THE BELL SYSTEM TECHNICAL JOURNAL
Vol. 50, No. 7, September, 1971
Printed in U.S.A.

On the Intersymbol Interference Problem for the Gaussian Channel

By A. D. WYNER

(Manuscript received March 31, 1971)

In this paper we are concerned with the Holsinger–Gallager model for the continuous-time Gaussian channel. Gallager[1] *proved a coding theorem for this channel, and Cordaro and Wagner*[2] *showed that the theorem remains valid when the effect of intersymbol interference from previous channel uses is taken into account. We show here that the Cordaro–Wagner result holds under somewhat weaker hypotheses. Further, the proof here is more elementary, since it does not depend on reproducing kernel Hilbert space theory. Finally we pose what we feel is an important open problem concerning the stability of the model.*

I. INTRODUCTION

In this paper we are concerned with the Holsinger-Gallager model of the continuous-time Gaussian channel. Gallager[1] proved a coding theorem for this channel, and Corado and Wagner[2] showed that this theorem remains valid when the effect of intersymbol interference from previous channel uses is taken into account. We show here that the Cordaro-Wagner result holds under somewhat weaker hypotheses. Further, our proof is more elementary, since it does not depend on reproducing kernel Hilbert space theory. Finally, we pose what we feel is an important open problem concerning the stability of the model.

In the Holsinger-Gallager[1] model, the channel output is

$$y(t) = \int_{-\infty}^{t} h_1(t - \tau)x(\tau)\, d\tau + z(t), \qquad -\infty < t < \infty, \tag{1}$$

where $x(t)$ is the channel input, $h_1(t)$ is the impulse response of a causal linear filter, and $z(t)$ is a sample from a stationary Gaussian process with two-sided spectral density $N(f)$. A *code* (M, T, S, λ) for this channel is a set of M functions $\{x_i(t)\}_{i=1}^{M}$ with support on the interval $[0, T]$ which satisfy

$$\| x_i \|^2 = \int_0^T x_i^2(t)\, dt \leqq ST \tag{2}$$

(thus S is the allowable average signal "power"), together with a set of M disjoint Borel sets $\{B_i\}_1^M$ of functions defined on $[0, T]$, such that the error probabilities

$$P_{ei} \triangleq \Pr\{y_i \notin B_i\} \leqq \lambda, \qquad 1 \leqq i \leqq M. \tag{3}$$

The random function $y_i(t)(0 \leqq t \leqq T)$ is given by (1) with $x(t) = x_i(t)$.

Gallager made the following assumptions

$$\int_{-\infty}^{\infty} N(f)\, df < \infty,$$

$$\int_{-\infty}^{\infty} | H_1(f) |^2\, df < \infty, \qquad \int_{-\infty}^{\infty} \frac{| H_1(f) |^2}{N(f)}\, df < \infty, \tag{4}$$

where $H_1(f) = \int_0^\infty h_1(t) e^{-i2\pi f t}\, dt$ is the Fourier transform of $h_1(t)$. Subject to these conditions, Gallager shows that the capacity of the channel with allowable average signal power S is

$$C = C_S = \int_{-\infty}^{\infty} \max \left\{ \log \frac{B_S\, | H_1(f) |^2}{N(f)}, 0 \right\} df, \tag{5a}$$

where the number B_S is defined (uniquely) by

$$S = \int_{-\infty}^{\infty} \max \left\{ B_S - \frac{N(f)}{| H_1(f) |^2}, 0 \right\} df. \tag{5b}$$

Equations (5) are justified by the following:

Theorem 1 (Gallager): Let $E(R, S)$ be the error-exponent given by Gallager (Ref. 1, Section 8.5). $E(R, S) > 0$ and continuous, for $S > 0$, and $0 \leqq R < C_S$. Then for arbitrary $\epsilon > 0$, $S > 0$, $0 \leqq R < C_S$, there exist codes (M, T, S, λ) where (as $T \to \infty$)

$$M \geqq e^{RT} \quad \text{and} \quad \lambda \leqq \exp\{-(E(R, S) - \epsilon)T + o(T)\}.$$

