, g_i is the normalized prototype filter parameter, and Q_i is the unloaded Q of each filter element.

The unloaded Q_i of a microstrip element is

$$Q_i = \frac{20\pi}{\ln 10} \cdot \frac{1}{\alpha_o \lambda_o}, \tag{13}$$

where λ_o is the free-space wavelength, and α_o is the free-space attenuation in dB per unit length as derived by M. V. Schneider,[6] where

$$\alpha_o = \frac{10 R_s}{\pi h \ln 10} \frac{\left(\frac{8h}{w} - \frac{w}{4h}\right)\left(1 + \frac{h}{w} + \frac{h}{w}\frac{\partial w}{\partial t}\right)}{Z_o \exp\left(\frac{Z_o}{60}\right)} \qquad w/h \leqq 1 \tag{14}$$

or

$$\alpha_o = \frac{Z_o R_s}{720\pi^2 h \ln 10}\left[1 + \frac{0.44h^2}{w^2} + \frac{6h^2}{w^2}\left(1 - \frac{h}{w}\right)^5\right]\left(1 + \frac{w}{h} + \frac{\partial w}{\partial t}\right)$$

$$w/h \leqq 1. \tag{15}$$

Z_o is the characteristic impedance of the microstrip line:

$$Z_o = 60 \ln\left(\frac{8h}{w} + \frac{w}{4h}\right) \qquad w/h \leqq 1 \tag{16}$$

$$Z_o = \frac{240\pi}{\dfrac{2w}{r} + 5 - \dfrac{h}{w}} \qquad w/h \geqq 1. \tag{17}$$

R_s is the skin resistance of the metal conductor as shown in Fig. 9, h is the conductor height, and w is the conductor width. The partial derivative $\partial w/\partial t$ is given by:

$$\frac{\partial w}{\partial t} = \frac{1}{\pi}\ln\frac{4\pi w}{t} \qquad w/h \leqq \frac{1}{2\pi} \tag{18}$$

$$\frac{\partial w}{\partial t} = \frac{1}{\pi}\ln\frac{2h}{t} \qquad w/h \geqq \frac{1}{2\pi} \tag{19}$$

where t is the conductor thickness. The calculated and measured dissipative losses for the low-pass filter and for the band-pass filter are shown in Table III.

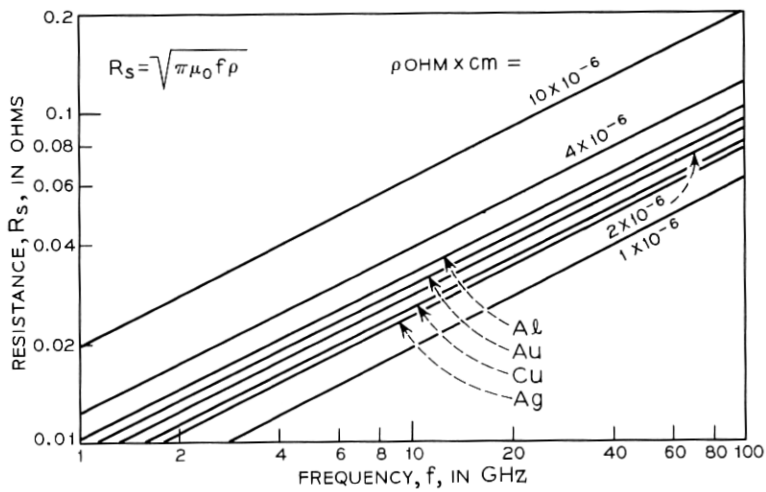

Fig. 9—Skin resistance of metals.

TABLE III—INSERTION LOSS MEASUREMENTS

Filter	Insertion Loss in dB	
	Calculated	Measured
Loss-Pass $f_c = 5.7$ GHz 5 sections	0.13	0.28
Band-Pass $f_0 = 8.2$ GHz $w' = 12\%$ 2 sections	0.11	0.16

Unloaded Q of Single Band-Pass Resonator	
Calculated Q	546
Measured Q	375

The calculated loss and the calculated Q factor are based on the assumption that the conductor resistivity is independent of frequency and equal to the dc resistivity of the metal.