Now suppose that we wish to use the channel in successive T-second intervals. If $\{x_i(t)\}_{i=1}^M$ are a set of M code functions with support on $[0, T]$, then the channel output for $0 \leqq t \leqq T$ will be given by

$$y_i^*(t) = \int_0^t h_1(t - \tau) x_i(\tau)\, d\tau + z(t) + \sum_{n=-\infty}^{-1} \int_{nT}^{(n+1)T} h_1(t - \tau) x_{i_n}(\tau - nT)\, d\tau, \tag{6}$$

where $1 \leqq i, i_n \leqq M$ $(n = -1, -2, \cdots)$. The term

$$\hat{r}(t) = \sum_{n=-\infty}^{-1} \int_{nT}^{(n+1)T} h_1(t - \tau)x_{i_n}(\tau - nT)\, d\tau$$

represents intersymbol interference in the interval $[0, T]$ due to signals corresponding to previous intervals. We use $y_i^*(t)$ instead of $y_i(t)$ to indicate the presence of intersymbol interference. With the channel output given by (6) we can redefine a code (M, T, S, λ) as a set of M pairs $\{(x_i(t), B_i)\}_{i=1}^{M}$ exactly as above. Here, however, we require [instead of (3)] that

$$\Pr\,\{y_i^*(t) \notin B_i\} \leqq \lambda, \qquad 1 \leqq i \leqq M, \tag{7}$$

for *all possible* $\hat{r}(t)$. The random function $y_i^*(t)$ is given by (6).

Cordaro and Wagner[2] succeeded in establishing the validity of Gallager's Theorem for this modified model by making the following additional assumptions. The first is that

$$\int_{-\infty}^{\infty} [|\log N(f)|/(1 + f^2)]\, df < \infty, \tag{8}$$

so that $N(f) = |G_1(f)|^2$ for some $G_1(f)$ with inverse Fourier transform $g_1(t) = 0$, $t < 0$. The second assumption is that $k_1(t)$, the inverse Fourier transform of $H_1(f)/G_1(f)$, is bounded by

$$|k_1(t)| \leqq Ae^{-ct}, \tag{9}$$

for some A, $c > 0$.

In Section II we show that the Cordaro-Wagner result holds with condition (9) replaced by the following essentially weaker condition: By (8) we can find a $G(f)$ such that $N(f) = |G(f)|^2$ and the linear filters corresponding to $G(f)$ and its inverse are causal.[3] Let $k(t)$ be the inverse Fourier transform of $H_1(f)/G(f)$. Then the new condition is that for t sufficiently large

$$|k(t)| \leqq \frac{A}{t^{1+\beta}}, \tag{9$'$}$$

for some A, $\beta > 0$. Note that by (4)

$$\int_{-\infty}^{\infty} [|H_1(f)|^2/N(f)]\, df = \int_0^{\infty} k^2(t)\, dt < \infty,$$

so that (9′) is not a very strong additional assumption.

An Open Problem: In order to achieve the error probability guaranteed by Theorem 1 or in fact simply a vanishing error probability (as $T \to \infty$) for $R < C_S$, Gallager's proof requires the receiver to make arbitrarily precise measurements (see for example Lemma 8.5.1 in Ref. 2). A practical system, however, imposes certain limitations on the accuracy with which we can make measurements. Therefore a reasonable requirement for the decoding regions $\{B_i\}_1^M$ is the following:

$$y_1 \varepsilon B_i , y_2 \varepsilon B_j \qquad (i \neq j) \Rightarrow \frac{1}{T} \int_0^T [(y_1(t) - y_2(t))]^2 \, dt > \nu, \tag{10}$$

for some $\nu > 0$. Thus ν is a measure of the accuracy of the measurements at the decoder. The channel capacity will therefore depend on ν, say $C_S(\nu)$. A quantity of interest might be $\lim_{\nu \to 0} C_S(\nu)$. Whether or not this quantity is the same as C_S in (5) is an open question.

II. PROOF OF THE MAIN RESULT

Let us consider first the problem with no interference from previous channel uses when the input $x(t)$ has support on the interval $[0, T]$. Gallager (Lemma 8.5.1, p. 413) shows that knowledge of the function $y(t)$, $0 \leqq t \leqq T$, is equivalent to knowledge of a certain vector $\mathbf{v} = (v_1 , v_2 , \cdots)$. This vector $\mathbf{v}$ can be represented by

$$\mathbf{v} = \mathbf{u} + \mathbf{z}, \tag{11}$$

where $\mathbf{z}$ is a sequence of statistically independent standard Gaussian variates, and the vector $\mathbf{u}$ is defined as follows. Let

$$u(t) = \int_0^T k(t - \tau)x(\tau) \, d\tau, \qquad t \geqq 0, \tag{12}$$

where $k(t)$ is defined in Section I and $x(t)$ is the channel input. Let $\mathcal{S}$ be the subspace of $\mathcal{L}_2(-\infty, \infty)$ spanned by the orthonomal functions $\{\theta_i(t)\}_1^\infty$ defined on p. 416 of Ref. 2. Let $P_\mathcal{S}(u)$ be the projection of $u(t)$ on the subspace $\mathcal{S}$. Then $\mathbf{u} = (u_1 , u_2 , \cdots)$, where u_k is the coefficient of $\theta_k(t)$ in the expansion of $P_\mathcal{S}(u)$ in the basis $\{\theta_i(t)\}_1^\infty$.

We will not need any properties of the $\theta_i(t)$ except for the fact [which follows from the causality of the filters corresponding to $G(f)$ and $H_1(f)/G(f)$] that $\theta_i(t)$ has support on the interval $[0, T]$.

Let $\{x_i(t)\}_1^M$ be a set of code signals with parameters $S = S_1$ and $T = T_1$. Let $\mathbf{u}_i$ be the $\mathbf{u}$ corresponding to $x(t) = x_i(t)$, and let $\mathbf{v}_i = \mathbf{u}_i + \mathbf{z}$. Then if the minimum-distance decoder is used,

$$\begin{aligned} P_{ei} = \Pr \{y_i \not\varepsilon B_i\} &= \Pr \bigcup_{j \neq i} [\| \mathbf{v}_i - \mathbf{u}_i \| \geqq \| \mathbf{v}_i - \mathbf{u}_j \|] \\ &= \Pr \bigcup_{j \neq i} [\| \mathbf{z} \| \geqq \| \mathbf{z} - (\mathbf{u}_j - \mathbf{u}_i) \|] \\ &= \Pr \bigcup_{j \neq i} [\langle \mathbf{z}, \mathbf{u}_j - \mathbf{u}_i \rangle \geqq \tfrac{1}{2} \| \mathbf{u}_j - \mathbf{u}_i \|^2], \end{aligned} \tag{13}$$

where "$\| \ \|$" denotes Euclidean norm and "$\langle \ , \ \rangle$" denotes inner product. In particular,

$$P_{ei} = \Pr \{y_i \notin B_i\} \geqq \Pr \{\langle \mathbf{z}, \mathbf{u}_j - \mathbf{u}_i \rangle \geqq \tfrac{1}{2} \| \mathbf{u}_j - \mathbf{u}_i \|^2\}$$
$$= \Phi_c(\tfrac{1}{2} \| \mathbf{u}_j - \mathbf{u}_i \|), \qquad j \neq i, \tag{14}$$

where $\Phi_c(\xi) = \int_\xi^\infty (2\pi)^{-\frac{1}{2}} e^{-\eta^2/2} \, d\eta$, the complementary error function.

Now, let us suppose that we are given a code (M, T_1, S_1, λ_1), $\{(x_i(t), B_i)\}_1^M$ for the no-interference model. We can assume that the B_i correspond to the minimum-distance decoder. We now form a new code $\{(x_i^*(t), B_i^*)\}_1^M$ with parameters $T = T_2 = (1 + \delta)T_1$ and $S = S_2 = \alpha S_1/(1 + \delta)$ for use on the channel with intersymbol interference (6). We set

$$x_i^*(t) = \begin{cases} \alpha x_i(t), & 0 \leqq t \leqq T_1, \\ 0, & T_1 < t \leqq (1 + \delta)T_1 = T_2, \end{cases} \tag{15}$$

where $\alpha > 1$ and $\delta > 0$ are arbitrary. Note that we have allowed a guard band of width δT_1 between channel inputs. We will specify the decoding sets B_i^* below, mentioning here only that the decoder will observe the received waveform $y^*(t)$ only for $0 \leqq t \leqq T_1$.