VIII. CONCLUSION

It is shown that scaling is a powerful tool for building and optimizing microwave integrated circuits. A five-element low-pass filter and a two-element band-pass filter have been built and measured in the 200- to 800-MHz frequency band. The low-pass filter has been scaled to 5.7 GHz and 10 GHz, and the band-pass filter has been scaled to 8.2 GHz and 30 GHz. Good agreement between measured and calculated losses has been obtained. Unloaded Q factors of 375 at 8.2 GHz are reported.

IX. ACKNOWLEDGMENTS

The author wishes to thank M. V. Schneider for his helpful suggestions throughout this project, and to thank S. R. Shah for building the accurate, scaled microcircuit filters on fused silica.

APPENDIX

Properties of High-Purity Synthetic Fused Silica

Table IV gives detailed data on the dielectric constant, loss tangent, and volume resistivity of high-purity synthetic fused silica produced by chemical vapor deposition (Dynasil No. 4000).

TABLE IV—MIT MEASUREMENTS OF DYNASIL NO. 4000 (JULY 1970)

Dielectric Constant	100 Hz	10 kHz	1 MHz	10 MHz	8.60 GHz	24.0 GHz
25°C	3.826	3.826	3.826	3.826	3.824	3.82
100°C	3.834	3.834	3.834	3.834	3.832	3.83
200°C	3.843	3.843	3.843	3.843	3.841	3.84
300°C	3.861	3.856	3.854	3.854	3.852	3.85
400°C	4.043	3.870	3.865	3.865	3.863	3.86
500°C	4.795	3.886	3.878	3.878	3.876	3.87
Loss Tangent						
25°C	<.000004	<.000005	.000015	.00002	.00012	.00033
100°C	<.000004	<.00002	<.00005	<.0001	.00012	.00032
200°C	.00137	.00003	<.00005	<.0001	.00012	.00026
300°C	.0930	.00093	<.00005	<.0001	.00012	.00025
400°C	2.03	.0207	.00021	<.0001	.00012	.00022
500°C	11.36	.140	.00140	.00014	.00012	.00020
Volume Resistivity	$\log_{10} \rho$ ohm-cm					
25°C	>15.0	>12.9	10.5	9.34	5.65	
100°C	>15.0	>12.4	>9.95	8.65	5.65	
200°C	12.5	12.2	>9.95	>8.65	5.65	
300°C	10.9	10.9	>9.95	>8.65	5.65	
400°C	9.34	9.34	9.34	>8.65	5.65	
500°C	8.52	8.52	8.52	8.52	5.65	

Source: A. R. von Hippel and W. Westphal, Laboratory of Insulation Research, MIT.

REFERENCES

1. Chinchillo, A. R., and Perry, R. W., "Microstrip Filter Loss Reduction Techniques," NEREM 70 Record, *12* (November 1970), pp. 72–73.
2. Dell-Imagine, R. A., "A Parallel Coupled Microstrip Filter Design Procedure," G-MTT 1970 Int. Microwave Symp. Digest, May 1970, pp. 29–32.
3. Schneider, M. V., and Snell, W. W., "A Scaled Hybrid Integrated Multiplier from 10 to 30 GHz," B.S.T.J., this issue, pp. 1933–1942.
4. Stratton, J. A., *Electromagnetic Theory,* New York: McGraw-Hill, 1941, pp. 488–490.
5. Cohn, S. B., "Dissipation Loss in Multiple-Coupled-Resonator Filters," Proc. IEEE, *47,* No. 8 (August 1959), pp. 1342–1348.
6. Schneider, M. V., "Microstrip Lines for Microwave Integrated Circuits," B.S.T.J., *48,* No. 5 (May–June 1969), pp. 1421–1444.