We can discretize the channel exactly as above and consider the channel output (when $x_i^*(t)$ is the input) to be given by the vector

$$\mathbf{v}_i^* = \alpha \mathbf{u}_i + \mathbf{z} + \hat{\mathbf{u}}, \tag{16}$$

where $\mathbf{u}_i$ and $\mathbf{z}$ are exactly as in (11) and $\hat{\mathbf{u}}$ (which represents the effect of previous channel uses) is the vector whose coordinates are the coefficients in the expansion of $P_S(\hat{u}(t))$ in the $\{\theta_i(t)\}$, and

$$\hat{u}(t) = \sum_{n=-\infty}^{-1} \int_{nT}^{(n+1)T} k(t - \tau) x_{i_n}^*(\tau - nT) \, d\tau = \sum_{n=-\infty}^{-1} \hat{u}_n(t), \tag{17}$$

where $1 \leqq i_n \leqq M$.

The decoding regions B_i^* will correspond to the minimum-distance decoder, i.e., $y^* \in B_i^*$ if the corresponding $\mathbf{v}$ is closer in Euclidean norm to $\alpha \mathbf{u}_i$ than to all $\alpha \mathbf{u}_j$ $(j \neq i)$. Thus for a given $\hat{\mathbf{u}}$,

$$\Pr \{y_i^* \notin B_i^*\} = \Pr \bigcup_{j \neq i} \{\| \mathbf{v}_i^* - \alpha \mathbf{u}_i \| \geqq \| \mathbf{v}_i^* - \alpha \mathbf{u}_j \|\}.$$

Now let ϵ, $S_1 > 0$, and $R(0 \leqq R < C_{S1})$ be given, and let the (M, T_1, S_1, λ_1) code $\{(x_i, B_i)\}_1^M$, discussed above, be a set of codes which satisfy Theorem 1; that is,

$$M \geqq e^{RT_1} \quad \text{and} \quad \lambda_1 \leqq \exp \{-(E(R, S_1) - \epsilon)T_1 + o_1(T_1)\}.$$

We will show that for T_1 sufficiently large, the derived code has parameter $\lambda \leqq \lambda_1$. Thus we will have found a set of codes (M, T_2, S_2, λ) for the model with intersymbol interference with

$$S_2 = \frac{\alpha^2 S_1}{(1 + \delta)}, \qquad M \geqq \exp\left\{\frac{R}{(1 + \delta)} T_2\right\},$$

and

$$\lambda \leqq \exp\left\{-\frac{(E(R, S_1) - \epsilon)}{(1 + \delta)} T_2 + \frac{o_1(T_2)}{(1 + \delta)}\right\}.$$

Since $E(R, S)$ is continuous and α may be chosen arbitrarily close to 1 and δ arbitrarily close to zero, we will have established Theorem 1 for the intersymbol interference case which is our main result. Thus it remains to establish that the error probability for the derived code $\leqq \lambda_1$. We will do this by showing that (for T_1 sufficiently large) for each $i = 1, 2, \cdots, M$ and all possible $\hat{r}(t)$,

$$\Pr\{y^*(t) \notin B_i^*\} \leqq \Pr\{y(t) \notin B_i\}. \tag{18}$$

Inequality (18) will follow directly from the following lemmas (the proofs of which conclude this section).

Lemma 1: Inequality (18) is satisfied, if

$$\|\hat{\mathbf{u}}\| \leqq \frac{(\alpha - 1)}{2} \min_{i \neq j} \|\mathbf{u}_i - \mathbf{u}_j\|. \tag{19}$$

Lemma 2: For the codes $\{(x_i, B_i)\}_{i=1}^{M}$, as $T_1 \to \infty$,

$$\min_{i \neq j} \|\mathbf{u}_i - \mathbf{u}_j\|^2 \geqq O(T_1).$$

Lemma 3: As $T_1 \to \infty$, $\|\hat{\mathbf{u}}\|^2 \leqq O(T_1^{1-2\beta})$.

From Lemmas 2 and 3, condition (19) in Lemma 1 will be satisfied for T_1 sufficiently large. Thus so will (18) be satisfied for T_1 sufficiently large.

Proof of Lemma 1: Since B_i and B_i^* are the minimum-distance decoders, the left member of (18) is

$$\Pr\{y_i^* \notin B_i^*\} = \Pr \bigcup_{j \neq i} \{\|\mathbf{v}_i^* - \alpha\mathbf{u}_i\| \geqq \|\mathbf{v}_i^* - \alpha\mathbf{u}_j\|\}. \tag{20}$$

The right member of (18), $\Pr\{y_i \notin B_i\}$ is given by (13). Consider the event

$$\{\|\mathbf{v}_i^* - \alpha\mathbf{u}_i\| \geqq \|\mathbf{v}_i^* - \alpha\mathbf{u}_j\|\}$$
$$= \left\{\langle \mathbf{z} + \hat{\mathbf{u}}, \mathbf{u}_j - \mathbf{u}_i\rangle \geqq \frac{\alpha}{2}\|\mathbf{u}_j - \mathbf{u}_i\|^2\right\}. \tag{21}$$

But by the hypothesis of Lemma 1 (19),

$$| \langle \hat{\mathbf{u}}, \mathbf{u}_j - \mathbf{u}_i \rangle | \leqq || \hat{\mathbf{u}} || \cdot || \mathbf{u}_j - \mathbf{u}_i || \leqq \frac{(\alpha - 1)}{2} || \mathbf{u}_j - \mathbf{u}_i ||^2.$$

Thus the event in (21)

$$\subseteq \left\{ \langle \mathbf{z}, \mathbf{u}_j - \mathbf{u}_i \rangle \geqq \frac{\alpha}{2} || \mathbf{u}_j - \mathbf{u}_i ||^2 - \frac{(\alpha - 1)}{2} || \mathbf{u}_j - \mathbf{u}_i ||^2 \right\}$$

$$= \{ \langle \mathbf{z}, \mathbf{u}_j - \mathbf{u}_i \rangle \geqq \tfrac{1}{2} || \mathbf{u}_j - \mathbf{u}_i ||^2 \},$$

Lemma 1 now follows from (20), (13) and the above.

Proof of Lemma 2: For the codes $\{(x_i , B_i)\}_1^M$,

$$\Pr \{ y_i \notin B_i \} \leqq \exp \{ - (E(R, S) - \epsilon) T_1 + o(T_1)) \},$$

so that from (14)

$$\Phi_c(\tfrac{1}{2} || \mathbf{u}_j - \mathbf{u}_i ||) \leqq \exp \{ - (E(R, S) - \epsilon) T_1 + o(T_1) \}.$$

Since, as $\xi \to \infty$, $\Phi_c(\xi) = e^{-(\xi^2/2)(1+o(1))}$, we have

$$|| \mathbf{u}_j - \mathbf{u}_i ||^2 \geqq 8(E(R, S) - \epsilon) T_1 + o(T_1),$$

which implies Lemma 2.

Proof of Lemma 3: Let $\hat{\theta}(t) \in \mathcal{S}$ be the function colinear with $P_{\mathcal{S}}(\hat{u}(t))$ with unit length in $\mathfrak{L}_2$ norm. That is $\hat{\theta}(t) = P_{\mathcal{S}}(\hat{u}) / || P_{\mathcal{S}}(\hat{u}) ||$, where "|| ||" applied to functions is the $\mathfrak{L}_2$ norm. Then

$$|| \hat{\mathbf{u}} || = || P_{\mathcal{S}} \hat{u} || = \left| \int_{-\infty}^{\infty} \hat{\theta}(t) \hat{u}(t) \, dt \right|$$

But since all functions in $\mathcal{S}$ and therefore $\hat{\theta}(t)$ have support on $[0, T_1]$,

$$|| \hat{\mathbf{u}} || = \left| \int_0^{T_1} \hat{\theta}(t) \hat{u}(t) \, dt \right| \leqq || \hat{\theta} || \left[\int_0^{T_1} \hat{u}^2(t) \, dt \right]^{\frac{1}{2}}$$

$$= \left[\int_0^{T_1} \hat{u}^2(t) \, dt \right]^{\frac{1}{2}} \leqq \sum_{n=-\infty}^{-1} \left[\int_0^{T_1} \hat{u}_n^2(t) \, dt \right]^{\frac{1}{2}}, \tag{22}$$

where $\hat{u}_n(t)$ is defined in (17).

Consider first the $n = -1$ term in the above summation. For $0 \leqq t \leqq T_1$,

$$| \hat{u}_{-1}(t) |^2 = \alpha^2 \left| \int_{-T_2}^{-\delta T_1} k(t - \tau) x_{i-1}(\tau + T_2) \, d\tau \right|^2$$

$$\leqq \alpha^2 \int_{-T_2}^{-\delta T_1} k^2(t - \tau) \, d\tau \int_{-T_2}^{-\delta T_1} x_{i-1}^2(\tau + T_2) \, d\tau$$

$$\leqq \alpha^2 S_1 T_1 \int_{-T_2}^{-\delta T_1} k^2(t - \tau) \, d\tau.$$

Changing the variable of integration to $w = t - \tau$, we have

$$= \alpha^2 S_1 T_1 \int_{t+\delta T_1}^{t+T_2} k^2(w)\, dw.$$

From condition (9′), for sufficiently large T_1,

$$\leqq \alpha^2 S_1 T_1 \int_{t+\delta T_1}^{t+T_2} \frac{A^2}{w^{2(1+\beta)}}\, dw \leqq \alpha^2 S_1 T_1 \int_{\delta T_1}^{\infty} \frac{A^2}{w^{2(1+\beta)}}\, dw$$

$$= \frac{\alpha^2 S_1 T_1 A^2}{1 + 2\beta} (\delta T_1)^{-1-2\beta} = \frac{\alpha^2 A^2 S_1}{(1 + 2\beta)(\delta)^{1+2\beta}} T_1^{-2\beta},$$

and therefore

$$\int_0^{T_1} \hat{u}_{-1}^2(t)\, dt \leqq \frac{\alpha^2 A^2 S_1}{(1 + 2\beta)\delta^{1+2\beta}} T_1^{1-2\beta}. \tag{23}$$

Next consider $-2 \geqq n > -\infty$. For $0 \leqq t \leqq T_1$, paralleling the above steps we obtain

$$| \hat{u}_n(t) |^2 \leqq \alpha^2 \int_{nT_2}^{(n+1)T_2} k^2(t - \tau)\, d\tau \int_{nT_2}^{(n+1)T_2} x_{in}^2(\tau - nT_2)\, d\tau$$

$$\leqq \alpha^2 S_1 T_1 \int_{t+n'T_2}^{t+(n'+1)T_2} k^2(w)\, dw,$$

where $n' = -n$. Again applying condition (9′), we have

$$| \hat{u}_n(t) |^2 \leqq \alpha^2 S_1 T_1 \int_{n'T_2}^{(n'+1)T_2} \frac{A^2}{w^{2(1+\beta)}}\, dw$$

$$= \frac{\alpha^2 S_1 T_1 A^2}{1 + 2\beta} \left[\frac{1}{(n'T_2)^{1+2\beta}} - \frac{1}{[(n' + 1)T_2]^{1+2\beta}} \right]$$

$$= \frac{\alpha^2 S_1 T_1 A^2}{(1 + 2\beta)(1 + \delta)^{1+2\beta} T_1^{1+2\beta}} \left[\frac{1}{(n')^{1+2\beta}} - \frac{1}{(n' + 1)^{1+2\beta}} \right]$$

$$= \frac{\alpha^2 S_1 A^2}{(1 + 2\beta)(1 + \delta)^{1+2\beta} T_1^{2\beta}} b_{n'} .$$

The important fact here is that as $n' \to \infty$, $b_{n'} = O(n'^{-(2+2\beta)})$. Therefore, the integral

$$\int_0^{T_1} \hat{u}_n^2(t)\, dt \leqq \frac{\alpha^2 S_1 A^2}{(1 + 2\beta)(1 + \delta)^{1+2\beta}} T_1^{1-2\beta} b_{n'} . \tag{24}$$

Substituting (23) and (24) into (22) we have,

$$|| \hat{\mathbf{u}} || \leqq T_1^{\frac{1}{2}-\beta} \alpha A \left(\frac{S_1}{1 + 2\beta} \right)^{\frac{1}{2}} \left[\delta^{-\frac{1}{2}-\beta} + (1 + \delta)^{-\frac{1}{2}-\beta} \sum_{n'=2}^{\infty} b_{n'}^{\frac{1}{2}} \right].$$

Since the summation converges, we have $|| \hat{\mathbf{u}} ||^2 \leqq O(T_1^{1-2\beta})$, which is Lemma 3.

REFERENCES

1. Gallager, R., *Information Theory and Reliable Communication,* New York: John Wiley, 1968, Chapter 8.
2. Cordaro, J. T., and Wagner, T. J., "Intersymbol Interference on a Continuous-Time Gaussian Channel," IEEE Trans. Inform. Theory, *IT-16,* July 1970, pp. 422–429.
3. Zadeh, L., and Desoer, C., *Linear System Theory,* New York: McGraw-Hill, 1963, pp. 423–428.